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Gradient-based compressive sensing for noise image and video reconstruction

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Abstract: In this study, a fast gradient-based compressive sensing (FGB-CS) for noise image and video is proposed. Given a noise image or video, the authors first make it sparse by orthogonal transformation, and then reconstruct it by solving a convex optimisation problem with a novel gradient-based method. The main contribution is twofold. Firstly, they deal with the noise signal reconstruction as a convex minimisation problem, and propose a new compressive sensing based on gradient-based method for noise image and video. Secondly, to improve the computational efficiency of gradient-based compressive sensing, they formulate the convex optimisation of noise signal reconstruction under Lipschitz gradient and replace the iteration parameter by the Lipschitz constant. With this strategy, the convergence of our FGB-CS is reduced from $O(1/k)$ to $O(1/k^2)$. Experimental results indicate that their FGB-CS method is able to achieve better performance than several classical algorithms.

1 Introduction

The well-known Nyquist/Shannon sampling theorem that the sampling rate must be at least twice the maximum frequency of the signal is a golden rule used in visual and audio electronics, medical imaging devices and so on. Compressive sensing (CS) is a sampling paradigm that provides the signal compression at a rate significantly lower than the Nyquist rate [1, 2].

CS has successfully been applied in a wide variety of applications in recent years, including photography [3], shortwave infrared cameras, optical system research [4], audio and music processing [5], MRI [6, 7] and so on. For example, Jorgensen *et al.* [8] developed an iterative image reconstruction method in X-ray computed tomography based on CS. Bhattacharya *et al.* [9] proposed a fast encoding method for synthetic aperture radar raw data compressing and reconstruction based on CS theory. Although CS achieves good application in most cases, there are still many difficulties for a noise signal processing based on CS theory [10, 11].

In this paper, we develop a fast gradient-based CS (FGB-CS) method for noise images and videos. Specially, we consider the CS for noise image and video as a convex optimisation problem, and present a gradient-based CS method to solve it. Moreover, to reduce the computational cost, we replace the traditional iteration parameter by Lipschitz constant. Experimental results show that our method outperforms several classical algorithms both in computational cost and performance.

The rest of this paper is organised as follows. In Section 2, we introduce some related works on CS. CS theory and gradient-based methods for convex optimisation problems are described in Section 3. In Section 4, we present the FGB-CS method for noise images and videos reconstruction. Experimental results are shown in Section 5. Finally, we conclude our paper in Section 6.

2 Related work

The major challenge in CS is to approximate a signal given a vector of samples. In recent years, many methods have been proposed which can be roughly divided into six categories:

(i) *Convex optimisation algorithms*: These techniques solve a convex problem which is used to approximate the target signal, including basis pursuit [12], greedy basis pursuit [13], basis pursuit de-noising [14], projected gradient method [15], least absolute shrinkage and selection operator [16] and least angle regression [17]. This type of algorithms solves a convex optimisation problem through linear programming to obtain reconstruction. The number of measurements in these algorithms required for exact reconstruction is small but the algorithms are complex in calculation.
(ii) *Greedy iterative algorithms*: These methods build up an approximation by making locally optimal choices step by step. The main advantages of these methods contain low implementation cost and high precision recovery. However, when the signal is not very sparse, recovery becomes costly. Examples include matching pursuit (MP), orthogonal matching pursuit (OMP) [18], regularised OMP [19], stagewise OMP [4], compressive sampling matching pursuit (CoSaMP) [20] and subspace pursuit (SP) [21].
(iii) *Iterative thresholding algorithms*: Iterative approaches for CS recovery problem are faster than the convex optimisation method. For this type of algorithms, correct measurements are recovered by soft or hard thresholding [16, 22] starting from noise measurements given the signal is sparse. The thresholding function depends upon the number of iterations and the problem configurations. Message passing (MP) algorithm [23] is an important modification of iterative thresholding algorithms in which basic variables (messages) are associated with directed graph edges. Expander matching pursuits [24], sparse matching pursuit [25] and sequential sparse matching pursuits [26], belief propagation [27] belong to this type. These approaches have many advantages such as low computational complexity and easy implementation in parallel or distributed manner.
(iv) *Combinatorial/sublinear algorithms*: This type of algorithms recovers sparse signal through group testing. They are extremely fast and efficient, as compared to convex relaxation or greedy algorithms but require specific pattern in the measurements. Representative algorithms are Fourier sampling algorithm, chaining pursuits, heavy hitters on steroids [28] and so on.
(v) *Non-convex minimisation algorithms*: Non-convex local minimisation techniques recover CS signals from far less measurements by replacing l_1-norm and l_p-norm, where $p \le 1$ [29]. There are many algorithms proposed in the literature that use this techniques such as focal underdetermined system solution [30], iterative re-weighted least squares [31], Monte-Carlo-based algorithms [5], sparse Bayesian learning algorithms [32] and so on. Non-convex optimisation is mostly utilised in medical imaging tomography, network state inference and streaming data reduction.

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(vi) *Bregman iterative algorithms*: When is applied to CS problems, the iterative approach using Bregman distance regularisation achieves reconstruction in four to six iterations [33]. These algorithms provide a simple and efficient way of solving l_1 minimisation problem. The computational speed of these algorithms is particularly appealing compared to that available with other existing algorithms.

Different from all the above methods, we propose a FGB-CS method for noise images and videos.

3 CS and gradient-based method

3.1 CS for noise signal

CS is based on the assumption of the sparse property of signal and incoherency between the bases of sparse domain and the bases of measurement vectors. It has three major steps: the construction of k-sparse representation, the compression and the reconstruction. The first step is the construction of k-sparse representation, where k is the number of the non-zero entries of sparse signal. Most natural signal can be made sparse by applying orthogonal transforms such as wavelet transform, fast Fourier transform and discrete cosine transform (DCT) [11]. This step is represented as

$$\boldsymbol{s} = \boldsymbol{\Psi}^{\mathrm{T}} x \tag{1}$$

where x is an N-dimensional non-sparse signal; $\boldsymbol{s}$ is a weighted N-dimensional vector (sparse signal with k non-zero elements) and $\boldsymbol{\Psi}$ is an $N \times N$ orthogonal basis matrix. The second step is compression. In this step, the random measurement matrix is applied to the sparse signal according to the following equation

$$y = \boldsymbol{\Phi s} = \boldsymbol{\Phi \Psi}^{\mathrm{T}} x \tag{2}$$

where $\boldsymbol{\Phi}$ is an $M \times N$ random measurement matrix ($M < N$). Let M be the number of measurements (the row dimension of y) sufficient for high probability of successful reconstruction, and M is determined by

$$M \geq= Cu^2(\boldsymbol{\Phi}, \boldsymbol{\Psi}) k \log N \tag{3}$$

For some positive constant C, $u^2(\boldsymbol{\Phi}, \boldsymbol{\Psi})$ is the coherence between $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$, and defined by

$$u(\boldsymbol{\Phi}, \boldsymbol{\Psi}) = \sqrt{N} \max_{i,j} |\langle \phi_i, \psi_j \rangle| \tag{4}$$

If the elements in ϕ and ψ are correlated, the coherence is large. Otherwise, it is small. From linear algebra, it is known that $u(\boldsymbol{\Phi}, \boldsymbol{\Psi}) \in [1, \sqrt{N}]$.

In the measurement process, the noise may occur. The noise is added into the compressed measurement vector as follows

$$y = \boldsymbol{\Phi s} + \text{noise} \tag{5}$$

where noise is an M-dimensional vector. As expected, signal x in (2) may be estimated from noise measurement y by solving the following minimisation problem

$$\begin{aligned} &\text{minimise } \|x\|_1 \\ &\text{subject to } \|\boldsymbol{\Phi \Psi}^{\mathrm{T}} x - y\|_2 \leq \varepsilon \end{aligned} \tag{6}$$

where ε is a bound of the amount of noise in the data. The robustness of the CS heavily relies on a notion called *restricted isometry property* (RIP) [11]. RIP is defined as follows

$$(1 - \delta_k) \| s \|_2^2 \leq \| \boldsymbol{\Phi s} \|_2^2 \leq (1 + \delta_k) \| s \|_2^2 \tag{7}$$

where $\| \ \|_2^2$ defines the l_2 norm, and δ_k is the k-restricted isometry constant of a matrix. RIP is used to ensure that all subsets of k columns taken from $\boldsymbol{\Phi}$ are nearly orthogonal. It should be noted that $\boldsymbol{\Phi}$ has more column than rows; thus $\boldsymbol{\Phi}$ cannot be exactly orthogonal.

3.2 Gradient-based method for convex optimisation problems

The convex optimisation problem we want to deal with is one of the form

$$\min\{g(x): x \in R^n\} \tag{8}$$

One of the simplest methods for solving (8) is the gradient-based algorithm which generates a sequence x_k via

$$x_0 \in R^n, \ \ x_k = x_{k-1} - t_k \nabla g(x_{k-1}) \tag{9}$$

where $t_k > 0$ is a suitable step size. It is very well known that the gradient iteration (9) can be viewed as a proximal regularisation [34] of the linearised function g at x_{k-1}, and written equivalently as

$$x_k = \arg\min_x \left\{ g(x_{k-1}) + \langle (x - x_{k-1}), \nabla g(x_{k-1}) \rangle + \frac{1}{2t_k} \| x - x_{k-1} \|_2^2 \right\} \tag{10}$$

Applying the same idea to the l_1 regularised problem

$$\min \{ g(x_{k-1}) + \lambda \| x \|_1 : x \in R^n \} \tag{11}$$

leads to the iterative scheme (see (12))

$$x_k = \arg\min_x \left\{ g(x_{k-1}) + \langle x - x_{k-1} \rangle, \nabla g(x_{k-1}) + \frac{1}{2t_k} \| x - x_{k-1} \|_2^2 + \lambda \| x \|_1 \right\} \tag{12}$$

Ignoring constant terms yields

$$x_k = \arg\min_x \left\{ \frac{1}{2t_k} \| x - (x_{k-1} \|_2^2 - t_k \nabla(x_{k-1})) \|_2^2 + \lambda \| x \|_1 \right\} \tag{13}$$

For (13), the convergence property has been developed and analysed by many researches through various techniques. But unfortunately, those computations are complex and the convergence for x_k is $O(1/k)$ [34]. Next, we focus on using gradient-based method to solve the problem of noise signal reconstruction, and improving the non-asymptotic global rate of convergence.

4 Noise image and video reconstruction based on CS

4.1 Noise signal optimisation with Lipschitz gradient

For a noise signal in (5), let us think about an objective function $F(x) = g(x) + n(x)$, which is a composite type convex function. In our method, (6) is more natural to study the closely related problem

$$\text{minimise } \| \boldsymbol{\Phi \Psi}^{\mathrm{T}} x - y \|_2^2 + \lambda \| x \|_1 \tag{14}$$

In order to improve the efficiency of signal reconstruction, we think about the Lipschitz gradient [35] is named

$$\| \nabla g(x) - \nabla g(y) \| \leq L(g) \| x - y \| \quad \text{for very } x, y \tag{15}$$

where $L(g) > 0$ is a (Lipschitz) constant and $\| \, \|$ denotes the standard Euclidean norm and $L(g) > 0$ is the Lipschitz constant of ∇g. At point x_{k-1}, the function $F(x)$ can be approximated by the following quadratic function

$$Q_L(x, x_{k-1}) = \left\{ g(y) + \langle x - x_{k-1}, \nabla g(x_{k-1}) \rangle + \frac{L}{2} \| x - x_{k-1} \|_2^2 + n(x) \right\} \tag{16}$$

which admits a unique minimiser

$$\mathrm{PL}(x_{k-1}) = \arg\min_x \{ Q_L(x, x_{k-1}), x \in R^n \} \tag{17}$$

Simple algebra shows that (ignoring constant terms)

$$\mathrm{PL}(x_{k-1}) = \arg\min_x \left\{ \frac{L}{2} \| x - (x_{k-1} - \frac{1}{L} \nabla g(x_{k-1})) \|_2^2 + n(x) \right\} \tag{18}$$

Clearly, the basic step in (9) is replaced by

$$x_k = \mathrm{PL}(x_{k-1}) \tag{19}$$

with L set to $1\backslash t_k$. Apparently, as long as the constant L in (16) is taken to be no less than Lipschitz constant, it follows that

$$\begin{aligned} g(x) + n(x) \leq g(x_{k-1}) + \langle \nabla g(x_{k-1}), x - x_{k-1} \rangle \\ + \frac{L}{2} \| x - x_{k-1} \|_2^2 + n(x) \end{aligned} \tag{20}$$

In our procedure, we replace $1/t_k$ by a constant L which will be related to the Lipschitz constant $L(g)$. We can find that the right-hand side of (20) is precisely equal to $Q_L(x, y)$ in (16). In other words, $Q_L(x, y)$ is an easier-to-deal-with convex upper bound of the objective function $F(x)$ and by minimising the upper bound, $Q_L(x, y)$ with x_k given by (19) offers a tight upper bound of $F(x)$, provided that $L \geq L(f)$.

4.2 CS for noise image and video reconstruction

Let us begin with considering the problem of (14). Assumed (14) is convex with smooth Lipschitz gradient. For any $L > 0$, CS for a noise image and video formulated by (14) becomes

$$x_k = \arg\min_x \left\{ \frac{L}{2} \| x - x_{k-1} \|_2 + \lambda \| x \|_1 \right\} \tag{21}$$

where $x_k = \mathrm{PL}(x_{k-1})$, so

$$x_k = \arg\min_x \left\{ \frac{L}{2} \| x - \left(x_{k-1} - \frac{1}{L} \nabla f(x_{k-1}) \right) \|_2^2 + \lambda \| x \|_1 \right\} \tag{22}$$

or equivalently

$$x_k = \arg\min_x \left\{ \frac{L}{2} \| x - d_k \|_2^2 + \lambda \| x \|_1 \right\} \tag{23}$$

where $d_k = x_{k-1} - 1/L \nabla f(x_{k-1})$. According to (14), this d_k can be rewritten as

$$d_k = x_{k-1} - \frac{1}{L} (\mathbf{\Phi} \mathbf{\Psi}^{\mathrm{T}})^{\mathrm{T}} (\mathbf{\Phi} \mathbf{\Psi}^{\mathrm{T}} x_{k-1} - y) \tag{24}$$

as both the l_1-norm and square of the l_2-norm are separable. And each of these terms involves only a single (scalar) variable, the iterate x_k in (23) can be computed exactly by a straightforward shrinkage step [assuming d_k in (24) has been calculated] as

$$x_k = \Gamma_{\lambda L}(d_k) \tag{25}$$

where Γ_α is a shrinkage operator which maps R^n to R^n with the ith entry of the output vector given by

$$\Gamma_\alpha(d)|_i = (|d_i| - \alpha)_+ \mathrm{sgn}(d_i) \tag{26}$$

where $(u)_+ = \max(u, 0)$.

4.3 FGB-CS algorithm

As per the fact described above, a FGB-CS algorithm for noise images and videos reconstruction method is proposed, the details are shown in Algorithm 1 (see Fig. 1).

Compared with other reconstruction algorithms, the proposed algorithm has several characteristic as follows:

(i) The CS for a noise signal may be estimated as a convex minimisation problem, and gradient-based method is used to solve the problem.
(ii) The problem of noise signal reconstruction is assumed to be the convex with Lipschitz gradient. A iteration parameter $1/t_k$ is replaced by a constant $1/L$ which is related to the Lipschitz constant $L(f)$.
(iii) As described in [35], we can easily compute that the convergence of FGB-CS is $O(1/k^2)$.

Algorithm 1:

Input : $L = L(g) - a$ Lipschitz constant of $\nabla g(x)$ in Eq.(15);
a signal s;
a signal sparse transform matrix $\Psi \in R^{N \times N}$;
a measurement matrix $\Phi \in R^{N \times N}$;
the iteration counter K and noise parameter λ ;
$s_p = \Psi s; x = \Phi s_p$.
Output: A sparse approximation x_k of the target signal then reconstruction result signal $s' = \Psi^T x_k$.
Initialisation: $y_1 = x_0 \in R^n, t_1 = 1, k = 1$
while $k < K$ **do**
  $x_k = PL(y_k)$ by solving the problem in Eq.(19);
  $t_{k+1} = \frac{1 + \sqrt{1 + 4t_k^2}}{2}$;
  $y_{k+1} = x_k + \frac{t_k - 1}{t_{k+1}} (x_k - x_{k-1})$.
end

Fig. 1 *FGB-CS algorithm*

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5 Experimental results

In order to evaluate the quality of the reconstructed results, the mean square error and peak signal–noise ratio (PSNR) are used for the comparison. They are defined as

$$\text{MSE} = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{M} (\hat{f}(i,j) - f(i,j))^2 \quad (27)$$

$$\text{PSNR} = 10\lg\left(\frac{255^2}{\text{MSE}}\right) \quad (28)$$

where M and N are the image dimensions, $\hat{f}$ is the denoised image and f is the original image. In (28) 255 means the pixel values is 0–255 in an optical grey image. Many researchers used PSNR to estimate the result in image processing [3]. In our study, the PSNR is also used to compare the experiment results. The experiments were implemented on a Pentium IV with 3.2 GHz CPU and 2048 MB RAM.

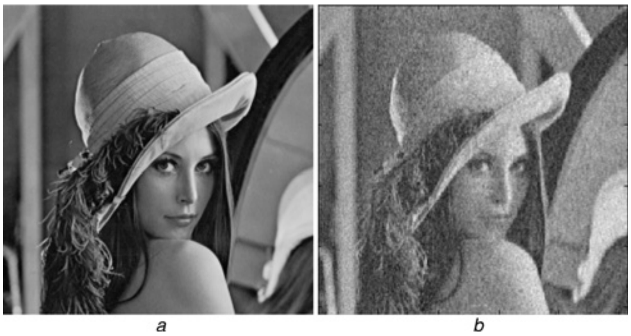


Fig. 2 *Original image and noise image*

a Lena
b Noise image ($\sigma = 15$)

5.1 Noise image reconstruction

'Lena' (size 256 × 256) is used as a test image in Fig. 2*a*. It is degraded by rand noise ($\sigma = 15$) in Fig. 2*b*. The DCT matrix [11] is used in sparsing image. The noise image reconstruction result based on different CS algorithms with matrix Rs rows $M = 200$ is shown in Figs. 3*a–e*, and the reconstruction result based on FGB-CS with $\lambda = 20$ and $K = 40$ is shown in Fig. 3*f*. The reconstruction result in PSNR and runtime based on different methods is shown in Table 1.

More experiments are carried out to compare reconstruction performances with different rows of measurement matrix, and the results are shown in Fig. 4. We can see from Table 1 and Fig. 4 that

(i) the PSNR in noise image reconstruction raises with the increasing of measurement matrix rows;
(ii) the method based on FGB-CS can obtain best result than those other methods, and iterative re-weighted least squares (IRLS), CoSaMP, SP, greedy basis pursuit (GBP) and OMP are in the second, third, fourth and fifth;
(iii) FGB-CS also spends the second least runtime than those other methods, and almost equals what is spent in OMP. IRLS, SP, CoSaMP and GBP spend more time in noise image reconstruction with the increase of the measurement matrix rows.

Table 1 Reconstruction result in PSNR and runtime by using the different methods

Methods	PSNR	Runtime, s
OMP	24.70	5.625
SP	25.64	49.25
CoSaMP	25.97	92.14
GBP	25.28	108.391
IRLS	27.70	294.08
FGB-CS	28.89	6.128

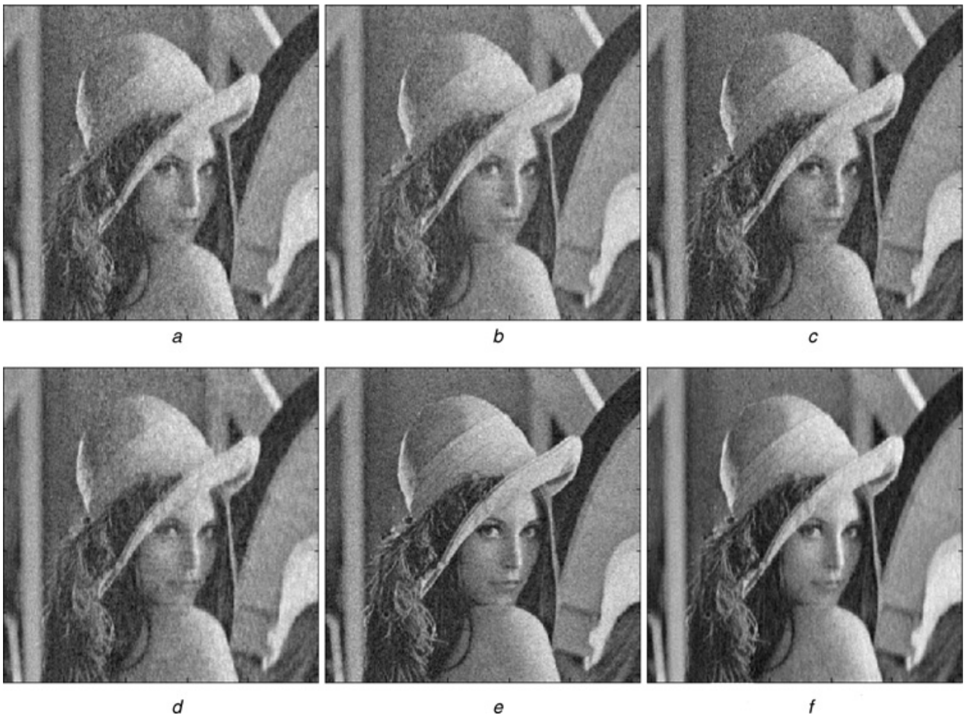
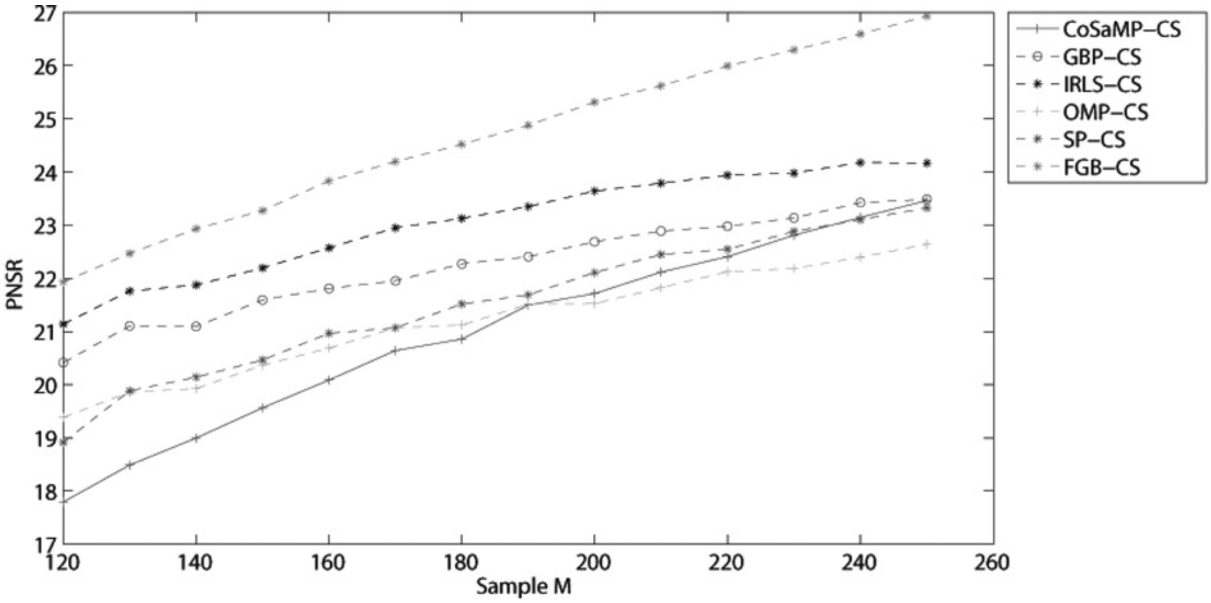
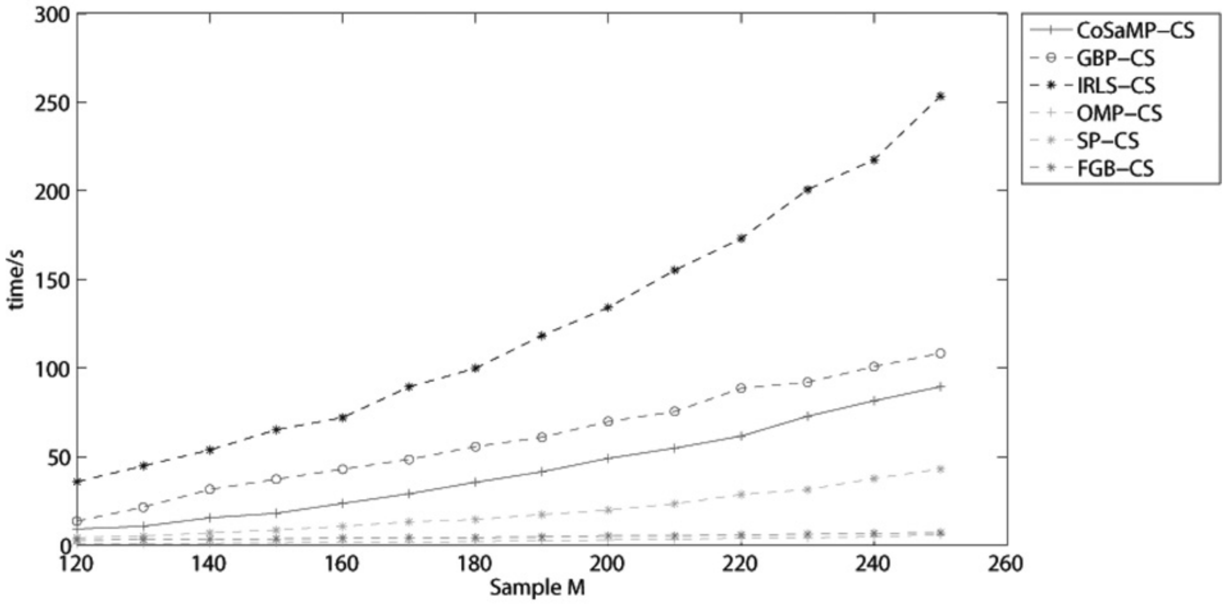


Fig. 3 *Noise image reconstruction based on CS*

a OMP
b SP
c CoSaMP
d GBP
e IRLS
f FGB-CS



a



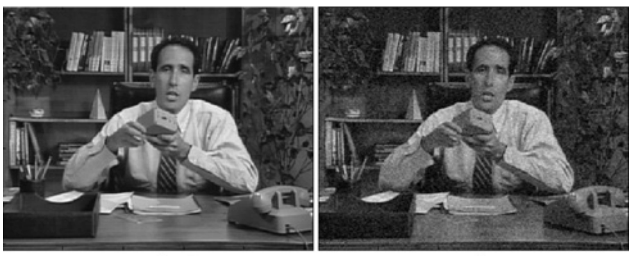
b

Fig. 4 *Quantisation comparisons in noise image reconstruction*

a PSNR comparisons
b Runtime comparisons

5.2 Noise video reconstruction

A video gsalesmang15.avi (total 48 frames) is used as a test data in Fig. 5. The video is degraded by rand noise ($\sigma = 15$). The some reconstruction results based on CS with the matrix rows $M = 230$ are shown in Figs. 5*b*–*f*. In FGB-CS, $\lambda = 20$ and $K = 40$. The reconstruction result in PSNR and runtime based on different methods are shown in Table 2 (Fig. 6).



a *b*

Fig. 5 *Original video and noise video*

a Frame = 10
b Noise frame ($\sigma = 15$)

More experiments are carried out to compare noise video reconstruction performances. The reconstruction average time and PSNR with different rows of measurement matrix are shown in Figs. 7*a* and *b*.

Table 2 Noise video reconstruction results in average PSNR and runtime by using the different methods

Methods	PSNR	Runtime, s
OMP	24.70	5.625
SP	25.64	49.25
CoSaMP	25.97	92.14
GBP	25.28	108.391
IRLS	27.70	294.08
FGB-CS	28.89	6.128

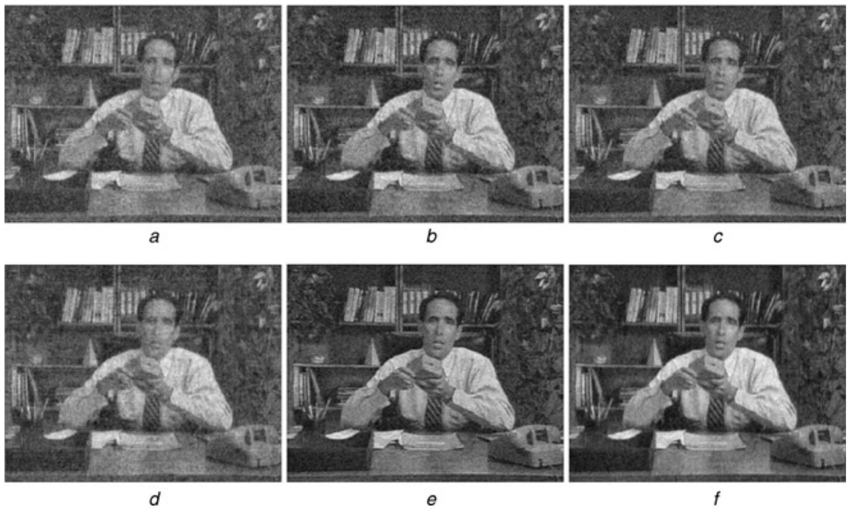


Fig. 6 *Noise video reconstruction results by using the different method*

a OMP
b SP
c CoSaMP
d GBP
e IRLS
f FGB-CS

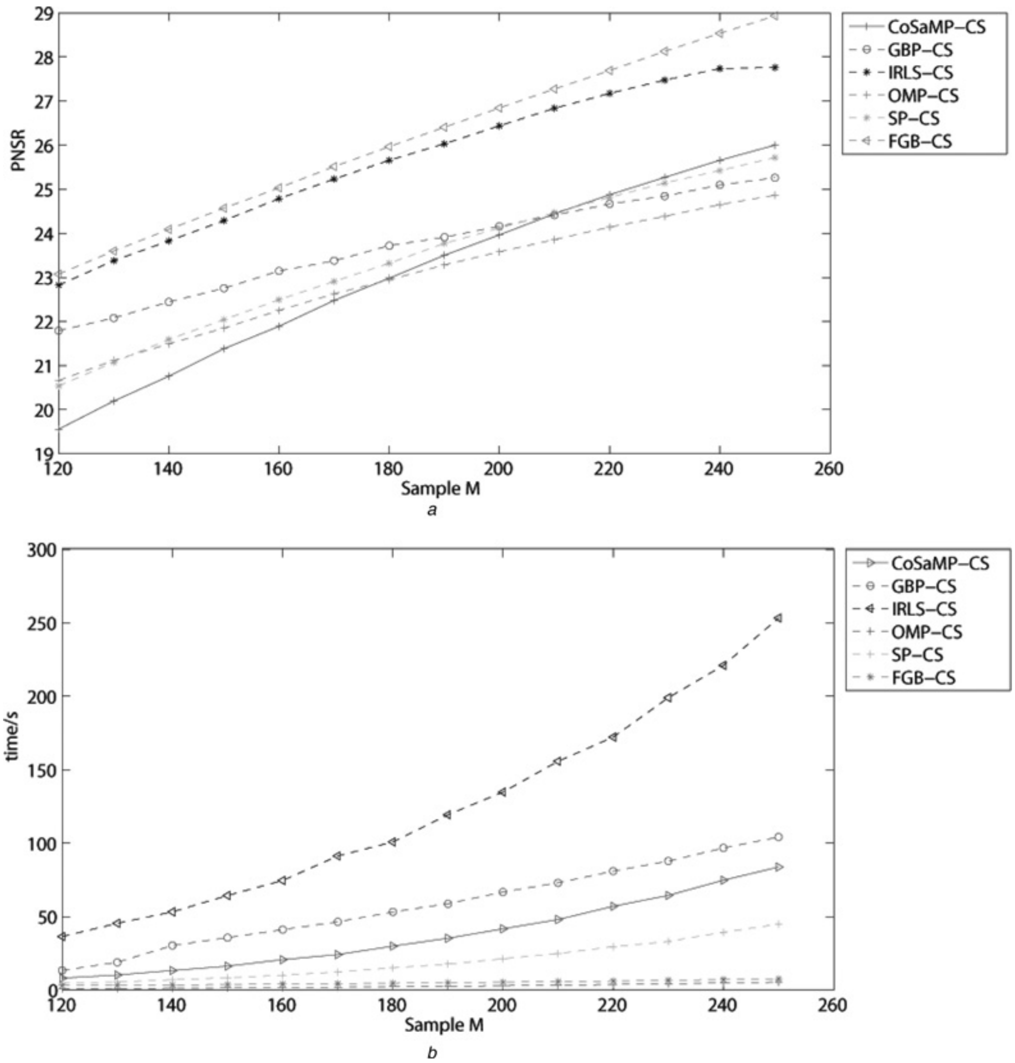


Fig. 7 *Quantisation comparisons in noise video reconstruction*

a Average PSNR comparisons
b Average runtime comparisons

We can see from Table 2 and Fig. 7 that

(i) the noise video reconstruction accuracy decreases with the increase of measurement matrix rows. And among those methods, FGB-CS algorithm can obtain best result than those other methods, and the second is IRLS algorithm;
(ii) the method based on OMP and FGB-CS algorithm can run the fastest than other methods in noise video reconstruction, and SP, CoSaMP, GBP and IRLS are in the third, fourth and fifth. Among them, the runtime decreases with the increasing of measurement matrix rows. IRLS method can obtain good reconstruction result, while spends the most time.

6 Conclusion and future work

In this paper, CS for noise images and video methods based on gradient-based algorithms (FGB-CS) is proposed. On one hand, we deal with the noise imagery and video reconstruction as a convex minimisation problem, and provide a new method based on gradient-based method. On the other hand, in order to improve the efficiency, we consider the problem of noise signal reconstruction assumed to be convex with Lipschitz gradient. The step size in gradient iteration is replaced by a constant $1/L$ which is related to the Lipschitz constant. The experiments have been shown that

(i) among those methods, the FGB-CS can obtain best reconstruction result in terms of PSNR comparing with OMP, SP, CoSaMP, GBP and IRLS;
(ii) the proposed method can run as fast as OMP methods in noise imagery and video reconstruction, and the fastest than SP, CoSaMP, GBP and IRLS methods;
(iii) with the increasing measurement matrix rows, the proposed method can obtain better reconstruction accuracy with only a few runtime changes.

In future studies, the more relationship between parameters λ and K in FGB-CS will be researched.

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