Tuesday	Dec 1	Go over homework, Section 9.5
Thursday	Dec 3	Test #4 Ch 7, 8, 9.1-9.4
Tuesday	Dec 8	Course Evaluations (bring pencil) Wrap up/review
Tuesday 10:15 - 12:00	Dec 15	Final Exam

Final exam practice problems will be posted on Blackboard. Can be turned in for bonus HW points (up to 30 pts).

Comments on Recent HW Problems

7.24 Let A be a set. If $[A \cap B = \emptyset$ for every set $B]$ then $[A = \emptyset.]$

① ~~This statement is false. Consider $A = \{1,2\}$ and $B = \{3,4\}$. Then $A \cap B = \emptyset$ but $A \neq \emptyset$.~~

~~$B = \{2,4\}$~~ $\quad A \cap B = \{2\} \neq \emptyset$

② ~~This statement is true. Let A be a set such that $A \cap B = \emptyset$ for every set B. Since $A \cap B = \emptyset$ for every B, then in particular, $A \cap B = \emptyset$ for the set $B = \emptyset$. When $B = \emptyset$,~~

$$A \cap \emptyset = \emptyset.$$

[This does not tell us if $A = \emptyset$ or $A \neq \emptyset$]

③ This statement is true. Let A be a set such that $A \cap B = \emptyset$ for every set B. Since $A \cap B = \emptyset$ for every set ~~set~~ B, then in particular, $A \cap B = \emptyset$ when we pick $B = A$.

$$A \cap \overset{B}{A} = \emptyset$$

$$A = \emptyset.$$

Therefore $A = \emptyset$. $\blacksquare$

If $A \cap B = \emptyset$ for every set B, then $A = \emptyset$.

If $A \neq \emptyset$, then there exists a set B such that $A \cap B \neq \emptyset$.

④ Suppose $A \neq \emptyset$. Then there must be some $x \in A$.
Let $B = \{x\}$. Then $A \cap B = A \cap \{x\} = \{x\} \neq \emptyset$. $\blacksquare$

8.12 $A = \{1,2,3,4\}$

(a) Reflexive, Symmetric but not Transitive

$\cancel{\{(1,1),(2,2),(3,3),(4,4)\}}$

"If xRy and yRz, then xRz."

$(1,\underline{1})$ $(\underline{2},2)$

$(1,\underline{1})$ $(\underline{1},1)$ $(1,1)$

$\cancel{\{(1,1),(2,2),(3,3),(4,4),(1,2),(2,1)\}}$

$1R2$ $2R1$ $1R1$

$\{(1,1),(2,2),(3,3),(4,4),(1,2),(2,1),(2,4),(4,2)\}$

$\underline{1}R\underline{2}$ $\underline{2}R\underline{4}$ $\times$ $\underline{1}\not R\underline{4}$

(c) Symmetric, Transitive but not Reflexive

$\{(1,2),(2,1)\}$ $1R2$ $2R1$ $1\not R1$?

$R = \{(1,1),(1,2),(2,1),(2,2)\}$

$(3,3) \notin R$

8.30 Let R be the relation on $\mathbb{Z}$ by aRb if $a+b \equiv 0 \pmod 3$. Show R is not an equivalence relation.

~~Let $a \in \mathbb{Z}$. Then $a+a = 2a$. But $3 \nmid 2a$, so $\boxed{2a \not\equiv 0 \pmod 3}$ therefore, R is not reflexive, so R is not an equivalence relation~~

Consider $a=1$. Then $a+a=2$, so $3 \nmid 2a$. Therefore $a+a \not\equiv 0 \pmod 3$ when $a=1$. so R is not reflexive.

9.5 $\boxed{\text{Composition of Functions}}$

Preliminary Example – College Algebra/Calculus

$$f(x) = x^2 \qquad g(x) = \sin(x)$$

$$f(g(x)) = f(\sin(x)) = (\sin(x))^2 = \sin^2(x)$$

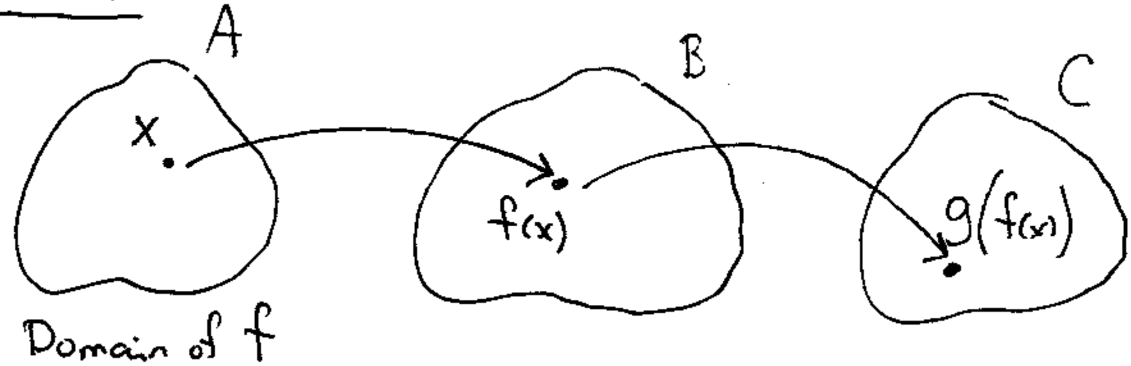
$$g(f(x)) = g(x^2) = \sin(x^2)$$

These are compositions of f and g.

Important in calculus:

$$\frac{d}{dx}\left(\sin(x^2)\right) = \cos(x^2) \cdot 2x$$

Picture



$f: A \to B \qquad g: B \to C$

$h: A \to C$ by $h(x) = g(f(x))$

Definition Suppose A, B, C are nonemty sets, and $f: A \to B$, $g: B \to C$ are functions. The function $h: A \to C$ defined by $h(x) = g(f(x))$ for each $x \in A$ is called the composition of f and g. In this case, we write $h = g \circ f$.

<u>EX</u> $A = \{1,2,3\}$ $B = \{10,20,30\}$ $C = \{x,y\}$

$f = \{(1,10), (2,20), (3,30)\}$

$g = \{(10,x), (20,y), (30,x)\}$

$f: A \to B$ and $g: B \to C$. Find $h = g \circ f$

$h: A \longrightarrow C$

$h = \{(1,x), (2,y), (3,x)\}$

<u>Theorem</u> Let $f: A \to B$ and $g: B \to C$ be two functions. If f and g are injective, then $g \circ f$ is also injective.

<u>Proof</u> Suppose f and g are injective. We will show $g \circ f$ is also injective. Let $a, b \in A$ and suppose $g \circ f(a) = g \circ f(b)$. Then

$$g(\underline{f(a)}) = g(\underline{f(b)})$$

Since g is one-to-one, $f(a) = f(b)$.

Since f is injective, $a=b$. Therefore $g\circ f$ is injective. $\blacksquare$

<u>Theorem</u> Let $f: A \to B$ and $g: B \to C$ be two functions. If f and g are surjective, then $g\circ f$ is also surjective.

Proof. See Book.

<u>Corollary</u> If $f: A \to B$ and $g: B \to C$ are bijective, then $g\circ f$ is bijective.