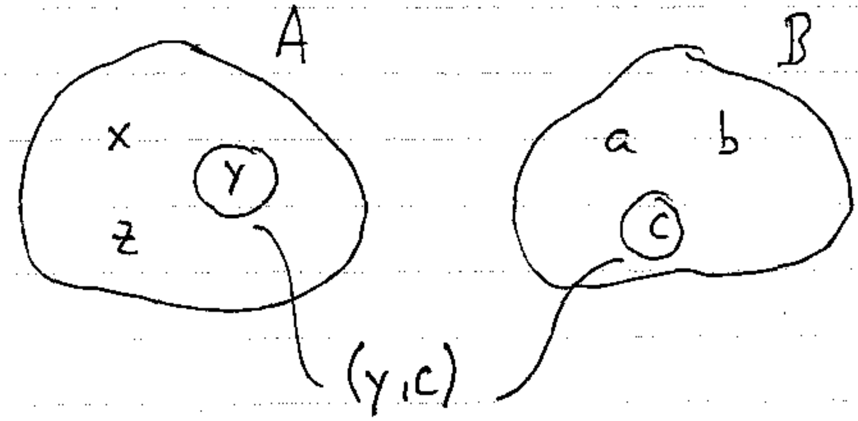
# 8.1 Relations




Relations are a way of connecting <u>some</u> elements of one set A to another set B.

<u>Definition</u> Suppose A and B are two sets.
A relation from A to B is a set R that is a subset of $A \times B$.

__EX__ $A = \{1,2\}$ $B = \{10,15\}$

Find four different relations from $A$ to $B$.

$$A \times B = \{(1,10), (1,15), (2,10), (2,15)\}$$

$R_1 = \{(1,10), (1,15)\}$ $\mathrm{dom}\, R_1 = \{1\}$ $\mathrm{ran}\, R_1 = \{10,15\}$

$R_2 = \{(2,10), (2,15)\}$

$R_3 = \{(1,10), (2,10)\}$

$R_4 = \{(1,10), (1,15), (2,10)\}$

$R_5 = \{(1,10), (1,15), (2,10), (2,15)\}$

$R_6 = \{(1,10)\}$

$R_7 = \{\ \}$

# Relations on a Set

Suppose $B$ is a set. A <u>relation on $B$</u> is a relation from $B$ to $B$. In other words, a relation on $B$ is a subset $R$ of $B \times B$.

<u>EX</u> $B = \{ 10, 15 \}$

$$B \times B = \{ (10,10), (10,15), (15,10), (15,15) \}$$

A relation on $B$

$$R = \{ (10,10), (15,15) \}$$

<u>Note</u> When $(x,y) \in R$, we write $x R y$ and say "$x$ is related to $y$."

## More Examples

(a) Relation on $\mathbb{R}$ : $R = \{(x, y) : x \in \mathbb{R}, y \in \mathbb{R}, y = x^2\}$

real #'s

Is $2R5$? $[(2,5) \in R?]$ $5 \neq 2^2$, so $2 \not R 5$.

Is $5R2$? $(5,2) \in R$? $2 \neq 5^2$ $5 \not R 2$.

Is $-3R9$? $(-3,9) \in R$? $9 = (-3)^2$ $-3R9$.

Is $9R(-3)$? $(9,-3) \in R$ $-3 \neq 9^2$ $9 \not R (-3)$

Note: We will abbreviate and say the relation $R$ on $\mathbb{R}$ is $xRy$ if $y = x^2$.

(b) Relation on $\mathbb{Z}$ : $R = \{(m,n) : m \in \mathbb{Z}, n \in \mathbb{Z}, m \mid n.\}$

Is $2 R 5$? $2 \nmid 5$, so $2 \not R 5$.

Is $3 R 12$? $3 \mid 12$, so $3 R 12$.

Is $12 R 3$? $12 \nmid 3$, so $12 \not R 3$.

Is $10 R 10$? $10 \mid 10$, so $10 R 10$.

8.2 $\boxed{\text{Properties of Relations}}$

EX Relation on $\mathbb{Z}$ : $xRy$ if $x+y$ is even.

(a) ~~Is it~~ Prove or disprove: For all $x \in \mathbb{Z}$, $xRx$.

Proof. This statement is true. Suppose $x$ is any integer. Since

$$x + x = 2x,$$

~~and~~ this shows $x+x$ is even. Therefore $xRx$. ∎

Reflexive Property

(b) Prove or disprove: If $xRy$, then $yRx$.

Proof. This statement is true. Suppose $x \in \mathbb{Z}$, $y \in \mathbb{Z}$ and $xRy$. Since $xRy$, this means $x+y$ is even. So $y+x$ is even, therefore $yRx$. ∎

Symmetric Property