1.6 Cartesian Products of Sets

Suppose A and B are two sets. The <u>Cartesian product</u> of A and B is the set

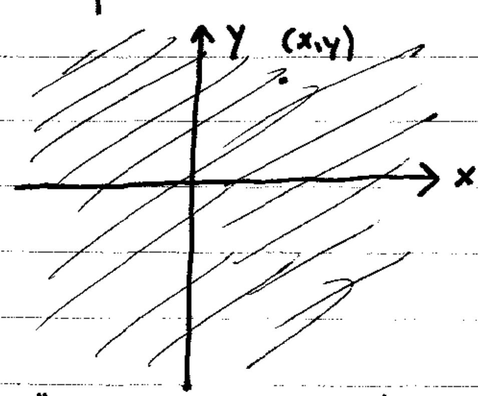
$$A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$$

<u>EX</u> (a) $A = \{1,2\}$, $B = \{-1,0,1\}$

$$A \times B = \{(1,-1), (1,0), (1,1), (2,-1), (2,0), (2,1)\}$$

(b) $\mathbb{R} \times \mathbb{R} = \{(x,y) : x \in \mathbb{R}, y \in \mathbb{R}\}$

all ordered pairs of real numbers



"The Cartesian Plane"

Chapter 2 - Logic

The underlying concepts in constructing logically valid proofs.

2.1 $\boxed{\text{Statements}}$

A delarative sentence or assertion that is either true or false. Every statement has a truth value, TRUE or FALSE.

EX (a) $5 + 8 = 12$ Statement. FALSE

(b) Sit down, please. Not a statement.

(c) The ZIP code for Yakima, WA is 98902. Statement (TRUE)

(d) Luis Pujols is a better baseball player than Johnny Damon. ← opinion Not a statement

(e) $x^2 = 9$ Not a statement,.. but.... see next page....

An <u>open sentence</u> is a declarative sentence that contains a variable, which becomes a statement when the variable is given a value.

$x^2 = 9$ open sentence

"For $x = 4$, $x^2 = 9$" statement FALSE

"For $x = 3$, $x^2 = 9$" statement TRUE

The value(s) that we consider is the <u>domain</u>.

<u>EX</u> Consider $x^2 = 9$ over the domain $\{-3, -2, 1, 0, 1\}$

For which values of the domain is $x^2 = 9$ a true statement?

$x = -3$

Notation

We will use $P, Q, R, \ldots$ or $P_1, P_2, P_3, \ldots$

to denote statements.

Not "equal sign"

$P:$ The integer 3 is odd.

We will use $P(\cdot)$ to denote open statements

$P(x): \quad (x-3)^2 \leq 1$

$P(x,y): \quad |x| + |y| = 1$

Truth Tables

A listing of the possible truth values for statements.

P
T
F

P	Q
T	T
T	F
F	T
F	F

Four ways to combine statements into new statements:

not , and, or, implies

2.2 $\boxed{\text{The Negation of a Statement}}$

Suppose P is a statement. The <u>negation</u> of P, written $\sim P$, is another statement such that the truth value of $\sim P$ is opposite the truth value of P.

<u>EX</u> (a) P: The integer 3 is odd.

$\sim P$: The integer 3 is not odd.
or, The integer 3 is even.

(b) Q: Fifi is not a black cat.

$\sim Q$: Fifi is a black cat.

<u>Truth Table</u>

P	$\sim P$
T	F
F	T

2.3 The Conjunction of Two Statements

Suppose P, Q are two statements.

P and Q

is a new statement (True or False), written

$$P \wedge Q$$

EX P: 3 is prime , Q: 7 is even

$P \wedge Q$: 3 is prime and 7 is even.

For $P \wedge Q$ to be true, both P and Q have to be true. Otherwise $P \wedge Q$ is false. So

3 is prime and 7 is even

is FALSE because "7 is even" is FALSE.

Truth Table

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Summarizes the truth value of $P \wedge Q$

2.3 $\boxed{\text{The Disjunction of Two Statements}}$

Suppose P and Q are two statements.

$$P \text{ or } Q$$

is a new statement, written $P \vee Q$.
The "or" means one, the other, or both.

EX $\quad A = \emptyset \quad B = \{\emptyset\}$

(a) $|A| = 0$ or $|B| = 0$

$|A| = 0$ is true, so "$|A|=0$ or $|B|=0$" is True.

(b) $A \subseteq B$ or $A = B$

$A \subseteq B$ True (the empty set is a subset of all sets)

So "$A \subseteq B$ or $A = B$" is true.

Truth Table

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

<u>EX</u> $A = \{1,2,3,4,5\}$ $B = \{2,4,6\}$

$P: A \cap B = \emptyset$ $Q: A \cup B = A$

Find the truth value of $(\sim P) \vee Q$

" $A \cap B \neq \emptyset$ or $A \cup B = A$ "

$A \cap B = \{1,2,3,4,5\} \cap \{2,4,6\} = \{2,4\}$

" $A \cap B \neq \emptyset$ " is TRUE

" $A \cap B \neq \emptyset$ or $A \cup B = A$ " is TRUE.

Before discussing Implications, consider this.

Betty is a candidate for Congress.

She states

" If (I am elected), your (taxes will go down)."

P: Betty is elected Q: Your taxes go down

Did Betty lie (FALSE statement), or not (TRUE)?

(Did Betty win?) P	(Did taxes go down?) Q	Did Betty lie? If P... then Q	
Betty wins (T)	Taxes go down (T)	True	
Betty win (T)	Taxes do not go down (F)	False	
Betty lost (F)	Taxes go down (T)	True	Since Betty lost in these two cases, her pre-election claim cannot be judged to be a false statement.
Betty Lost (F)	Taxes do not go down (F)	True	

2.4 $\boxed{\text{The Implication}}$

Suppose P and Q are two statements. We can form a new statement

If P, then Q,

called an implication, written $P \Rightarrow Q$ (P implies Q).

<u>Truth Table</u>

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

<u>EX</u> P: 16 is even Q: π is rational.

Find the truth value:

(a) $P \Rightarrow Q$ "If (16 is even) then (π is rational)"

False T F

(b) $Q \Rightarrow P$ "If (π is rational), then (16 is even)"

True False True

2.5 $\boxed{\text{Open Sentences}}$

Not, And, Or, Implies also apply to open sentences.

EX $P(x): x > 1$ $\quad Q(x): 3x - 8 \geq 0$ $\quad$ Domain $\{0, 1, 2, 3\}$

Which values of the domain, is $P(x) \Rightarrow Q(x)$ true?

$x = 0$ $\quad P(0): 0 > 1$ $\quad Q(0): -8 \geq 0$

"If $\underbrace{(0 > 1)}_{\text{False}}$, then $\underbrace{(-8 \geq 0)}_{\text{False}}$"

$P(0) \Rightarrow Q(0)$ True

$x = 1$ $\quad P(1): 1 > 1$ $\quad Q(1): 3 \cdot 1 - 8 \geq 0$

"If $\underbrace{(1 > 1)}_{\text{False}}$, then $\underbrace{(-5 \geq 0)}_{\text{False}}$"

$P(1) \Rightarrow Q(1)$ True

(Continued on Thursday)