1.2 $\boxed{\text{Subsets}}$

Set A is called a <u>subset</u> of set B if every element in A is also an element of B. In this case, we write $A \subseteq B$.

<u>EX</u>

$$\{1,5\} \subseteq \{1,2,3,4,5\}$$
$$\{1\} \subseteq \{1,2,3,4,5\}$$
$$\{1,2,3,4,5\} \subseteq \{1,2,3,4,5\}$$
$$\emptyset \subseteq \{1,2,3,4,5\}$$

<u>EX</u> $\{1, \{2\}, \{1,2\}\}$

Which of the following are subsets?

1 not a subset (not a set - its an element)

$\{1\} \subseteq \{1, \{2\}, \{1,2\}\}$

$\{1,2\}$ not a subset $\qquad \{\{1,2\}\}$

$\{1, \{2\}\} \subseteq \{1, \{2\}, \{1,2\}\}$

Set A is <u>not</u> a subset of B if there is at least one element of A that is <u>not</u> an element of B. $A \not\subseteq B$

<u>EX</u> $\{10,1,2\} \not\subseteq \{1,2,3,4,5\}$

Two sets are equal $A = B$ if $A \subseteq B$ and $B \subseteq A$. A and B have exactly the same elements.

Set A is a <u>proper subset</u> of B if $A \subseteq B$ but $A \neq B$. (Every element in A is an element of B, but not all elements of B are elements of A.) $A \subset B$

Subsets of Real Numbers

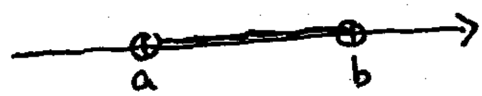
- $\mathbb{N} \quad \mathbb{Z} \quad \mathbb{Q} \quad \mathbb{I}_r \quad \mathbb{R} \quad \mathbb{C}$

  $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$ $\qquad$ $5 = \frac{5}{1}$

  $\mathbb{I}_r \subset \mathbb{R}$

- Intervals

  $(a,b) = \{x \in \mathbb{R} : a < x < b\}$ open interval



  $[a,b] = \{x \in \mathbb{R} : a \leq x \leq b\}$ closed interval

  $(a,b] = \{x \in \mathbb{R} : a < x \leq b\}$ half open/closed

  $(a,\infty) = \{x \in \mathbb{R} : a < x\}$

  $(-\infty,a) = \{x \in \mathbb{R} : x < a\}$

Power Set

Suppose A is a set. The set consisting of <u>all</u> subsets of A is called the <u>power set</u> of A, $\mathcal{P}(A)$.

<u>EX</u> $A = \{5, 10\}$

$$\mathcal{P}(A) = \{ \{5\}, \{10\}, \{5,10\}, \emptyset \}$$

<u>EX</u> $B = \{ \emptyset, 1, \{2,4\} \}$

$$\mathcal{P}(B) = \{ \{\emptyset\}, \{1\}, \{\{2,4\}\}, \emptyset, \{\emptyset, 1, \{2,4\}\}, \{\emptyset, 1\}, \{\emptyset, \{2,4\}\}, \{1, \{2,4\}\} \}$$

Note: $|\mathcal{P}(A)| = 4$ $\quad |\mathcal{P}(B)| = 8$

$$|\mathcal{P}(C)| = 2^{|C|}$$

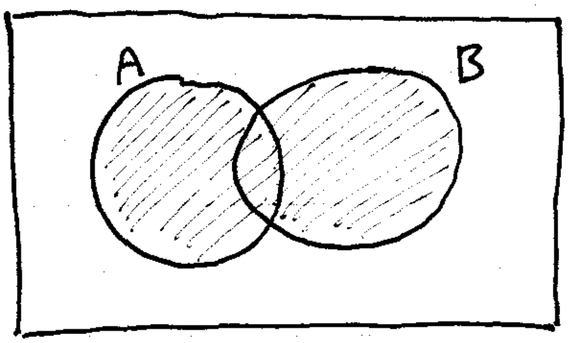
1.3 Set Operations

Suppose A and B are two sets.

- The union of A and B

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

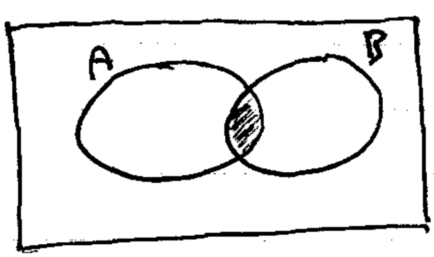
includes the possibility of both



Ex $A = \{1,3\}$ $B = \{3,5\}$ $A \cup B = \{1,3,5\}$

- The intersection of A and B

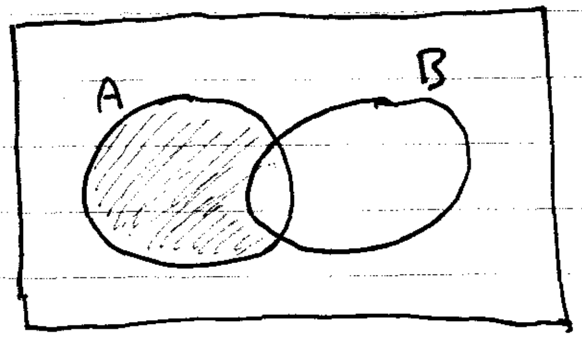
$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$



Ex $A = \{1,3\}$, $B = \{3,5\}$ $A \cap B = \{3\}$

- The difference

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$



<u>Ex</u> $A = \{1,3\}$ $B = \{3,5\}$ $A - B = \{1\}$

$B - A = \{5\}$

- Complement

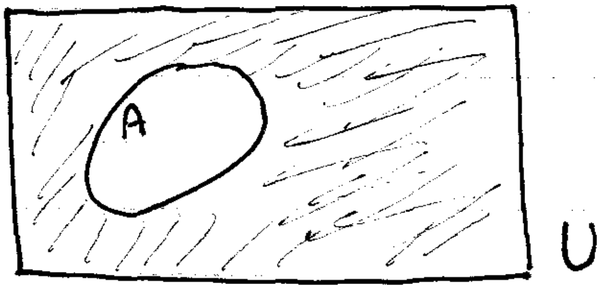
Start with a "universal" set, U.

The complement of a subset A of U is

$$\overline{A} = \{x : x \in U \text{ and } x \notin A\} = U - A$$

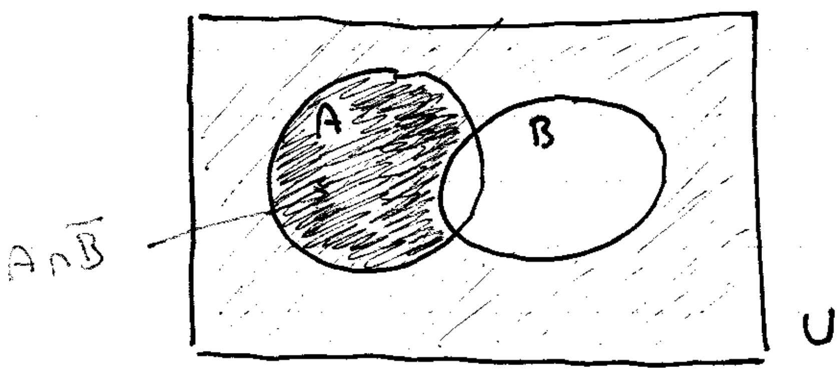
<u>Ex</u> $U = \mathbb{Z}$ (all integers) $A = \mathbb{N}$ (natural numbers)

$$\overline{A} = \{x : x \text{ is an integer } x \leq 0\}.$$



<u>EX</u> $U = \{1, 2, 3, 4, \ldots, 10\}$, $A = \{2, 3, 5, 7\}$, $B = \{2, 4, 6, 8, 10\}$

$$A \cap \overline{B} = \{2, 3, 5, 7\} \cap \{1, 3, 5, 7, 9\} = \{3, 5, 7\}$$



It appears

$A \cap \overline{B} = A - B$

1.4 Indexed Collection of Sets

$$\{A_1, A_2, A_3, \ldots, A_{10}\}$$

Indexed collection of sets.

$$\{A_i\}_{i=1}^{10} \quad \text{or} \quad \{A_i\}_{i\in S} \qquad S=\{1,2,3,\ldots,10\}$$

<u>EX</u> $\left\{ \left[-\frac{1}{n}, \frac{1}{n}\right] \right\}_{n\in \mathbb{N}}$

$$\left\{ [-1,1], \left[-\frac{1}{2}, \frac{1}{2}\right], \left[-\frac{1}{3}, \frac{1}{3}\right], \ldots \right\}$$

<u>EX</u> $\left\{ \left[\frac{-1}{n}, \frac{1}{n}\right] \right\}_{n\in S} \qquad S=\{1,3,5\}$

$$\left\{ [-1,1], \left[\frac{-1}{3}, \frac{1}{3}\right], \left[\frac{-1}{5}, \frac{1}{5}\right] \right\}$$

Unions and intersections of indexed collections of sets:

$$\{A_i\}_{i \in S}$$

$$\bigcup_{i \in S} A_i = \{x : x \text{ is in } \underset{\substack{\uparrow \\ \text{at least } 1}}{\text{some}} \ A_i \text{ for some } i \in S\}$$

$$\bigcap_{i \in S} A_i = \{x : x \text{ is in every } A_i \text{ for all } i \in S\}.$$

<u>EX</u> Let $A_i = \{i, i+1\}$ for $i = 1, 2, \dots, 10$

For example, $A_1 = \{1,2\}$, $A_2 = \{2,3\}$, $A_3 = \{3,4\}$, ..

(a)
$$\bigcup_{i \in \{1,3,5\}} A_i = A_1 \cup A_3 \cup A_5$$
$$= \{1,2\} \cup \{3,4\} \cup \{5,6\}$$
$$= \{1,2,3,4,5,6\}$$

(b)
$$\bigcap_{i \in \{9,10\}} A_i = A_9 \cap A_{10}$$
$$= \{9,10\} \cap \{10,11\}$$
$$= \{10\}$$

(c) $\bigcap_{i \in \{1,2,3\}} A_i = A_1 \cap A_2 \cap A_3$

$= \{1,2\} \cap \{2,3\} \cap \{3,4\}$

$= \{ \}$

$= \emptyset$