Chapter 1 Sets

A <u>set</u> is a collection of objects.

$\{1, 2, 3, 4\}$ ~~$1, 2, 3, 4$~~

{ Alan, Betty, Charles }

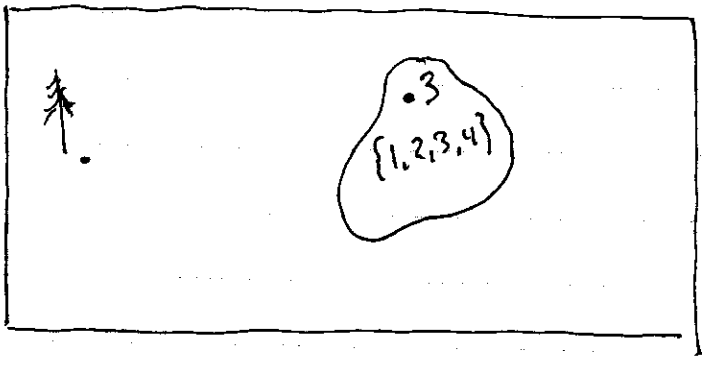
{  , 4,  }

Any object may or may not be a member of a given set.

$\{1,2,3,4\}$ 3 is "in" the set $3 \in \{1,2,3,4\}$

 is not an element  $\notin \{1,2,3,4\}$

Venn Diagrams



Denoting and Describing Sets

- Normally (not always) use capital letters to denote or represent a set.

  Let A be the set of even integers greater than 1 but less than 15.

- Different ways to describe a set.

List $A = \{2, 4, 6, 8, 10, 12, 14\}$

$A = \{4, 6, 2, 14, 12, 10, 8\}$

$A = \{2, 4, 6, 8, \cdots, 14\}$

$A = \{x : x \text{ is an even integer } 2 \leq x \leq 14\}$

(":" = such that)

$A = \{2x : x \text{ is an integer } 1 \leq x \leq 7\}$

- $\{x : p(x)\}$, $\{x \mid p(x)\}$

Special Sets

- Empty Set $\emptyset = \{\ \}$
- Positive Integers $N = \{1, 2, 3, 4, \dots\}$
  Natural Numbers
- All Integers $\mathbb{Z} = \{0, 1, -1, 2, -2, \dots\}$
  $= \{\dots, -2, -1, 0, 1, 2, \dots\}$
- All Rational Numbers $\mathbb{Q} = \{x : x = p/q$ for some integers $p, q\}$
- All Irrational Numbers $\mathbb{I} = \{x : x$ is not a rational numba$\}$
  $= \{x : x \notin \mathbb{Q}\}$
- All Real Numbers $\mathbb{R}$
- All Complex Numbers $\mathbb{C} = \{x + iy : x, y \in \mathbb{R}\}$.

Cardinality

The number of elements in a set.

EX $A = \{1, 3, 5, 7\}$ Cardinality of A $|A| = 4$

$B = \{$  $\}$ $|B| = 3$

$N = \{1, 2, 3, 4, \ldots\}$ N is infinite (Cardinality is <u>not</u> a number.)

$\emptyset$ $|\emptyset| = 0$

Examples

(a) $T = \{ 0, \{1,2,3\}, 4, 5 \}$

Elements are $0, \{1,2,3\}, 4, 5$

$0 \in T$, $\{1,2,3\} \in T$, $1 \notin T$, $3 \notin T$

$|T| = 4$

(b) $S = \{ \emptyset \} = \{ \{\} \}$

Element of S is $\emptyset$, $|S| = 1$

(c) $U = \{ 0 \}$ $|U| = 1$

(d) $D = \{ n \in \mathbb{N} : n \leq 9 \}$

$E = \{ x \in \mathbb{Q} : x \leq 9 \}$

$H = \{ x \in \mathbb{R} : x^2 = 2 \}$

$J = \{ x \in \mathbb{Q} : x^2 = 2 \}$

- Describe by listing if possible.

$D = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

E

$H = \{\sqrt{2}, -\sqrt{2}\}$

$J = \{\ \} = \emptyset$

- Cardinality

$|D| = 9$ $|E|$ infinite, $|H| = 2$, $|J| = 0$

(e) $E = \{2x : x \in \mathbb{Z}\}$

Listed form $E = \{\ldots, -4, -2, 0, 2, 4, \ldots\}$

(f) $D = \{\ldots, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \ldots\}$

Description $D = \{2^x : x \in \mathbb{Z}\}$