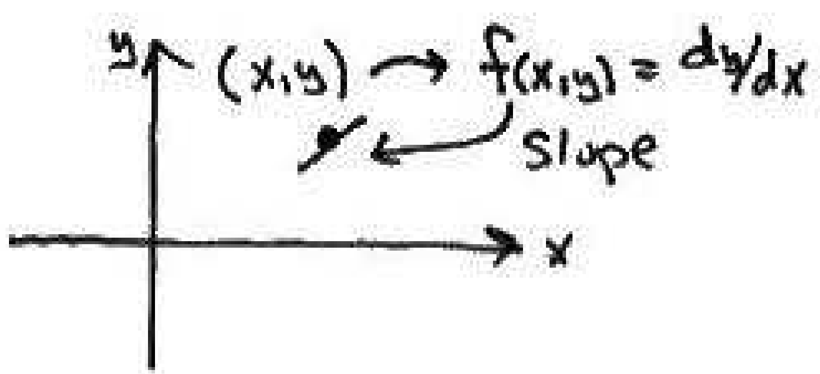
2.1 Direction Field and Autonomous DEs

2.1.1 Direction Fields

A graphical method to "solve" a 1^{st} order differential equation

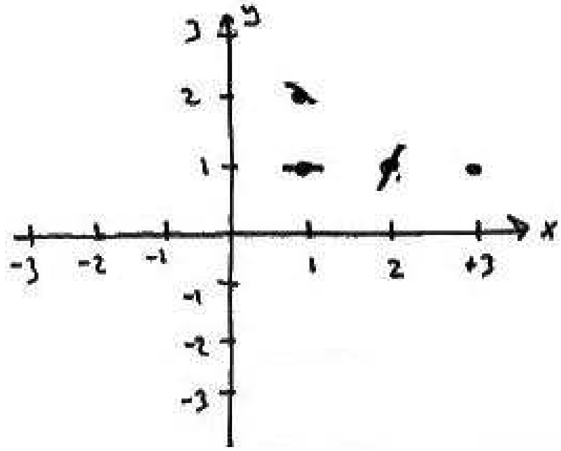
$$\frac{dy}{dx} = f(x, y)$$

$\frac{dy}{dx}$ is the slope of the line tangent to the graph of y(x).
So $\frac{dy}{dx} = f(x, y)$ gives us a formula for computing slopes.



<u>EX</u>

$$\frac{dy}{dx} = x^2 - y$$



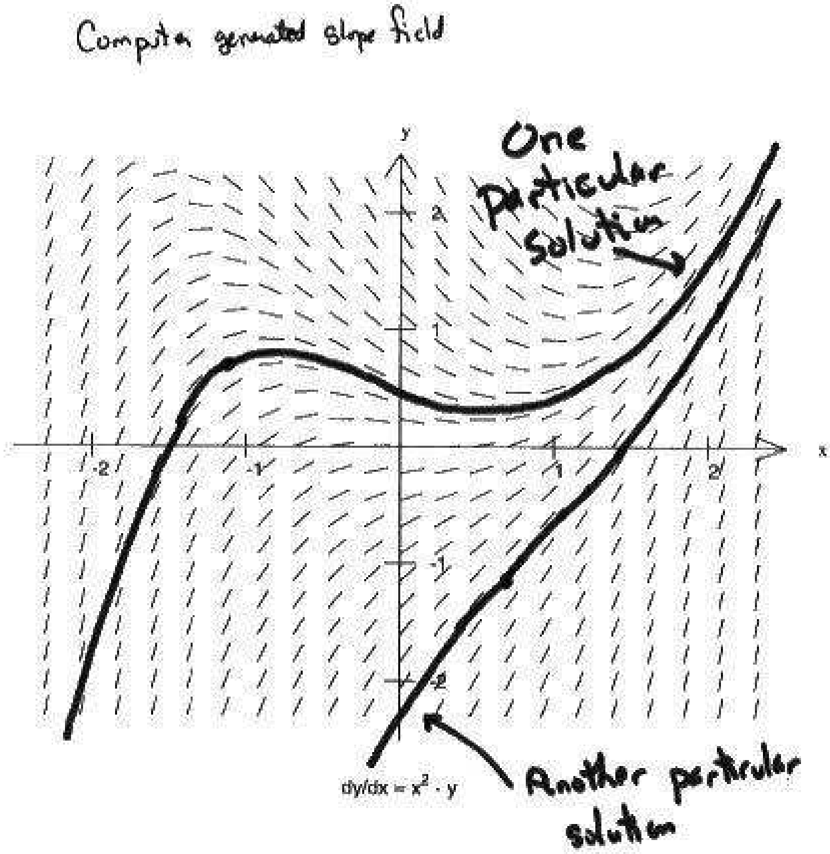
$$AT(1,1), \quad \frac{dy}{dx} = (1)^2 - 1 = 0$$

$$AT(2,1), \quad \frac{dy}{dx} = (2)^2 - 1 = 3$$

$$AT(1,2), \quad \frac{dy}{dx} = (1)^2 - 2 = -1$$

Constructing many slopes over an array of point generates a "field" of slopes that helps "see" what the solutions look like: (see next page)

Computer generate slope field



Another Particular Solution

$$dy/dx = x^2 - y$$

EX A model for the population p (in thousands) at time t years of a certain species is

$$\frac{dp}{dt} = p(1-p)$$

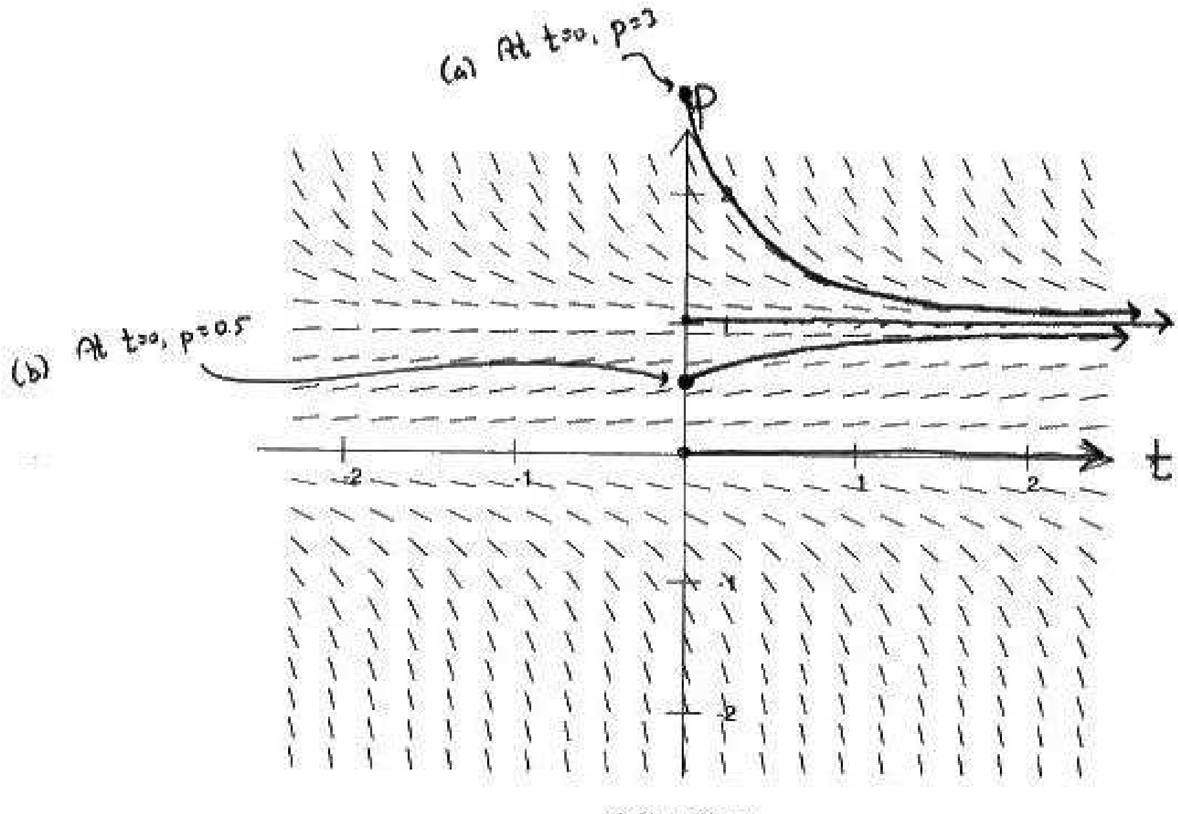
(a) If p(0) = 3 (at t=0, the population is 3 thousands), what happens to the population ∞ t$\rightarrow \infty$?

(b) If p(0) =.5 (at t=0, the population is 500), what happens to the population ∞ t$\rightarrow \infty$?

(a) At t=0, p=3

(b) At t=0, p=0.5

$dp/dt = p'(1-p)$



$$\frac{dp}{dt} = p(1-p)$$

Constant population p $\Rightarrow 0 = p(1-p) \Rightarrow p = 0 \; p = 1$

(a) The population decreases to a level of 1 thousand ?

(b) The population increases to a level of 1 thousand ?

Isocline: For $\dfrac{dy}{dx} = f(x, y)$, the curer of $f(x, y) = C$, C a constant, is called an isocline.

Method of isoclines: sketch a direction field by hand.

2.1.2 Autonomous DES

An ODE in which the independent variable does not appear explicitly is said to be autonomous.

$$\frac{dy}{dx} = f(y)$$

EX: $\dfrac{dy}{dt} = y - y^3$ autonomous

$\dfrac{dy}{dt} = y \sin t$ non autonomous

Critical points : $\dfrac{dy}{dx} = f(y)$,if $f(C) = 0$ then C is called a critical point.

If C is a critical point of $\dfrac{dy}{dt} = f(y)$, then $y(x) = C$ is a constant solution (equilibrium solution)

<u>EX</u>

$$\frac{dp}{dt} = p(a - bp)$$

$$f(p) = p(a - bp) = 0, \quad p = 0, p = \frac{a}{b}$$

Graph 2.4

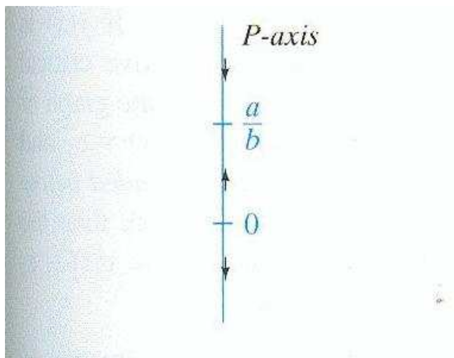


FIGURE 2.4 Phase portrait of $dP/dt = P(a - bP)$

phase portrait

Interval	Sign of $f(P)$	$P(t)$	Arrow
$(-\infty, 0)$	minus	decreasing	points down
$(0, a/b)$	plus	increasing	points up
$(a/b, \infty)$	minus	decreasing	points down

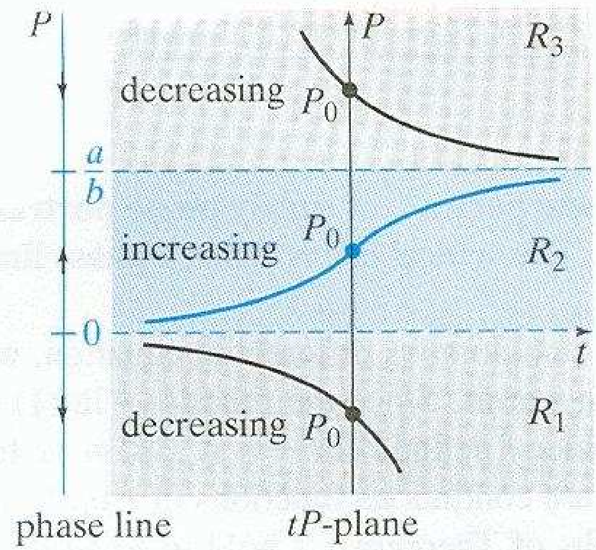


FIGURE 2.6 Phase portrait and solution curves in each of the three subregions

Arrectors and Repellers

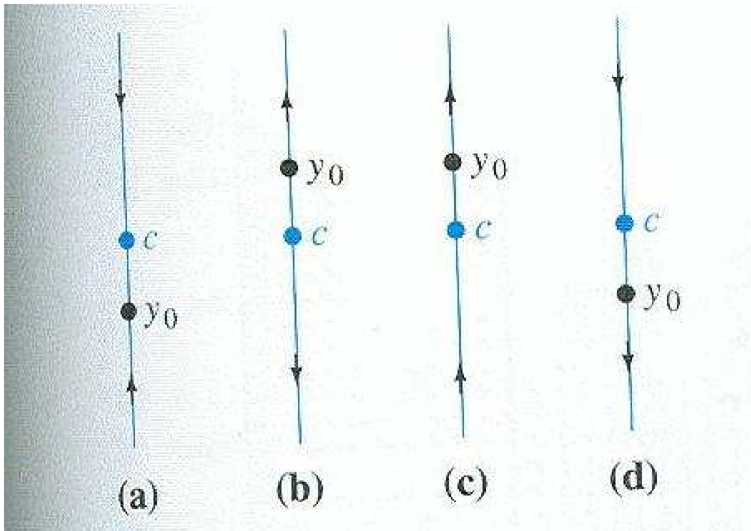


FIGURE 2.8 Critical point c is: an attractor in (a), a repeller in (b), and semi-stable in (c) and (d).

affector repeller semi-stabs