Math 6382 Homework 2
due on 10/9/2016

Show all work. You are welcome to discuss HW with anyone in the class, but submit your own solution. Submit both written answers and R codes.

1. (10 pts each) In the textbook, page 96 number 3.19, 3.20, 3.21

2. (6 pts each) Simulate the empirical distribution of the eigenvalues of the following block matrices in which A and B are 1000×1000 Wigner matrices

   (a)

   $$X = \begin{pmatrix} A & B \\ B & A \end{pmatrix}$$

   (b)

   $$Y = \begin{pmatrix} A & B & B \\ B & A & B \\ B & B & A \end{pmatrix}$$

   Using graphs, compare their probability histograms to the actual limiting probability distribution which are γ_2 and the mixture $\frac{2}{3}\gamma_2 + \frac{1}{3}\gamma_5$, respectively. The probability measure γ_{σ^2} is the semicircle law with pdf $f(x) = \dfrac{1}{2\pi\sigma^2}\sqrt{4\sigma^2 - x^2}$ for $x \in (-2\sigma, 2\sigma)$.

3. (10 pts) Consider a random walk on $\mathbb{N} \cup \{0\}$ started at stated 0 with a reflecting barrier at 0 (so $P_{0,1} = 1$). Estimate the probability distribution of the time till visiting state $n = 10$ if $p_i = P_{i,i+1} = \dfrac{1}{(1 + i)^2}$ by simulating the process 100 times and then plotting a probability histogram for the results.

4. (10 pts) Write a code to randomly scatter 100 Pokemon characters on earth. Then select one of them randomly to be the legendary character "Zekrom." Graph their locations on earth with special mark or color to Zekrom. Consider earth as a unit sphere.

Best wishes!