**Math 6382 Homework 5**
**due on 11/30/2016**

Show all work. You are welcome to discuss HW with anyone in the class, but submit your own solution. Submit both written answers and R codes.

1. (10 pts each) Generate 20 random numbers from the mixture $exp(\lambda_1 = 5)$ and $gamma(r = 2, \lambda_2 = 10)$ with weights $p = .2$ and $1 - p = .8$, respectively. Then:

   (a) Estimate $p, \lambda_1, r, \lambda_2$ using Maximum Likelihood method (with both optimization and root finding)

   (b) Estimate the bias and standard error of the estimates.

   (c) Estimate $p, \lambda_1, r, \lambda_2$ using the EM algorithm. Then based on it,

   (d) If you don't know the actual parameters' values, find estimates for bias and standard error of each of the parameters using bootstrap method.

   (e) Use the Monte Carlo likelihood ratio test to test whether the model of the data is $exp(\lambda_1 = 5)$ versus the mixture you have used originally to generate the data. Are they nested? How?

   (f) Do the same likelihood ratio test again but for another data of 20 points you generate randomly but with $p = .8$

Best wishes!