BEAM DEFLECTION

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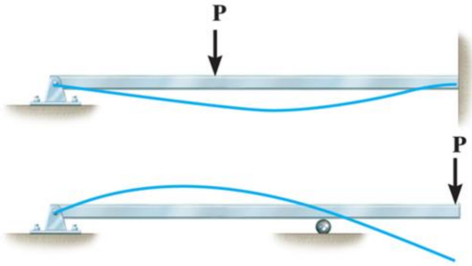
THE ELASTIC CURVE

The deflection diagram of the longitudinal axis that passes through the centroid of each cross-sectional area of a beam.

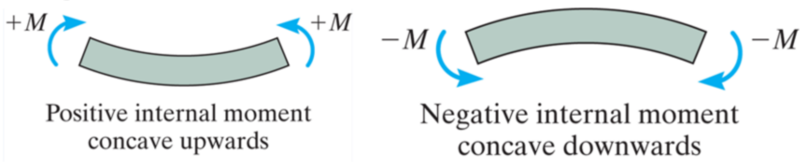


- Supports that apply a moment restrict displacement as well as rotation or slope.
- Supports that apply a force, restrict displacement.

TO FIND THE ELASTIC CURVE:

1. Draw the moment diagram
2. Apply the restriction at the connections
3. Take into account the curvature and determine the inflection points where M is zero.

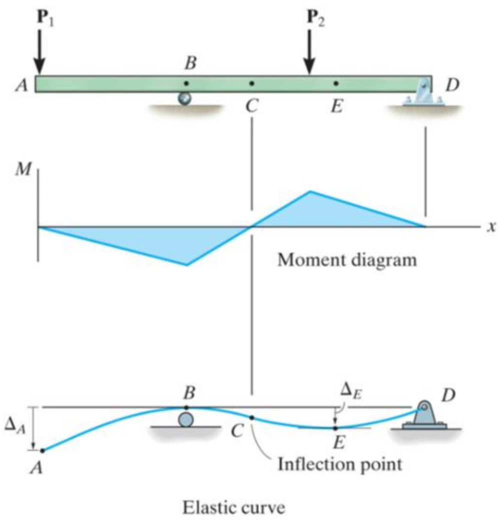
Sign Convention



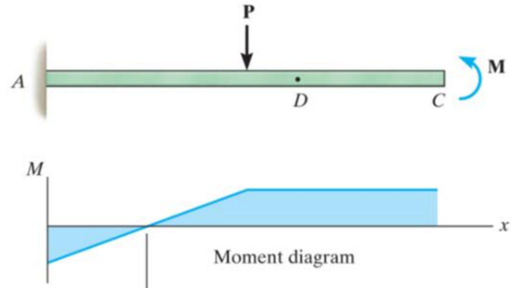
CASE 1

Note: Point C is an inflection point (change of curvature).

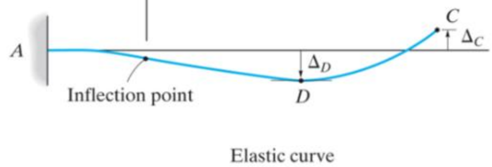
It is necessary to determine ΔA and ΔE to find the maximum deflection of the beam.



CASE 2

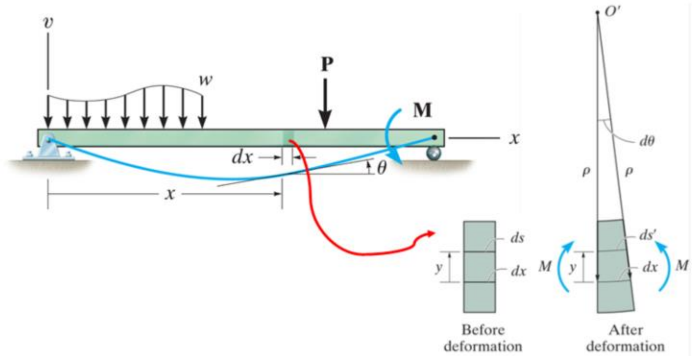


Note the inflection point (change of curvature) occurs when M=0.



It is necessary to determine ΔC and ΔD to find the maximum deflection of the beam.

MOMENT-CURVATURE RELATIONSHIP



For the element dx, O' is the center of curvature and ρ is the radius of curvature.

RADIUS OF CURVATURE

Combining Hooke's Law and the Flexure Formula,

$$\frac{1}{\rho} = \frac{M}{EI}$$

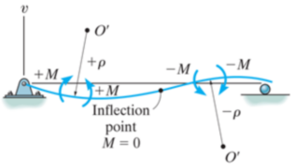
where

ρ: the radius of curvature at the point on the elastic curve (sign depends on M)

M: the internal moment in the beam at the point

E: the material's modulus of elasticity

I: the beam's moment of inertia about the neutral axis



"Flexural Rigidity" (EI) is always positive.

SLOPE & DISPLACEMENT BY INTEGRATION

From Calculus, the radius of curvature is defined where v=f(x)

$$\frac{1}{\rho} = \frac{\frac{d^2v}{dx^2}}{\left[1 + \left(\frac{dv}{dx}\right)^2\right]^{3/2}}$$

In many engineering applications, $\frac{dv}{dx} \ll 1$

$$\frac{1}{\rho} = \frac{d^2v}{dx^2}$$

Apply relation between radius of curvature and flexure formula

$$\frac{M}{EI} = \frac{d^2v}{dx^2}$$

SLOPE & DISPLACEMENT BY INTEGRATION

Equation (1) $$M(x) = EI\frac{d^2v}{dx^2}$$

Integrating the bending moment diagram twice will give you the deflection at any point along the beam.

Recall the relationship between a distributed load, shear force, and bending moment:

$$\frac{dM}{dx} = V \qquad \frac{dV}{dx} = -w(x)$$

Thus,

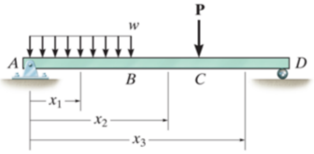
Equation (2) $$V(x) = EI\frac{d^3v}{dx^3}$$

Equation (3) $$-w(x) = EI\frac{d^4v}{dx^4}$$

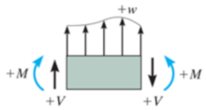
Note: Equations 1, 2, & 3 are only valid when EI is constant along the beam.

DISCONTINUITIES & SIGN CONVENTION

Discontinuities occur at the end or start of distributed loadings, concentrated forces, and concentrated moments.



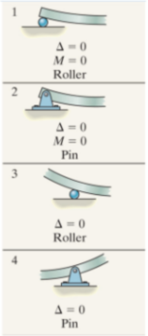
Recall, a positive sign convention for beam bending will cause the beam to "hold water"

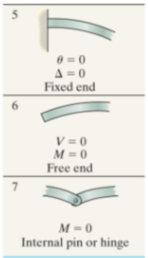


Positive sign convention

BOUNDARY CONDITIONS

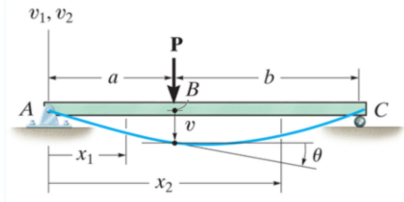
Boundary condition are required to determine the constants of integration of M, V, Displacement, and Slope curves.





CONTINUITY CONDITIONS

Continuity conditions are also used to determine constants of integration of the displacement and slope curves.



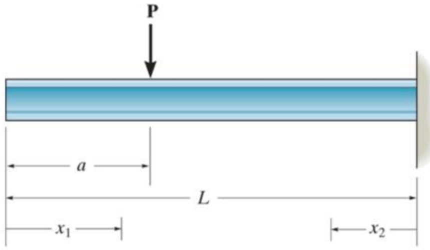
$$0 \le x_1 \le a$$
$$a \le x_2 \le a + b$$

Continuity Equations

$$v_1(a) = v_2(a)$$
$$\theta_1(a) = -\theta_2(a)$$

EXAMPLE 0

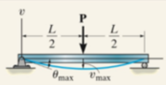
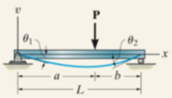
Determine the equations of the elastic curve using the x_1 and x_2 coordinates. EI is constant.



METHOD OF SUPERPOSITION

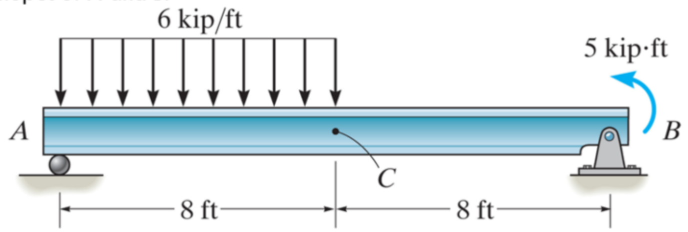
Superposition is used to determine the slope or deflection at certain points of a beam due to several loads whose effect is first separately computed and then added to find total values.

Simply Supported Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \frac{-PL^2}{16EI}$	$v_{max} = \frac{-PL^3}{48EI}$	$v = \frac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \frac{-Pab(L+b)}{6EIL}$ $\theta_2 = \frac{Pab(L+a)}{6EIL}$	$v\Big\rvert_{x=a} = \frac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \frac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$

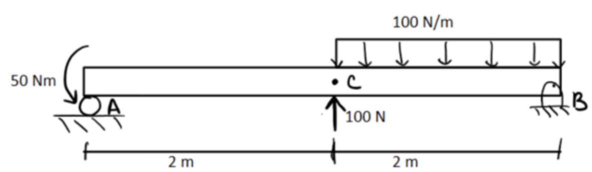
EXAMPLE 1

The W8x24 simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at C and the slopes of A and B.



EXAMPLE 2

The C180x22 beam is made of 2014-T6 aluminum and subjected to the loadings shown. Determine the deflection at C and the slope of A and B.



EXAMPLE 3

The W12x26 beam is made of Gray ASTM 20 cast iron and is subjected to the loadings shown. Determine the deflection and slope at the end of the beam (point B).