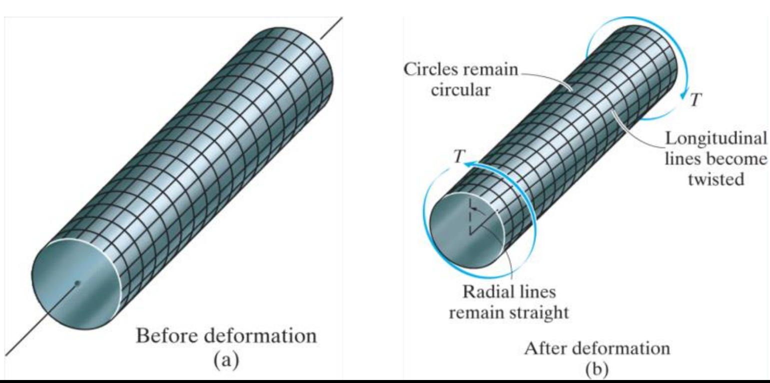
TORSIONAL DEFORMATION

Samantha Ramirez, MSE

TORSION

Torque

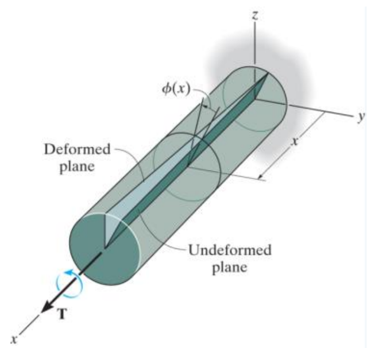
- A moment that tends to twist a member about its longitudinal axis



TORSIONAL DEFORMATION OF A CIRCULAR SHAFT

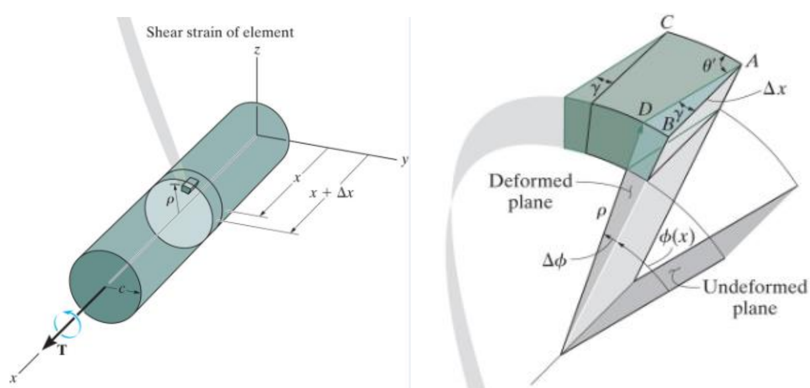
Assumption

- If the angle of twist is small, the length and radius of the shaft remain the same



The angle of twist $\phi(x)$ increases as x increases.

RELATION OF SHEAR STRAIN TO ANGLE OF TWIST



$$L_{BD} = \gamma\Delta x = \Delta\phi\rho$$

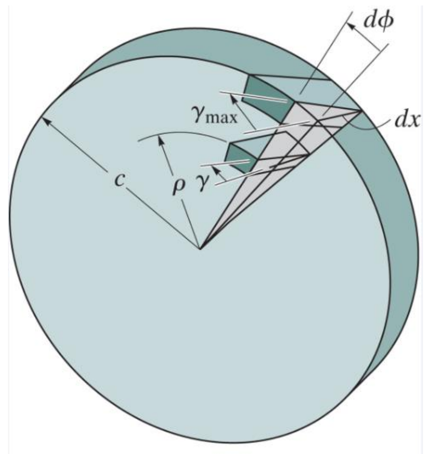
TORSIONAL DEFORMATION OF A CIRCULAR SHAFT: CROSS-SECTION

$$\gamma_{max} = c\frac{d\phi}{dx}$$

$$\gamma = \frac{\rho}{c}\gamma_{max}$$

$$\tau = G\gamma$$

$$\tau = \frac{\rho}{c}\tau_{max}$$



DERIVING ANGLE OF TWIST

Recall,

- Relationship between shear strain and angle of twist $\frac{d\phi}{dx} = \frac{\gamma}{\rho}$ (1)
- Hooke's Law $G = \frac{\tau}{\gamma}$ (2)
- Torsional Formula $\tau = \frac{T(x)\rho}{J(x)}$ (3)

Plugging (2) into (3) and then (1) into the resulting equation you get an equation for the angle of twist:

- $\phi = \int_0^L \frac{T(x)dx}{J(x)G(x)}$

ANGLE OF TWIST

Assuming a homogeneous material with a constant cross-sectional area and applied torque,

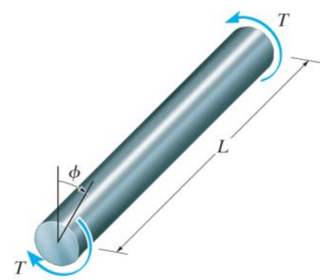
$$\phi = \frac{TL}{JG}$$

ϕ: the angle of twist of one end of the shaft with respect to the other end, measured in radians

T: the internal torque at the arbitrary position x

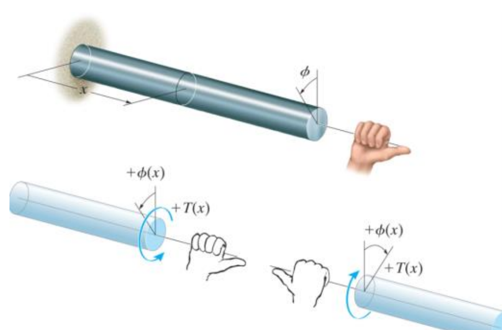
J: the shaft's polar moment of inertia

G: the shear modulus of elasticity or the modulus of rigidity



SIGN CONVENTION

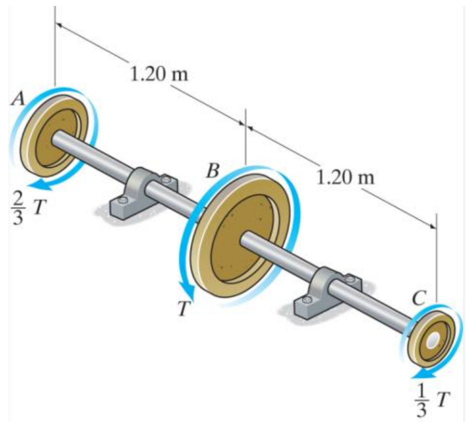
Using the right-hand rule, the torque and angle of twist will be positive, provided the thumb is directed outward from the shaft when the fingers curl to give the tendency for rotation.



Positive sign convention for T and ϕ.

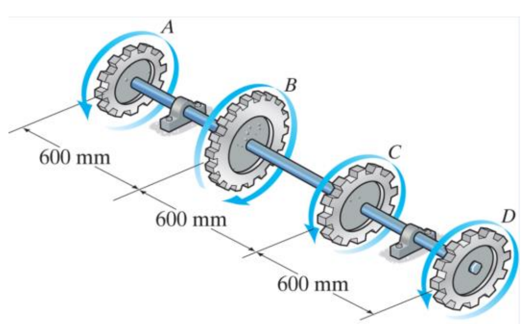
EXAMPLE 1

The 60 mm diameter shaft is made of 6061-T6 aluminum having an allowable shear stress of τ_{allow}=80 MPa. Determine the maximum allowable torque T. Also, find the corresponding angle of twist of disk A relative to disk C.



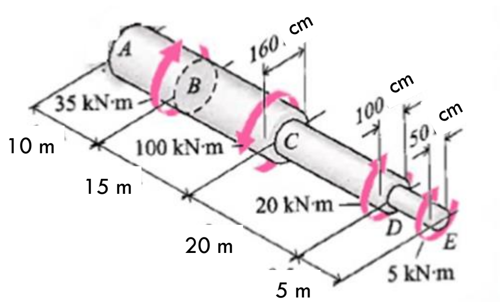
EXAMPLE 2

The shaft is made of A992 steel with the allowable shear stress of τ_{allow}=75 Mpa. If gear B supplies 15 kW of power, while gears A, C, and D withdraw 6 kW, 4 kW, and 5 kW, respectively, determine the required minimum diameter d of the shaft to the nearest millimeter. Also find the corresponding angle of twist of gear A relative to gear D. The shaft is rotating at 600 rpm.

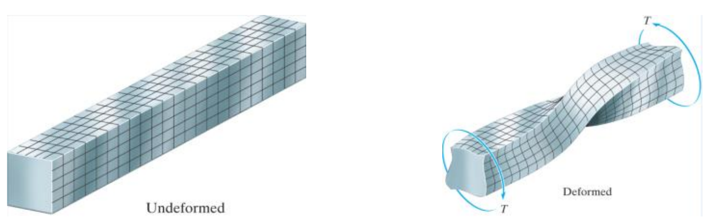


EXAMPLE 3

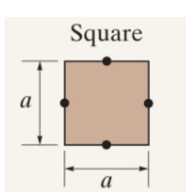
A series of torques are applied to the 6061-T6 aluminum alloy shaft. The diameter and length of each section are shown in the figure. Determine absolute maximum shear stress in the shaft and the angle of twist of point E relative to point A. Note: Point A is fixed to a wall.

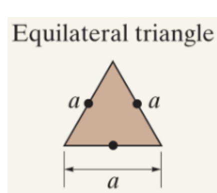


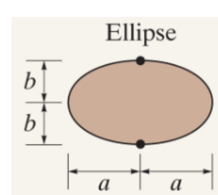
NON-CIRCULAR SHAFTS



Maximum shear stress occurs at a point on the edge of the cross section that is closest to the center axis of the shaft.







$\tau_{max} = \frac{4.81T}{a^3}$ $\phi = \frac{7.10TL}{a^4G}$

$\tau_{max} = \frac{20T}{a^3}$ $\phi = \frac{46TL}{a^4G}$

$\tau_{max} = \frac{2T}{\pi ab^2}$ $\phi = \frac{(a^2 + b^2)TL}{\pi a^3 b^3 G}$

EXAMPLE 4

The aluminum rod has a square cross section of 10 mm by 10 mm. If it is 8 m long, determine the torque T that is required to rotate one end relative to the other end by 90°. (G_{al}= 28 GPa, τ_{allow}= 240 MPa)