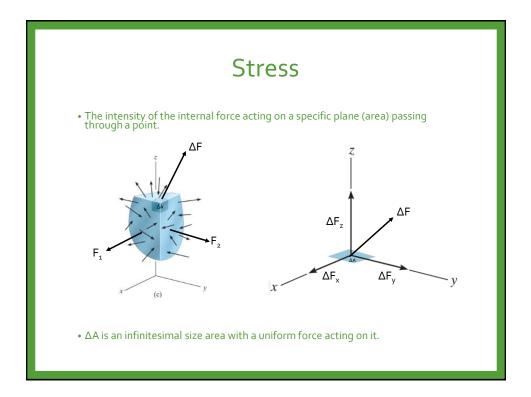
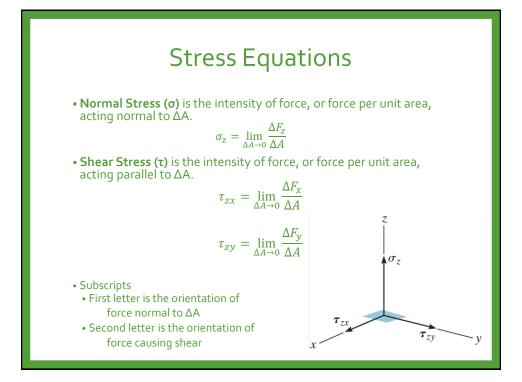
NORMAL AND SHEAR STRESS

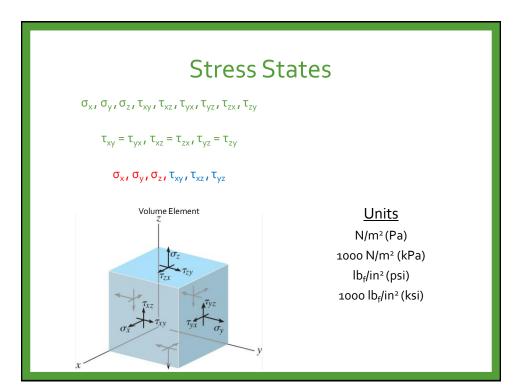
Samantha Ramirez, MSE

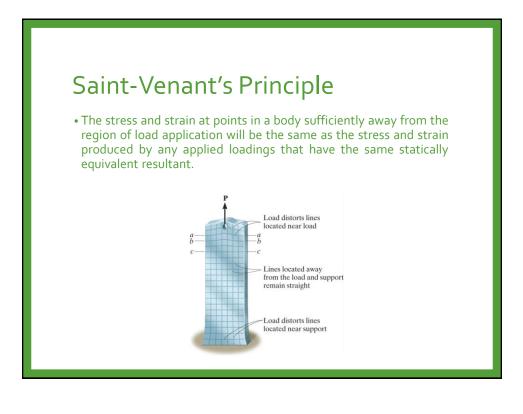
Objectives

- 1. Define stress, normal stress, direct shear stress, and factor of safety.
- 2. Design members subjected to an axial load while considering allowable stress.
- 3. Design members subjected to direct shear while considering allowable stress.







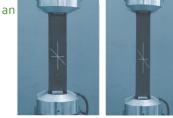


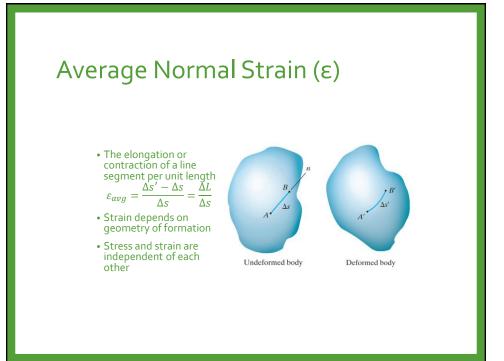
Saint-Venant's Principle

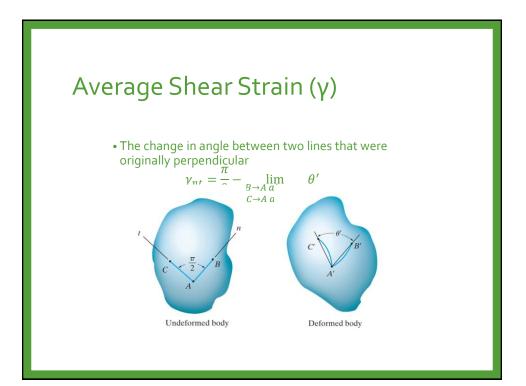
• "If the forces acting on a small portion of the surface of an elastic body are replaced by another statically equivalent system of forces acting on the same portion of the surface, this redistribution of loading produces substantial changes in the stresses locally, but has a negligible effect on the stresses at distances which are large in comparison with the linear dimensions of the surface on which the forces are changed." (B. Saint-Venant, Mém. savants étrangers, vol. 14, 1855.)

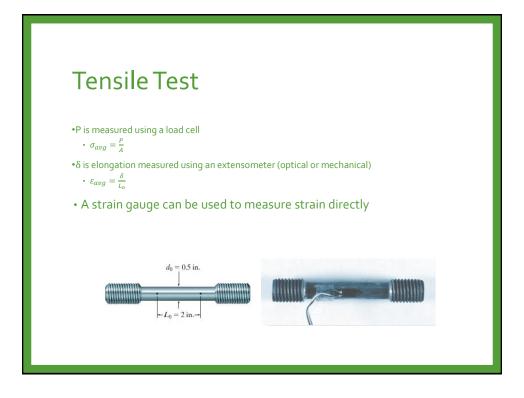
Deformation

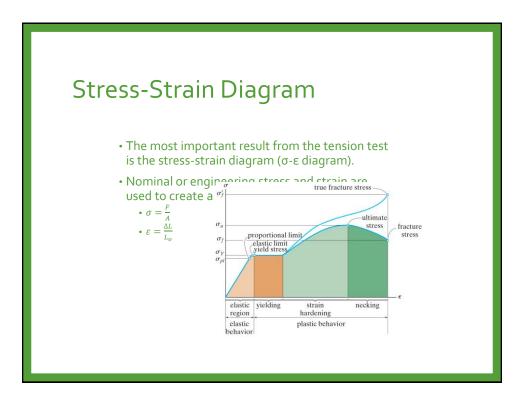
- Deformation is defined as the changes to a body's shape and size when a load is applied.
- Deformation is specified by normal and showed of loads

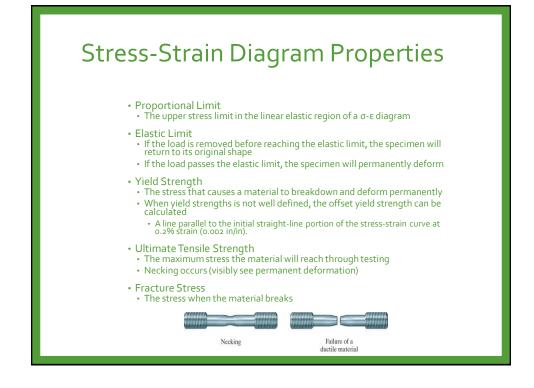


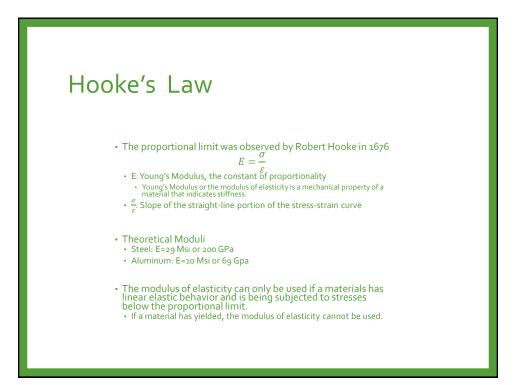








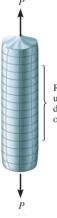




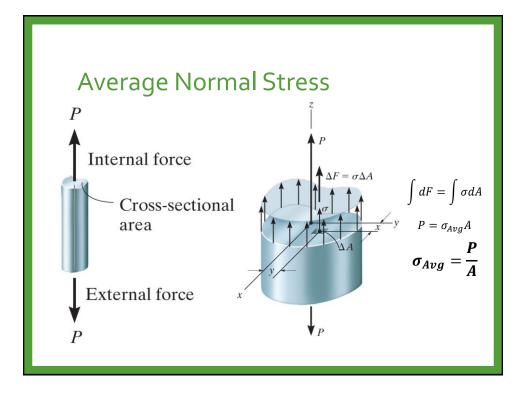
AVERAGE NORMAL STRESS

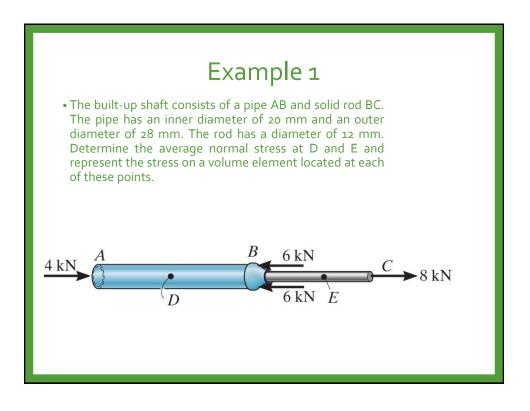
Uniaxial Tensile Test

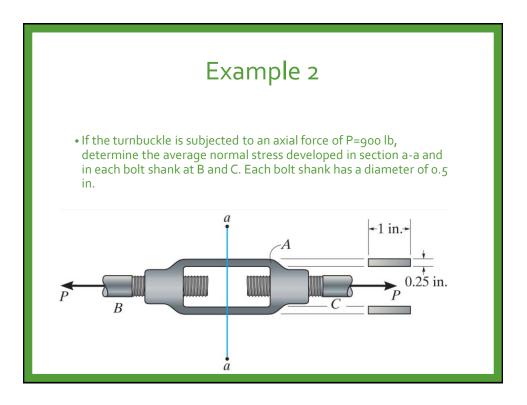
- Axially Loaded Bar (Uniaxial Tensile Test)
 - Assumptions
 - Homogeneous material
 - Isotropic material
 - Bar remains straight and cross-section flat
 - P is applied along the centroid axis



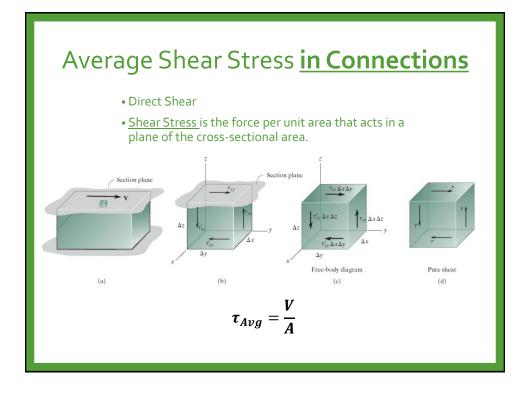
Region of uniform deformation of bar

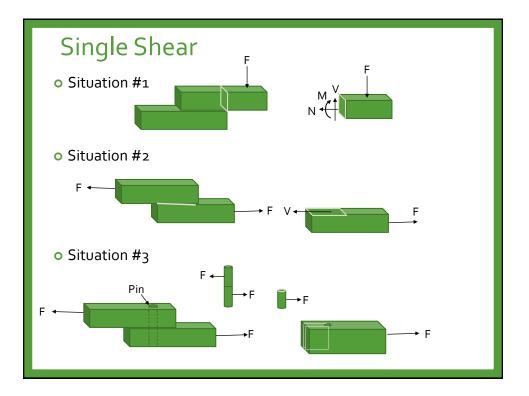


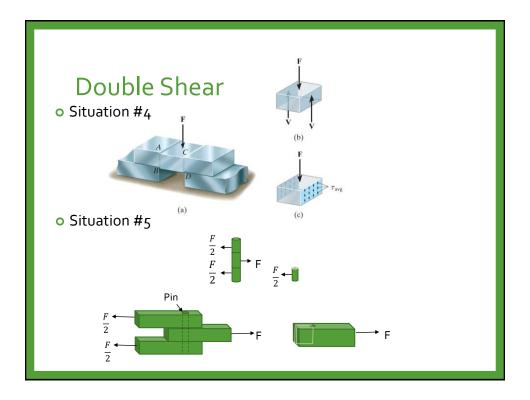


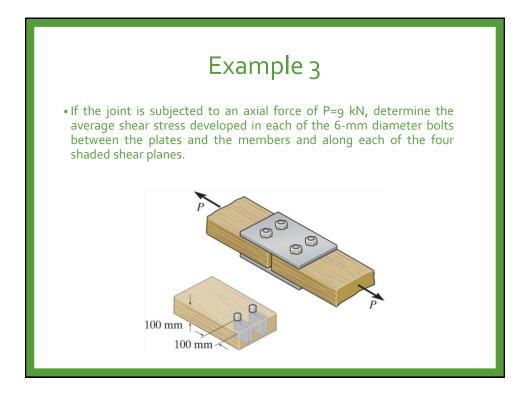


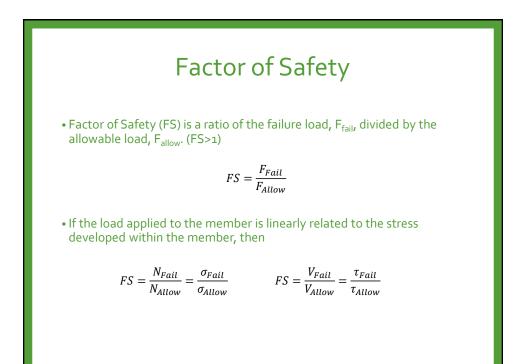


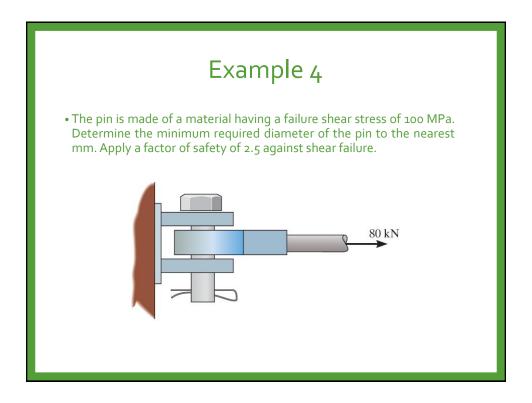


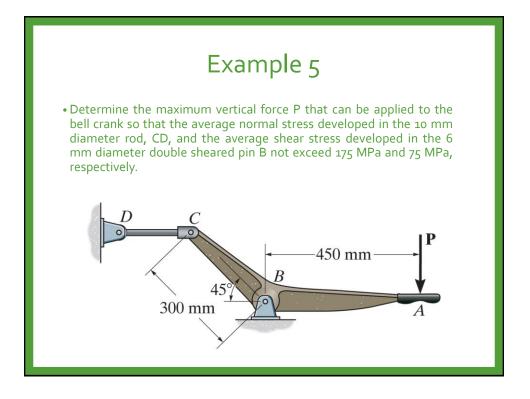


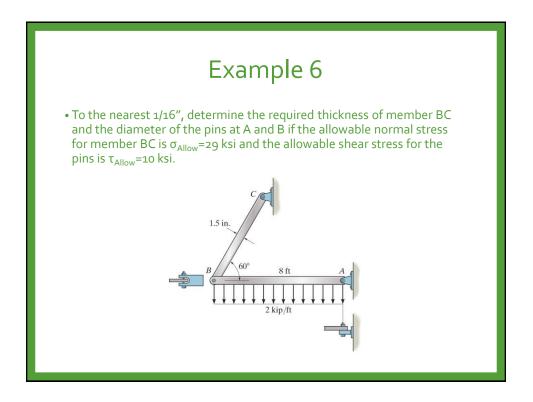


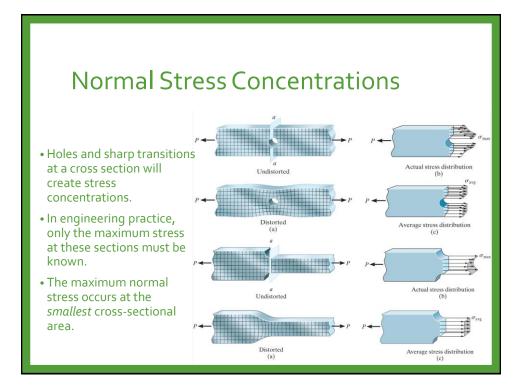










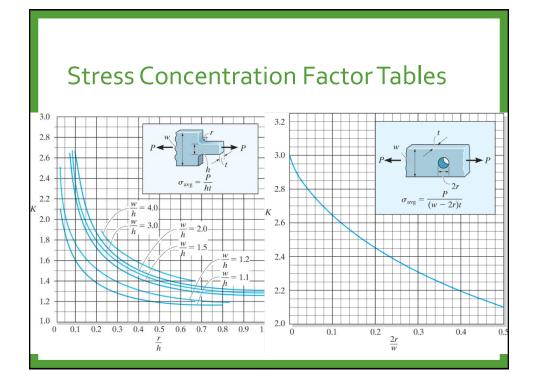


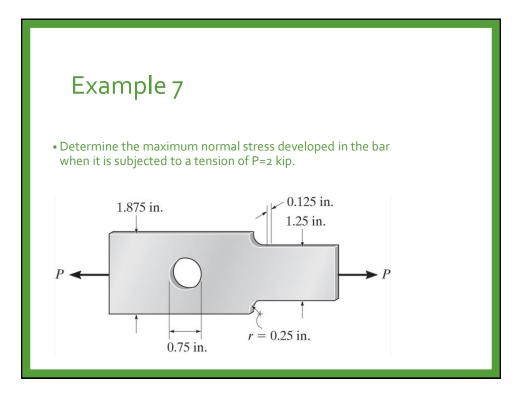
Stress Concentration Factor

• A stress concentration factor "K" is used to determine the maximum stress at sections where the cross-sectional area changes.

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

- K is found from graphs in handbooks of stress analysis
- $\bullet\,\sigma_{_{avg}}$ is the average normal stress at the smallest cross section

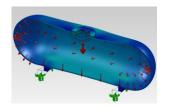




Thin-Walled Pressure Vessels

- Cylindrical or spherical vessels are commonly used in industry to serve boilers or tanks.
- A "thin wall" refers to a vessel having an inner radius to wall thickness ratio of 10 or more $\left(\frac{r}{t} \ge 10\right)$.
- We will assume a uniform or constant stress distribution throughout the thickness because it is thin.
- Pressure vessels are subjected to loadings in all directions.







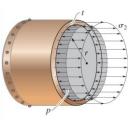
Circumferential or Hoop Stress (σ_1)

Longitudinal or Axial Stress (σ_2)

$$\sigma_1 = \frac{pr}{t}$$



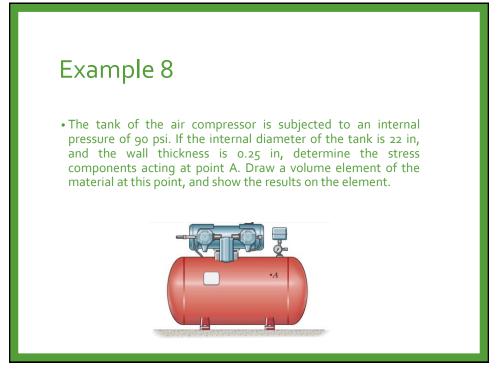
 $\sigma_2 = \frac{pr}{2t}$



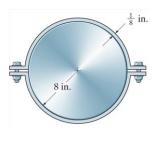
Spherical Vessels

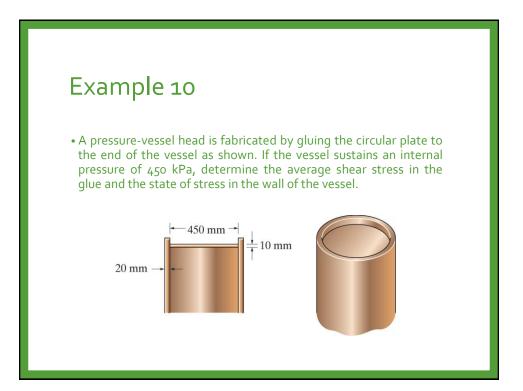
$$\sigma_2 = \frac{pr}{2t}$$

- σ_{1}, σ_{2} : the normal stress in the hoop and longitudinal directions, respectively.
 - Each is assumed to be constant throughout the wall of the cylinder, and each subjects the material to tension.
- p: the internal gauge pressure developed by the contained gas
- r: the inner radius of the cylinder
- t: the wall thickness (r/t≥10)



• The A-36 steel band is 2 in wide and is secured around the smooth rigid cylinder. If the bolts are tightened so that the tension in them is 400 lb, determine the normal stress in the band, the pressure exerted on the cylinder, and the distance half the band stretches.



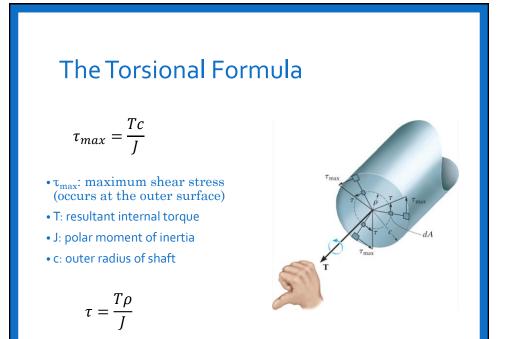


SHEAR STRESS DUE TO TORSION

Samantha Ramirez, MSE

Objective

- 1. Define shear stress due to an internal torsional moment, polar moment of inertia, and power transmitted by a torsional shaft.
- 2. Design torsional-loaded members including noncircular shafts.



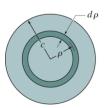
Section Property: Polar Moment of Inertia

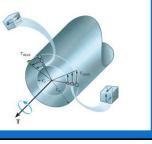
Solid Circular Shaft

$$J = \frac{\pi}{2}c^4$$

• Tubular Shaft

$$J = \frac{\pi}{2} \left(c_o^4 - c_i^4 \right)$$





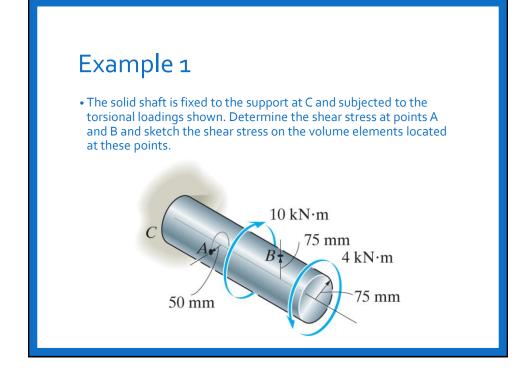
Recall: How to Determine Internal Resultant Torque

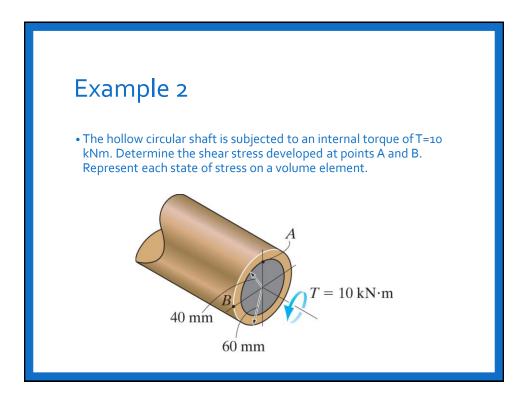
- If necessary, determine the reactions on the shaft
- Section (cut) the shaft perpendicular to its axis at the point where the shear stress is to be determined
- Draw a free-body diagram of the shaft on either side of the cut
- Use a static-equilibrium equation and the following sign convention to obtain the internal torque at the section

Positive sign conve for T and ϕ .

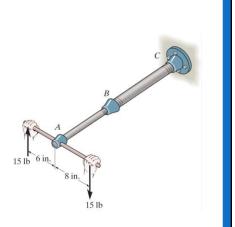
- Sign Convention
 - Using the right-hand rule, the torque and angle of twist will be positive, provided the thumb is directed outward from the shaft when the fingers curl to give the tendency for rotation.

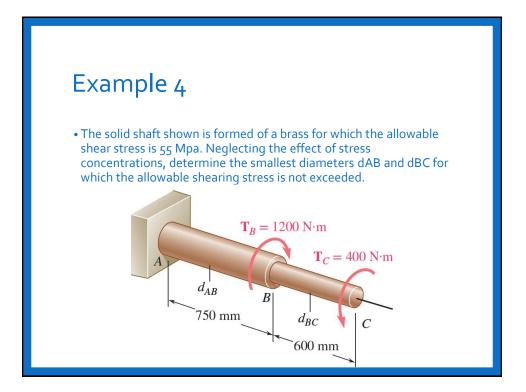
Torque Diagram		
 A torsion diagram is a graphical representation of the internal resultant torque at any point along a shaft. 		
Torque (Nm or Ibin)		Distance (m or in)

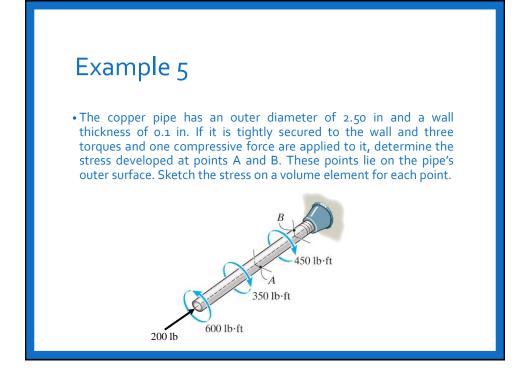




• The assembly consists of two sections of galvanized steel pipe connected together using a reducing coupling at B. The smaller pipe has an outer diameter of 0.75 in and an inner diameter of 1 in and an inner diameter of 1 in and an inner diameter of 0.86 in. If the pipe is tightly secured into the wall at C, determine the maximum shear stress developed in each section of the pipe when the couple shown is applied to the handles of the wrench.







Power Transmission

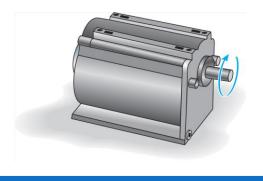
- Power
 - The work performed per unit of time
- The power transmitted by a shaft subjected to a T and angular velocity " ω " is:

$$P = T\omega$$

• The size of the shaft can be determined using the allowable shear stress:

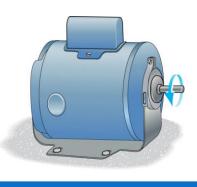
$$\tau_{allow} = \frac{Tc}{J}$$

• The gear motor can develop 3 hp when it turns at 150 rev/min. If the allowable shear stress for the shaft is τ_{allow} =12 ksi, determine the smallest diameter of the shaft to the nearest 1/8 in that can be used.

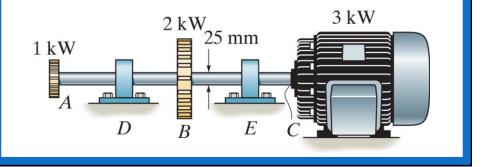


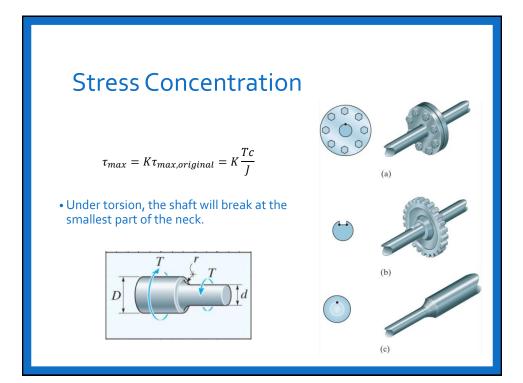
Example 7

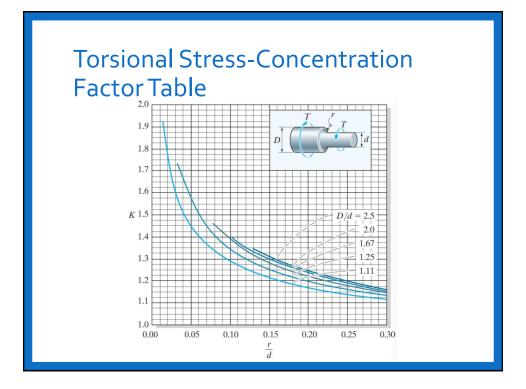
• The 25 mm diameter shaft on the motor is made of a material having an allowable shear stress of τ_{allow} =75 MPa. If the motor is operating at its maximum power of 5 kW, determine the minimum allowable rotation of the shaft.

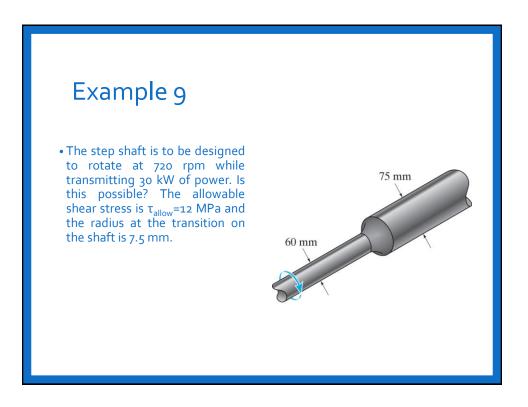


• The solid steel shaft AC has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at C, which delivers 3 kW of power to the shaft while it is turning at 50 rev/s. If gears A and B remove 1 kW and 2 kW, respectively, determine the maximum shear stress developed in the shaft within regions AB and BC. The shaft is free to turn in its support bearing D and E.





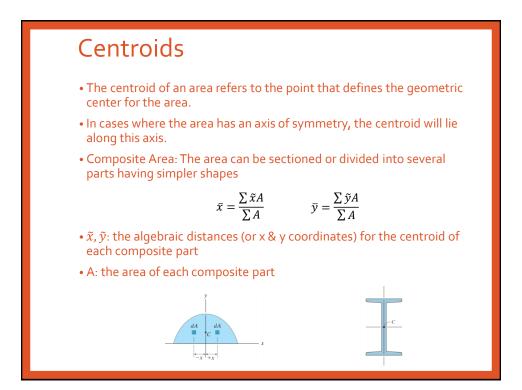


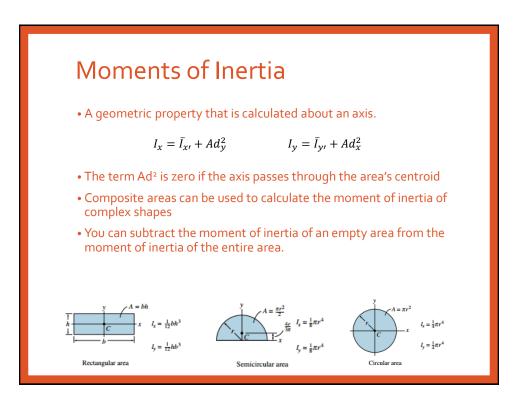


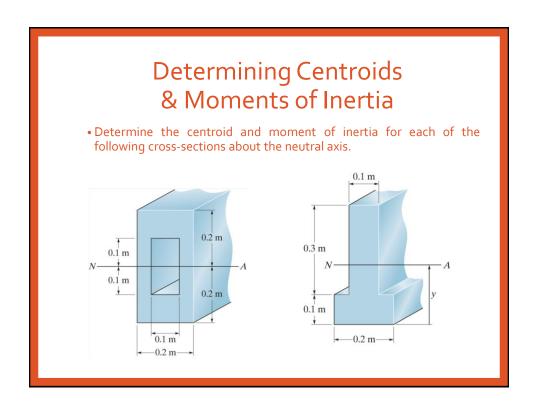
MODULE 2C: NORMAL STRESS DUE TO BENDING MOMENT

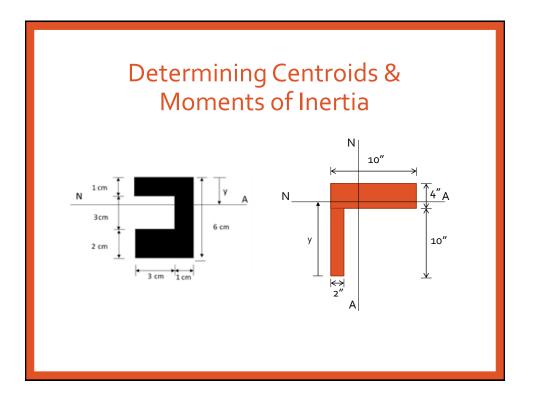
Samantha Ramirez, MSE







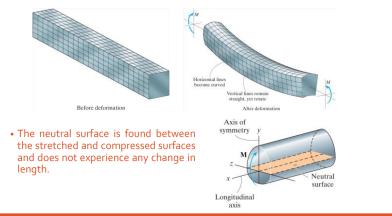




NORMAL STRESS DUE TO BENDING MOMENT

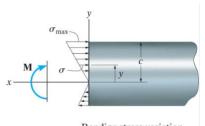
Bending Deformation of a Straight Member

• When a bending moment is applied, the longitudinal lines become curved and the vertical transverse lines remain straight and undergo a rotation.





• Assuming a homogeneous material and linear elastic deformation, the stress also varies in a linear fashion over the cross-sectional area.

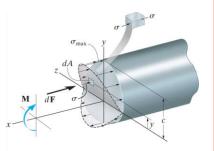


Bending stress variation (profile view)

Maximum Flexure Formula

$$\sigma_{max} = \frac{Mc}{I}$$

- σ_{max} : the maximum normal stress in a member which occurs at a point on the cross-sectional area farthest away from the neutral axis
- M: the resultant internal moment
- c: the perpendicular distance from the neutral axis to where σ_{max} acts (point farthest from the neutral axis)
- I: the moment of inertia of the cross-sectional area about the neutral axis



 $\sigma = -\left(\frac{y}{c}\right)\sigma_{max}$

Bending stress variation

The Flexure Formula

• Similarly,

$$\sigma = -\frac{My}{I}$$

• where y is the perpendicular distance from the neutral axis to the point of interest

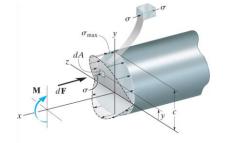
- This equation is valid for beams with cross-sectional areas symmetric about the y-axis.
- Note: For linear elastic materials, the neutral axis passes through the centroid



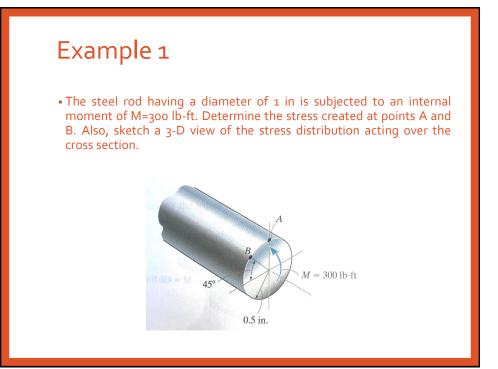
1.Internal Moment

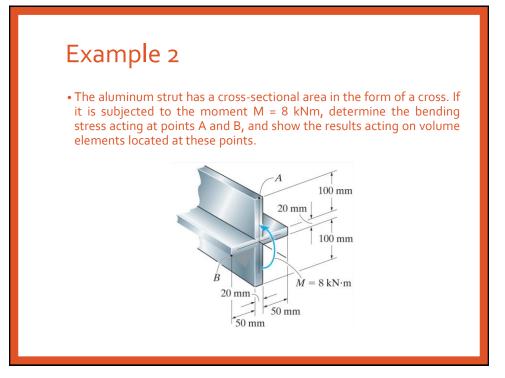
- FBD/Statics
- FBD/Solids
- 2.Section Property

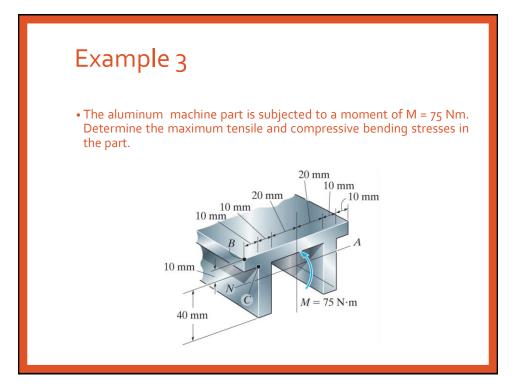
 Moment of Inertia
- 3.Normal Stress
 - Specify the location y
 - Apply Flexure Formula

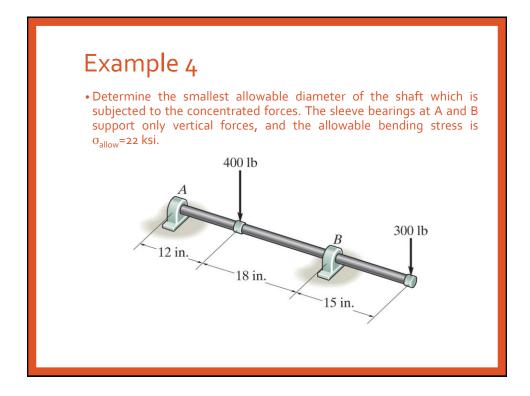


Bending stress variation

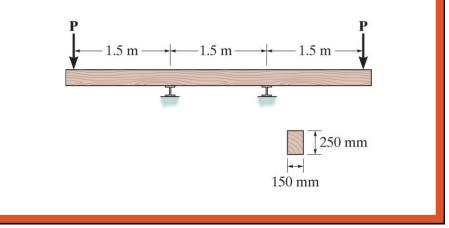








• The beam has a rectangular cross section as shown. Determine the largest load P that can be supported on its overhanging ends so that the bending stress does not exceed σ_{max} =10 MPa.

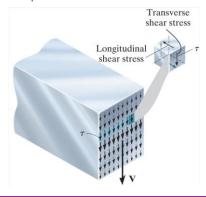


SHEAR STRESS DUE TO SHEAR FORCE

Samantha Ramirez, MSE

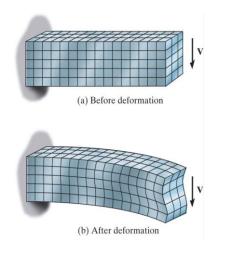
Transverse Shear Stress

• A beam will support both shear forces and bending moments. Due to the complementary property of shear, this stress will create corresponding longitudinal shear stresses which will act along longitudinal planes of the beam.



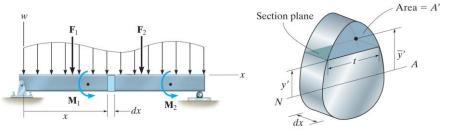
Shear in Straight Members

- Transverse loadings will generate both bending moments and shear forces along the beam.
- When a beam is subjected to both bending and shear, the cross section will not remain plane.
 - Assuming small cross-sectional warping allows it to be neglected.



Transverse Shear Stress

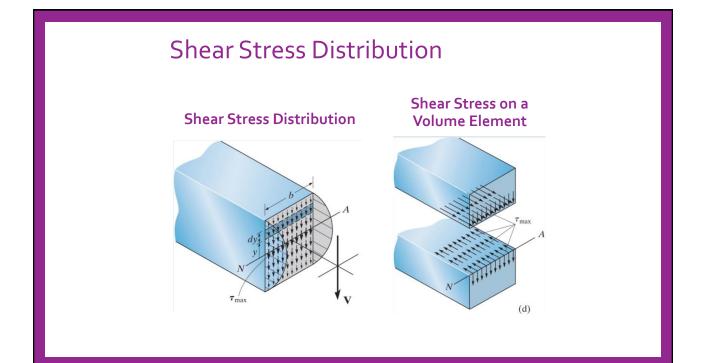
- The shear formula was derived by horizontal force equilibrium of the longitudinal shear stress and bending stress distributions acting on a portion of a differential segment of a beam.
- The shear formula is valid for homogeneous materials and the internal resultant shear force is directed along an axis of symmetry.

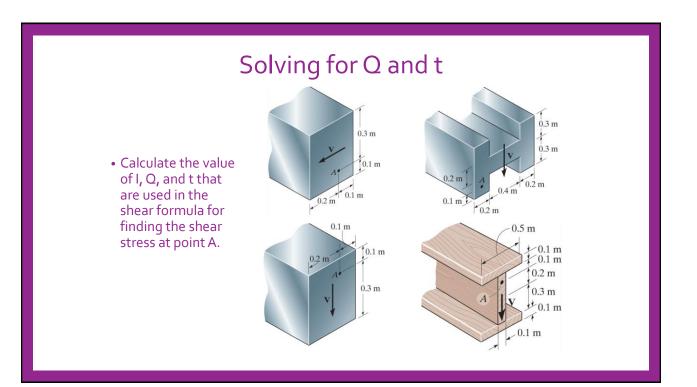


The Shear Formula

$$\tau = \frac{VQ}{It}$$

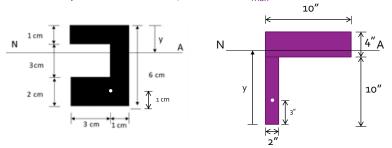
- τ : the shear stress in the member at the point located a distance y' from the neutral axis
- V: the internal resultant shear force
- Q: $\bar{y}'A'$, where A' is the area of the top (or bottom) portion of the member's cross-sectional area, above (or below) the section plane where t is measured, and \bar{y}' is the distance from the neutral axis to the centroid of A'
- I: the moment of inertia of the entire cross-sectional area calculated about the neutral axis
- t: the width of the member's cross-sectional area, measured at the point where $\boldsymbol{\tau}$ is to be determined



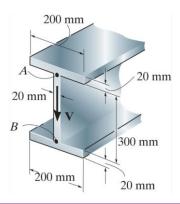


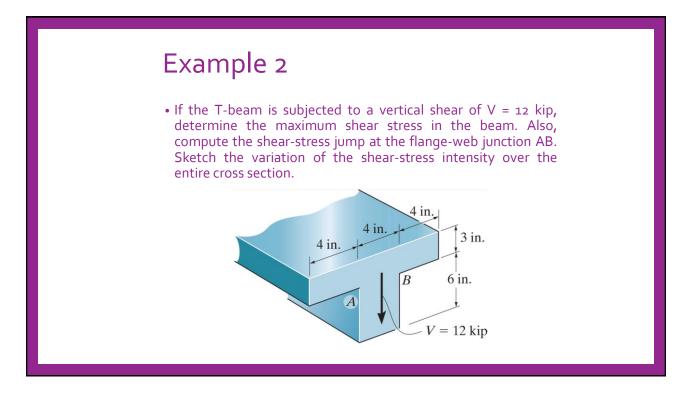


- Calculate the value of Q and t that are used in the shear formula for finding the shear stress at the point shown. Also, calculate $Q_{\rm max}$.



• If the wide-flange beam is subjected to a shear of V=20 kN, determine the shear stress on the web at A. Indicate the shear-stress components on a volume element located at this point.

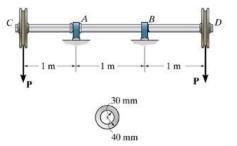




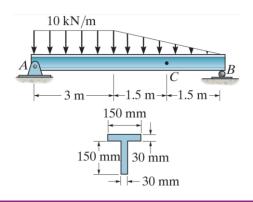
Example 3 • The beam has a square cross-section and is made of wood having an allowable shear stress of 1.4 ksi. If it is subjected to a shear force of 1.5 kip, determine the smallest dimension a of its sides. V = 1.5 kip

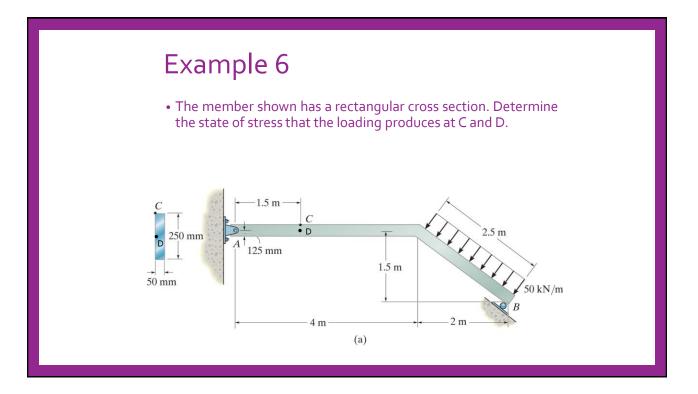
Example 4

• The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. If P=26 kN, determine the absolute maximum shear and bending stress in the shaft.



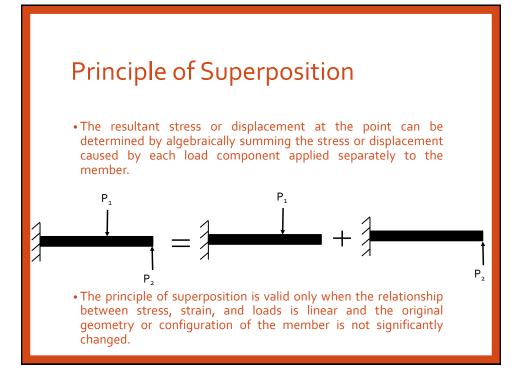
• Determine the maximum shear stress and normal stress in the T-beam at the critical section where the internal shear force is maximum and internal bending moment is maximum.





STRESS COMBINED LOADING

Samantha Ramirez, MSE



State of Stress Caused by Combined Loading

- The cross section of a member is subjected to several loadings simultaneously.
- The method of superposition can be used to determine the resultant stress distribution on the cross section as long as:
 A linear relationship exists between the stress and the loads
 - The geometry of the member should not undergo significant changes when the loads are applied





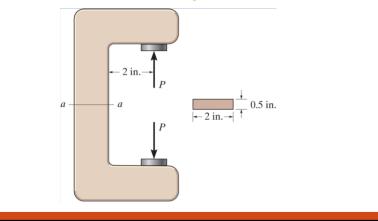
Procedure for Analysis

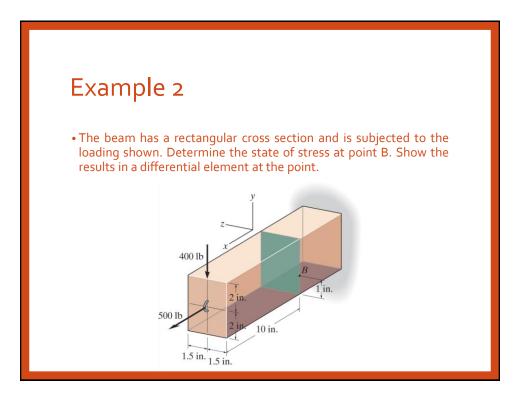
- 1. Internal Loadings
 - Cut member perpendicular to its axis at the point where the stress is to be determined.
 - Obtain the resultant internal normal force, shear force, bending moment, and torsional moment.
 - Force components act through the centroid of the cross section
 - Moment components are computed about centroidal axes
- 2. Stress Components
 - Determine the stress component associated with each internal loading
 - Normal Force: $\sigma = \frac{F}{A}$
 - Shear Force: $\tau = \frac{VQ}{It}$
 - Bending Moment: $\sigma = -\frac{My}{r}$

• Torsional Moment:
$$\tau = \frac{T\rho}{T}$$

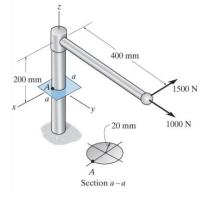
- 3. Superposition
- Determine the resultant normal and shear stress components and represent using a volume element or stress distribution.

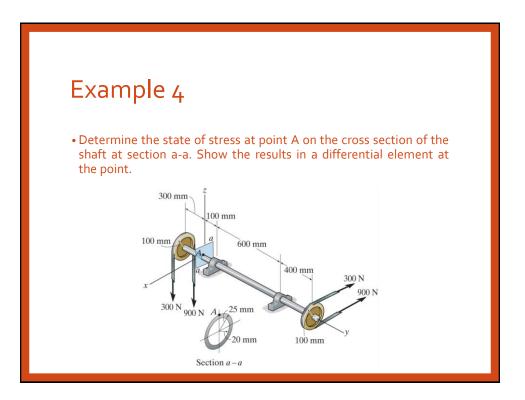
• Determine the magnitude of the load P that will cause a maximum normal stress of 30 ksi in the link along section a-a.

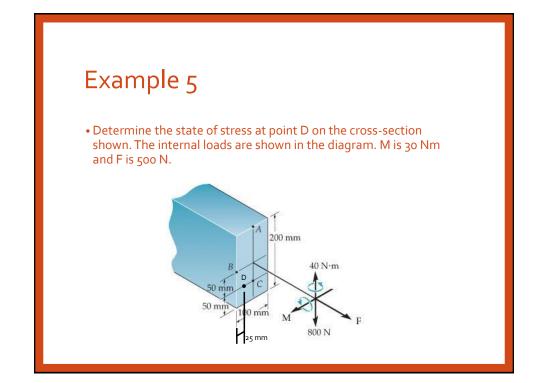


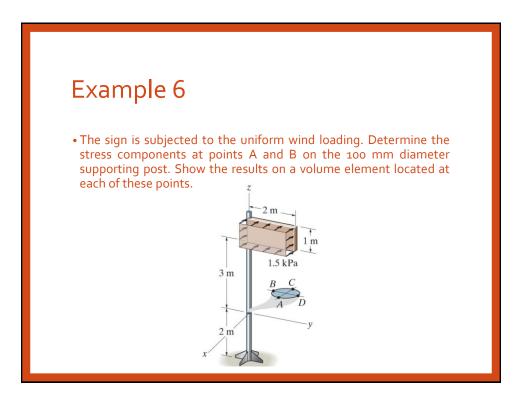


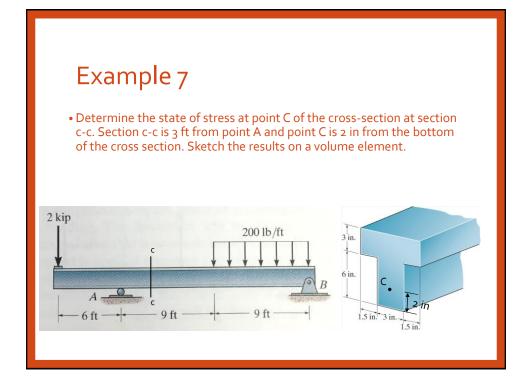
• Determine the state of stress at point A on the cross section of the pipe assembly at section a-a. Show the results in a differential element at the point.











• Several forces are applied to the pipe assembly. Knowing that each section of pipe has inner and outer diameters equal to 36 and 42 mm, respectively, determine the normal and shear stresses at point H located at the top of the outer surface of the pipe.

