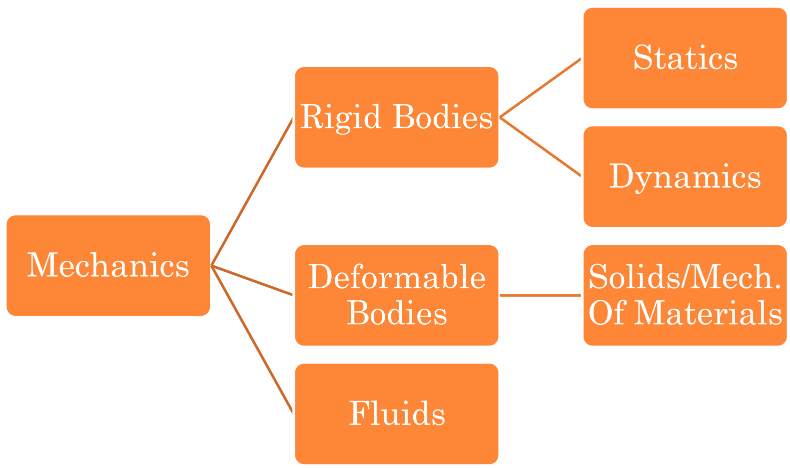
Internal Loads
Free-Body Diagrams

Samantha Ramirez, MSE

Objective

1. Define Mechanics of Materials
2. Identify and categorize internal resultant loadings
3. Solve for internal resultant loadings for 3-D and 2-D problems

WHAT IS MECHANICS?

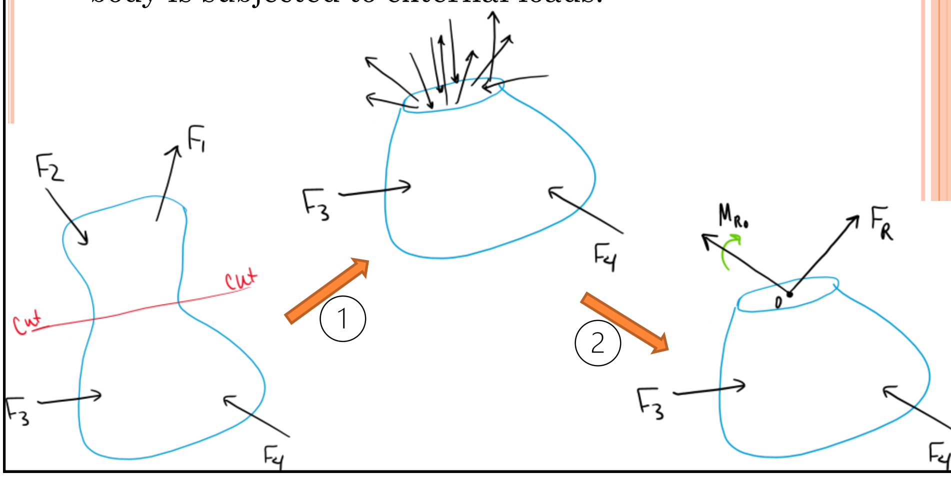


WHAT IS MECHANICS OF MATERIALS?

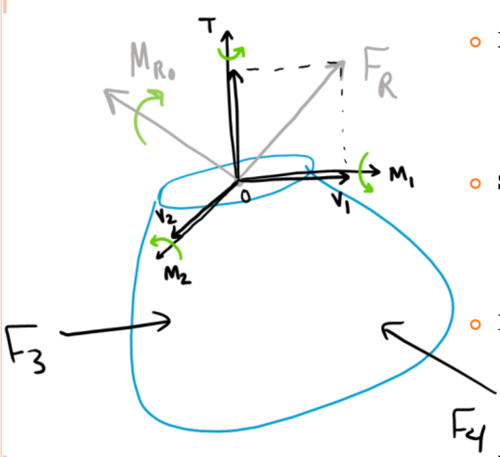
- Mechanics
  - The branch of physics concerned with the state of rest or motion of material objects that are subjected to the action of forces or by thermal disturbances.
- Mechanics of Materials
  - A branch of mechanics that studies the relationship between **external loads** applied to a deformable body and the intensity of **internal forces**.
  - Involves computing the deformation of the body based on the determination and understanding of the mechanical behavior of the materials being used.

Internal Resultant Loadings

- The resultant force and moment which are necessary to hold the body together when the body is subjected to external loads.



Internal Resultant Loadings Cont'd

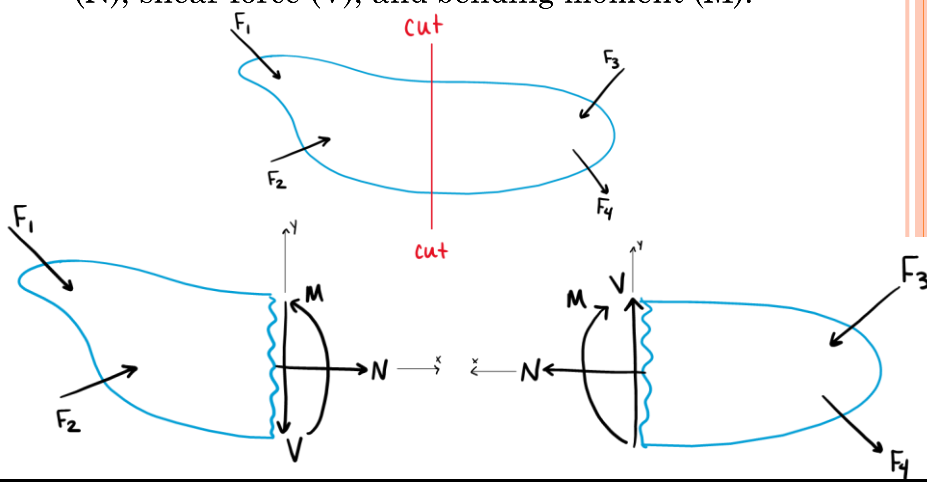


- **Normal Force (N)**
  - Acts **perpendicular** to the area due to external loads **pushing or pulling** on the two segments of the body
- **Shear Force (V_1, V_2)**
  - Lies **in the plane** of the area due to external loads causing the two segments of the body to **slide** over one another (2 forces)
- **Bending Moment (M_1, M_2)**
  - Caused by the external loads **bending** the body **about an axis lying within the plane** of the area (2 moments)
- **Torsional Moment or Torque (T)**
  - Caused by the external loads **twisting** one segment of the body with respect to the other **about an axis perpendicular** to the area

- 3-D Force & Moment
  - Yields 3 force & 3 moment components

Coplanar Loadings – 2-D

- When only coplanar forces act on the body, the internal resultant loadings are the normal force (N), shear force (V), and bending moment (M).



Important Notes About Internal Loadings

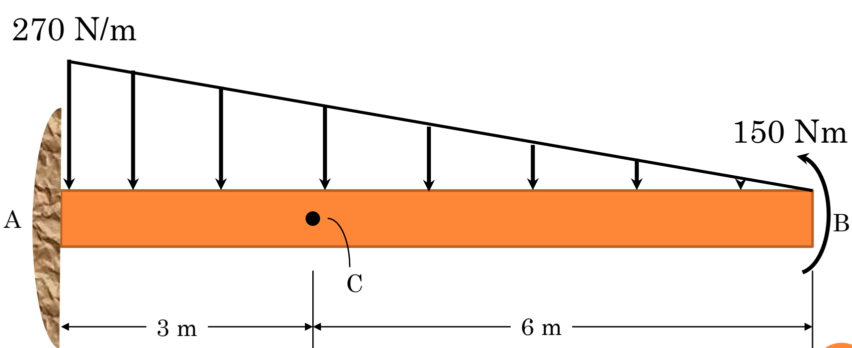
- When analyzing a body that is a member or part of a machine or structure, the machine or structure might have to be disassembled to determine the forces and moments acting on the body before computing the internal loadings.
- The internal loads of a structure only change when an external load is applied.
- The method of sections is used to determine the internal resultant loadings acting on the surface of a sectioned body.

How to Find Internal Loadings

- Procedure
  1. Draw a FBD and determine the reactions at the body's connections (Statics)
  2. Pass an imaginary section through the body (to cut the body) at the location where the internal loadings are to be determined.
  3. Draw a FBD of one of the segments of the cut body
     - Establish theh x, y, and z coordinate axes with origin at the centroid
  4. Clearly indicate the internal resultant loadings (N, V, M, T) acting on the cross-section.
  5. Use the static equilibrium equations to find the internal resultant loadings.

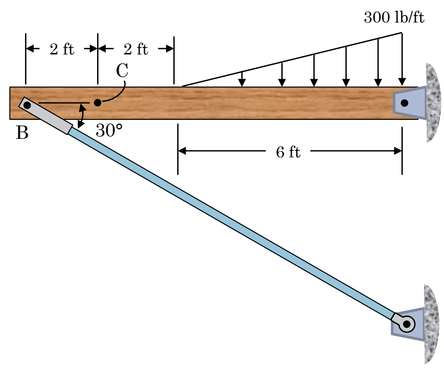
Example 1

- Determine the resultant internal loadings acting on the cross section at C of the cantilevered beam.



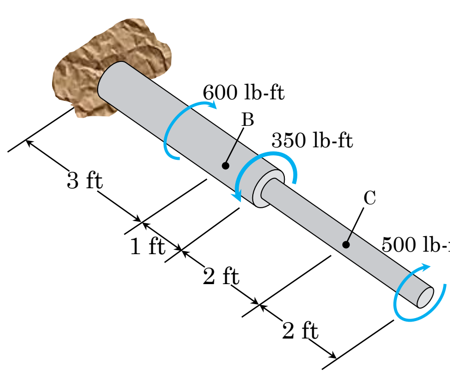
Example 2

- Determine the resultant internal loadings acting on the cross section at C of the beam shown below.



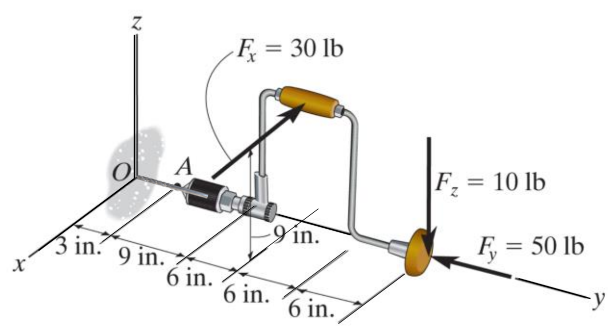
Example 3

- Determine the resultant internal torque acting on the cross section through point B and C.



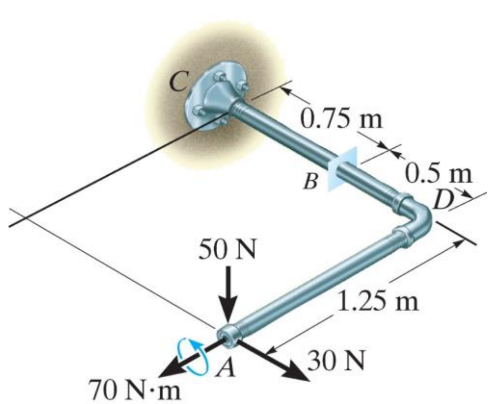
EXAMPLE 4

- The brace and drill bit is used to drill a hole at O. If the drill bit jams when the brace is subjected to the forces shown, determine the resultant internal loadings acting on the cross section of the drill bit at A.



EXAMPLE 5

- Determine the internal loads at point B.



Internal Loads
Normal Force and Torque Diagrams

Samantha Ramirez, MSE

Objective

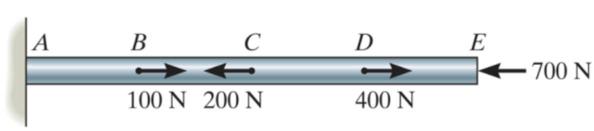
- Synthesize internal normal force diagrams and internal torque diagrams while following a defined sign convention.

How to determine internal Resultant Normal Force

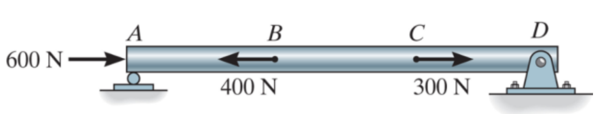
- If necessary, determine the reactions on the shaft
- Section (cut) the shaft perpendicular to its axis at the point where the normal stress is to be determined
- Draw a free-body diagram of the shaft on either side of the cut
  - Sign Convention: Assume the normal internal load is in tension.
- Use a static-equilibrium equation and the appropriate sign convention to obtain the internal normal force at the section

Internal Normal Force Calculation: FBD

- Calculate the normal internal loading at each section using free-body diagrams.



(a)



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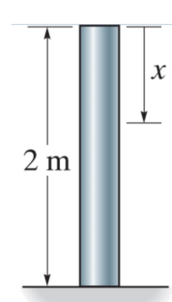


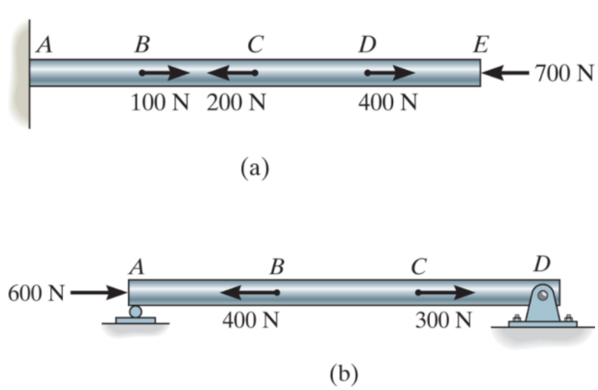
(c)

NORMAL FORCE DIAGRAM

- A normal force diagram is a graphical representation of the internal resultant normal force at any point along a shaft.

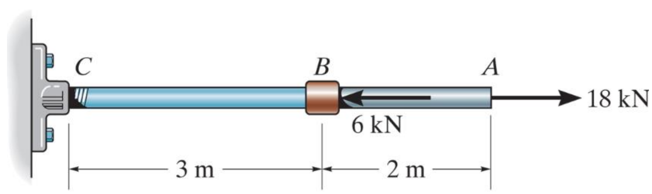
INTERNAL NORMAL FORCE CALCULATION: FORCE DIAGRAM

- Draw the normal force diagram for the following shafts.



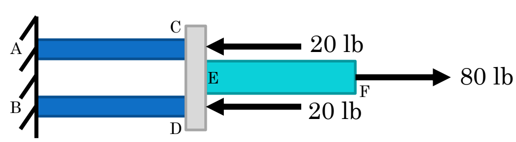
Example 1

- Draw the normal force diagram for the following shaft.



Example 2

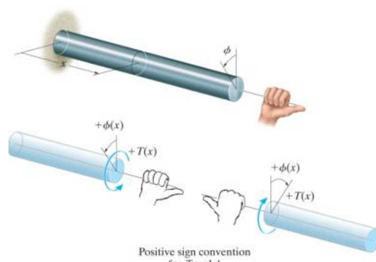
- Draw the normal force diagram for the following component. Determine the internal normal force in each member: AC, BD, EF.



How to determine internal Resultant torque

- If necessary, determine the reactions on the shaft
- Section (cut) the shaft perpendicular to its axis at the point where the shear stress is to be determined
- Draw a free-body diagram of the shaft on either side of the cut
- Use a static-equilibrium equation and the following sign convention to obtain the internal torque at the section

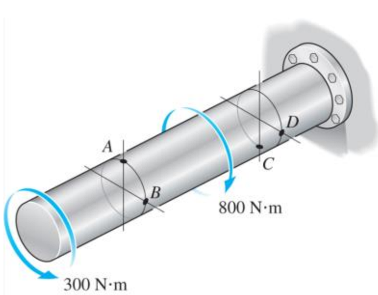
  - Sign Convention
    - Using the right-hand rule, the torque and angle of twist will be positive, provided the thumb is directed outward from the shaft when the fingers curl to give the tendency for rotation.

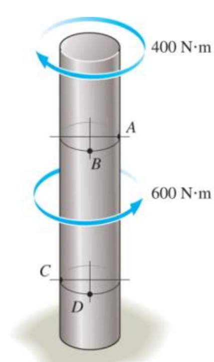


Positive sign convention for T and ϕ.

Internal Torque Calculation using Free-Body Diagrams

- Determine the internal torque for each section of the following shafts.



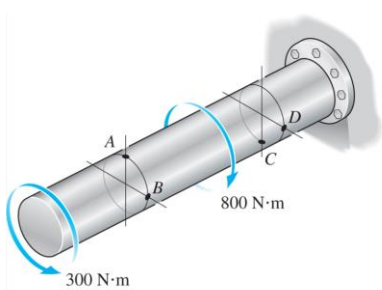


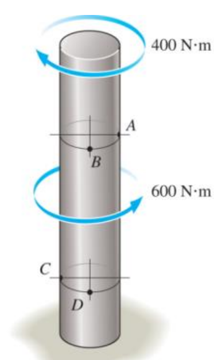
Torque Diagram

- A torsion (torque) diagram is a graphical representation of the internal resultant torque at any point along a shaft.

Internal Torque Calculation using Torque Diagram

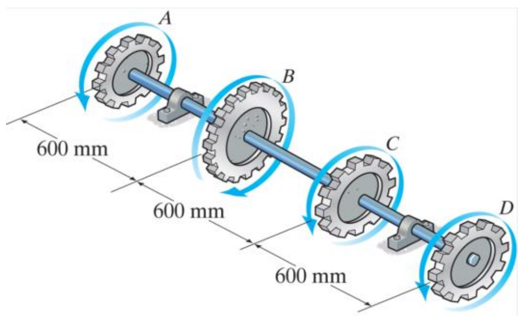
- Draw the torque diagram for each shaft.





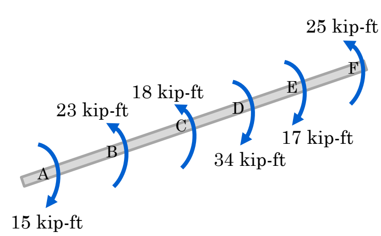
Example 3

- Draw the internal torque diagram for the shaft. The external torque at A is 50 Nm, at B is 150 Nm, at C is 75 Nm, and at D is 25 Nm



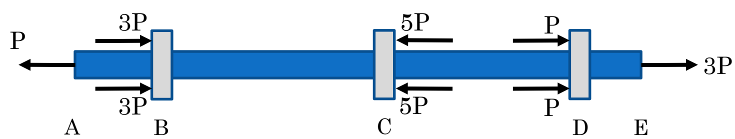
Example 4

- Draw the torque diagram for the following shaft.



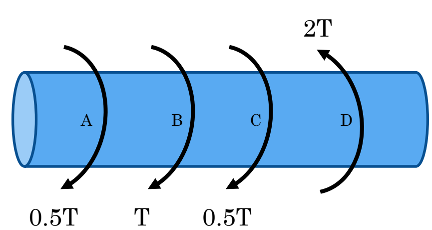
Internal Normal Force Challenge

- Rank the sections from greatest to least based on the internal normal force in each section. Explain your reasoning. How sure are you of your reasoning?



Internal Torque Challenge

- Rank the sections from greatest to least based on the internal torque in each section. Explain your reasoning. How sure are you of your reasoning?



Internal Loads Shear Force & Bending Moment Diagrams

Samantha Ramirez, MSE

Objective

- Synthesize shear and bending moment diagrams using analytical and graphical methods.

Beams

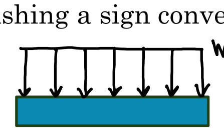
- **<u>Beams</u>** are long straight members that carry loads perpendicular to their longitudinal axis
- Beams are classified by the way they are supported
  - Simply Supported Beam

  - Cantilever Beam

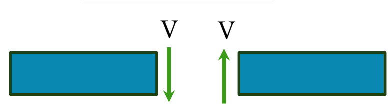
  - Overhanging Beam

Shear Force & Bending Moment Diagrams

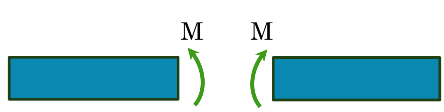
- **<u>Shear and bending moment diagrams</u>** are graphical representations of the internal shears and moments within a beam.
- They can be constructed by establishing a sign convention.
  - Positive Distributed Load



  - Positive Internal Shear
    - Causes a clockwise rotation



  - Positive Internal Moment
    - Compression on top
      - Can hold water



Graphical Method for Constructing V & M Diagrams

$$\frac{dV}{dx} = -w(x)$$

- The slope of the shear curve is equal to the negative of the intensity of the distributed load.

$$\frac{dM}{dx} = V(x)$$

- The slope of the moment curve is equal to the intensity of the shear force.

$$\Delta V = \int -w(x)dx$$

- The change of shear is equal to the negative of the area under the distributed load.

$$\Delta M = \int V(x)dx$$

- The change of moment is equal to the area under the shear diagram.

Shear Force & Bending Moment Diagrams: Method Comparison

Equation Method

- Reactions
- Free-Body Diagrams
- Best for very complex beam loadings

Graphical Method

- Reactions
- Derivative & Integral Relationships
- Quick method for simpler beam loadings

How to Analyze V & M Diagrams: the Equation Method

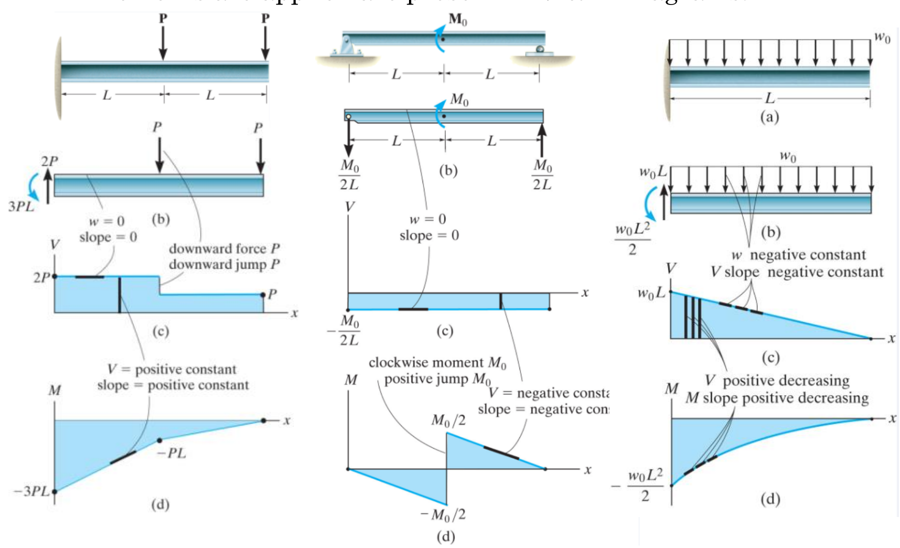
1. Statics
   - FBD
   - Reactions
2. Solids
   - Cut between concentrated forces or moments
     - Note distance, x, from the beam's left end
   - FBD of each section
   - Solve for V and M
3. Shear & Moment Diagrams
   - Table of coordinates for x, V, and M
   - Plot the shear diagram (V vs x)
   - Plot the moment diagram (M vs x)

How to Analyze V & M Diagrams: the Graphical Method

1. Statics
   - FBD
   - Reaction Forces
2. Establish V & M at the ends of the member
3. Use 4 relations to draw the diagrams
   - V vs x
   - M vs x

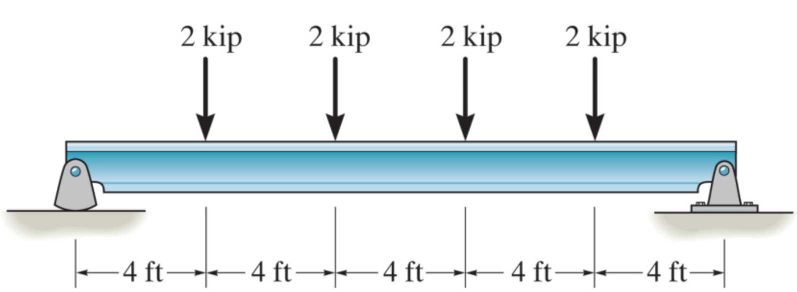
DISCONTINUITIES

- Discontinuities (jumps) at points where concentrated loads or moments are applied are present in V & M diagrams.



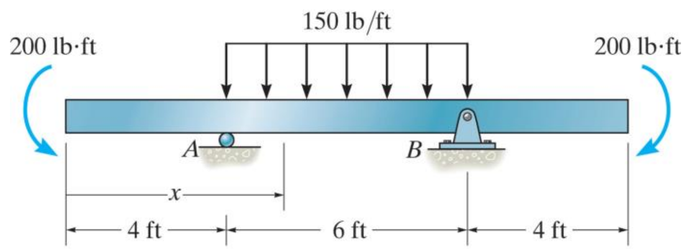
EXAMPLE 1

- Draw the shear and moment diagrams for the beam.



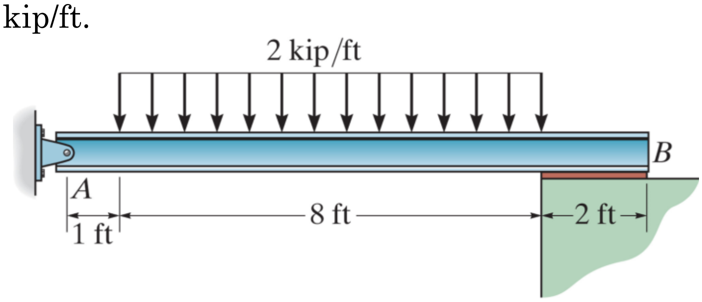
EXAMPLE 2

- Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x, where 4 ft < x < 10 ft.



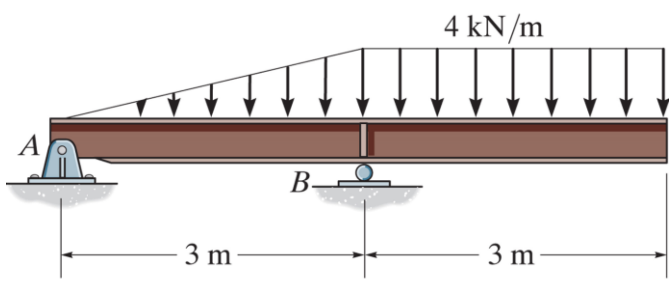
EXAMPLE 3

- The beam is bolted or pinned at A and rests on a bearing pad at B that exerts a uniform distributed loading on the beam over its 2-ft length. Draw the shear and moment diagrams for the beam if it supports a uniform loading of 2 kip/ft.



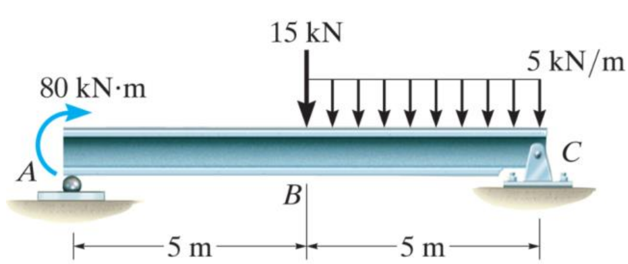
EXAMPLE 4

- Draw the shear and moment diagrams for the overhang beam.



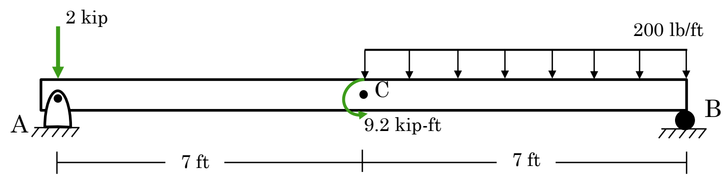
EXAMPLE 5

- Draw the shear and moment diagrams for the beam shown.



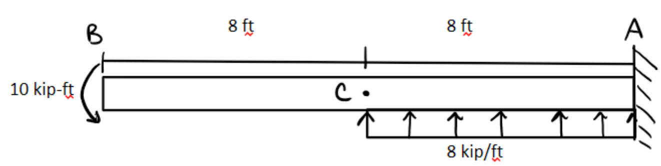
Example 6

- Draw the shear and moment diagrams for the beam shown.



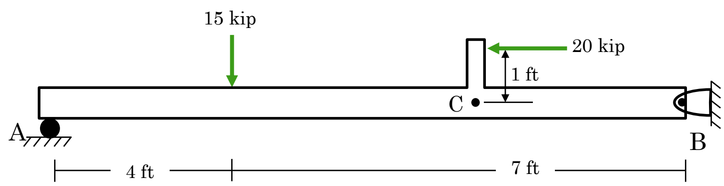
Example 7

- Draw the shear and moment diagrams for the beam shown.



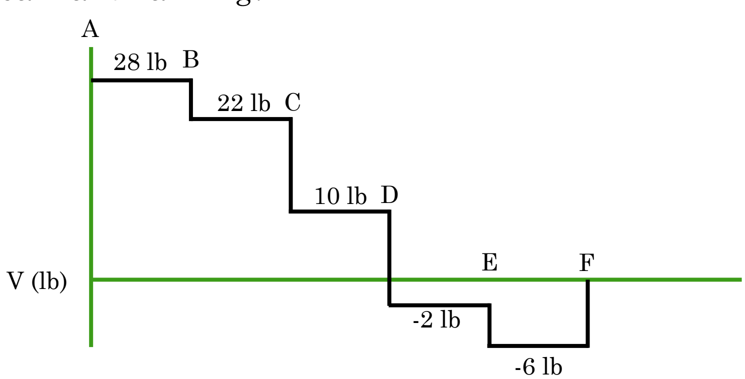
EXAMPLE 8

- Draw the shear and moment diagrams for the beam.



SHEAR FORCE DIAGRAM CHALLENGE

- The figure below is a shear diagram for a simply supported beam with six different external loads applied to it. Draw the beam with all external loads that would create this shear diagram. Then, rank the internal loads in each section and indicate how sure are you of your beam and ranking?



Bending Moment Diagram Challenge

- From the shown bending moment diagrams, rank each one, from greatest to least, on the basis of the absolute maximum internal bending moment. Explain your reasoning. How sure are you of your ranking?