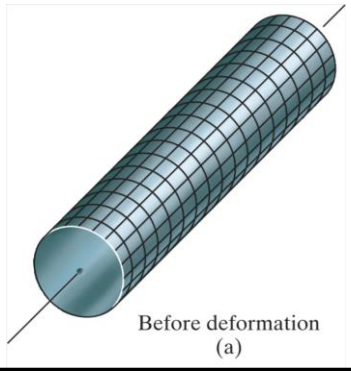
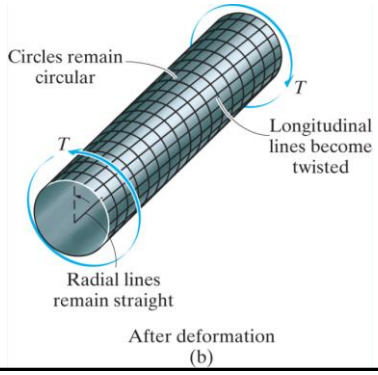
# MECE 3321: MECHANICS OF SOLIDS CHAPTER 5

SAMANTHA RAMIREZ

## TORSION

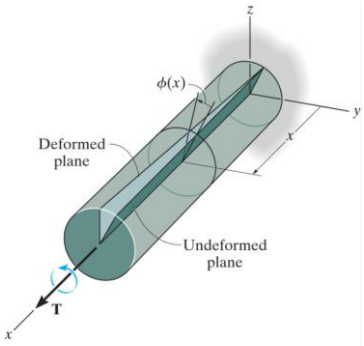
- Torque
  - A moment that tends to twist a member about its longitudinal axis



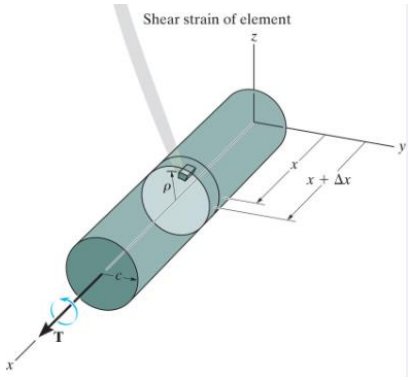
# TORSIONAL DEFORMATION OF A CIRCULAR SHAFT

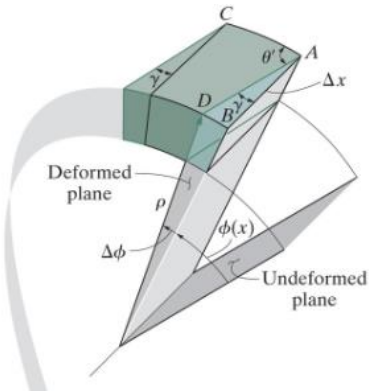
- Assumption
  - If the angle of twist is small, the length and radius of the shaft remain the same




The angle of twist $\phi(x)$ increases as $x$ increases.

# RELATION OF SHEAR STRAIN TO ANGLE OF TWIST




$$L_{BD} = \gamma \Delta x = \Delta \phi \rho$$

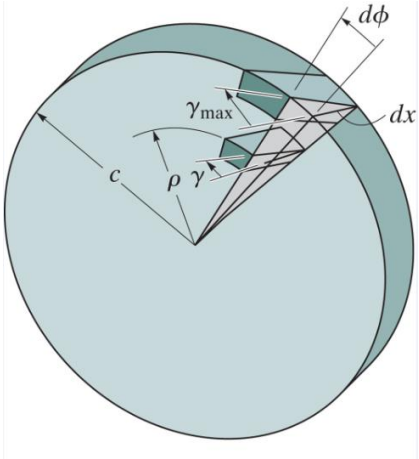
## TORSIONAL DEFORMATION OF A CIRCULAR SHAFT

$$\gamma_{max} = c\frac{d\phi}{dx}$$

$$\gamma = \frac{\rho}{c}\gamma_{max}$$

$$\tau = G\gamma$$

$$\tau = \frac{\rho}{c}\tau_{max}$$

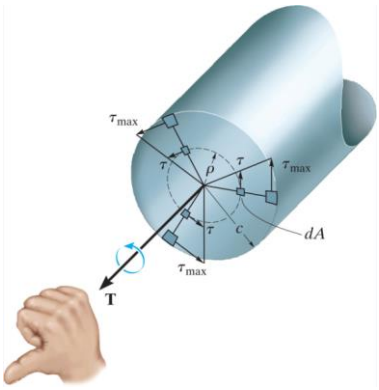



## THE TORSIONAL FORMULA

$$\tau_{max} = \frac{Tc}{J}$$

- $\tau_{max}$: maximum shear stress (occurs at the outer surface)
- T: resultant internal torque
- J: Polar moment of inertia
- c: outer radius of shaft

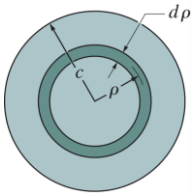
$$\tau = \frac{T\rho}{J}$$



## POLAR MOMENT OF INERTIA

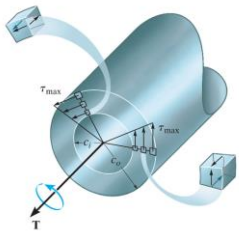
- Solid Circular Shaft

$$J = \frac{\pi}{2} c^4$$



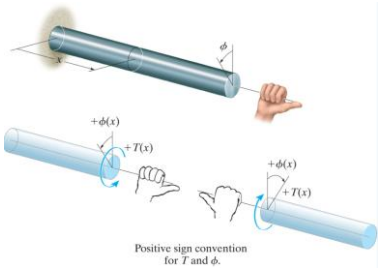

- Tubular Shaft

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right)$$



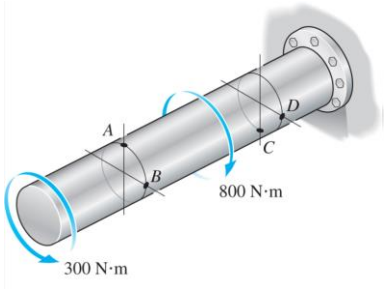

## HOW TO DETERMINE INTERNAL RESULTANT TORQUE

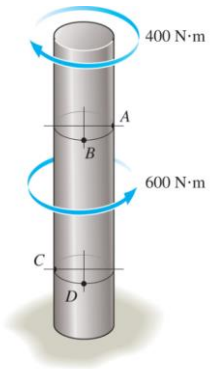
- If necessary, determine the reactions on the shaft
- Section (cut) the shaft perpendicular to its axis at the point where the shear stress is to be determined
- Draw a free-body diagram of the shaft on either side of the cut
- Use a static-equilibrium equation and the following sign convention to obtain the internal torque at the section
  - Sign Convention
    - Using the right-hand rule, the torque and angle of twist will be positive, provided the thumb is directed outward from the shaft when the fingers curl to give the tendency for rotation.



# EXAMPLES

- Determine the internal torque at each section.



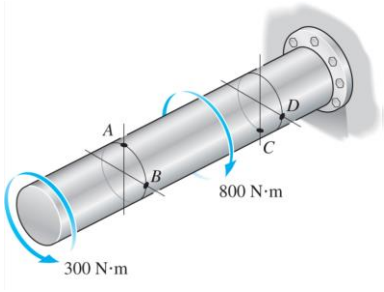



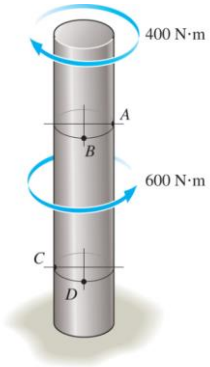

# TORSION DIAGRAM

- A torsion diagram is a graphical representation of the internal resultant torque at any point along a shaft.

## EXAMPLES

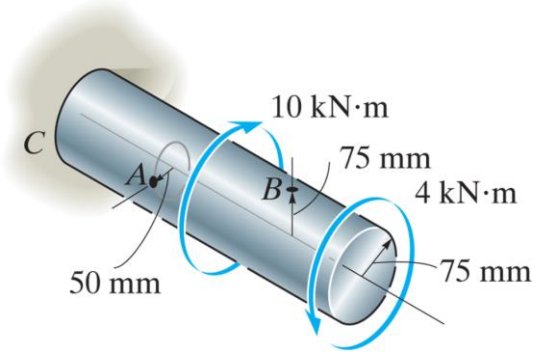
- Draw the torque diagram for each shaft.



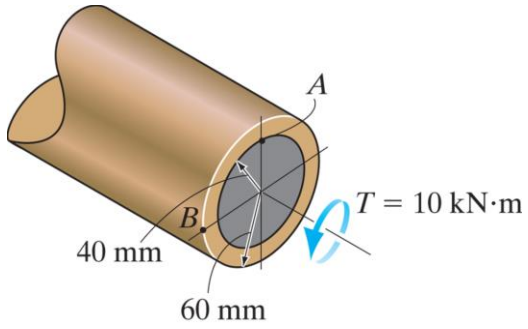

## PROBLEM 5-3

- The solid shaft is fixed to the support at C and subjected to the torsional loadings shown. Determine the shear stress at points A and B and sketch the shear stress on the volume elements located at these points.



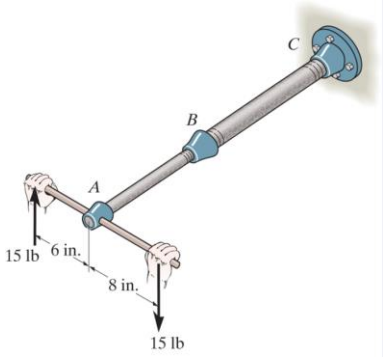
## PROBLEM F5-2

- The hollow circular shaft is subjected to an internal torque of T=10 kNm. Determine the shear stress developed at points A and B. Represent each state of stress on a volume element.



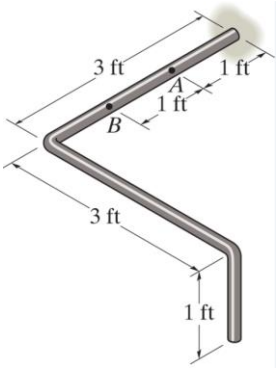

## PROBLEM 5-11

- The assembly consists of two sections of galvanized steel pipe connected together using a reducing coupling at B. The smaller pipe has an outer diameter of 0.75 in and an inner diameter of 0.68 in, whereas the larger pipe has an outer diameter of 1 in and an inner diameter of 0.86 in. If the pipe is tightly secured into the wall at C, determine the maximum shear stress developed in each section of the pipe when the couple shown is applied to the handles of the wrench.



## PROBLEM 5-44

- The rod has a diameter of 0.5 in and weight of 5 lb/ft. Determine the maximum torsional stress in the rod at a section at A due to the rod's weight.




## POWER TRANSMISSION

- Power
  - The work performed per unit of time
- The power transmitted by a shaft subjected to a T and angular velocity "ω" is:

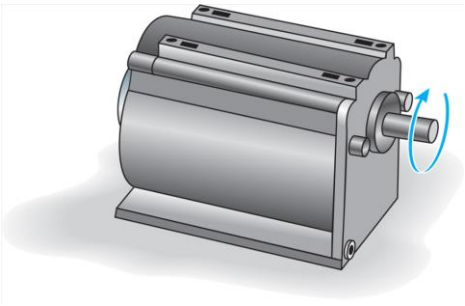
$$P = T\omega$$

- The size of the shaft can be determined using the allowable shear stress:

$$\tau_{allow} = \frac{Tc}{J}$$

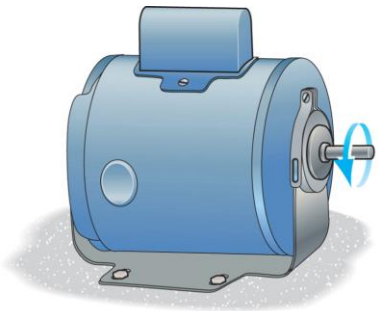
## PROBLEM F5-8

- The gear motor can develop 3 hp when it turns at 150 rev/min. If the allowable shear stress for the shaft is $\tau_{allow}$=12 ksi, determine the smallest diameter of the shaft to the nearest 1/8 in that can be used.



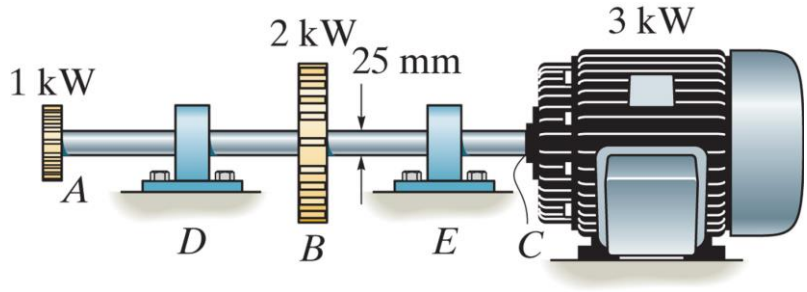
## PROBLEM 5-35

- The 25 mm diameter shaft on the motor is made of a material having an allowable shear stress of $\tau_{allow}$=75 MPa. If the motor is operating at its maximum power of 5 kW, determine the minimum allowable rotation of the shaft.



## PROBLEM 5-31

- The solid steel shaft AC has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at C, which delivers 3 kW of power to the shaft while it is turning at 50 rev/s. If gears A and B remove 1 kW and 2 kW, respectively, determine the maximum shear stress developed in the shaft within regions AB and BC. The shaft is free to turn in its support bearing D and E.




## ANGLE OF TWIST

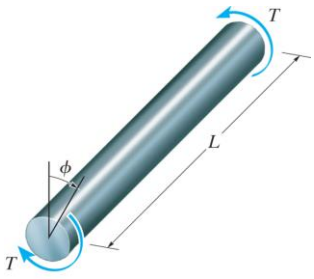
- Recall,
  - Relationship between shear strain and angle of twist $\frac{d\phi}{dx} = \frac{\gamma}{\rho}$ (1)
  - Hooke's Law $G = \frac{\tau}{\gamma}$ (2)
  - Torsional Formula $\tau = \frac{T(x)\rho}{J(x)}$ (3)
- Plugging (2) into (3) and then (1) into the resulting equation you get an equation for the angle of twist:
  - $\phi = \int_0^L \frac{T(x)dx}{J(x)G(x)}$

# ANGLE OF TWIST

- Assuming a homogeneous material with a constant cross-sectional area and applied torque,

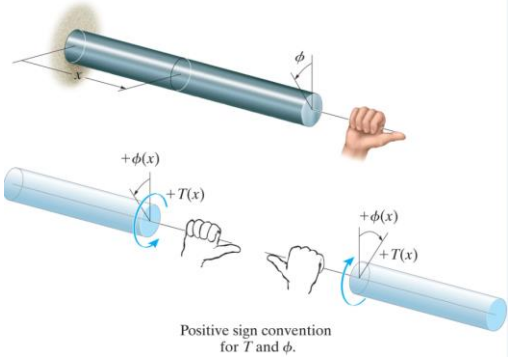
$$\phi = \frac{TL}{JG}$$

- ϕ: the angle of twist of one end of the shaft with respect to the other end, measured in radians
- T: the internal torque at the arbitrary position x
- J: the shaft's polar moment of inertia
- G: the shear modulus of elasticity or the modulus of rigidity




# SIGN CONVENTION

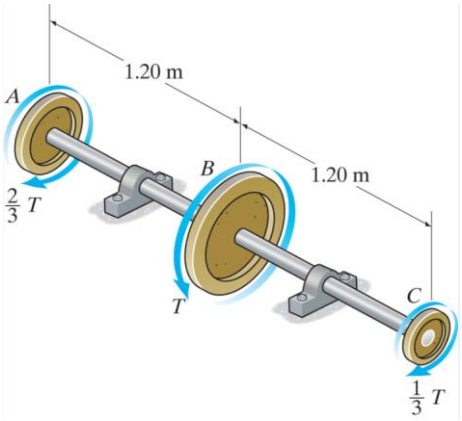
- Using the right-hand rule, the torque and angle of twist will be positive, provided the thumb is directed outward from the shaft when the fingers curl to give the tendency for rotation.




Positive sign convention for $T$ and $\phi$.

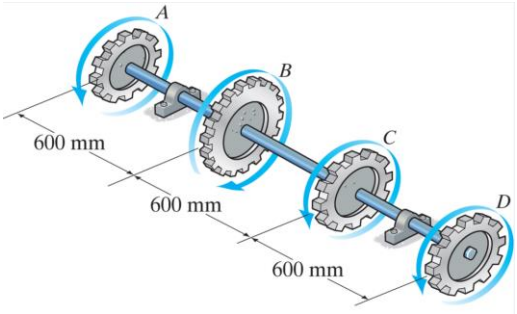
# PROBLEM 5-51

- The 60 mm diameter shaft is made of 6061-T6 aluminum having an allowable shear stress of $\tau_{allow}$=80 MPa. Determine the maximum allowable torque T. Also, find the corresponding angle of twist of disk A relative to disk C.



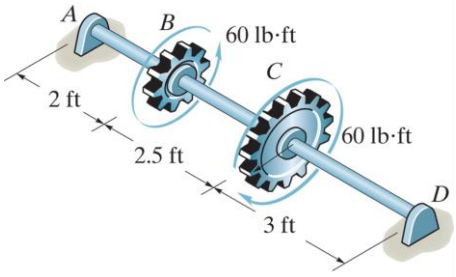

# PROBLEM 5-54

- The shaft is made of A992 steel with the allowable shear stress of $\tau_{allow}$=75 Mpa. If gear B supplies 15 kW of power, while gears A, C, and D withdraw 6 kW, 4 kW, and 5 kW, respectively, determine the required minimum diameter d of the shaft to the nearest millimeter. Also find the corresponding angle of twist of gear A relative to gear D. The shaft is rotating at 600 rpm.



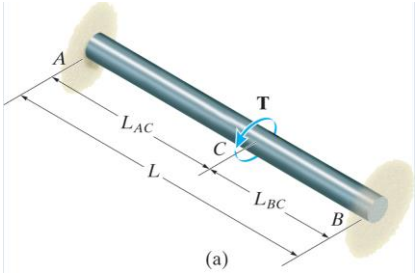
# PROBLEM 5-59

- The shaft is made of A992 steel. It has a diameter of 1 in, and is supported by bearings at A and D, which allow free rotation. Determine the angle of twist of B with respect to D.




# STATICALLY INDETERMINATE TORQUE-LOADED MEMBERS

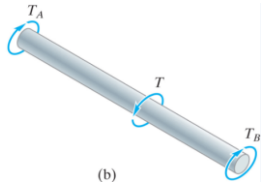
- A torsionally loaded shaft may be classified as statically indeterminate if the moment equation of equilibrium is not adequate to determine the unknown torques acting on the shaft.



# ANALYZING STATICALLY INDETERMINATE TORQUE-LOADED MEMBERS

1. Free Body Diagram

$$\sum M = 0 = T - T_A - T_B \qquad (1)$$



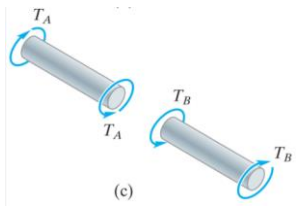

2. Geometry of Deformation

$$\phi_{A/B} = 0 = \phi_{A/C} + \phi_{C/B}$$

3. Plug in resultant loads or displacements

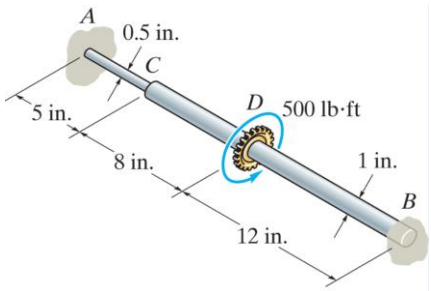
$$0 = \frac{T_{AC}L_{AC}}{JG} + \frac{T_{BC}L_{BC}}{JG}$$

$$0 = \frac{T_A L_{AC}}{JG} - \frac{T_B L_{BC}}{JG} \qquad (2)$$



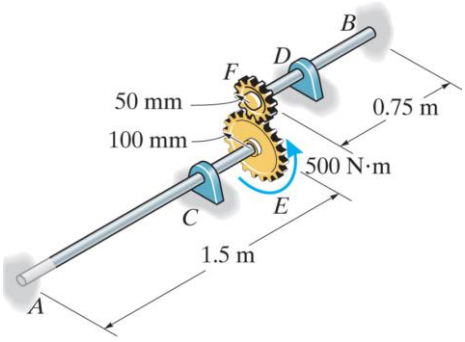

# PROBLEM 5-79

- The steel shaft is made from two segments: AC has a diameter of 0.5 in and CB has a diameter of 1 in. If the shaft is fixed at its ends A and B and subjected to a torque of 500 lbft, determine the maximum shear stress in the shaft. $G_{st}$=10.8Msi.

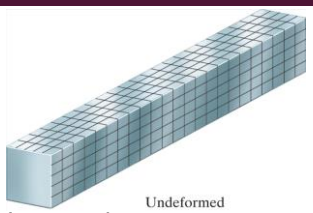


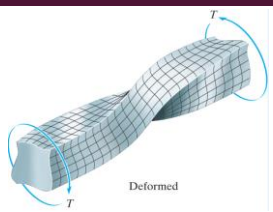
# PROBLEM 5-86/87

- The two shafts are made of A-36 steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by journal bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 500 Nm is applied to the gear at E as shown, determine the reactions at A and B. Determine the rotation of the gear at E.



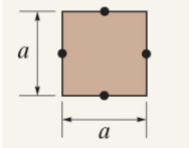

# NON-CIRCULAR SHAFTS



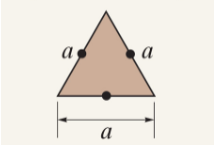

- Maximum shear stress occurs at a point on the edge of the cross section that is closest to the center axis of the shaft.

Square



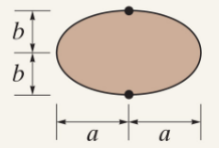

$\tau_{max} = \frac{4.81T}{a^3}$ $\phi = \frac{7.10TL}{a^4G}$

Equilateral triangle




$\tau_{max} = \frac{20T}{a^3}$ $\phi = \frac{46TL}{a^4G}$

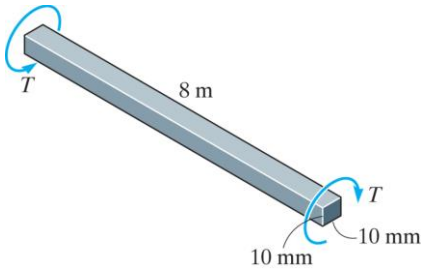
Ellipse




$\tau_{max} = \frac{2T}{\pi ab^2}$ $\phi = \frac{(a^2+b^2)TL}{\pi a^3 b^3 G}$

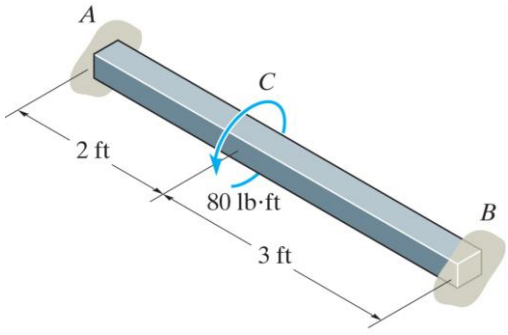
## PROBLEM 5-95

- The aluminum rod has a square cross section of 10 mm by 10 mm. If it is 8 m long, determine the torque T that is required to rotate one end relative to the other end by 90°. ($G_{al}$= 28 GPa, $\tau_{allow}$= 240 MPa)




## PROBLEM 5-102

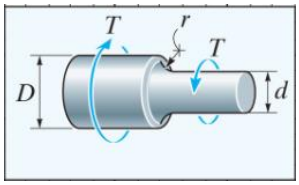
- The aluminum strut is fixed between the two walls at A and B. If it has a 2 in by 2 in cross section, and it is subjected to the torque of 80 ftlb at C, determine the reactions at the fixed supports. Also, what is the angle of twist at C? $G_{al}$= 3.8 Msi

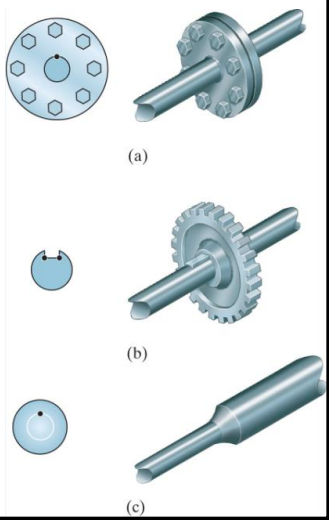


# STRESS CONCENTRATION

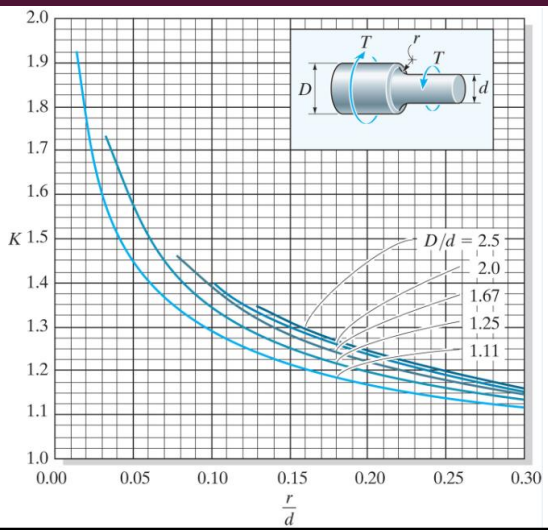
$$\tau_{max} = K\tau_{max,original} = K\frac{Tc}{J}$$

- Under torsion, the shaft will break at the smallest part of the neck.




# TORSIONAL STRESS-CONCENTRATION FACTOR



# PROBLEM 5-121

- The step shaft is to be designed to rotate at 720 rpm while transmitting 30 kW of power. Is this possible? The allowable shear stress is $\tau_{allow}$=12 MPa and the radius at the transition on the shaft is 7.5 mm.