CHAPTER 2: BASIC BOND GRAPH ELEMENTS

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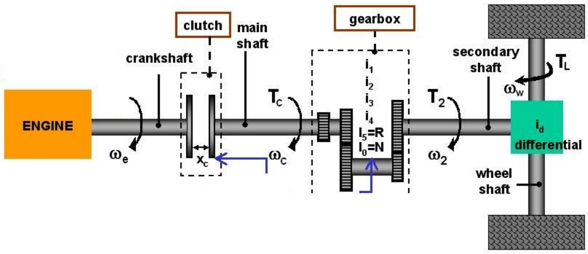
Challenge

- Identify the elements of the following dynamic system based on what they do with energy.



http://www.gipsa-lab.fr/~mazen.alamir/images/AMT_system.jpg

2.1 Introduction

Objectives:

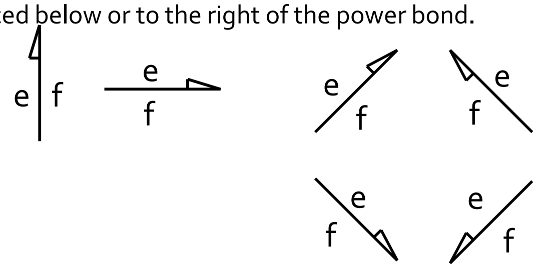
- To be able to decompose dynamic systems into more basic elements that facilitate mathematical modeling,
- To understand how energy usage and conversion are utilized to categorize basic elements, and
- To be able to model the constitutive relations of basic dynamic system elements based energy.

Outcomes: Upon completion of this chapter, you will

- be able to categorize basic elements of dynamic systems,
- be able to derive the mathematical input-output relations for each element,
- begin to draw analogies between basic elements in different power domains, and
- begin to understand the flow of "mathematical information" within dynamic system models.

Labeling Power Bonds

- A power bond is expressed by a half-arrow which indicates the direction of power flow.
- Power is the product of an effort and a flow.
- Efforts are placed above or to the left of the power bond.
- Flows are placed below or to the right of the power bond.

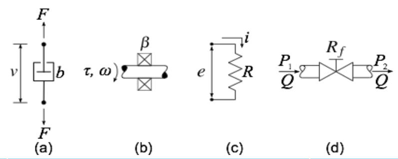


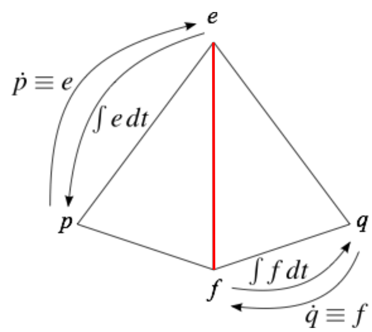
2.2 Basic 1-Port Elements

- 1-Port elements store or dissipate energy
  - R-elements dissipate energy
  - C-elements store potential energy
  - I-elements store kinetic energy
- These elements are referred to as 1-ports due to their single energy port
- Power generally flows **from the system to the 1-ports** because the opposite would imply that the 1-port supplies energy to the system.

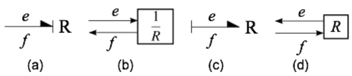
R-Elements

- Dissipate energy





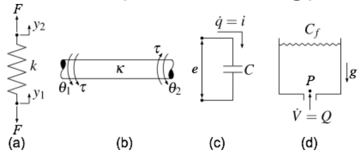
Domain	Parameters	SI Units
Generalized	R=e/f	N/A
Translational	b, damping constant	N-s/m
Rotational	β, rotational damping constant	N-m-s/rad
Electrical	R, resistance	Ω (ohms)
Hydraulic	R_f, hydraulic resistance	Pa-s/m^3

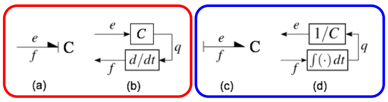


Nonlinear	Linear
$e = \phi_R(f)$	$f = \frac{e}{R}$
$f = \phi_R^{-1}(e)$	$e = Rf$

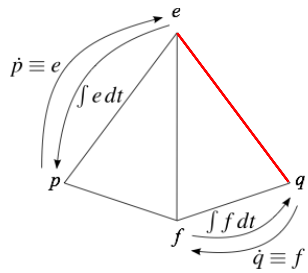
C-Elements

- Stores potential energy





Derivative Causality   Integral Causality

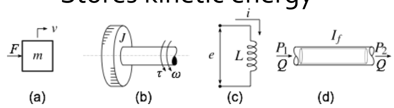


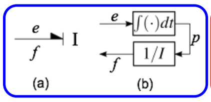
Domain	Parameters	SI Units
Generalized	R=q/e	N/A
Translational	1/k, spring compliance	m/N
Rotational	1/κ, rotational compliance	Rad/N-m
Electrical	C, capacitance	F (farad)
Hydraulic	C_f, hydraulic capacitance	m^3/Pa

Integral	Derivative
$e = \frac{1}{C}\int f dt$	$f = \frac{d}{dt}(Ce)$
Nonlinear	**Linear**
$q = \phi_C(e)$	$q = Ce$
$e = \phi_C^{-1}(q)$	$e = \frac{q}{C}$

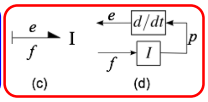
I-Elements

- Stores kinetic energy

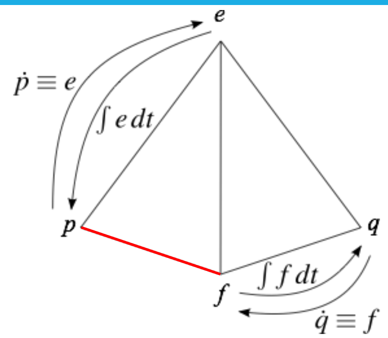




Integral Causality



Derivative Causality



Nonlinear	Linear
$p = \phi_I(f)$	$p = If$
$f = \phi_I^{-1}(p)$	$f = \dfrac{p}{I}$

Integral	Derivative
$f = \dfrac{1}{I}\int edt$	$e = \dfrac{d}{dt}(If)$

Domain	Parameters	SI Units
Generalized	I=p/f	N/A
Translational	m, mass	Kg
Rotational	J, rotational inertia	Kg-m²
Electrical	L, inductance	H (henrys)
Hydraulic	I_f, hydraulic inertia	kg/m⁴

The Tetrahedron of State

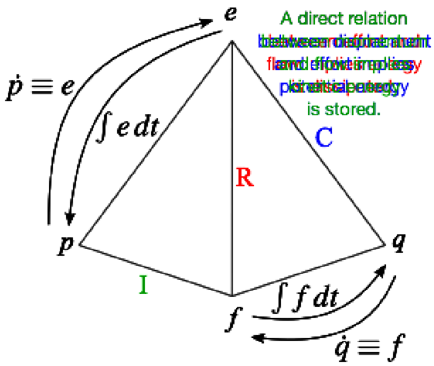


Figure 2.7

1-Port Constitutive Relations

Linear, 1-port R-elements

DOMAIN	EFFORT-IN RELATION	FLOW-IN RELATION
GENERALIZED	$f = \frac{e}{R}$	$e = Rf$
TRANSLATIONAL	$v = \frac{F}{b}$	$F = bv$
ROTATIONAL	$\omega = \frac{\tau}{B}$	$\tau = B\omega$
ELECTRICAL	$i = \frac{e}{R}$	$e = Ri$
HYDRAULIC	$Q = \frac{P}{R_f}$	$P = R_f Q$

Linear, 1-port C-elements.

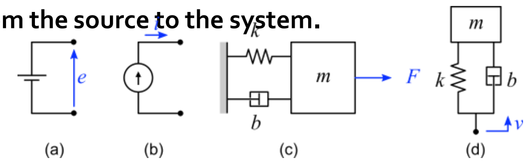
DOMAIN	LINEAR RELATION	INTEGRAL RELATION	DERIVATIVE RELATION
GENERALIZED	$e = \frac{q}{C}$	$e = \frac{\int f\,dt}{C}$	$f = \frac{d}{dt}(Ce)$
TRANSLATIONAL	$F = kx$	$F = k\int v\,dt$	$v = \frac{d}{dt}\left(\frac{f}{k}\right)$
ROTATIONAL	$\tau = \kappa\theta$	$\tau = \kappa\int \omega\,dt$	$\omega = \frac{d}{dt}\left(\frac{\tau}{\kappa}\right)$
ELECTRICAL	$e = \frac{q}{C}$	$e = \frac{\int i\,dt}{C}$	$i = \frac{d}{dt}(Ce)$
HYDRAULIC	$P = \frac{V}{C_f}$	$P = \frac{\int Q\,dt}{C_f}$	$Q = \frac{d}{dt}(C_f P)$

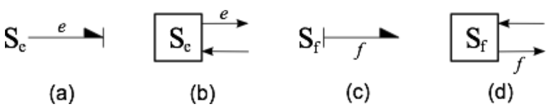
Linear, 1-port I-elements

DOMAIN	LINEAR RELATION	INTEGRAL RELATION	DERIVATIVE RELATION
GENERALIZED	$f = \frac{p}{I}$	$f = \frac{\int e\,dt}{I}$	$e = \frac{d}{dt}(If)$
TRANSLATIONAL	$v = \frac{p}{m}$	$v = \frac{\int F\,dt}{m}$	$F = \frac{d}{dt}(mv)$
ROTATIONAL	$\omega = \frac{h}{J}$	$\omega = \frac{\int \tau\,dt}{J}$	$\tau = \frac{d}{dt}(J\omega)$
ELECTRICAL	$i = \frac{\lambda}{L}$	$i = \frac{\int e\,dt}{L}$	$e = \frac{d}{dt}(\lambda i)$
HYDRAULIC	$Q = \frac{\Gamma}{I_f}$	$Q = \frac{\int P\,dt}{I_f}$	$P = \frac{d}{dt}(I_f Q)$

Effort and Flow Sources

- Supply energy
- Effort sources specify effort as an input to the system
- Flow sources specify flow as an input to the system
- Power goes **from the source to the system.**





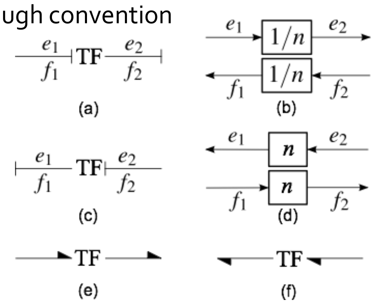
2.3 Basic 2-Port Elements

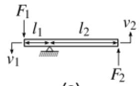
- 2-port elements transmit energy from one element or junction to another
  - Transformer (TF)
  - Gyrator (GY)
- 2-ports can serve as an interface between various energy domains
- Power generally flows **through** 2-ports

Transformers

$$e_1 = ne_2$$
$$nf_1 = f_2$$

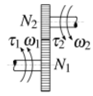
- Converts energy
- Energy-conserving
- Efforts are algebraically related
- Flows are algebraically related
- Power through convention





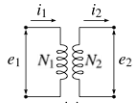
(a)

$$\frac{F_1}{F_2} = \frac{v_2}{v_1} = \frac{l_2}{l_1}$$



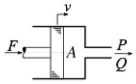
(b)

$$n = \frac{\tau_1}{\tau_2} = \frac{\omega_2}{\omega_1}$$



(c)

$$\frac{e_1}{e_2} = \frac{i_2}{i_1} = \frac{N_1}{N_2}$$

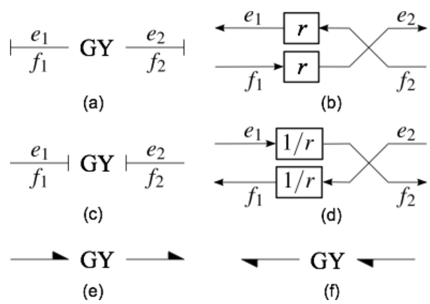


(d)

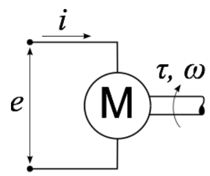
$$A = \frac{F}{P} = \frac{Q}{v}$$

Gyrators

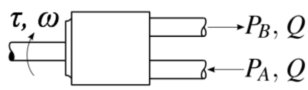
- Converts energy
- Energy-conserving
- Effort on one side related to flow on the other
- Power through convention



$e_1 = rf_2$
$rf_1 = e_2$

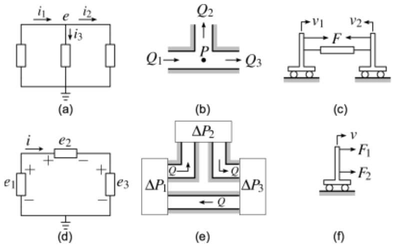


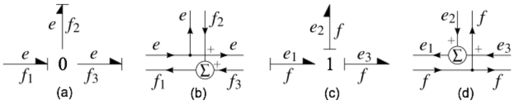
$k_m = \frac{\tau}{i} = \frac{e}{\omega}$



2.4 Junctions

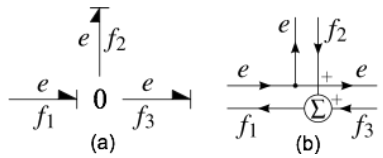
- Interconnect basic elements
- Energy-conserving
- Characterized by two conditions
  - Primary Condition: Commonality
  - Secondary Condition: Zero summation
- Half-arrow direction specifies power direction (sign of efforts & flows)





o-Junctions

- Common effort
  - One bond specifies the effort into the junction
- Summation of flows
  - Solve for flow out of the junction
    - Flow out is caused by the bond specifying effort in



$$\mathcal{P}_{in} = \mathcal{P}_{out} \Longrightarrow \mathcal{P}_{in} - \mathcal{P}_{out} = 0$$

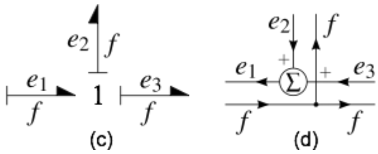
$$\sum_{j=1}^{n} \mathcal{P}_j = \sum_{j=1}^{n} e_j f_j = 0$$

$$e_1 = e_2 = e_3 = \cdots = e$$

$$\sum_{j=1}^{n} \mathcal{P}_j = \sum_{j=1}^{n} e_j f_j = e \sum_{j=1}^{n} f_j = 0 \Longrightarrow \sum_{j=1}^{n} f_j = 0$$

1-Junctions

- Common flow
  - One bond specifies the flow into the junction
- Summation of efforts
  - Solve for effort out of the junction
    - Effort out is caused by the bond specifying flow in



(c) (d)

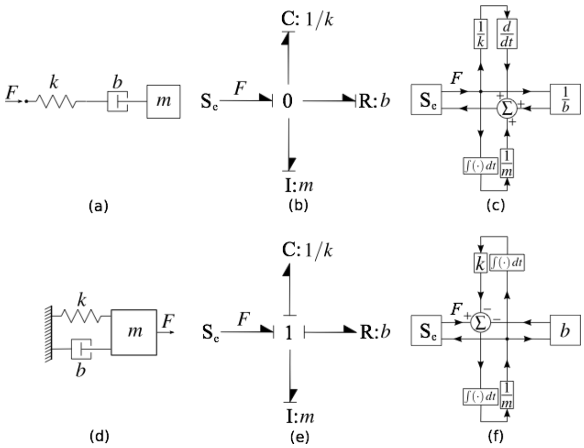
$$\mathcal{P}_{in} = \mathcal{P}_{out} \Longrightarrow \mathcal{P}_{in} - \mathcal{P}_{out} = 0$$

$$\sum_{j=1}^{n} \mathcal{P}_j = \sum_{j=1}^{n} e_j f_j = 0$$

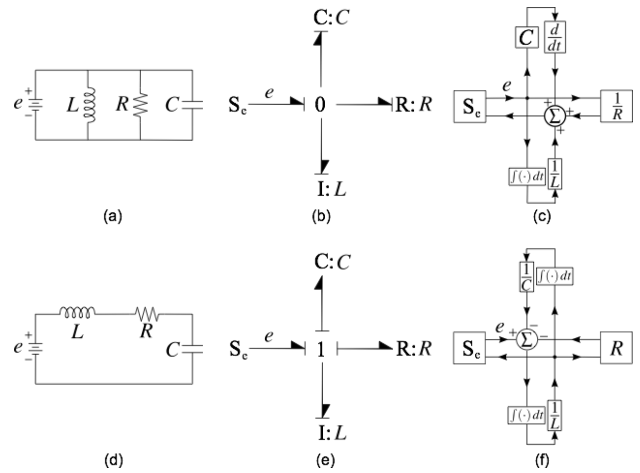
$$f_1 = f_2 = f_3 = \cdots = f$$

$$\sum_{j=1}^{n} \mathcal{P}_j = \sum_{j=1}^{n} e_j f_j = f \sum_{j=1}^{n} e_j = 0 \Longrightarrow \sum_{j=1}^{n} e_j = 0$$

Mechanical System Examples

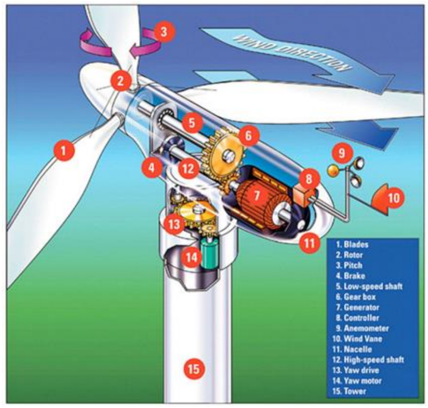


Electrical System Examples



Challenge

- Identify the elements of the following dynamic system. Recollect your word bond graph.



2.6 Linear vs Nonlinear Systems and Linearization

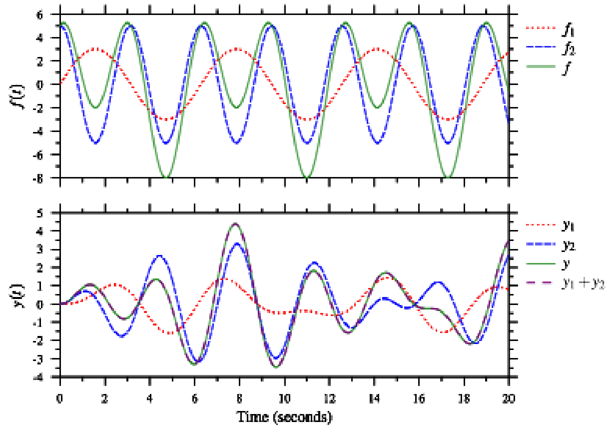
- The responses of linear systems obey the properties of superposition (or additive property) and homogeneity.

- Superposition
  - $y\left(x_1(t) + x_2(t)\right) = y_1(t) + y_2(t)$

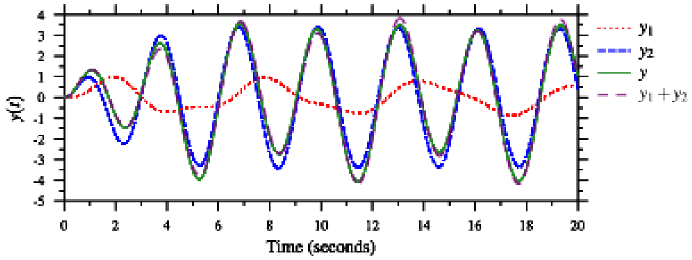
- Homogeneity
  - $ay\left(x(t)\right) = y(ax(t))$

- Superposition and Homogeneity
  - $y\left(a_1 x_1(t) + a_2 x_2(t)\right) = a_1 y_1(t) + a_2 y_2(t)$

Additive and Homogenous Properties of Linear Systems



Nonlinear System Responses



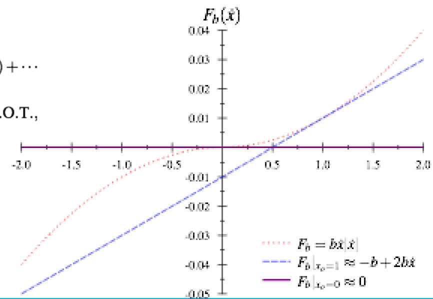
Linearization

$$m\ddot{x} + b\dot{x}|\dot{x}| + kx = f(t)$$

$$f(x) = f(\hat{x}) + \frac{f'(\hat{x})}{1!}(x-\hat{x}) + \frac{f''(\hat{x})}{2!}(x-\hat{x})^2 + \frac{f^{(3)}(\hat{x})}{3!}(x-\hat{x})^3 + \cdots$$
$$= \sum_{n=0}^{\infty} \frac{f^{(n)}(\hat{x})}{n!}(x-\hat{x})^n .$$

$$f(x) = f(\hat{x}) + \left.\frac{\partial f}{\partial x}\right|_{\hat{x}} (x-\hat{x}) + \text{H.O.T.}$$

$$f(\mathbf{x}) = f(\hat{\mathbf{x}}) + \left.\frac{\partial f}{\partial x_1}\right|_{\hat{\mathbf{x}}} (x_1-\hat{x}_1) + \left.\frac{\partial f}{\partial x_2}\right|_{\hat{\mathbf{x}}} (x_2-\hat{x}_2) + \cdots$$
$$+ \left.\frac{\partial f}{\partial x_n}\right|_{\hat{\mathbf{x}}} (x_n-\hat{x}_n) + \text{H.O.T.},$$



Summary

- R-elements dissipate energy. They have a constitutive relation that directly relates effort to flow. They can exhibit one of two causalities – effort-in-flow-out or flow-in-effort-out. Power is generally assumed to flow from the system to the R-element.
- C-elements store potential energy. They have a constitutive relation that directly relates effort to displacement. They can exhibit one of two causalities – integral causality where flow is an input to the C- element and derivative causality where effort is an input to the C- element. Power is generally assumed to flow from the system to the C-element.
- I-elements store kinetic energy. They have a constitutive relation that directly relates momentum to flow. They can exhibit one of two causalities – integral causality where effort is an input to the I-element and derivative causality where flow is an input to the I-element. Power is generally assumed to flow from the system to the I-element.
- Effort sources provide an external effort as an input to the system.
- Flow sources provide an external flow as an input to the system.

Summary Continued

- Transformers transmit and/or change the form of energy. They con- serve power (i.e., the power in is equal to the power out). The efforts on either side are directly related, and the flows on either side are directly related. Only one bond attached to a transformer can specify effort as an input.
- Gyrators also transmit and change the energy form, and they also conserve power. The effort on one side is directly related to the flow on the other. Either both bonds must specify effort as an input, or both bonds must specify flow as an input.
- 0-junctions have common effort and sum flows. Only one bond can specify effort as input. The power direction specifies whether the flows on the attached bonds are positive or negative relative to the junction.
- 1-junctions have common flow and sum efforts. Only one bond can specify flow as input. The power direction specifies whether the efforts on the attached bonds are positive or negative relative to the junction.
- Linearization can be used to approximate nonlinear functions and systems. This can be accomplished by using the first few terms of the Taylor Series Expansion of the nonlinear terms.