

# Chapter 1 Introduction to System Dynamics

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# Introduction

## What is System Dynamics?

The synthesis of mathematical models to represent dynamic responses of physical systems for the purpose of analysis, design, and/or control.

System Dynamics draws on a variety of engineering specialties to form a unified approach to study dynamic systems.

Typically, building a prototype system and conducting experimental tests are either infeasible or are too expensive for a preliminary design. Therefore, mathematical modeling, analysis, and simulation of engineering systems aid the design process immensely.

# Challenge



[https://www.youtube.com/watch?v=MLEikyXbjlc&feature=player\\_embedded](https://www.youtube.com/watch?v=MLEikyXbjlc&feature=player_embedded)

## Challenge Questions

When modeling a system

- ▶ What aspects of the system must you consider?
- ▶ What tools, models, or information will you need?
- ▶ How do you design or optimize the system to ensure reasonable performance?
- ▶ What metrics do you use to measure the system's performance?
- ▶ How do you automate or control a system?
- ▶ Where do you get started?

- 1) What is the system? Rider, bike, suspension (energy absorption), spring (store energy), shock absorber (dissipates energy),
- 2) Answers to all questions are dependent on use of analysis.

## Objectives & Outcomes



In this chapter you will:

come to a deeper understanding of the art of System Dynamics and the purpose it serves in the design, analysis, and control of physical systems, and begin to conceptualize how a system is broken down into subsystems and components to enable synthesis of mathematical models that represent the dynamics.



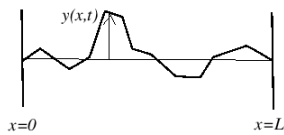
After completing this chapter, you will be able to:

identify systems, subsystems, and components,  
identify potential applications of system dynamics in design and analysis of mechanisms, and  
recognize and/or recall concepts used to represent dynamic responses in other engineering courses you are or have previously taken.

# Classification of Dynamic Systems

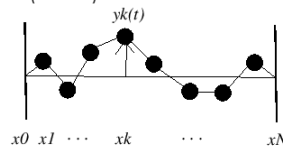
## Distributed System

- ▶ Requires an infinite number of "internal" variables
- ▶ Variables are functions of time and at least one spatial variable
- ▶ Governed by partial differential equations (PDEs)



## Lumped System

- ▶ Involves a finite number of "internal" variables
- ▶ Variables are functions of time alone
- ▶ Governed by ordinary differential equations (ODEs)

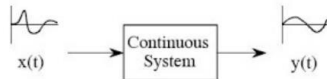


[https://ccma.stanford.edu/~jos/NumericalInt/Lumped\\_vs\\_Distributed\\_Systems.html](https://ccma.stanford.edu/~jos/NumericalInt/Lumped_vs_Distributed_Systems.html)

# Classification of Dynamic Systems

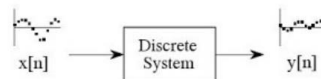
## Continuous-Time Systems

- ▶ Variables and functions defined for all time
- ▶ Similar to variables in the "analog" domain
- ▶ Described by differential equations



## Discrete-Time Systems

- ▶ Variables defined only at discrete time points
- ▶ Similar to variable in the "digital" domain
- ▶ Described by difference equations



<http://signalsworld.blogspot.com/2009/11/continuous-time-and-discrete-time.html>



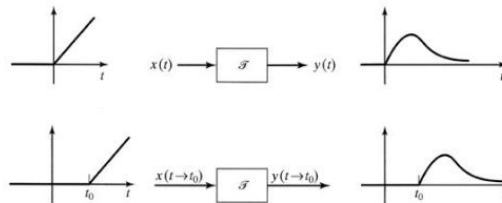
# Classification of Dynamic Systems

## Time-Varying Systems

- ▶ System parameters vary with time

## Time-Invariant Systems

- ▶ System parameters remain constant

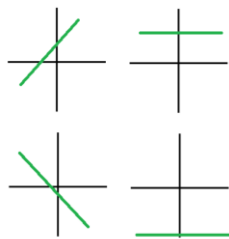


<http://slideplayer.com/slide/10398142/>

# Classification of Dynamic Systems

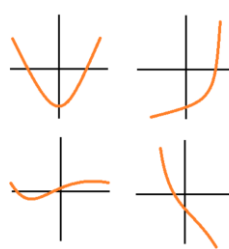
## Linear System

- ▶ Obeys superposition
- ▶ Has homogeneity



## Non-linear System

- ▶ Does not obey superposition
- ▶ Does not have homogeneity



<https://study.com/academy/lesson/how-to-recognize-linear-functions-vs-non-linear-functions.html>

Superposition:  $f(x+y)=f(x)+f(y)$

Homogeneity:  $f(ax)=af(x)$

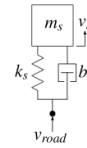
# System Decomposition and Model Complexity

## A Quarter-Car Suspension Model

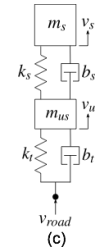
- ▶ To formulate a model we must identify the pertinent components and formulate mathematical representations for each.
- ▶ The complexity of the model depends on its intended use.



(a)



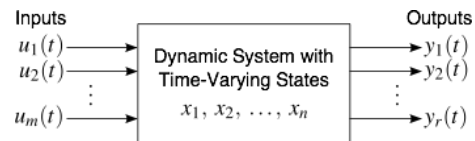
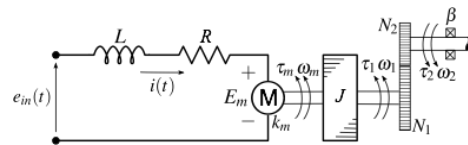
(b)



(c)

# Mathematical Modeling and Dynamic Systems

- ▶ Basic physical laws, theorems, etc. are often modeled using mathematical formulations.
  - ▶ Newton's Second Law
  - ▶ Kirchhoff's Equations
- ▶ In System Dynamics, differential and algebraic equations are used to represent dynamic responses.
- ▶ If the system of differential equations are linear, we can take advantage of this to use Linear Algebra or Laplace Transforms.



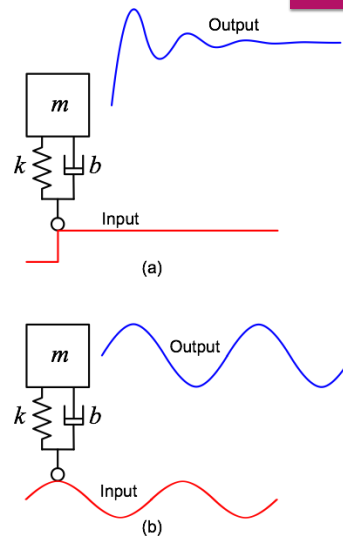
**Inputs** are variables that change the condition of the dynamic system and can include things like external force, voltage sources, pressure sources, etc.

**Outputs** are variables that are measured or observed to assess the dynamic condition of the system

**States** are variables that are used to mathematically model the dynamic behavior of the system.

## Analysis and Design of Dynamic Systems

- ▶ We analyze systems to determine what makes them function or respond as they do so that we might be able to alter or optimize their responses.
- ▶ Analyses are commonly conducted in the time- or frequency-domains.
  - ▶ Step responses usually entails time-domain analysis
  - ▶ Cyclic inputs entails frequency domain analysis

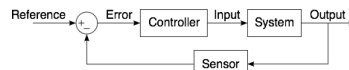


Use a single input to determine the dynamic response.

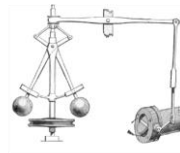
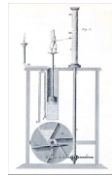
Dynamic systems are often characterized in the time or frequency domain.

# Control of Dynamic Systems

- ▶ Control systems are used to automate dynamic responses; that is to achieve the desired dynamic and static characteristics.



- ▶ Automata or self-operating machines like the water clock and fly-ball governor have existed for centuries.

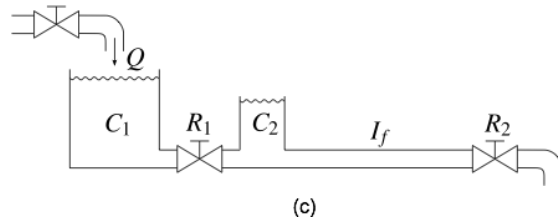
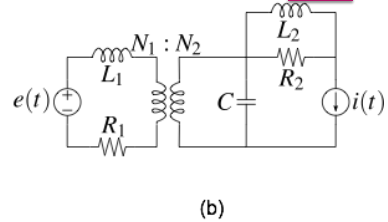
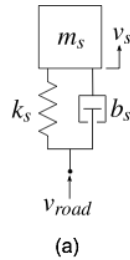


Each block in a block diagram is a dynamic system.

## Diagrams of Dynamic Systems

▶ A number of graphical approaches are used to represent or model dynamic systems in a variety of energy domains:

- ▶ Free-body diagrams
  - ▶ Electric circuit diagrams
  - ▶ Hydraulic circuit diagrams
- ▶ Bond graphs are a generalized graphical method based on power and energy.



## A Graph-Centered Approach to Modeling

- ▶ Bond graphs are a graphical approach for diagramming the distribution and flow of power and energy within a dynamic system.
- ▶ Originally developed by Dr. Henry M. Paynter at MIT in 1959.
- ▶ Bond graphing is a unified approach that accounts for the storage, dissipation, and conversion of energy within a dynamic system.
  - ▶ The bond graph accounts for the input/output relations between elements and subsystems of the model that leads to computer simulation of the dynamic response.



## Power and Energy Variables

- ▶ Energy is defined as the capacity for doing work.
- ▶ Power is defined as the rate of doing work or the amount of energy consumed per unit time.
- ▶ Power ( $\mathcal{P}$ ) is also defined as the multiplication of an effort and a flow
  - ▶ Effort ( $e$ ): force-like variable
  - ▶ Flow, ( $f$ ): velocity-like variable
- ▶ Effort and flow can be related to generalized momentum ( $p$ ) and displacement ( $q$ ), respectively.

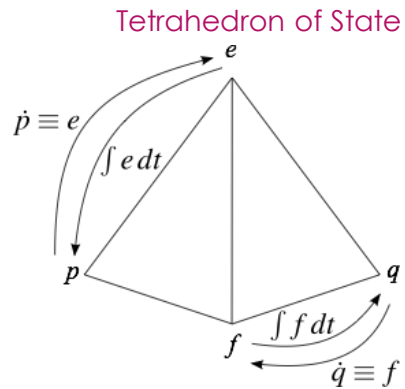
Effort: Force ( $F$ ), torque ( $\tau$ ), voltage ( $e$ ), pressure ( $P$ )

Flow: linear velocity ( $v$ ), angular velocity ( $\omega$ ), current ( $i$ ), volume flow rate ( $Q$ )

# How is Energy Accounted For?

What can we do with energy?

- ▶ Storage of Potential Energy
- ▶ Storage of Kinetic Energy
- ▶ Dissipation of Energy
- ▶ Transformation of Energy
- ▶ Energy Sources



Tetrahedron: 4-sided pyramid (5 sides: 4-triangular, 1-rectangular)

# Effort, Flow, Momentum, and Displacement Variables

## Effort and flow variables

<b>Domain</b>	<b>Effort</b>	<b>Flow</b>	<b>Power</b>
<i>Translational</i>	$F$ , force (N)	$v$ , velocity (m/s)	$\mathcal{P} = F v$
<i>Rotational</i>	$\tau$ , torque (N-m)	$\omega$ , angular velocity (rad/s)	$\mathcal{P} = \tau \omega$
<i>Electrical</i>	$e$ , voltage (V)	$i$ , current (A)	$\mathcal{P} = e i$
<i>Hydraulic</i>	$P$ , pressure (Pa)	$Q$ , flowrate (m <sup>3</sup> /s)	$\mathcal{P} = P Q$

## Momentum and displacement variables

<b>Domain</b>	<b>Momentum</b>	<b>Displacement</b>
<i>Translational</i>	$p$ , linear (N-s)	$x$ , displacement (m)
<i>Rotational</i>	$h$ , angular (N-m-s)	$\theta$ , angle (rad)
<i>Electrical</i>	$\lambda$ , flux linkage (V-s)	$q$ , charge (C)
<i>Hydraulic</i>	$\Gamma$ , hydraulic (N-s/m <sup>2</sup> )	$V$ , volume (m <sup>3</sup> )

# Momentum, Effort, Displacement, and Flow

## Generic Relationships for Effort and Flow

▶ Power	$\mathcal{P}(t) = e(t)f(t)$
▶ Generalized Momentum	$p(t) = \int e(t)dt$
▶ Effort	$e(t) = \frac{dp}{dt} = \dot{p}$
▶ Generalized Displacement	$q(t) = \int f(t)dt$
▶ Flow	$f(t) = \frac{dq}{dt} = \dot{q}$

Momentum:  $p=mv$

$$\dot{p} = \dot{m}v + m\dot{v}$$

$\dot{m}v$  goes to 0 if it isn't losing mass

$$m\dot{v} = ma = F$$

$$ma = \dot{p} = F$$

$$p = \int Fdt$$

Displacement:  $x$

$$\dot{x} = v$$

$$\int vdt = x$$

## Potential and Kinetic Energy

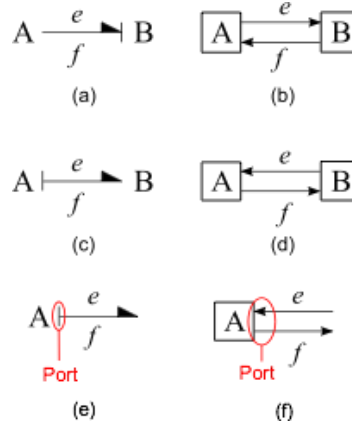
► Energy  $E(t) = \int \mathcal{P}(t)dt = \int e(t)f(t)dt$

► Potential Energy  $E(t) = \int e(t)\frac{dq}{dt}dt = \int e(q)dq$

► Kinetic Energy  $E(t) = \int \frac{dp}{dt}f(t)dt = \int f(p)dp$

# Bonds, Ports, Signals, Inputs, and Outputs

- ▶ **Bonds** connect elements at power **ports** and represent an effort-flow pair.
- ▶ Efforts or flows individually can be represented by a **signal** in a block diagram.
- ▶ Each element and port has an **input** and **output**.
- ▶ Causal strokes are used in bond graphs to indicate what end of the bond has effort as in input.
- ▶ **Port** is a connection to something else.



Effort on top or left. Flow on bottom or right.

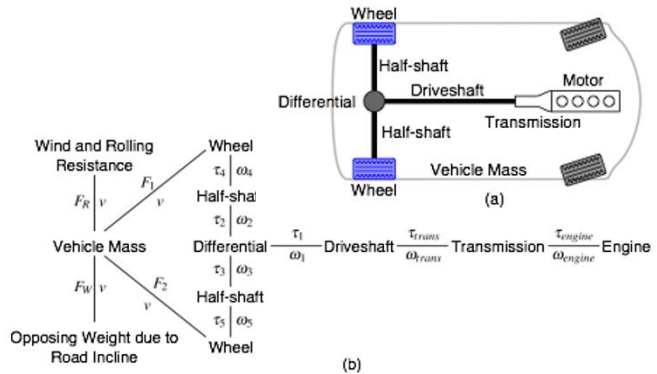
Power direction is shown in bond graph not in block diagram.

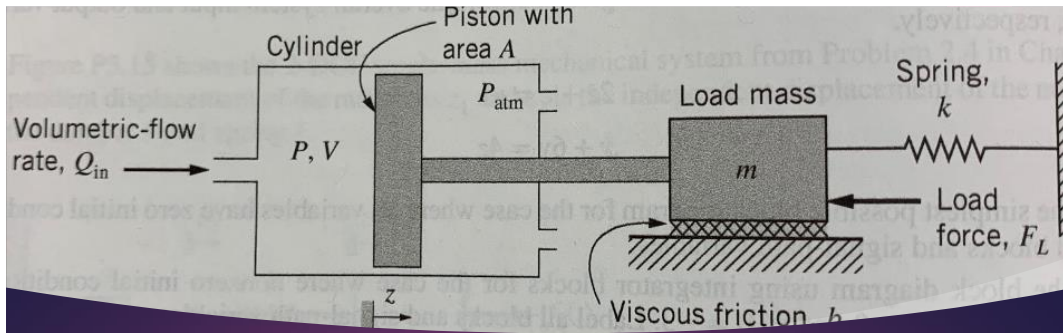
Causal stroke shows where effort is going.

Port is a connection to something else. 1-port is shown in e, f.

# Word Bond Graphs

- ▶ A method for decomposing a system by identifying the more basic components in words and sketching the connections.
- ▶ Process
  - ▶ Identify the basic components of the system
  - ▶ Connect interacting components
  - ▶ Identify the effort-flow pairs

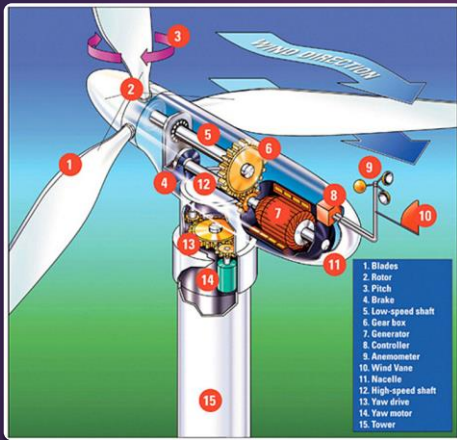




## Word Bond Graphs

- ▶ Create a word bond graph for the hydraulic actuator shown.





## Challenge Problem

CREATE A WORD BOND GRAPH FOR THE SYSTEM SHOWN THROUGH PART 7.

## Summary

- ▶ System decomposition consists of breaking down the system into basic components that can be readily characterized to enable modeling and mathematical representation.
- ▶ Model complexity depends on use of the system representation and the necessary accuracy of the predicted dynamic response.
- ▶ Mathematical models of dynamic systems commonly take the form of differential and algebraic equations. As such, mathematical methods such as Linear Algebra and Laplace Transforms are commonly used to analyze and design dynamic systems.
- ▶ Analysis is used to study dynamic systems and to characterize their responses. It can be used to determine how changes in system parameters vary the dynamic response.
- ▶ When the desired dynamic response cannot be achieved through parametric optimization, automatic control systems can be employed to compensate and alter the system response. Automatic controls are used to modify and/or automate dynamic responses.

## Summary Continued

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