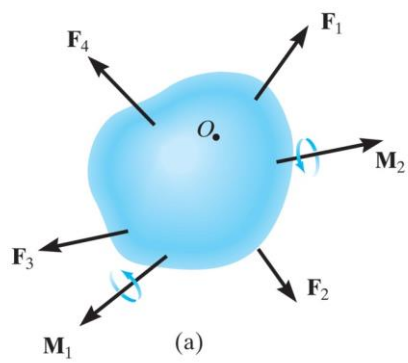
EQUILIBRIUM OF RIGID BODIES

Rigid-Body in Equilibrium

- Force on a rigid-body are not usually concurrent and may cause rotation of the body
- For a rigid-body to be in equilibrium, the net force, as well as the net moment, about any arbitrary point, *O*, must be equal to zero.

$$\Sigma F = 0 \quad \text{(No Translation)}$$

$$\Sigma M = 0 \quad \text{(No Rotation)}$$

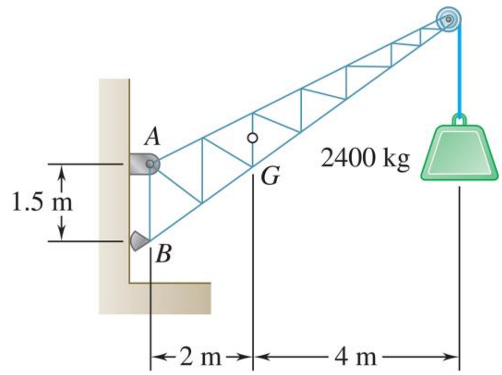


(a)

Figure: 05_001a

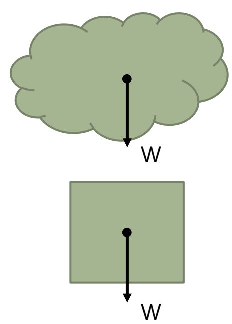
Rigid-Body in Equilibrium

- For equilibrium, we need to create a free-body diagram of the problem.
- We cannot apply equilibrium to the problems we have attempted so far until we have "freed" the body from its restraints.
- To free the body from its restraints or supports, we need to consider how to simulate the reactions so that we may replace each support with an appropriate force or moment that does the same thing.

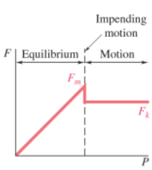


Common 2-D Reactions at Supports and Connections

- **Weight**: The line of action of the force **W** that passes through the center of gravity of the body and is directed toward the center of the Earth.

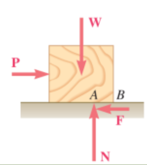


- **Friction:** Tangential forces that develop when two surfaces in contact slide against each other. Friction can be broken up into two types: dry friction and fluid friction. In this class we will be discussing a part of dry friction called **Static Friction.**



(c)

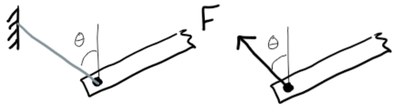
$$F_f = \mu_s N$$



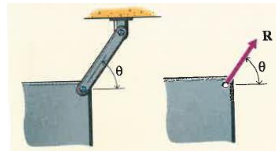
(b)

Common 2-D Reactions at Supports and Connections

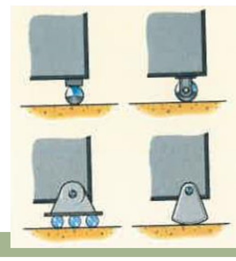
- **Flexible cord, rope, chain, or cable**: Always exerts a tensile force **F** on the body whose line of action is known and tangent to the connection at the point of attachment. **1 Reaction**

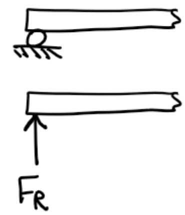


- **Rigid Link**: Can exert either a tensile or compressive force **R** on the body whose line of action is know and directed along the axis of the link. **1 Reaction**



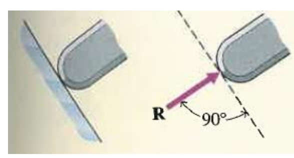
- **Ball, Roller, or Rocker**: Can exert a compressive for **F** on the body whose line of action is perpendicular to the surface supporting the ball, roller, or rocker. **1 Reaction**



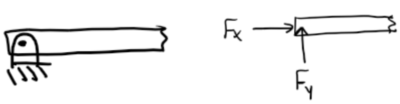


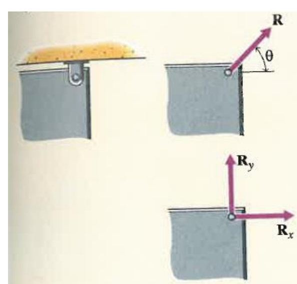
Common 2-D Reactions at Supports and Connections

- **Smooth Surface**: Flat or curved, can exert a compressive force **R** on the body whose line of action is perpendicular to the smooth surface at the point of contact between the body and the smooth surface. **1 Reaction**



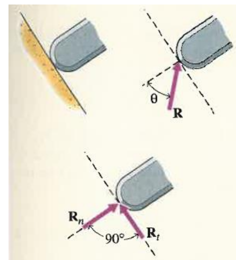
- **Smooth Pin**: Usually represented by the rectangular components of the reaction force **R** directed at an angle θ. **2 Reactions**



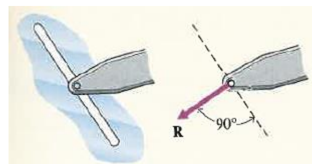


Common 2-D Reactions at Supports and Connections

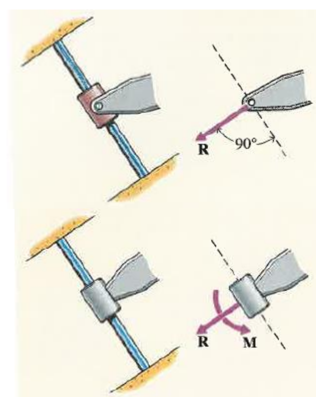
- **Rough Surface**: Capable of supporting a tangential frictional force $\mathbf{R_t}$ as well as a compressive normal force $\mathbf{R_n}$ which are components of a compressive force **R** at an angle θ. **2 Reactions**



- **Pin in a Smooth Guide**: Can transmit only a force **R** perpendicular to the surfaces of the guide. **1 Reaction**

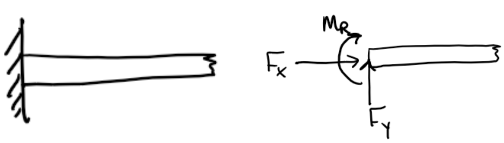


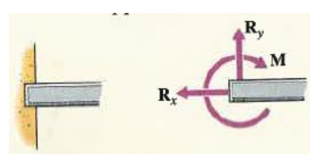
Common 2-D Reactions at Supports and Connections



- **Collar on a Smooth Shaft**: If pin connected, can transmit only a force **R** perpendicular to the axis of the shaft. If fixed, can transmit both a force **R** and a moment **M** perpendicular to the axis of the shaft. **1 or 2 Reactions**

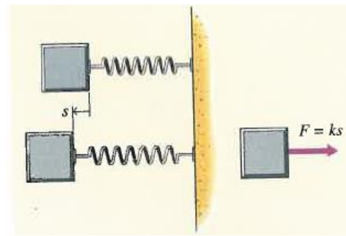
- **Fixed Support**: Represented by the component forces of a force **F** and by a moment **M**. **3 Reactions**



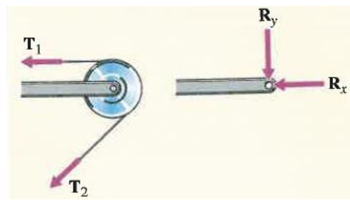


Common 2-D Reactions at Supports and Connections

- **Linear Elastic Spring**: The force **R** exerted on a body by a linear elastic spring is proportional to the change in length of the spring and acts along the axis of the spring. **1 Reaction**



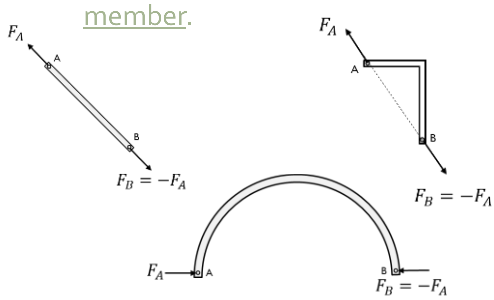
- **Ideal Pulley**: The pin connecting an ideal pulley to a member exerts reactions like a smooth pin. Since the pin is frictionless, the tension **T** in the cable must remain constant. **2 Reactions**



Special Cases 2-Force & 3-Force Members

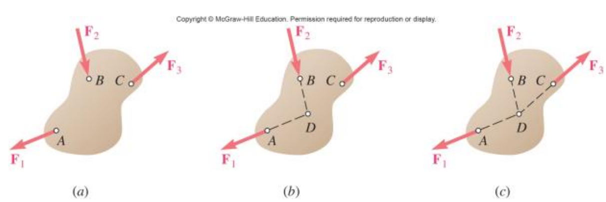
Two-force member

- A member that has pin supports at both ends and is subjected to no load in between is called a <u>two-force member</u>.



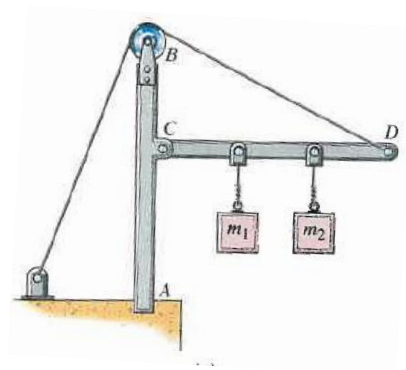
Three-force member

- The lines of action of the three forces must be either concurrent or parallel.



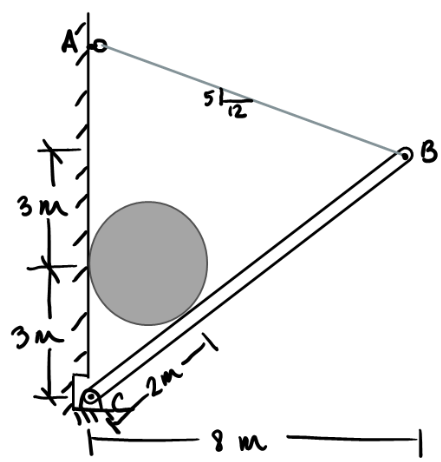
FBD - Example 1

- Draw a free-body diagram for the following:
  - The pulley at B
  - The post AB
  - The beam CD



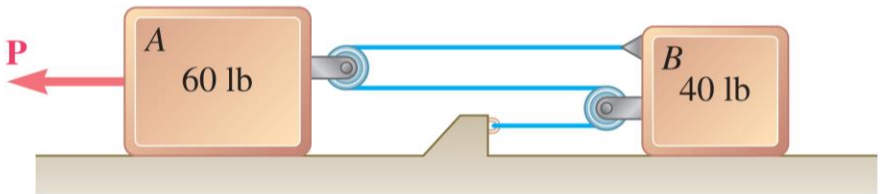
FBD - Example 2

- A barrel is being supported by bar BC pinned at C and a steel guy wire at B. Draw a free-body diagram and solve for unknown reactions for the barrel and the bar if the barrel has a mass of 275 kg and the bar has a mass of 70 kg.



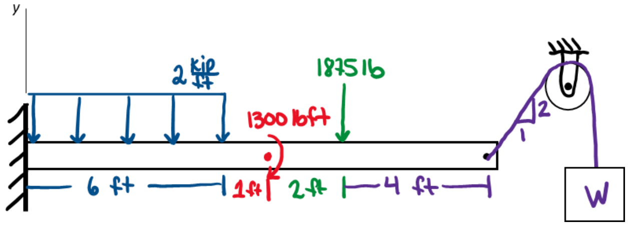
FBD – Example 3

- The blocks A and B are connected by a cable as shown. Knowing that the coefficient of static friction at all surfaces of contact is μ_s=0.30, and neglecting the friction of the pulleys, draw the free-body diagrams needed to determine the smallest force P required to move the blocks.



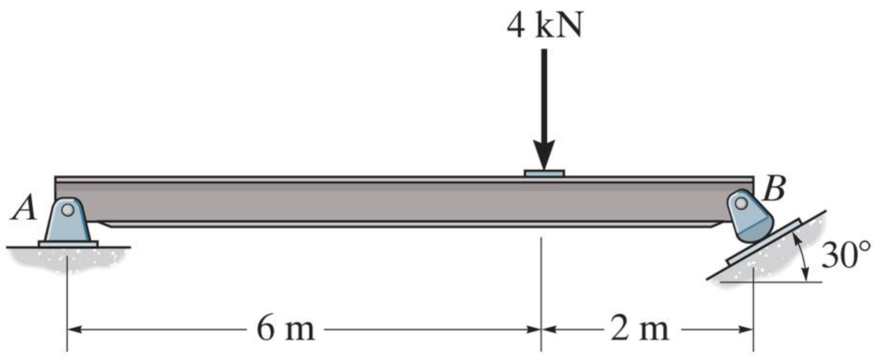
Example 1

- A wall and block with a weight, W, of 900 lb is supporting a loaded beam. Draw a free-body diagram of the beam shown and determine the unknown reactions. The weight of the beam is 1400 lb.



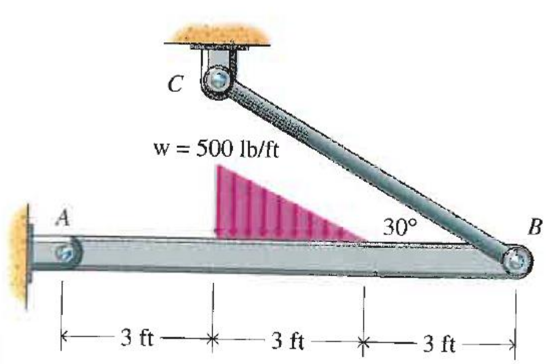
Example 2

- Determine the horizontal and vertical components of reaction at the pin *A* and the reaction of the rocker *B* on the beam.



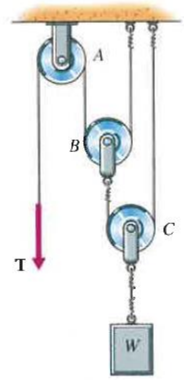
Example 3

- A beam is loaded and supported as shown. Determine the force in member BC and the reaction at support A.



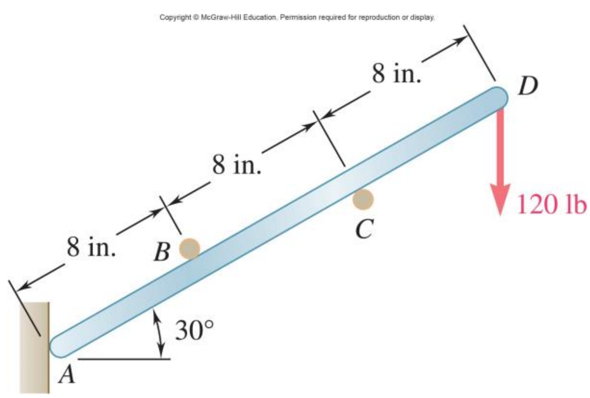
Example 4

- A rope and pulley system is used to support a body W as shown. Each pulley is free to rotate. One rope is continuous over pulleys A and B; the other is continuous over pulley C. Determine the tension T in the rope over pulleys A and B required to hold body W in equilibrium if the mass of body W is 175 kg.



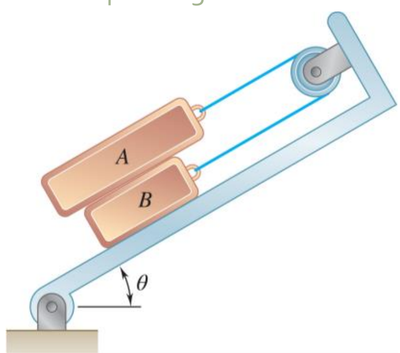
Example 5

- A light rod AD is supported by frictionless pegs at B and C and rests against a frictionless wall at A.



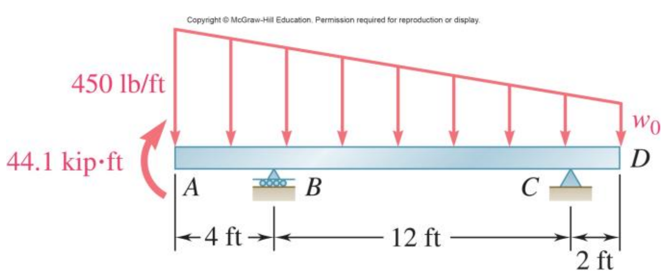
Example 6

- The 50-lb block A and the 25-lb block B are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is μ_s=0.15 between the two blocks and zero between block B and the incline, determine the value of θ for which motion is impending.



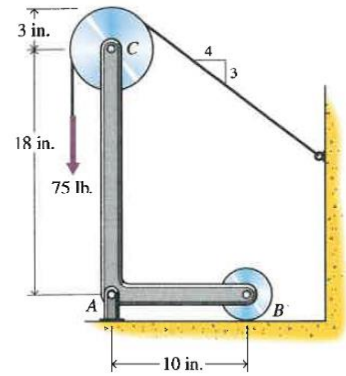
Example 7

- Determine the reactions at the beam supports for the given loading when w_O = 150 lb/ft.

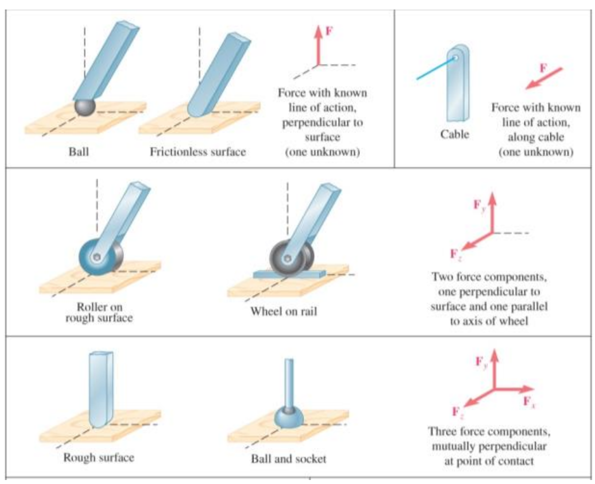


Example 8

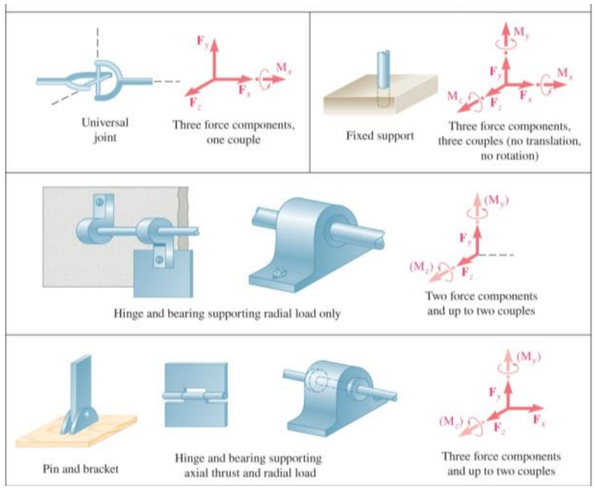
- A 75-lb load is supported by an angle bracket, pulley, and cable as shown. Determine the force exerted on the bracket by the pin at C and the reactions at supports A and B of the bracket.



Common 3-D Reactions at Supports and Connections

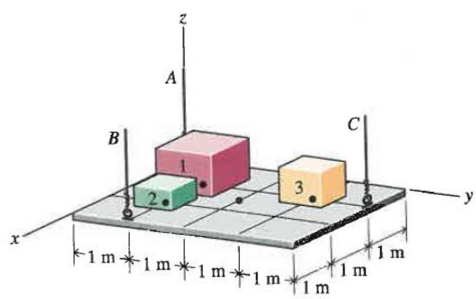


Common 3-D Reactions at Supports and Connections



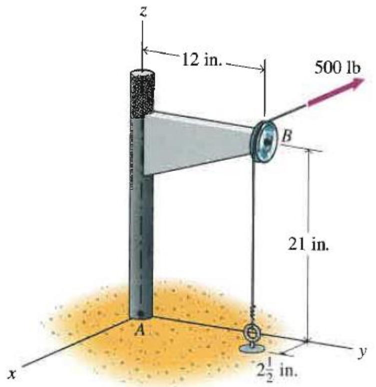
Example 9

- The masses of cartons 1, 2, and 3, which rest on the platform shown, are 300 kg, 100 kg, and 200 kg, respectively. The mass of the platform is 500 kg. Determine the tensions in the three cables A, B, and C that support the platform.



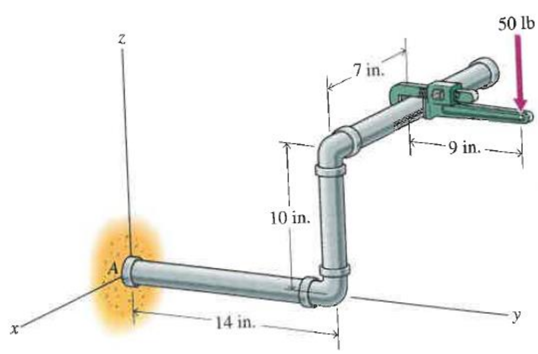
Example 10

- A post and bracket is used to support a pulley. A cable passing over the pulley transmits a 500-lb load as shown. Determine the reaction at support A of the post.



Example 11

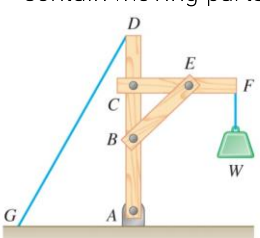
- Determine the reaction at support A of the pipe system shown when the force applied to the pipe wrench is 50 lb.



Analysis of Structures

Frames & Machines

- Frames and machines are two types of structures which are often composed of pin-connected multi-force members.
- Frames: designed to support loads and are usually stationary, fully constrained structures
- Machines: designed to transmit and modify forces, may or may not be stationary, always contain moving parts



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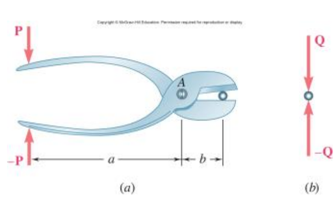

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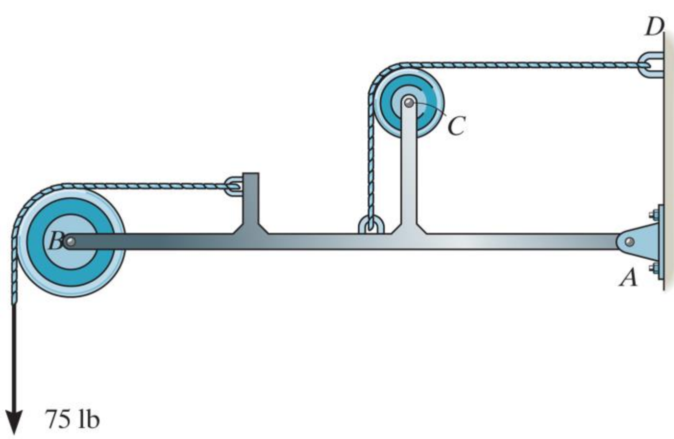
Analyzing Frames & Machines

- Step 1: Draw a FBD of the frame or machine and its members, as necessary
  - Hints:
    - Identify any 2-force members
    - Note that forces on contacting surfaces (usually between a pin and a member) are equal and opposite
    - For a joint with more than 2 members or an external force, it is advisable to draw a FBD of the pin.
- Step 2: Develop a strategy to apply the equations of equilibrium to solve for the unknowns.
  - Look for ways to form single equations and single unknowns.

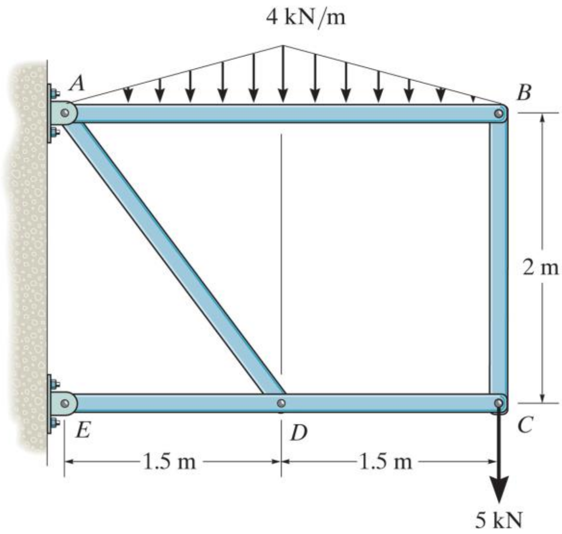
FBD – Example 1

- For the frame shown, draw the FBD of
  - (a) the entire frame including the pulleys and cords,
  - (b) the frame without the pulleys and cords, and
  - (c) each of the pulleys.



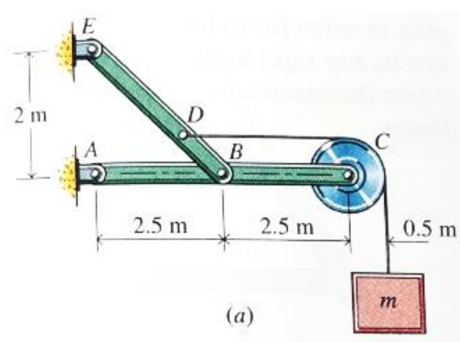
FBD – Example 2

- Draw the appropriate free-body diagrams to determine the components of reaction at *E*.



Example 1

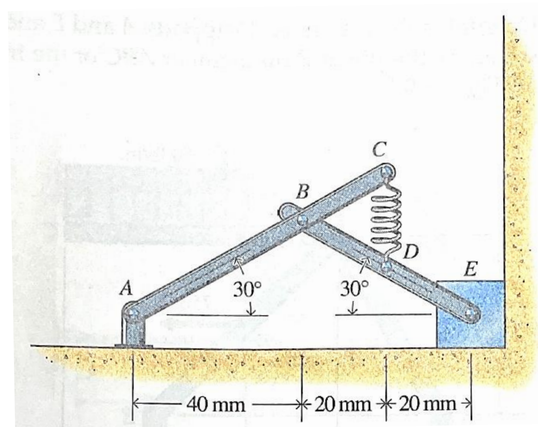
- The cord is wrapped around a frictionless pulley and supports a body with a mass, m, of 100 kg. Determine the reaction components at supports A and E and the forces exerted on the bar ABC by the pin at B.



(a)

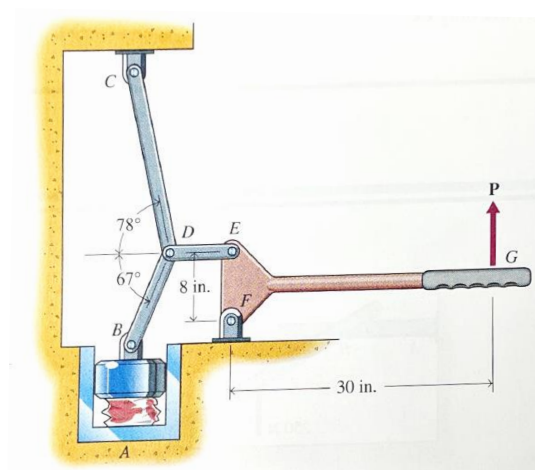
Example 2

- The spring clamp is used to hold block E into the corner. The force in the spring is F=k(l-lo), where l is the present length of the spring, lo=15 mm is the unstretched length of the spring, and k=5000 N/m is the spring constant. Determine all forces acting on member ABC of the spring clamp and the force exerted by the spring clamp on the block E.



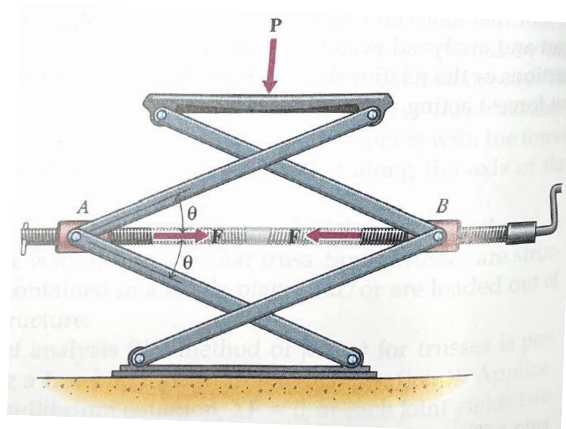
Example 3

- A pin-connected system of levers and bars is used as a toggle for a press as shown. Determine the force **F** exerted on the can at A when a force P=100 lb is applied to the lever at G.



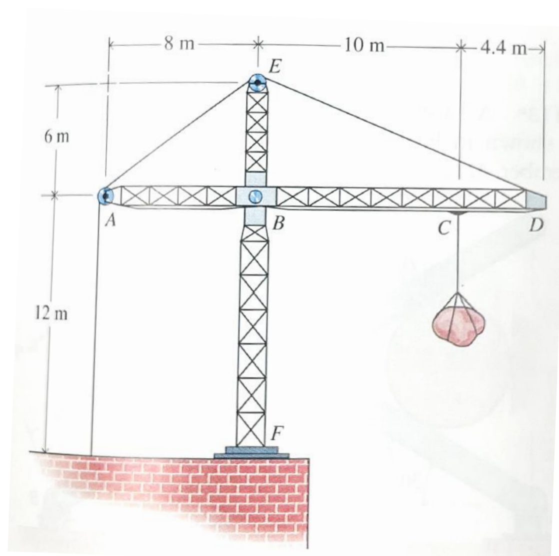
Example 4

- A scissors jack for an automobile is shown. The screw threads exert a force **F** on the blocks at joints A and B. Determine the force **P** exerted on the automobile if **F**=800 N and θ=30°.



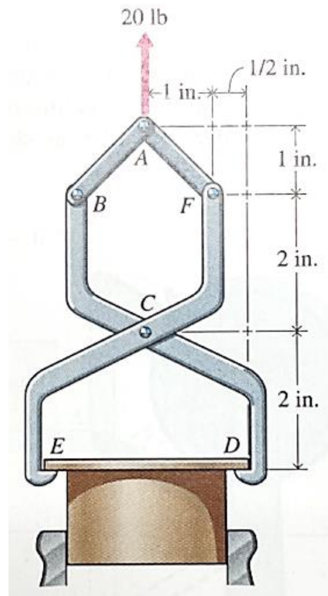
Example 5

- The tower crane is rigidly attached to the building at F. A cable is attached at D and passes over small frictionless pulleys at A and E. The object suspended from C weighs 1500 N. Determine all forces acting on member ABCD.



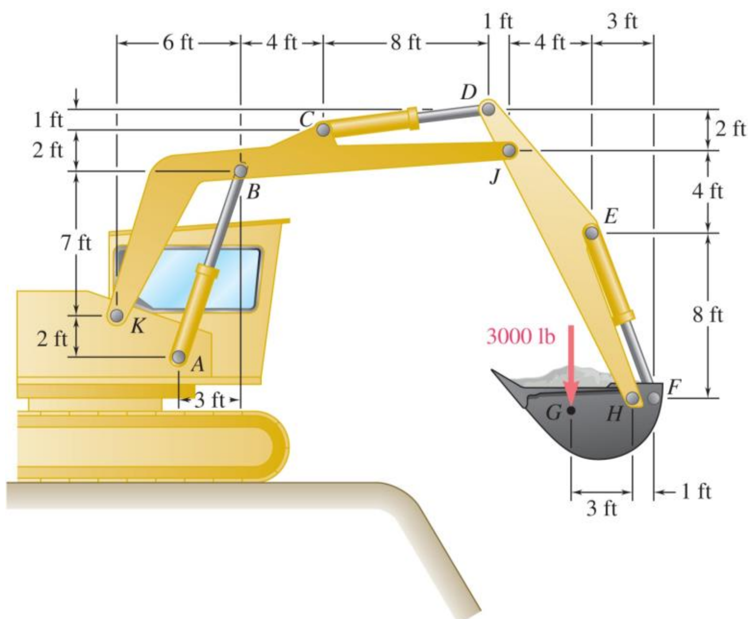
Example 6

- A force of 20 lb is required to pull the stopper DE. Determine all forces acting on member BCD.



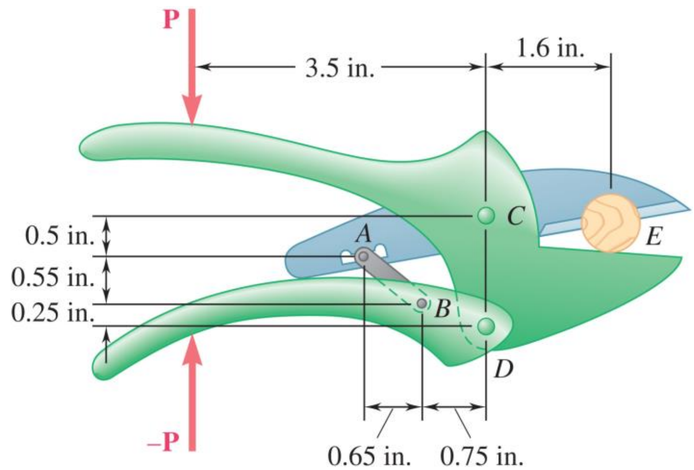
Example 7

- The action of the backhoe bucket is controlled by the three hydraulic cylinders shown. Determine the force exerted by each cylinder in supporting the 3000 lb load shown.



Example 8

- The compound-lever pruning shears shown can be adjusted by placing pin A at various ratchet positions on blade ACE. Knowing that 300-lb vertical forces are required to complete the pruning of a small branch, determine the magnitude P of the forces that must be applied to the handles when the shears are adjusted as shown.

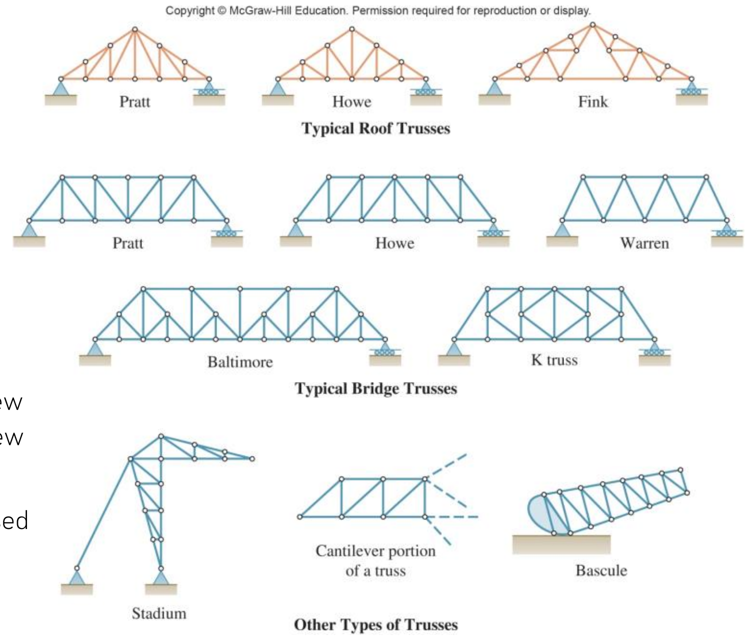


Trusses

- Designed to support loads and are usually stationary, fully constrained structures
- Consist exclusively of straight, slender members connected at joints located at the ends of each member
  - Two-Force Members: A member that has pin supports at both ends and is subjected to no load in between

Different Types of Trusses

- Rigid Truss: A truss that will not collapse
- Simple Truss: A truss made up of a sequence of triangles, with each new triangle formed by attaching one new joint by two new members
- Planar Truss: A truss and its imposed loads lie in a single plane.



Rigidity and Determinacy

- Valid for systems consisting only of 2-force members.
- m is the number of members, r is the number of reactions at the supports, and n is the number of joints

$If\ m + r > 2j: Statically\ Indeterminate$

- Stop, this is a Dynamics or Solids problem where we cannot arbitrarily say that $\Sigma\vec{F} = m\vec{a} = 0$

$If\ m + r = 2j: Statically\ Determinate/"Just\ Right"$

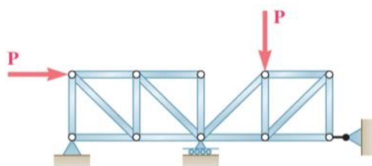
- Proceed, all internal loads can be determined

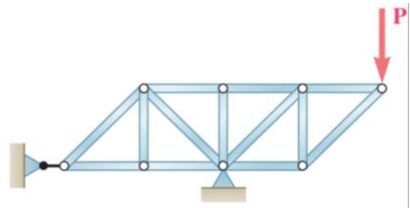
$If\ m + r < 2j: Redundant\ Structure, Partially\ Constrained$

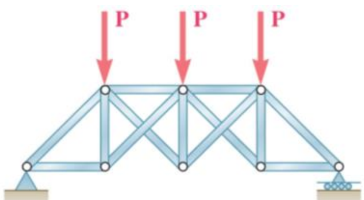
- Careful, some loads can be determined, and some cannot. Map out a strategy to see if you can get what you want before trying to solve.

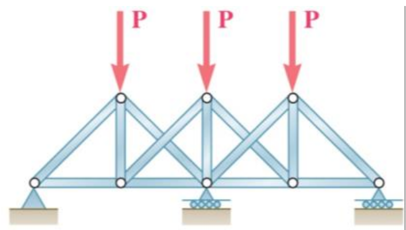
Example 9

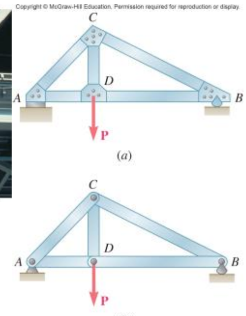
- Solve for the determinacy of each structure.

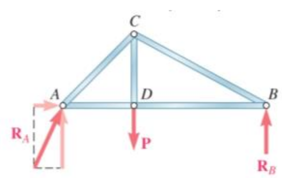








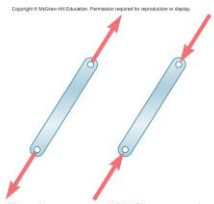




Assumptions made in the Force Analysis of a Truss

- Truss members are connected together at their ends only.
- Truss members are connected together by frictionless pins.
- The truss structure is loaded only at the joints.
- The weights of the slender members may be neglected.

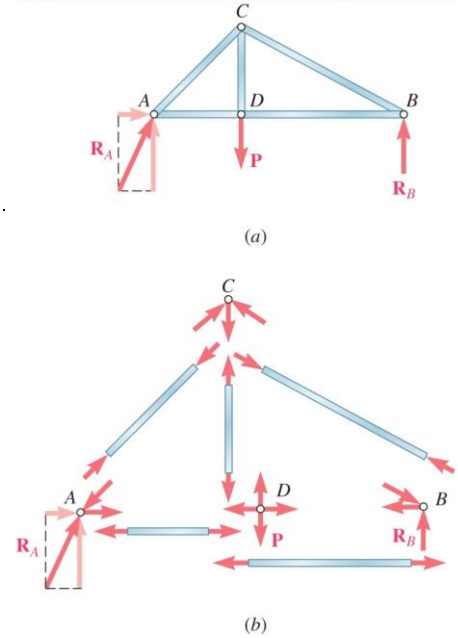
- Result of these 4 assumptions:
  - All members are Two-Force Members.



(a) Tension (b) Compression

Method of Joints

- Comparable to static equilibrium equations for a particle.
- Equilibrium of a joint is considered.
- All forces acting at the joint are shown in an FBD.
- Equations of equilibrium ($\Sigma F_x = 0\ and\ \Sigma F_y = 0$) are used to solve for the unknown forces acting at the joints.

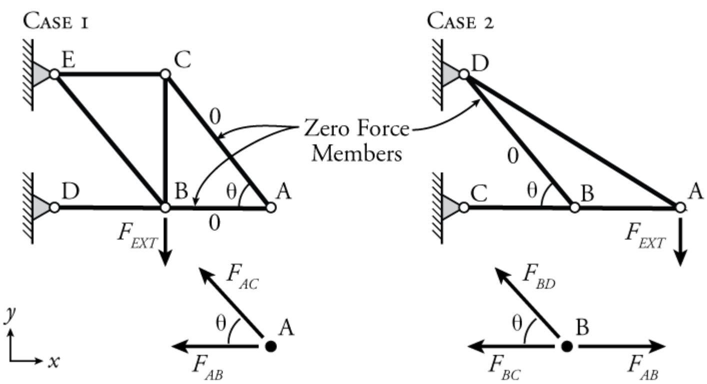


Method of Joints

- Step 1: If support reactions are unknown, draw a FBD of the entire truss and determine the support reactions.
- Step 2: Draw a FBD of a joint with one or two unknowns.
  - Assume all unknown member forces act in tension.
- Step 3: Apply the scalar equations, $\Sigma F_x = 0\ and\ \Sigma F_y = 0$, to determine the unknown(s).
- If the answer is positive, then the assumed direction (tension) is correct, otherwise it is in the opposite direction (compression).
- Step 4: Repeat steps 2 and 3 at each joint in succession until all required forces are determined.

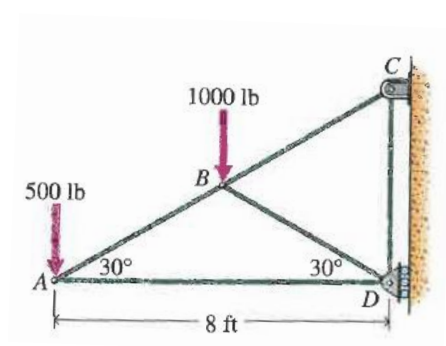
Special Case – Zero-Force Members

- Zero-Force Member: members in a truss that are neither in tension nor compression
- Can be identified by:
  - At a 2-member joint: If those members are NOT parallel AND there are no other external loads (or reactions) at the joint THN both of those members are zero-force members.
  - In a 3-member joint: If TWO of those members ARE parallel AND there are no other external loads (or reactions at the joint THEN the member that is not parallel is a zero-force member.



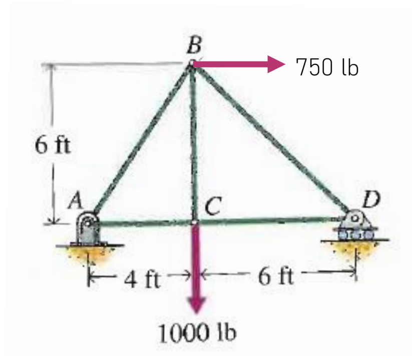
Example 10

- Use the method of joints to determine the force in each member of the truss shown.



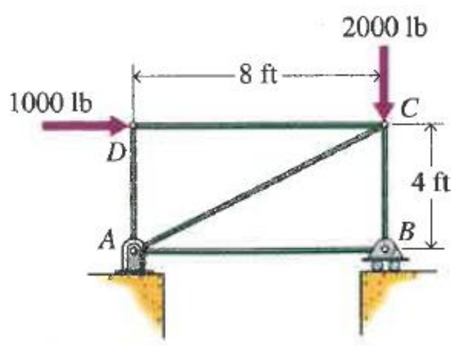
Example 11

- Use the method of joints to determine the force in each member of the truss shown.



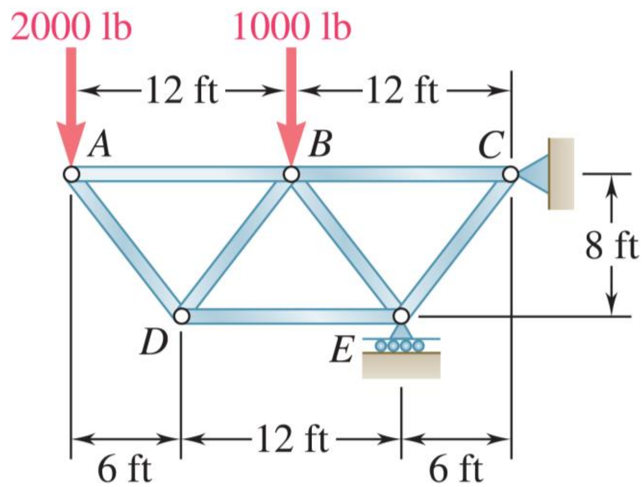
Example 12

- Use the method of joints to determine the force in each member of the truss shown.



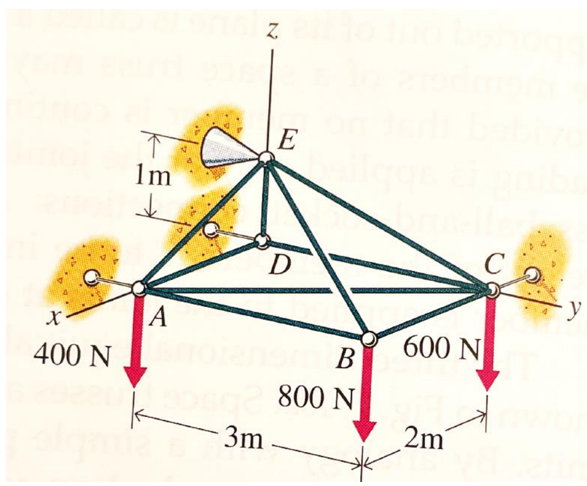
Example 13

- Determine the force in each member of the truss shown. Determine if each member is in tension or compression.



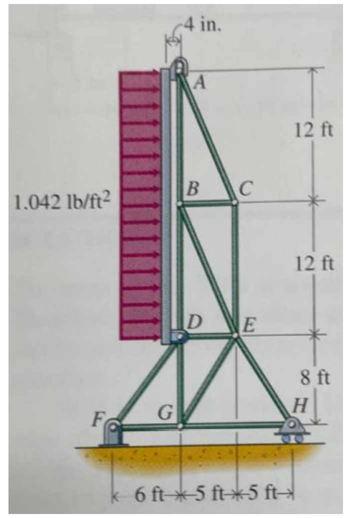
Space Truss Example 14

- The simple space truss is supported by a ball-and-socket joint at E and by short links at A, D, and C. Determine the forces in each of the members.



Challenge in PLTL

- The truss shown supports one end of a 40-ft wide, 24-ft high outdoor movie screen, which weighs 7000 lb. An identical truss supports the other end of the screen. A 20-mph wind blowing directly at the screen creates a wind loading of 1.042 lb/ft^2 on the screen. Calculate the maximum tensile and compressive forces in the members of the truss and indicate in which members they occur.

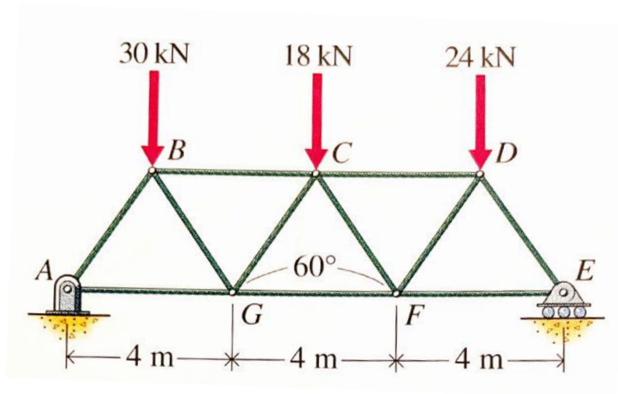


Method of Sections

- Instead of focusing on each individual joint, you can isolate at a section of a truss to analyze
- Step 0: Solve for external reactions (if needed)
- Step 1: Cut the structure through three members (at most)
- Step 2: Draw a FBD of one side of the cut truss
- Step 3: Apply the static equilibrium equations to solve for internal reactions.

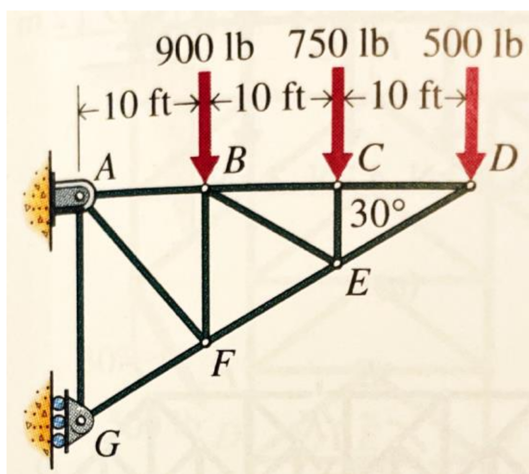
Example 15

- Each truss member has a length of 4 m. Use the method of sections to find the forces in members BC, CG, and FG.



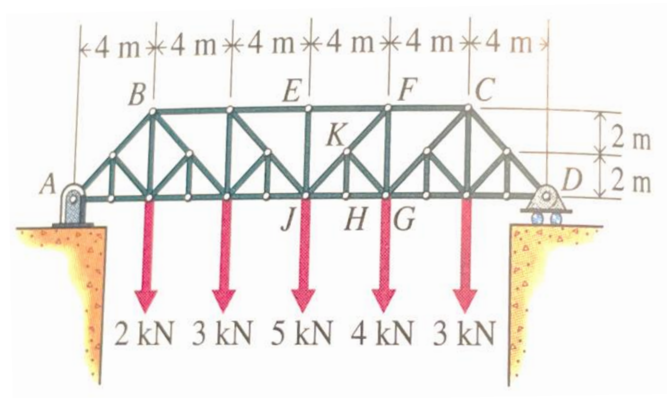
Example 16

- Use the method of sections to find the forces in members AB, AF, and FG of the truss shown.



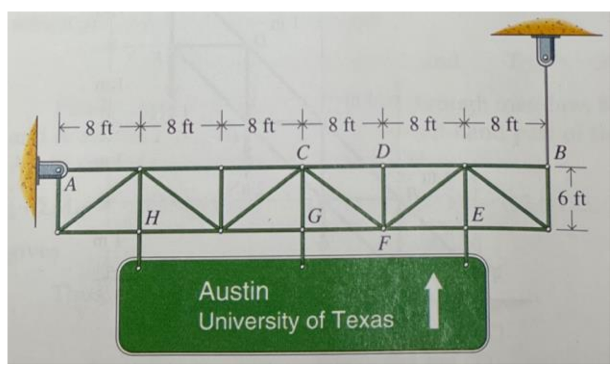
Example 17

- Use the method of sections to find the forces in members EF, JK, and HJ of the Baltimore truss shown.



Example 18

- The truss shown supports a sign that weighs 3000 lb. The sign is connected to the truss at joints E, G, and H, and the connecting links are adjusted so that each joint carries 1/3 of the load. Determine the forces in members CD, CF, and FG.

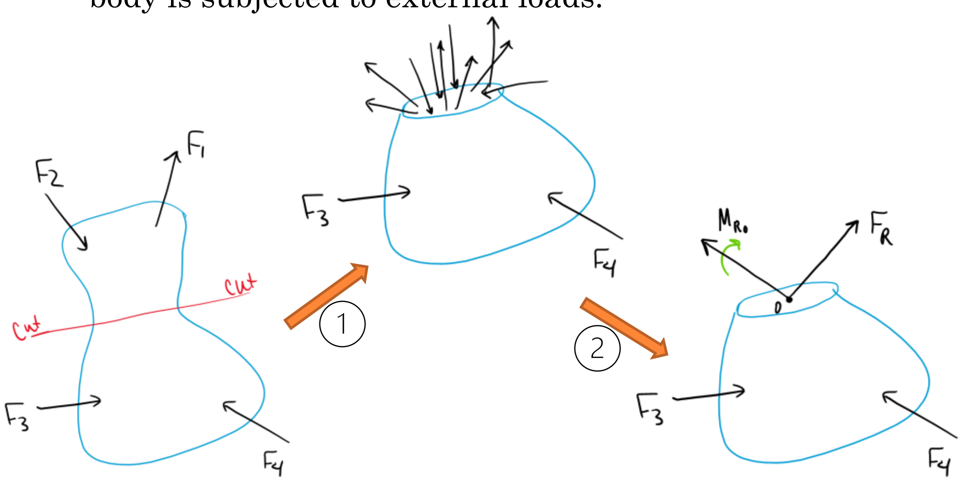


Internal Loads
Free-Body Diagrams

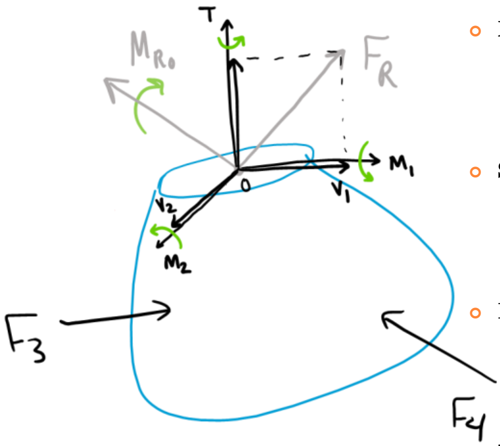
Samantha Ramirez, MSE

Internal Resultant Loadings

- The resultant force and moment which are necessary to hold the body together when the body is subjected to external loads.



Internal Resultant Loadings Cont'd

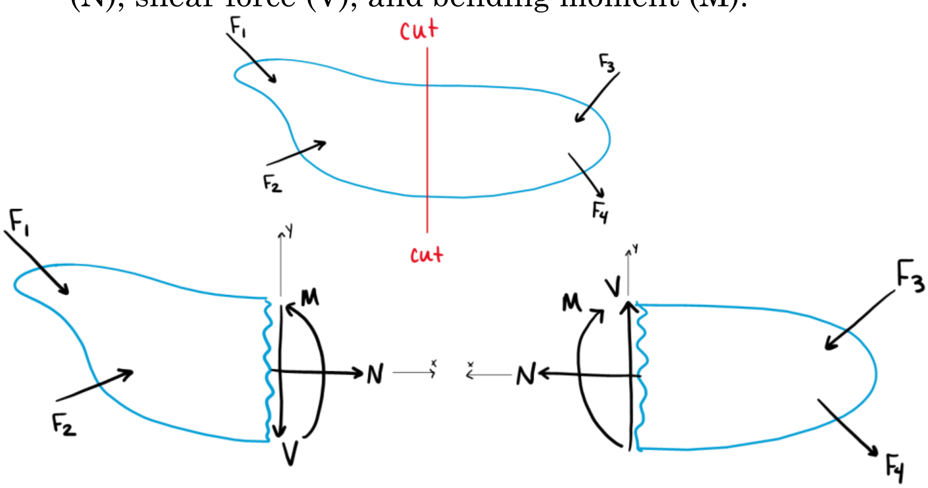


- 3-D Force & Moment
  - Yields 3 force & 3 moment components

- **Normal Force (N)**
  - Acts **perpendicular** to the area due to external loads **pushing or pulling** on the two segments of the body
- **Shear Force (V_1, V_2)**
  - Lies **in the plane** of the area due to external loads causing the two segments of the body to **slide** over one another (2 forces)
- **Bending Moment (M_1, M_2)**
  - Caused by the external loads **bending** the body **about an axis lying within the plane** of the area (2 moments)
- **Torsional Moment or Torque (T)**
  - Caused by the external loads **twisting** one segment of the body with respect to the other **about an axis perpendicular** to the area

Coplanar Loadings – 2-D

- When only coplanar forces act on the body, the internal resultant loadings are the normal force (N), shear force (V), and bending moment (M).



Important Notes About Internal Loadings

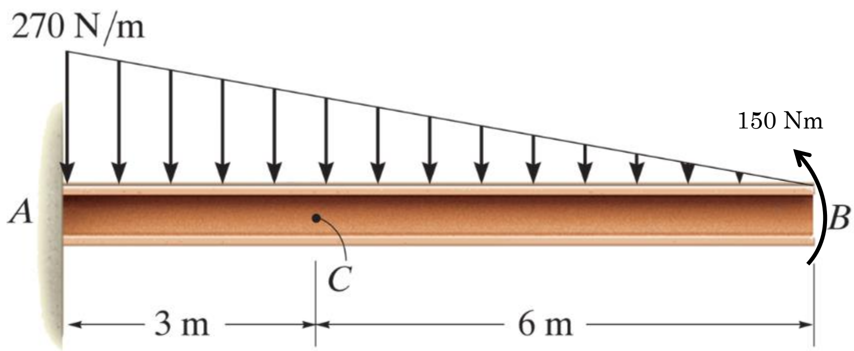
- When analyzing a body that is a member or part of a machine or structure, the machine or structure might have to be disassembled to determine the forces and moments acting on the body before computing the internal loadings.
- The internal loads of a structure only change when an external load is applied.
- The method of sections is used to determine the internal resultant loadings acting on the surface of a sectioned body.

How to Find Internal Loadings

- Procedure
  1. Draw a FBD and determine the reactions at the body's connections (Statics)
  2. Pass an imaginary section through the body (to cut the body) at the location where the internal loadings are to be determined.
  3. Draw a FBD of one of the segments of the cut body
     - Establish theh x, y, and z coordinate axes with origin at the centroid
  4. Clearly indicate the internal resultant loadings (N, V, M, T) acting on the cross-section.
  5. Use the static equilibrium equations to find the internal resultant loadings.

EXAMPLE 1

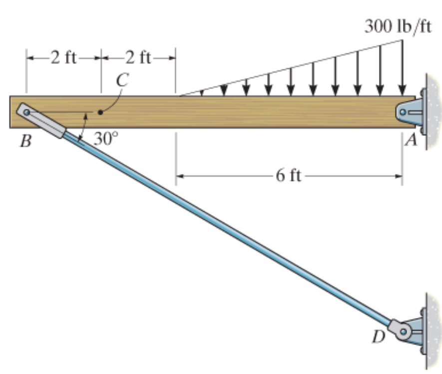
- Determine the resultant internal loadings acting on the cross section at C of the cantilevered beam.



(a)

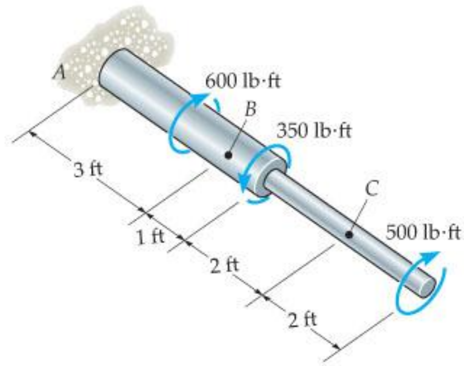
EXAMPLE 2

- Determine the resultant internal loadings acting on the cross section at C of the beam shown below.



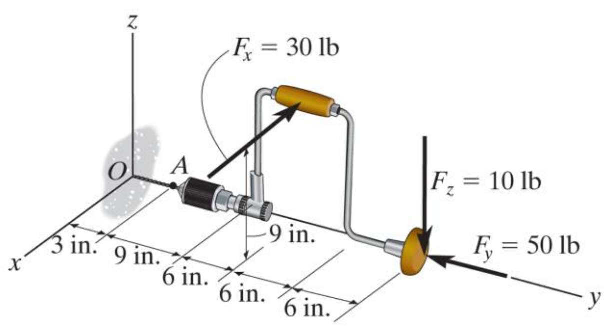
EXAMPLE 3

- Determine the resultant internal torque acting on the cross section through point B and C.



EXAMPLE 4

- The brace and drill bit is used to drill a hole at O. If the drill bit jams when the brace is subjected to the forces shown, determine the resultant internal loadings acting on the cross section of the drill bit at A.



Example 5

- Determine the internal loads at point B.