WELCOME TO MECE 2301 STATICS!

Samantha Ramirez, MSE

Syllabus Information

Course Information

- TR 12:30 PM – 1:45 PM
- PLTL: 2:00 PM – 3:15 PM
- Pre-requisites:
  - "C" or better in PHYS 2425 and MECE 1101 and "C" or better or concurrent enrollment in MATH 2414
- Required Materials
  - Vector Mechanics for Engineers Statics & Dynamics, Beer, 12th ed, Connect Access with eText

Instructor Contact Information

- Office: EENGR 3.261
- Office Hours
  - TR 10:00 AM – 11:00 AM
- Email:
  - samantha.Ramirez@utrgv.edu
- Website:
  - https://faculty.utrgv.edu/samantha.ramirez

Course Structure

- Pre-Lecture (Due before class – by 12:30 PM)
  - SmartBook assignment in MH Connect
    - Aimed to introduce topics before we cover them in class
- Lecture
  - 12:30 PM – Course content and example problems
    - In-class quizzes on Tuesdays covering content from previous week
  - 2:00 PM – Mandatory PLTL session
    - Session work due by 3:15 PM of same day
- Post-Lecture (Due by class time – 12:30 PM)
  - Homework in MH Connect
    - Late submissions accepted until 11:59 PM with a 10% penalty.
    - No submissions accepted after 11:59 PM.

Grading Policy

- 3 Midterm Exams – 45% (15% each)
  - Scheduled on Fridays from 8:30 AM – 10:30 AM in ESCNE 2.106
    - Exam 1: 2/16/24, Exam 2: 3/22/24, Exam 3: 4/26/24
  - To take a midterm exam, you cannot have more than 2 unexcused absences from class at 12:15 PM and 2 unexcused absences from PLTL at 2:00 PM
  - Once a graded exam is returned, you only have 24 hours to contest your exam grade
- Quizzes – 15%
  - Every Tuesday
  - SmartBook assignments – due at class time before content is covered
- Homework – 10%
  - McGraw-Hill Connect – link in Blackboard
    - 10% penalty for late work accepted until 11:59 PM on due date
- PLTL – 10%
  - Attendance is 4%
  - Session Work is 6% and graded following rubric in syllabus
- Cumulative Final Exam – 20%

Classroom Expectations

- Attendance is taken daily
- No live Zoom sessions
- Participation during class
  - Calculating answers, taking notes, etc.
- If you are going to miss class, email me before class detailing your situation. If you are sick, provide a doctor's excuse or test results (with name and DOB).
  - DO NOT WAIT UNTIL AFTER DUE DATES/EXAM DATES TO LET ME KNOW.

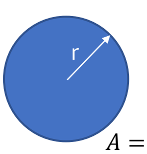
PRE-REQUISITE MATH

Pre-Requisite Math Topics

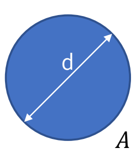
- Geometry
  - Pythagorean Theorem, areas, coordinate systems
- Trigonometry (Pre-Calculus)
  - SOH-CAH-TOA
  - Law of Sine & Law of Cosine
- Algebra
  - System of Linear Equations
  - Roots of a Quadratic Equation
- Calculus
  - Integrals
  - Derivatives

Areas of Basic Shapes

- Circle

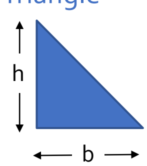


$A = \pi r^2$

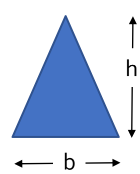


$A = \frac{\pi}{4} d^2$

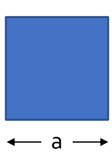
- Triangle



$A = \frac{1}{2} bh$

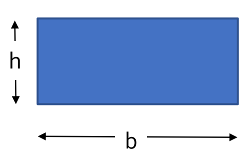


- Square



$A = a^2$

- Rectangle



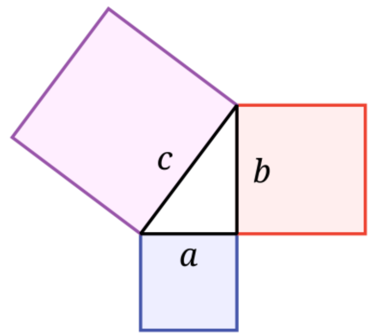
$A = bh$

Pythagorean Theorem

- The sum of the areas of the two squares on the legs (a and b) equals the sum of the area on the hypotenuse.

$$a^2 + b^2 = c^2$$

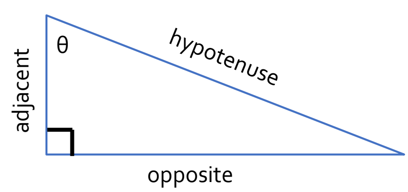
- Only applies to right triangles
  - Right triangles are triangles that have one corner equal to 90°



https://upload.wikimedia.org/wikipedia/commons/thumb/d/d2/Pythagorean.svg/1280px-Pythagorean.svg.png

Triangles – SOH-CAH-TOA

- $sin\theta = \frac{opposite}{hypotenuse}$ and $\theta = \sin^{-1}\frac{opposite}{hypotenuse}$
- $cos\theta = \frac{adjacent}{hypotenuse}$ and $\theta = \cos^{-1}\frac{adjacent}{hypotenuse}$
- $tan\theta = \frac{opposite}{adjacent}$ and $\theta = \tan^{-1}\frac{opposite}{adjacent}$

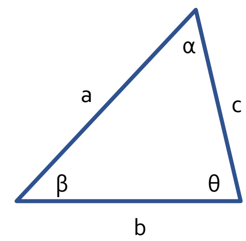


Only applies to right triangles

Law of Sine & Law of Cosine

- Applies to all triangles
- Law of Sine is used when you know either:
  - 2 angles and 1 side
  - 2 sides and 1 angle

$$\frac{\sin\theta}{a} = \frac{\sin\alpha}{b} = \frac{\sin\beta}{c}$$

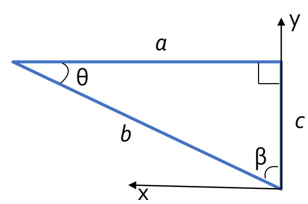


- Law of Cosine is used when you know either:
  - 3 sides
  - 2 sides and 1 angle

$$c^2 = a^2 + b^2 - 2abcos(\beta)$$
$$a^2 = c^2 + b^2 - 2cbcos(\theta)$$
$$b^2 = a^2 + c^2 - 2cacos(\alpha)$$

Example 1

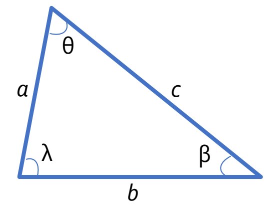
1. What are the distances for a and c if b=13 m and β=30°?



2. If a=4.9 ft and c=8.7 ft, what are b, θ, and β?

Example 2

1. What is θ if a=23 mm, b=50 mm, and c=48 mm?



2. What is c if a=67 in, λ=84°, and β=27°?

System of Linear Equations

- A collection of one or more equations with the same variables
  - The number of equations and the number of unknowns are the same

1. Rearrange equations so that variables are in the same order for all equations
2. For 2 equations/2 unknowns, you can use the graphing method, the substitution method, or the elimination method.
3. For 3 equations/3 unknowns, you can use the linear combination method.
   a) Take 2 equations at a time to eliminate one variable. Then use the resulting equations with 2 variables to eliminate another variable.

Example 3

- Solve for a and b in the following equations using the substitution method and the elimination method.

$$7a - 6b = 19$$
$$34 + 9a = 12b$$

Example 4

- Solve for x, y, and z for the following equation system:

$$9x - 6y + 7z = 62$$
$$6z - 7x = 87 - 3y$$
$$56 + 8x = 2z - 9y$$

- Solution in Blackboard

Roots of a Quadratic Equation

- Quadratic Formula

$$x = \frac{-b \pm \sqrt[2]{b^2 - 4ac}}{2a}$$

- Use the quadratic formula to solve for the roots (x) of a quadratic equation of the following form:

$$ax^2 + bx + c = 0$$

Example 5

- Find the roots of the following equations:

$$5x^2 + 23x - 67 = 0$$

$$23 - 45x + 2x^2 = 3x^2 - 10$$

Derivatives – Product Rule

- Product (Chain) Rule

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Example 6

- Find the derivative of $5xcos(x)$ and evaluate it at x=0.

Derivatives – Quotient Rule

- Quotient Rule

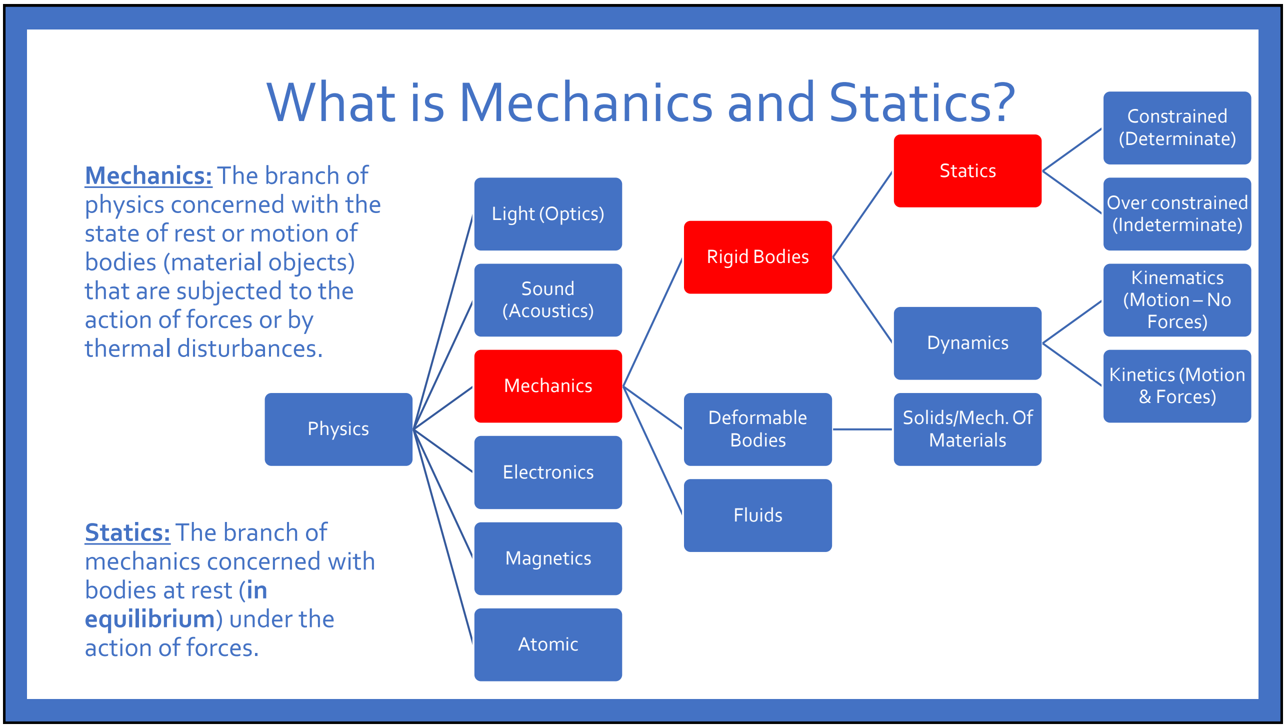
$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'^{(x)}g(x) - f(x)g'(x)}{[g(x)]^2}$$

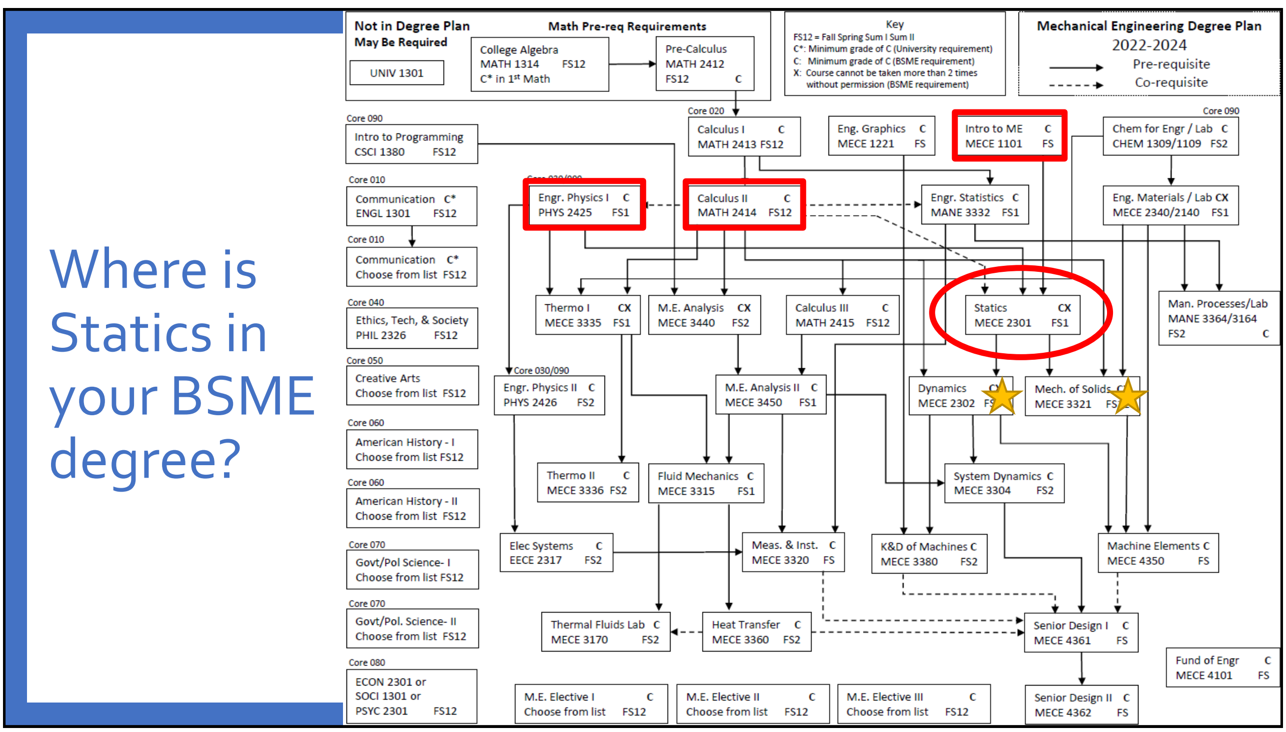
- Example 7
- Find the derivative of $\frac{8x}{2x+\cos(x)}$.

Example 8 - Integrals

- Find the integral of $\cos(x) + 4x^3$ from 0 to A.
- Find the integral of $16x^6 + \sin(x)$.

WHAT IS STATICS? & UNIT CONVERSIONS


What is Mechanics and Statics?
Mechanics: The branch of physics concerned with the state of rest or motion of bodies (material objects) that are subjected to the action of forces or by thermal disturbances.
Statics: The branch of mechanics concerned with bodies at rest (in equilibrium) under the action of forces.
Physics
Light (Optics)
Sound (Acoustics)
Mechanics
Electronics
Magnetics
Atomic
Rigid Bodies
Deformable Bodies
Fluids
Statics
Dynamics
Solids/Mech. Of Materials
Constrained (Determinate)
Over constrained (Indeterminate)
Kinematics (Motion – No Forces)
Kinetics (Motion & Forces)

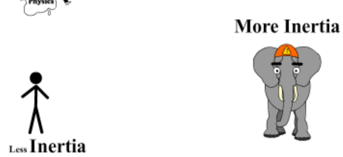

Where is Statics in your BSME degree?
Not in Degree Plan
May Be Required
UNIV 1301
Math Pre-req Requirements
College Algebra MATH 1314 FS12 C* in 1st Math
Pre-Calculus MATH 2412 FS12 C
Mechanical Engineering Degree Plan
2022-2024
Pre-requisite
Co-requisite
Intro to Programming CSCI 1380 FS12
Communication C* ENGL 1301 FS12
Communication C* Choose from list FS12
Ethics, Tech, & Society PHIL 2326 FS12
Creative Arts Choose from list FS12
American History - I Choose from list FS12
American History - II Choose from list FS12
Govt/Pol Science- I Choose from list FS12
Govt/Pol. Science- II Choose from list FS12
ECON 2301 or SOCI 1301 or PSYC 2301 FS12
Calculus I C MATH 2413 FS12
Eng. Graphics C MECE 1221 FS
Intro to ME C MECE 1101 FS
Chem for Engr / Lab C CHEM 1309/1109 FS2
Engr. Physics I C PHYS 2425 FS1
Calculus II C MATH 2414 FS12
Engr. Statistics C MANE 3332 FS1
Eng. Materials / Lab CX MECE 2340/2140 FS1
Thermo I CX MECE 3335 FS1
M.E. Analysis CX MECE 3440 FS2
Calculus III C MATH 2415 FS12
Statics CX MECE 2301 FS1
Man. Processes/Lab MANE 3364/3164 FS2 C
Engr. Physics II C PHYS 2426 FS2
M.E. Analysis II C MECE 3450 FS1
Dynamics MECE 2302
Mech. of Solids MECE 3321
Thermo II C MECE 3336 FS2
Fluid Mechanics C MECE 3315 FS1
System Dynamics C MECE 3304 FS2
Elec Systems C EECE 2317 FS2
Meas. & Inst. C MECE 3320 FS
K&D of Machines C MECE 3380 FS2
Machine Elements C MECE 4350 FS
Thermal Fluids Lab C MECE 3170 FS2
Heat Transfer C MECE 3360 FS2
Senior Design I C MECE 4361 FS
Fund of Engr C MECE 4101 FS
M.E. Elective I C Choose from list FS12
M.E. Elective II C Choose from list FS12
M.E. Elective III C Choose from list FS12
Senior Design II C MECE 4362 FS

Fundamental Concepts

- **Length**: used to locate the position of a point in space
- **Time**: the interval between two events and required with position to define an event
- **Mass**: the quantity of matter OR a measure of its inertia (resistance to a change in its motion)
- **Force**: the push or pull of one body on another which has a point of application, magnitude, and direction

Important Principles and Laws

- Newton's Three Laws
  1. If a particle is acted on by a balanced force system, it will continue with the same state of motion. The direction and magnitude of the motion will remain unchanged.
     - An object at rest will remain at rest unless acted upon by an unbalanced force.
       - $\sum \vec{F} = \vec{0}$
  2. The resultant force acting on a particle of mass is proportional to the acceleration of the particle.
     - $\sum \vec{F} = m\vec{a}$
     - When the acceleration of a particle is 0, Newton's second law becomes F = 0 which is the central focus of statics.
  3. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.
     - For every action, there is an equal and opposite reaction.



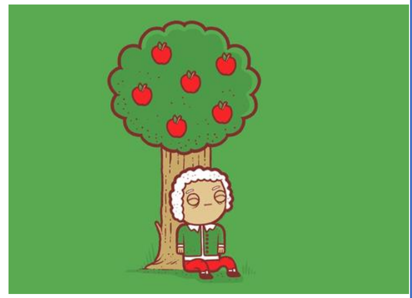
https://stickmanphysics.com/wp-content/uploads/2019/10/HumanElephant.gif https://qph.cf2.quoracdn.net/main-qimg-2dd0563c068c7927d6141e3fa85280ee https://qph.cf2.quoracdn.net/main-qimg-865c540650b4c9b38515ed20d949301b https://media3.giphy.com/media/3orieSP8hVdrlUGTT2/200w.gif?cid=6c09b95230e164bwn41txfbfir3hdymnvgr8kfwcnlrh4ntz&rid=200w.gif&ct=g

Important Principles and Laws

- Newton's Law of Gravitation
  - Two particles are attracted with equal and opposite forces

$$F = G\frac{m_1 m_2}{r^2}$$

    - m_1 and m_2 are masses of two particles attracted to each other
    - G is the constant of gravitation of 6.673×10^{-11} m^3/kgs^2
    - r is the distance between two particles



https://media0.giphy.com/media/l1Jgz5SmyhtdybtaU/giphy.gif

- What happens when one particle is Earth and the second particle is an object on Earth?

$$F = G\frac{m_1 m_2}{r^2} \rightarrow W = G\frac{m_e m}{r_e^2} = mg$$

g is Earth's gravitational constant
9.81 m/s^2 or 32.2 ft/s^2

Units

SI Units

- International System of Units
- Used mostly around the world

US Customary Units

- Used in the US only
  - Imperial units are used in England now

Quantity	SI Units	US Customary Units
Time	s	s
Length	m	ft
Mass	kg	slug

Challenge

- Derive the units for weight in both unit systems.

Weight

- Newton's Second Law states:

$$F = ma$$

- The weight of an object of mass m on the surface of Earth is

$$W = mg$$

$$W = m(kg) \times 9.81 \frac{m}{s^2}$$

$$W\left(\frac{kgm}{s^2}\right) = m(kg) \times 9.81 \frac{m}{s^2}$$

$$W(N) = m(kg) \times 9.81 \frac{m}{s^2}$$

$$W = m(slug) \times 32.2 \frac{ft}{s^2}$$

$$W\left(\frac{slugft}{s^2}\right) = m(slug) \times 32.2 \frac{ft}{s^2}$$

$$W(lb_f) = m(slug) \times 32.2 \frac{ft}{s^2}$$

Basic Units & Conversions

Quantity	SI Units	US Customary Units
Time	s	s
Length	m	ft
Mass	kg	slug
Force	N	lb

Quantity	Unit of Measurement	Equals	Unit of Measurement
Length	1 in	=	0.0254 m
	1 mi	=	5280 ft
Mass	1 slug	=	14.594 kg
	1 lb_m	=	0.4536 kg
Force	1 lb	=	4.448 N

Unit Prefixes

Multiplication Factor	Prefix	Symbol
1 000 000 000	Giga	G
1 000 000	Mega	M
1 000	Kilo	k
0.01	Centi*	c
0.001	Milli	m
0.000 0001	Micro	μ
0.000 000 001	Nano	n

*Unit should be avoided unless using for nontechnical purpose

Engineering Notation

- A version of the scientific notation in which the exponent must be a multiple of 3

Units in Engineering

- If you work a problem and your units are incorrect, or
- If you work a problem and you don't indicate the units,

Your answer is **WRONG**!

Numerical Calculations

- Dimensional Homogeneity
  - Dimensions must be the same on both sides of the equal sign
- Significant Figures
  - Determines the accuracy of the number
  - Do not record more significant figures than necessary
  - Practical Rule
    - Use 4 figures to record numbers starting with a "1"
    - Use 3 figures in all other cases
- Rounding off numbers
  - Necessary so the accuracy of the result will be the same as that of the problem data
  - Be consistent with the textbook

Fundamental Rules for Algebraic Manipulation

- You can multiply any number by 1

$$\overset{5}{\cancel{15}}\ \frac{kgm}{\cancel{s^2}} \times \frac{3\ \cancel{s}}{\cancel{3}\ s} = 15\ \frac{kgm}{s^2}$$

- Dimensional analysis is key

$$W(N) = m(kg) \times 9.81\,\frac{m}{s^2} \qquad N \approx \frac{kgm}{s^2}$$

- What you do on one side of the equal sign, you must do on the other side

Example 1

- Convert the velocity v=75 km/hr to units of mi/hr.

Example 2

- Convert the mass m=350 slug to units of kg.

Example 3

- Convert the torque T=76.8 Nm to units of lb-in.

Example 4

- Calculate the weight in lb_f of 120 lb_m.

Example 5

- Calculate the weight in lb_f of a body with a mass of 58 kg.

Example 6

- Calculate the area of a lot in m^2 if it measures 1575 ft^2.

Example 7

- Convert an acceleration of 9000 mi/h^2 to m/s^2.

Prefix Examples

8. Convert 15 km to m.
9. Convert 7.8 MPa to N/m^2.
10. Convert 4.3 μm to m.
11. Convert 2 N/mm^2 to MPa.

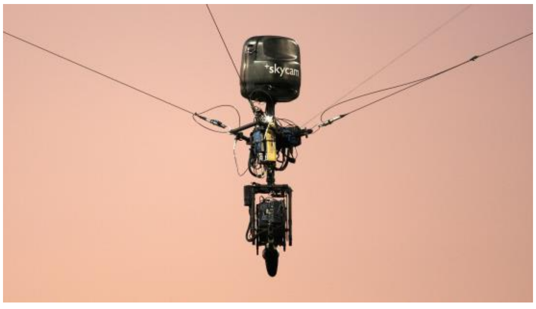
STATICS OF PARTICLES

Fundamental Concepts

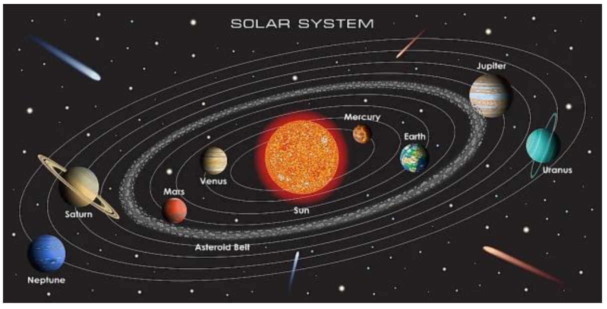
- **Body**: substance that occupies space and has a defined boundary
- **Space**: the geometric region in which the physical events of interest in mechanics occur
- **Scalar**: a quantity represented by magnitude alone
  - i.e. mass, temperature, pressure, time, length
- **Vector**: a quantity represented by both magnitude and direction
  - i.e. $\vec{v}$elocity, $\vec{a}$cceleration, $\vec{f}$orce, displacement ($\vec{x}$)

Particle

- A body with negligible dimensions but not necessarily small with respect to you
- Earth is a particle when studying its motion around the sun



https://upload.wikimedia.org/wikipedia/commons/5/5f/Skycam_Husky_Stadium.jpg



https://www.insightsonindia.com/wp-content/uploads/2021/09/Our-Solar-System.jpg

Rigid Body

- A large number of particles occupying fixed positions with respect to one another
- A body which does not deform
- Nothing is completely rigid, but assuming so is continually used in the applications of mechanics

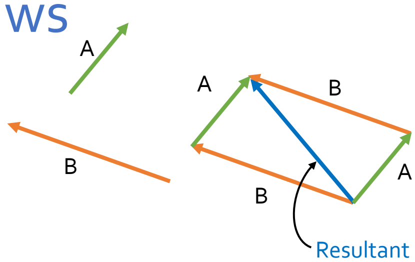
https://www.internationalcranes.media/Images/480xany/20220412-094711-photo-8.jpg

http://mobileimages.lowes.com/productimages/be8e19f4-4b8e-415a-93a8-5ec424888aa1/15122123.jpg

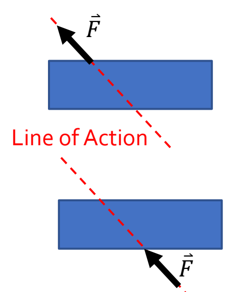
https://m.media-amazon.com/images/I/31w-1OaNC7L._AC_.jpg

Important Principles and Laws

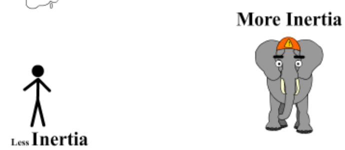
- Parallelogram Law for the Addition of Vectors
  - Two forces acting on a particle can be replaced by a single force (resultant) by drawing a parallelogram with the two forces as sides and the diagonal is the resultant.



- Principle of Transmissibility
  - A rigid body will remain unchanged if a force acting at a given point is replaced by a force with the same magnitude and direction at another point on the line of action



Important Principles and Laws



- Newton's Three Laws
  1. If a particle is acted on by a balanced force system, it will continue with the same state of motion. The direction and magnitude of the motion will remain unchanged.
     - An object at rest will remain at rest unless acted upon by an unbalanced force.
       - $\sum \vec{F} = \vec{0}$
  2. The resultant force acting on a particle of mass is proportional to the acceleration of the particle.
     - $\sum \vec{F} = m\vec{a}$
     - When the acceleration of a particle is 0, Newton's second law becomes F = 0 which is the central focus of statics.
  3. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.
     - For every action, there is an equal and opposite reaction.

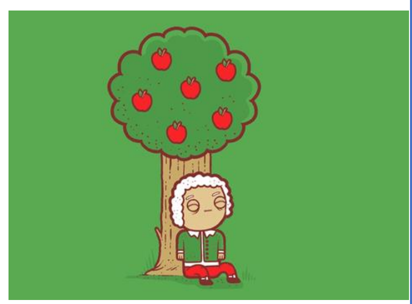
https://stickmanphysics.com/wp-content/uploads/2019/10/HumanElephant.gif https://qph.cf2.quoracdn.net/main-qimg-2dd0563c068c7927d6141e3fa85280ee https://media3.giphy.com/media/3orieSP8hVdrlUGTT2/200w.gif?cid=6c09b95230e164bwn41txfbfir3hdymnvgr8kfwcnlrh4ntz&rid=200w.gif&ct=g

Important Principles and Laws

- Newton's Law of Gravitation
  - Two particles are attracted with equal and opposite forces

$$F = G\frac{m_1 m_2}{r^2}$$

    - m_1 and m_2 are masses of two particles attracted to each other
    - G is the constant of gravitation of 6.673×10^{-11} m³/kgs²
    - r is the distance between two particles



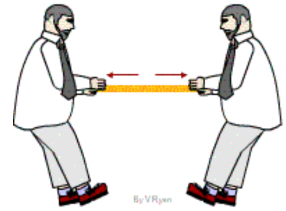
https://media0.giphy.com/media/l1Jgz5SmyhtdybtaU/giphy.gif

- What happens when one particle is Earth and the second particle is an object on Earth?

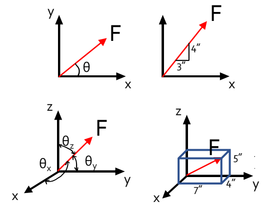
$$F = G\frac{m_1 m_2}{r^2} \rightarrow W = G\frac{m_e m}{r_e^2} = mg$$

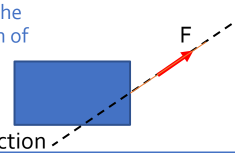
g is Earth's gravitational constant
9.81 m/s² or 32.2 ft/s²

Introduction



- Force: The action of one body on another body
  - Results from
    - Direct physical contact*
    - Gravitational*, electrical, magnetic effects
  - Effects on body
    - External*
      - Change its motion
      - Develop resisting forces (reactions)
    - Internal
      - Deform (change shape)
- Must have the following
  - Magnitude – positive numerical value
    - Amount or size
    - Length
  - Direction – slope and sense
    - Planar – 2D
    - Spatial – 3D
  - Point of application
    - Point of contact
    - Line of Action*
      - A straight line extending through the point of application in the direction of the force



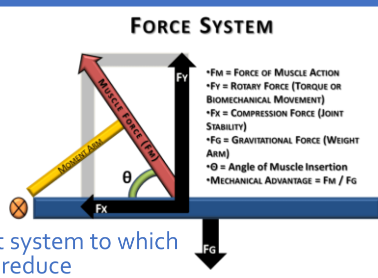


* of primary concern in Statics

https://i.gifer.com/origin/6c/6c3c264af2d297f981a3c3b209c8e717.gif

Introduction

- System of Forces – Multiple forces treated as a group
  - If no external effect
    - Forces are "in balance"
    - Body is "in equilibrium"
  - Otherwise
    - Forces are "unbalanced"
    - And have a non-zero "resultant"
- Equivalent Force Systems
  - Produce the same external effect
  - Have the same "resultant"

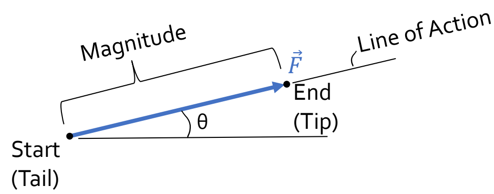


- Resultant
  - The simplest equivalent system to which the original system will reduce
- Reduction
  - The process of reducing a force system to a simpler equivalent system
- Resolution
  - The process of expanding a force or force system into a less simple equivalent system
- Component of a Force
  - One of the two or more forces into which the given force may be resolvedoo

http://samarpanphysioclinic.com/wp-content/uploads/2019/08/Force-System-1024x731.png

ADDITION OF PLANAR FORCES

Scalars vs Vectors



Scalars

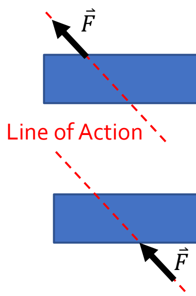
- Completely described with a magnitude (number) only
- Follow the rules of elementary algebra
- Examples
  - Mass
  - Density
  - Length
  - Area

Vectors

- Described by a magnitude (number) **AND** a direction
  - Orientation and sense
- Add according to Parallelogram Law or Triangle Rule
- Examples
  - Force
  - Moment
  - Acceleration
  - Velocity

Vector Types and Operations

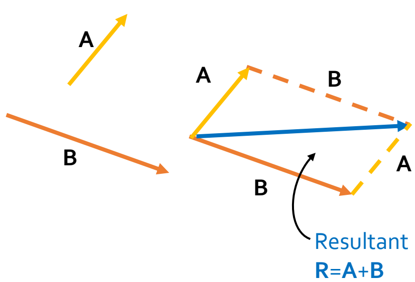
- Principle of Transmissibility
  - A rigid body will remain unchanged if a force acting at a given point is replaced by a force with the same magnitude and direction at another point on the line of action



- Free Vector
  - Has a specific magnitude, slope, and sense **BUT** its line of action does **not** pass through a unique point in space
- Sliding Vector
  - Has a specific magnitude, slope, and sense **AND** its line of action passes through a unique point in space
    - Can be anywhere along its line of action
- Fixed Vector
  - Has a specific magnitude, slope, and sense and its line of action passes through a unique point in space
    - Point of application is confined to a fixed point on its line of action

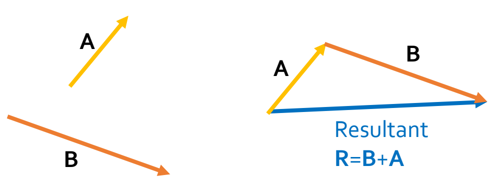
Vector Addition

Parallelogram Law



- Two forces acting on a particle can be replaced by a single force (resultant) by drawing a parallelogram with the two forces as sides and the diagonal is the resultant.

Triangle Rule (Tip-to-Tail)

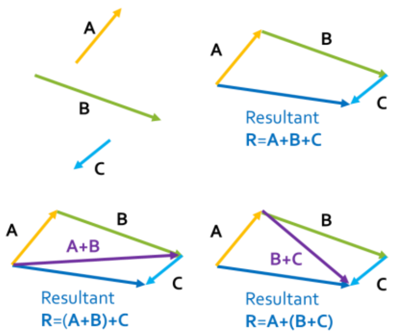


- Two forces acting on a particle can be replaced by a single force (resultant) by creating a triangle. Place the tip of one vector on the tail of the second. Draw a resultant vector from the tail of the first to the tip of the second.

Vector Addition

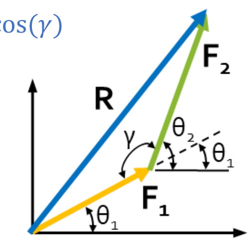
Polygon Rule

- For more than 2 vectors
- Vector addition is associative
  - **P+Q+S=(P+Q)+S=P+(Q+S)**



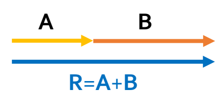
Graphical Method

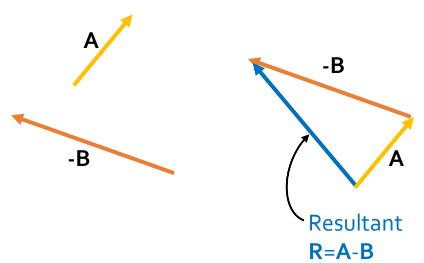
- Follow the Triangle Rule but use Law of Cosine to solve for resultant length.
  - $\gamma+\theta_2-\theta_1=180°$
  - $\gamma=180°-\theta_2+\theta_1$
- Law of Cosine
  - $R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos(\gamma)$

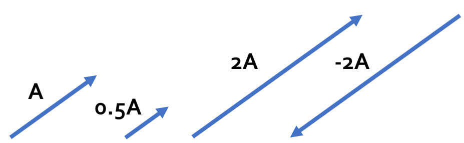


Vector Algebra

- If two or more vectors are collinear (on the same line of action), the resultant is formed by a scalar addition
- For vector subtraction,
  - Simply add the negative vector to the positive one.
  - $\boldsymbol{R}$ = A+(-B)
- Scaling
  - Product of a scalar and a vector

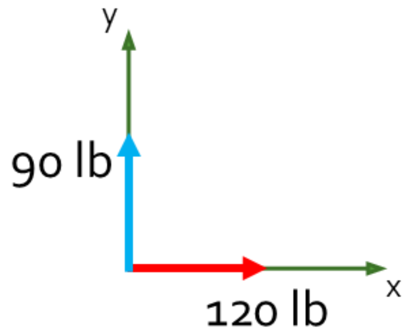






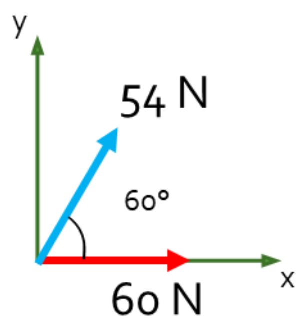
Example 1

- Determine the resultant force using a) the parallelogram law and b) the triangle rule.



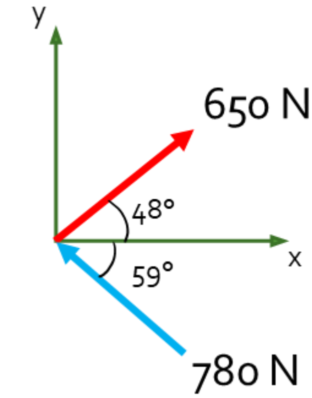
Example 2

- Determine the resultant force using a) the parallelogram law and b) the triangle rule.



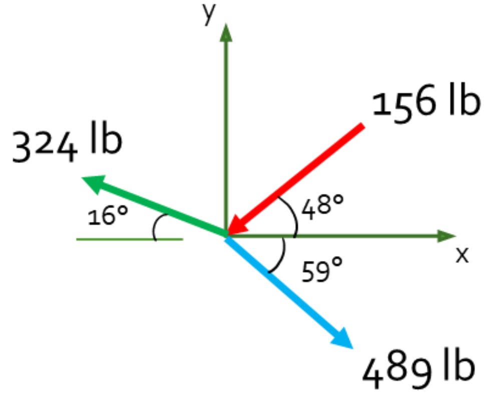
Example 3

- Determine the resultant force using the graphical method.



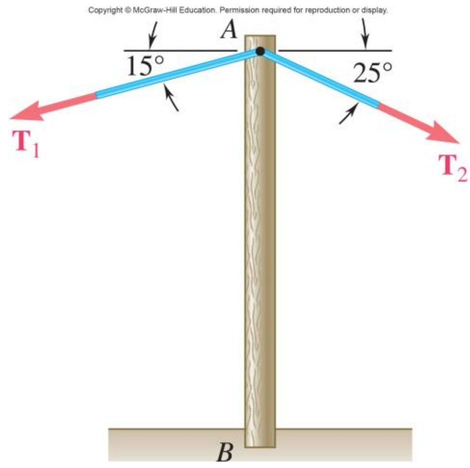
Example 4

- Determine the resultant force using the polygon rule.



Example 5

- A telephone cable is clamped at A to the pole AB. Knowing that the tension in the right-hand portion of the cable is T_2=1000 lb, determine by trigonometry (a) the required tension T_1 in the left-hand portion if the resultant **R** of the forces exerted by the cable at is to be vertical, and (b) the corresponding magnitude of **R**.



ADDING FORCES BY COMPONENTS

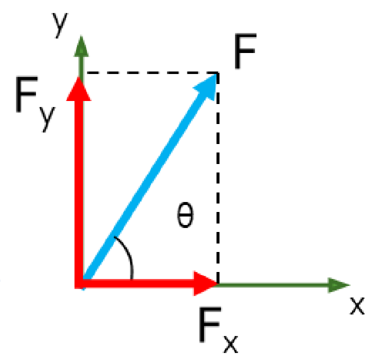
Vector Components

- From the resultant, use basic trigonometry functions to solve for the components along the x- and y-axes

$$F_x = F\cos(\theta)$$
$$F_y = F\sin(\theta)$$

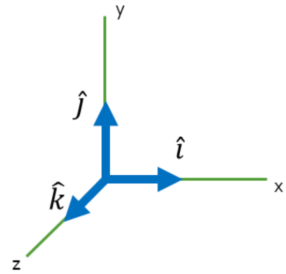
- From components, use Pythagorean Theorem to solve for the resultant and basic trigonometry functions to solve for the angle the resultant makes with the positive x-axis

$$F = \sqrt{F_x^2 + F_y^2} \qquad \tan\theta = \frac{F_y}{F_x}$$



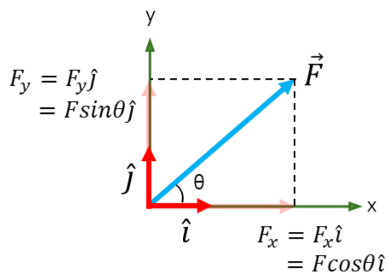
Unit Vectors & Cartesian Vector Form

- Vectors of unit magnitude, directed respectively along a positive axis direction
- Magnitude of unit vectors is always 1



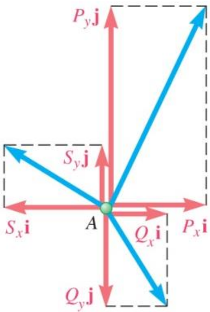
- Use Cartesian Vector Form (or Rectangular) to express the resultant

$$\left.\begin{aligned} F_x &= F\cos(\theta) \\ F_y &= F\sin(\theta) \end{aligned}\right\} \qquad \vec{F} = F_x\hat{\imath} + F_y\hat{\jmath}$$



Adding in Cartesian Vector Form

- When adding in Cartesian Vector Form to solve for a resultant, simply add the corresponding scalar components of the given forces.



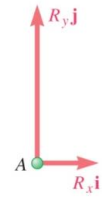
(b)

$$R_x = P_x i + Q_x i + S_x i$$

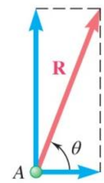
$$R_y = P_y j + Q_y j + S_y j$$

$$R = R_x + R_y$$

$$R = (P_x + Q_x + S_x)i + (P_y + Q_y + S_y)j$$



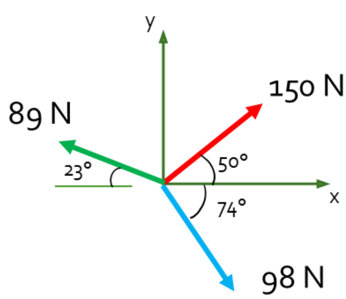
(c)



(d)

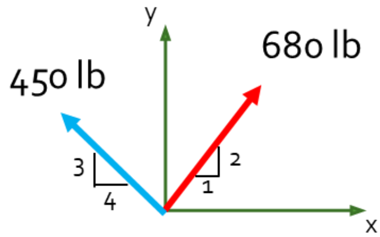
Example 6

- Determine the resultant force using Cartesian Vector form.



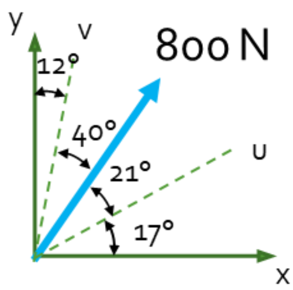
Example 7

- Determine the resultant force using Cartesian Vector form.



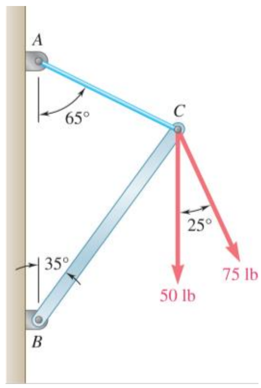
Example 8

- Determine the component forces along the a) x-axis and y-axis and b) the u-axis and v-axis.



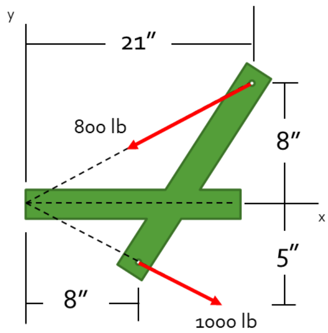
Example 9

- Cable AC has a tension of 40 lb. Determine the resultant of the cable and the two applied forces acting at point C on the short link.



Example 10

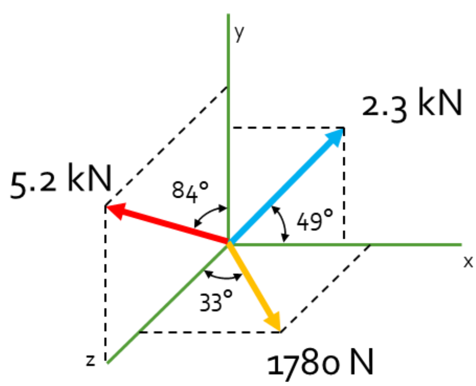
- Determine the resultant of the forces shown and the angle the resultant makes with the x-axis.



CHALLENGE PROBLEM

Example 11

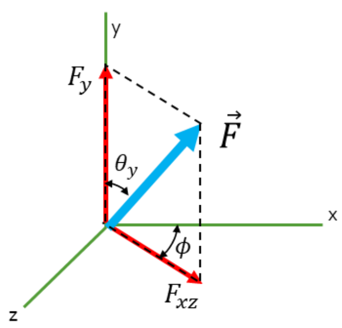
- Determine the resultant vector in Cartesian Vector form for the component forces shown.

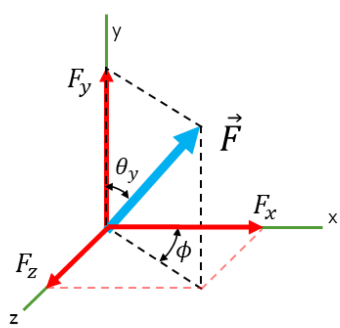


ADDING FORCES IN SPACE

Rectangular Components in Space (3-D)

If angles with some of the axes are given,





Magnitude of a force in space

$$\vec{F} = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F_y = F cos\theta_y$$

$$F_{xz} = F sin\theta_y$$

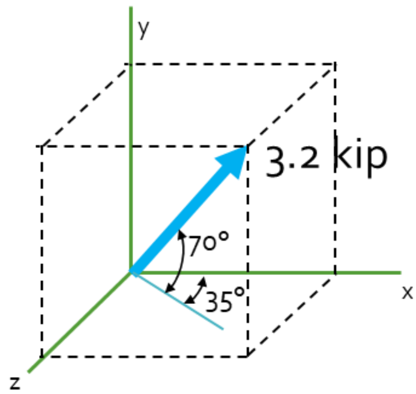
$$F_y = F cos\theta_y$$

$$F_x = F sin\theta_y cos\phi$$

$$F_z = F sin\theta_y sin\phi$$

Example 12

- Determine the component forces for the vector shown and place them in Cartesian Vector form.



Rectangular Components in Space (3-D)

If direction angles are given,

Scalar components of a force F

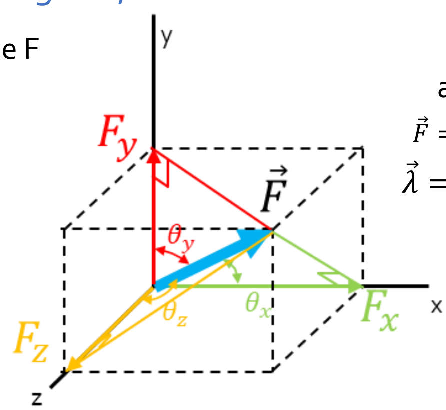
$$F_x = Fcos\theta_x$$

$$F_y = Fcos\theta_y$$

$$F_z = Fcos\theta_z$$

Vector expression of a force F

$$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$$

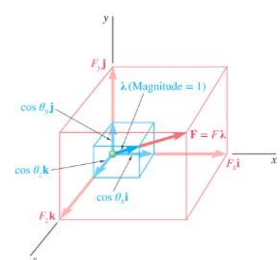


Unit vector along the line of action of **F**

$$\vec{F} = Fcos\theta_x\hat{\imath} + Fcos\theta_y\hat{\jmath} + Fcos\theta_z\hat{k}$$

$$\vec{\lambda} = cos\theta_x\hat{\imath} + cos\theta_y\hat{\jmath} + cos\theta_z\hat{k}$$

$$\vec{F} = F\vec{\lambda}$$



Example 13

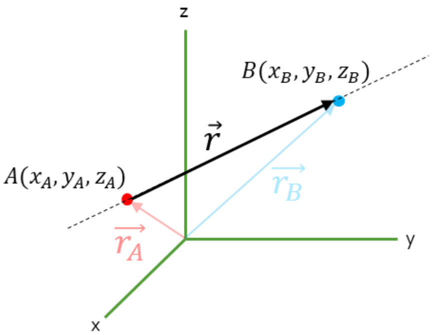
- A force of 500 N forms angles of 60°, 45°, and 120°, respectively, with the x, y, and z axes. Find the components F_x, F_y, and F_z of the force and express the force in terms of unit vectors.

Example 14

- A force has the components F_x=20 lb, F_y=-30 lb, and F_z=60 lb. Determine its magnitude F and the angles θ_x, θ_y, and θ_z it forms with the coordinate axes.

Unit Vector in Any Direction

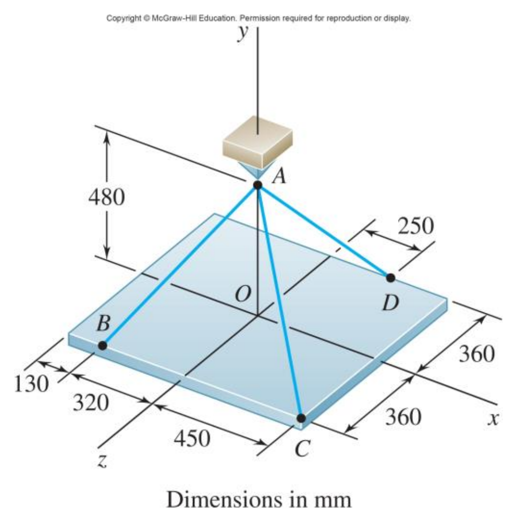
- The positions of A and B are known with respect to the origin of the coordinate system.



$$\vec{r} = \vec{r_B} - \vec{r_A}$$
$$\vec{r} = (x_B - x_A)\hat{\imath} + (y_B - y_A)\hat{\jmath} + (z_B - z_A)\hat{k}$$
$$\vec{r} = d_x\hat{\imath} + d_y\hat{\jmath} + d_z\hat{k}$$
$$|\vec{r}| = \sqrt{d_x^2 + d_y^2 + d_z^2}$$
$$\vec{\lambda} = \frac{\vec{r}}{|\vec{r}|} = \frac{d_x\hat{\imath} + d_y\hat{\jmath} + d_z\hat{k}}{\sqrt{d_x^2 + d_y^2 + d_z^2}} \qquad \vec{F} = F\vec{\lambda}$$

Example 15

- A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AD is 429 N, determine the components of the force exerted on the plate at D.



Adding Concurrent Forces in Space

- When adding concurrent vectors in space, add like components of vectors in Cartesian Vector form.

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$
$$\vec{R} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$

- To determine the resultant, take the square root of the sum of the squares of the components.

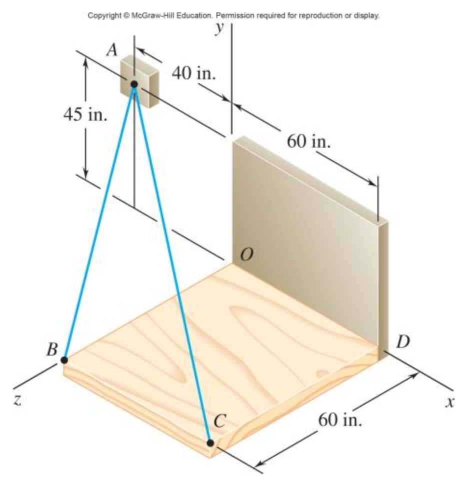
$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

- Use the cosine function to determine the direction angles of the resultant vector.

$$cos\theta_x = \frac{R_x}{R} \qquad cos\theta_y = \frac{R_y}{R} \qquad cos\theta_z = \frac{R_z}{R}$$

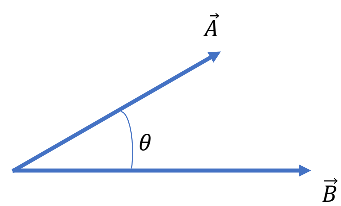
Example 16

- Knowing that the tension is 425 lb in cable AB and 510 lb in AC, determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.



PROJECTION OF A VECTOR

Scalar (Dot) Product



- Use the dot product to find:
  - Angle formed by two given vectors or intersecting lines.
  - The components of a vector parallel and perpendicular to a given axis or line.
    - Also called the projection of a vector
- By definition:

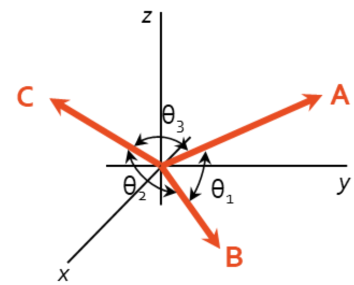
$$\vec{A} \cdot \vec{B} = ABcos\theta = A_x B_x + A_y B_y + A_z B_z$$

- The dot product is commutative and distributive

$$\vec{P} \cdot \vec{Q} = \vec{Q} \cdot \vec{P} \qquad \vec{P} \cdot (\overrightarrow{Q_1} + \overrightarrow{Q_2}) = \vec{P} \cdot \overrightarrow{Q_1} + \vec{P} \cdot \overrightarrow{Q_2}$$

Example 17

- If $\vec{A} = 20\hat{\imath} + 30\hat{\jmath} + 10\hat{k}$, $\vec{B} = 4\hat{\imath} + 6\hat{\jmath} - 5\hat{k}$, and $\vec{C} = 7\hat{\imath} - 15\hat{\jmath} + 12\hat{k}$, calculate the angle between the following vectors:
  - $\vec{A}$ and $\vec{B}$
  - $\vec{C}$ and $\vec{B}$
  - $\vec{A}$ and $\vec{C}$



Projection of a Vector on an Axis

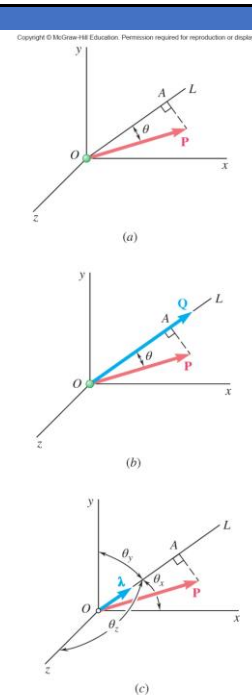
1. Find the unit vector of **P** in the direction of the axis or directed line
2. Calculate the dot product between the vector **P** and unit vector **λ** to find the component of **P** on **OL**.

$$P_{OL} = \vec{P} \cdot \vec{\lambda}_P$$

3. In vector cartesian form, write the vector of **P** along **OL.**

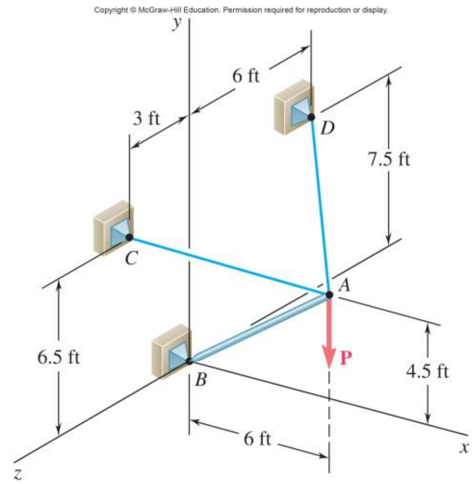
$$\overrightarrow{P_{OL}} = P_{OL}\vec{\lambda}_{OL} = (\vec{P} \cdot \vec{\lambda}_P)\vec{\lambda}_{OL}$$

Magnitude: $(\vec{P} \cdot \vec{\lambda}_P)$ — Direction: $\vec{\lambda}_{OL}$



Example 18

- Knowing that the tension in cable AC is 280 lb, determine:
  - The angle between cable AC and the boom AB
  - The projection on AB of the force exerted by cable AC at point A



EQUILIBRIUM OF A PARTICLE

Equilibrium of a Particle

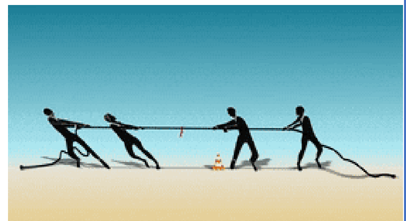
- When the resultant of all the forces acting on a particle is zero, the particle is in equilibrium.

$$\vec{R} = \Sigma\vec{F} = 0$$

- Scalar form for equilibrium of a particle

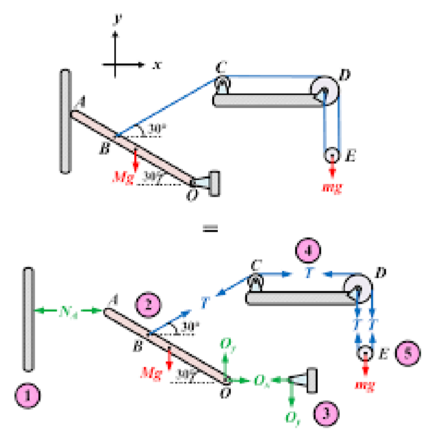
$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$$

- Newton's First Law
  - If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight link (if originally in motion).



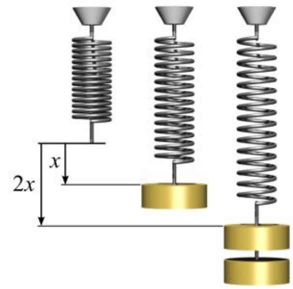
Free-Body Diagrams (FBD)

- Space Diagram
  - A sketch showing the physical conditions of the problem
- Free-Body Diagram
  - A sketch showing a particle (or component) and all the forces acting on it
  - Free from all other bodies in the actual situation



https://encrypted-tbn0.gstatic.com/images?q=tbn:ANd9GcRSozH-Yn2iAUCMyoWTjMgJjNx_lgmuTNLyvA&usqp=CAU

Common Connections for Particles

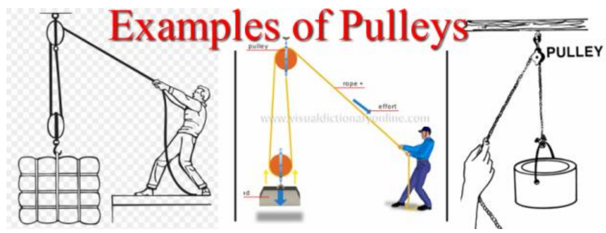


- Springs

$$F = ks$$

  - s is determined from the difference in the spring's deformed length and its undeformed length
  - k is the spring constant or stiffness of the spring

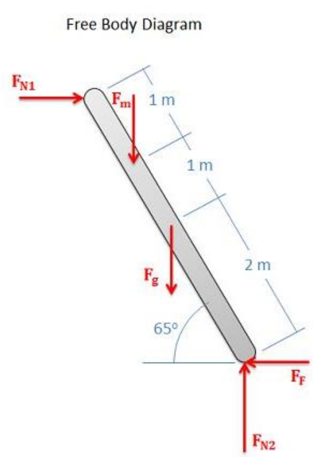
- Cables and Pulleys
  - Assume cables (or cords) have negligible weight and cannot stretch
  - A cable can support only tension which always acts in the direction of the cable.
  - Pulleys are always frictionless



https://upload.wikimedia.org/wikipedia/commons/f/fc/Hookes-law-springs.png

How to Construct a Free-Body Diagram

1. Decide which body (or combination of bodies) is to be isolated and analyzed (shown in the free-body diagram)
2. Draw a figure of the particle(s) isolated from its "environment"
3. Replace al physical contacts between the particle and the environment with forces (of assumed direction)
   - Contact forces occur from bodies, connections, friction in physical contact with the particle
   - Body forces is a force that acts throughout the volume of a body
     - i.e. the earth-pull on (or weight of) a body
4. Choose the set of coordinate axes to be used in solving the problem and indicate their directions on the free-body diagram.
   - Place any dimensions required for solution of the problem on the diagram.



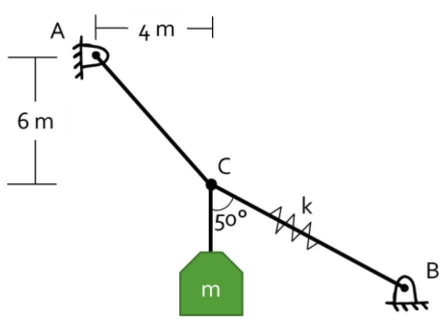
https://qph.cf2.quoracdn.net/main-qimg-c3cb612f1d758be783fdde67e9f2d783-pjlq

Tips for Drawing a Correct FBD

- **Always** establish the x, y, and z axes
- **Always** draw your FBD first
- Label **ALL** known and unknown force magnitudes and directions on the diagram
- Label **ALL** dimensions
- **Calculate** important angles
- The sense of a force having an unknown magnitude can be assumed
  - If the solution is negative, then the assumed direction is incorrect (opposite of what you assumed)

Example 19

- A cable and spring are tied together at C and are loaded with a mass of 120 kg. Determine the tension in cable AC and the change in length (in mm) of the spring if the spring stiffness, k, is 1.35 kN/mm and the system is in equilibrium.



Example 20

- The pulley system is used to keep a 30-lb weight suspended. If θ=10°, what is the force, F, required to keep the system in equilibrium?

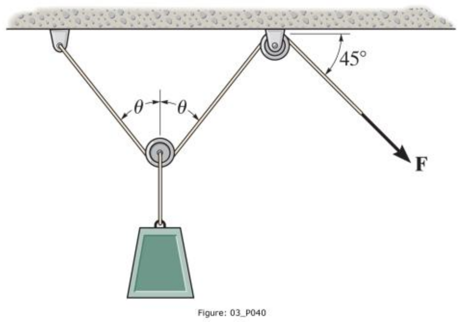
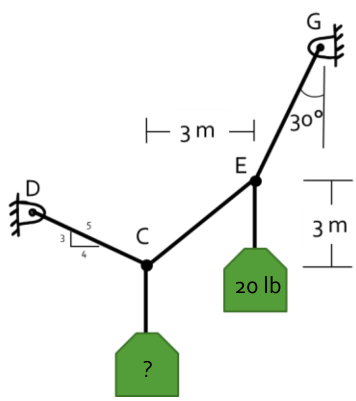


Figure: 03_P040

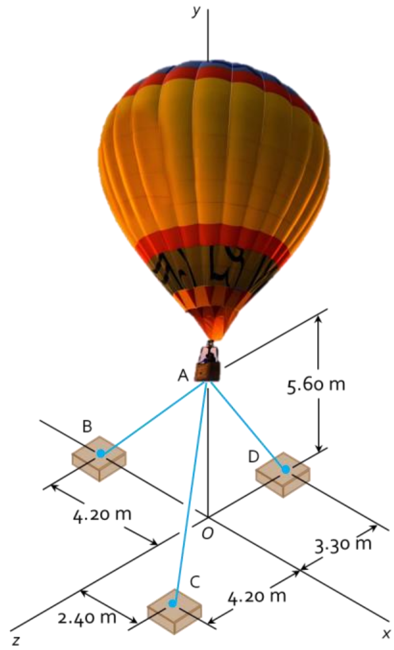
Example 21

- Determine the weight, in lb, attached at C for the system to be in static equilibrium.



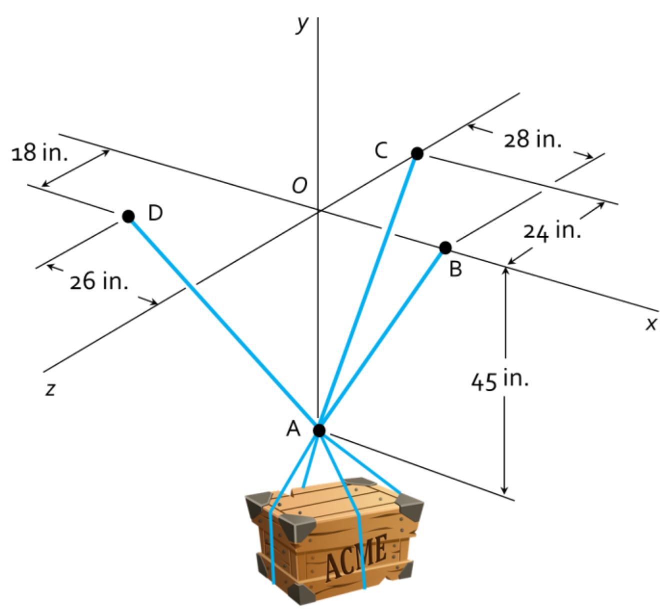
Example 22

- Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800 N vertical force at A, determine the tension in each wire.



Example 23

- A crate is supported by three cables as shown. Determine the weight, W, of the crate knowing that the tension in cable AD is 924 lb.



https://gamepedia.cursecdn.com/worldofmayhem_gamepedia_en/5/51/Free_Crate.png