

CHAPTER 2

STATICS OF PARTICLES

Fundamental Concepts

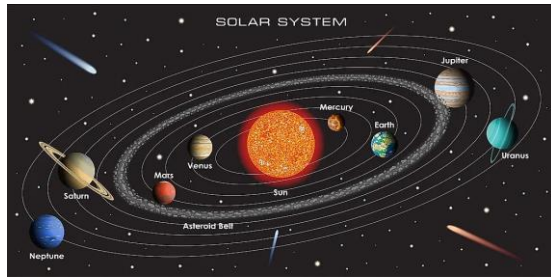
- **Body**: substance that occupies space and has a defined boundary
- **Space**: the geometric region in which the physical events of interest in mechanics occur
- **Scalar**: a quantity represented by magnitude alone
 - i.e. mass, temperature, pressure, time, length
- **Vector**: a quantity represented by both magnitude and direction
 - i.e. \vec{v} elocity, \vec{a} cceleration, \vec{f} orce, displacement (\vec{x})

Particle

- A body with negligible dimensions but not necessarily small with respect to you
- Earth is a particle when studying its motion around the sun



https://upload.wikimedia.org/wikipedia/commons/5/5f/Skycam_Husky_Stadium.jpg



<https://www.insightsonindia.com/wp-content/uploads/2022/09/Our-Solar-System.jpg>

Rigid Body

- A large number of particles occupying fixed positions with respect to one another
- A body which does not deform
- Nothing is completely rigid, but assuming so is continually used in the applications of mechanics



<http://www.internationalcranes.com/images/160xany/20220412-094711-photo-8.jpg>



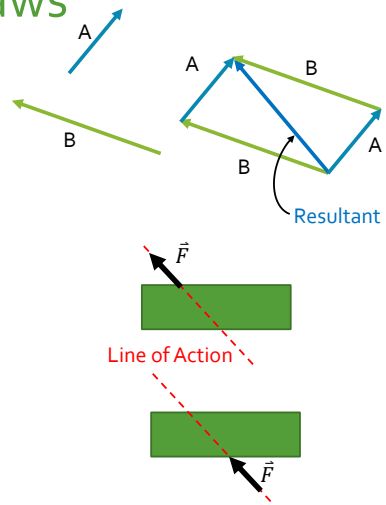
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Important Principles and Laws

- Parallelogram Law for the Addition of Vectors
 - Two forces acting on a particle can be replaced by a single force (resultant) by drawing a parallelogram with the two forces as sides and the diagonal is the resultant.
- Principle of Transmissibility
 - A rigid body will remain unchanged if a force acting at a given point is replaced by a force with the same magnitude and direction at another point on the line of action



Important Principles and Laws

- Newton's Law of Gravitation
 - Two particles are attracted with equal and opposite forces

$$F = G \frac{m_1 m_2}{r^2}$$

- m_1 and m_2 are masses of two particles attracted to each other
 - G is the constant of gravitation of $6.673 \times 10^{-11} \text{ m}^3/\text{kg s}^2$
 - r is the distance between two particles
- What happens when one particle is Earth and the second particle is an object on Earth?

$$F = G \frac{m_1 m_2}{r^2} \rightarrow W = G \frac{m_e m}{r_e^2} = mg$$

g is Earth's gravitational constant
 9.81 m/s^2 or 32.2 ft/s^2

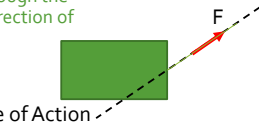
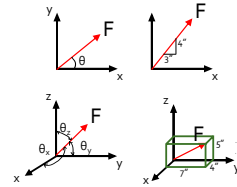
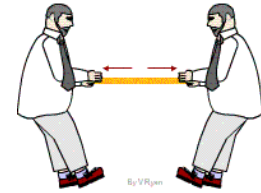


<https://mediao.giphy.com/media/l1Jz25myhtdybtaU/giphy.gif>

Introduction

- Force: The action of one body on another body
- Results from
 - Direct physical contact*
 - Gravitational*, electrical, magnetic effects
- Effects on body
 - External*
 - Change its motion
 - Develop resisting forces (reactions)
 - Internal
 - Deform (change shape)

- Must have the following
 - Magnitude – positive numerical value
 - Amount or size
 - Length
 - Direction – slope and sense
 - Planar – 2D
 - Spatial – 3D
 - Point of application
 - Point of contact
 - Line of Action*
 - A straight line extending through the point of application in the direction of the force



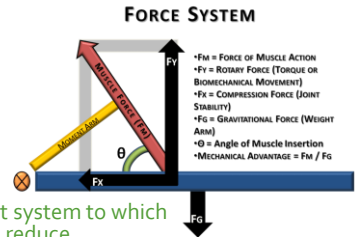
* of primary concern in Statics

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Introduction

- System of Forces – Multiple forces treated as a group
 - If no external effect
 - Forces are “in balance”
 - Body is “in equilibrium”
 - Otherwise
 - Forces are “unbalanced”
 - And have a non-zero “resultant”
- Equivalent Force Systems
 - Produce the same external effect
 - Have the same “resultant”

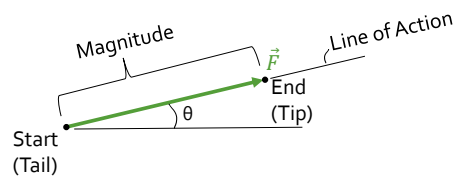
- Resultant
 - The simplest equivalent system to which the original system will reduce
- Reduction
 - The process of reducing a force system to a simpler equivalent system
- Resolution
 - The process of expanding a force or force system into a less simple equivalent system
- Component of a Force
 - One of the two or more forces into which the given force may be resolved



<http://samarpanphysioclinic.com/wp-content/uploads/2019/08/Force-System-1024x735.png>

ADDITION OF PLANAR FORCES

Scalars vs Vectors



Scalars

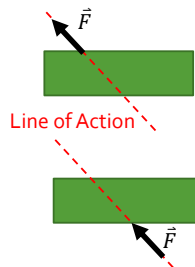
- Completely described with a magnitude (number) only
- Follow the rules of elementary algebra
- Examples
 - Mass
 - Density
 - Length
 - Area

Vectors

- Described by a magnitude (number) **AND** a direction
 - Orientation and sense
- Add according to Parallelogram Law or Triangle Rule
- Examples
 - Force
 - Moment
 - Acceleration
 - Velocity

Vector Types and Operations

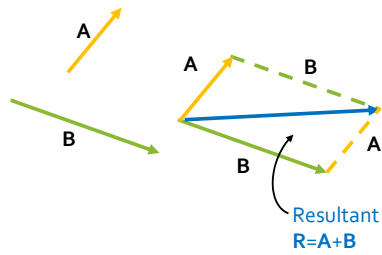
- Principle of Transmissibility
 - A rigid body will remain unchanged if a force acting at a given point is replaced by a force with the same magnitude and direction at another point on the line of action



- Free Vector
 - Has a specific magnitude, slope, and sense **BUT** its line of action does **not** pass through a unique point in space
- Sliding Vector
 - Has a specific magnitude, slope, and sense **AND** its line of action passes through a unique point in space
 - Can be anywhere along its line of action
- Fixed Vector
 - Has a specific magnitude, slope, and sense and its line of action passes through a unique point in space
 - Point of application is confined to a fixed point on its line of action

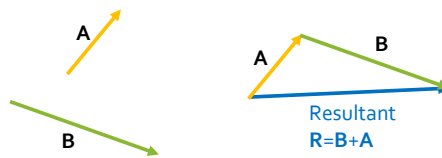
Vector Addition

Parallelogram Law



- Two forces acting on a particle can be replaced by a single force (resultant) by drawing a parallelogram with the two forces as sides and the diagonal is the resultant.

Triangle Rule (Tip-to-Tail)

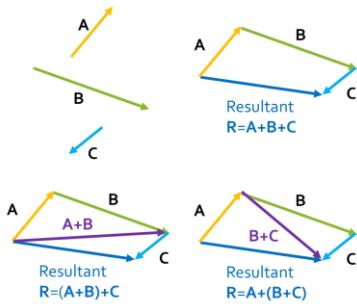


- Two forces acting on a particle can be replaced by a single force (resultant) by creating a triangle. Place the tip of one vector on the tail of the second. Draw a resultant vector from the tail of the first to the tip of the second.

Vector Addition

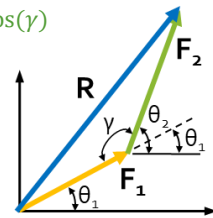
Polygon Rule

- For more than 2 vectors
- Vector addition is associative
 - $P+Q+S=(P+Q)+S=P+(Q+S)$



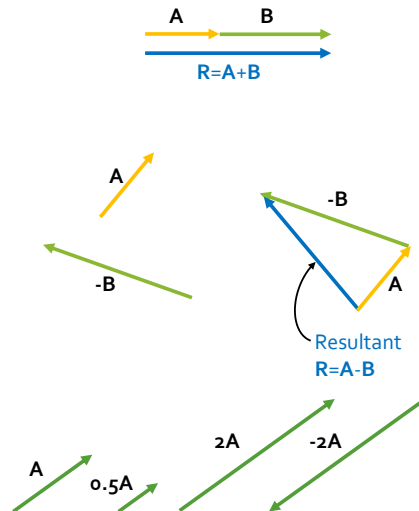
Graphical Method

- Follow the Triangle Rule but use Law of Cosine to solve for resultant length.
 - $\gamma + \theta_2 - \theta_1 = 180^\circ$
 - $\gamma = 180^\circ - \theta_2 + \theta_1$
- Law of Cosine
 - $R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos(\gamma)$



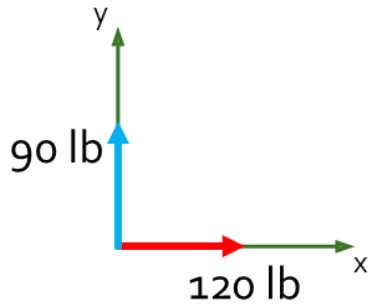
Vector Algebra

- If two or more vectors are collinear (on the same line of action), the resultant is formed by a scalar addition
- For vector subtraction,
 - Simply add the negative vector to the positive one.
 - $R = A + (-B)$
- Scaling
 - Product of a scalar and a vector



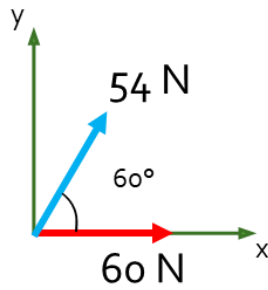
Example 1

- Determine the resultant force using a) the parallelogram law and b) the triangle rule.



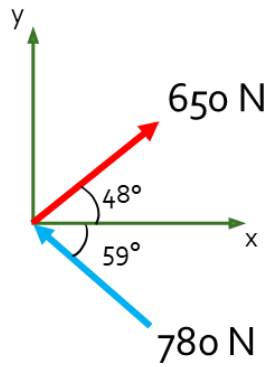
Example 2

- Determine the resultant force using a) the parallelogram law and b) the triangle rule.



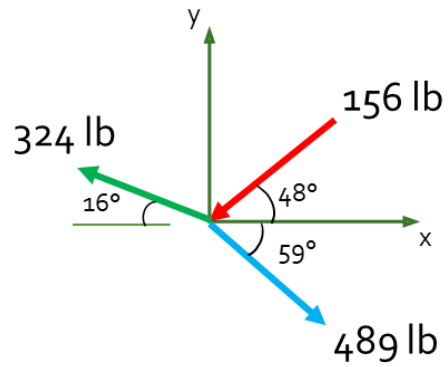
Example 3

- Determine the resultant force using the graphical method.



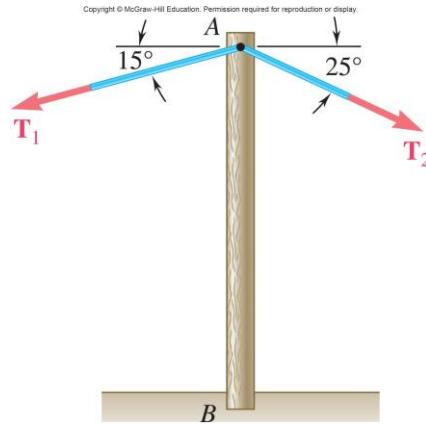
Example 4

- Determine the resultant force using the polygon rule.



Example 5

- A telephone cable is clamped at A to the pole AB. Knowing that the tension in the right-hand portion of the cable is $T_2=1000$ lb, determine by trigonometry (a) the required tension T_1 in the left-hand portion if the resultant \mathbf{R} of the forces exerted by the cable at A is to be vertical, and (b) the corresponding magnitude of \mathbf{R} .



ADDING FORCES BY COMPONENTS

Vector Components

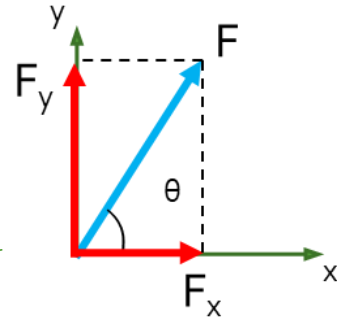
- From the resultant, use basic trigonometry functions to solve for the components along the x- and y-axes

$$F_x = F\cos(\theta)$$

$$F_y = F\sin(\theta)$$

- From components, use Pythagorean Theorem to solve for the resultant and basic trigonometry functions to solve for the angle the resultant makes with the positive x-axis

$$F = \sqrt{F_x^2 + F_y^2} \quad \tan\theta = \frac{F_y}{F_x}$$

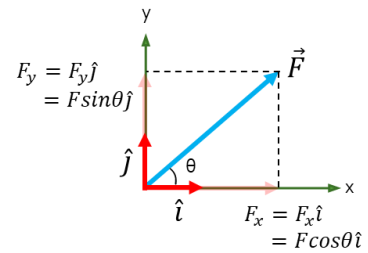
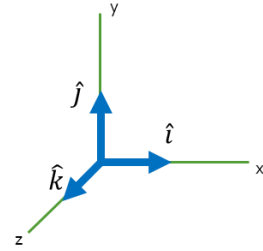


Unit Vectors & Cartesian Vector Form

- Vectors of unit magnitude, directed respectively along a positive axis direction
- Magnitude of unit vectors is always 1

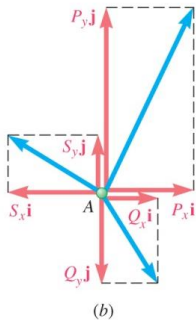
- Use Cartesian Vector Form (or Rectangular) to express the resultant

$$\left. \begin{array}{l} F_x = F \cos(\theta) \\ F_y = F \sin(\theta) \end{array} \right\} \vec{F} = F_x \hat{i} + F_y \hat{j}$$



Adding in Cartesian Vector Form

- When adding in Cartesian Vector Form to solve for a resultant, simply add the corresponding scalar components of the given forces.

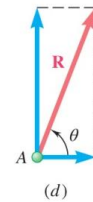
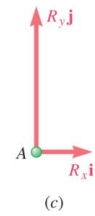


$$R_x = P_x i + Q_x i + S_x i$$

$$R_y = P_y j + Q_y j + S_y j$$

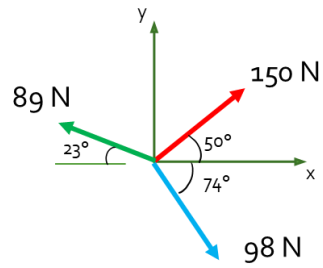
$$R = R_x + R_y$$

$$R = (P_x + Q_x + S_x)i + (P_y + Q_y + S_y)j$$



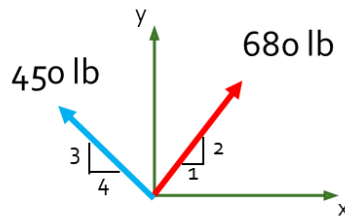
Example 6

- Determine the resultant force using Cartesian Vector form.



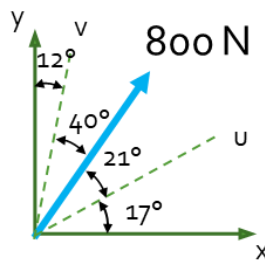
Example 7

- Determine the resultant force using Cartesian Vector form.



Example 8

- Determine the component forces along the a) x-axis and y-axis and b) the u-axis and v-axis.



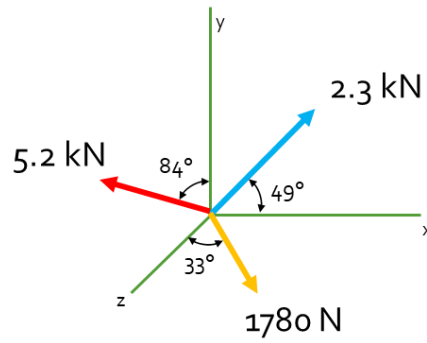
CHALLENGE PROBLEMS



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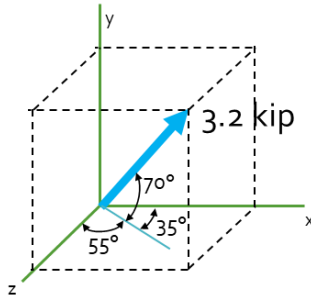
Example 9

- Determine the resultant vector in Cartesian Vector form for the component forces shown.



Example 10

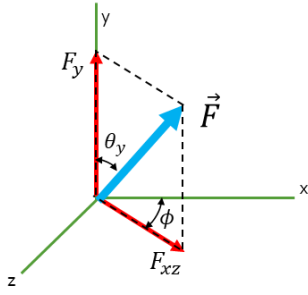
- Determine the component forces for the vector shown and place them in Cartesian Vector form.



ADDING FORCES IN SPACE

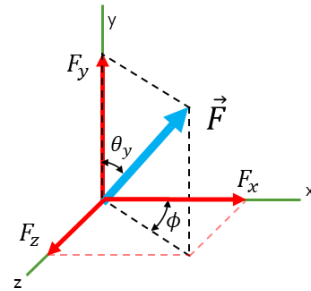
Rectangular Components in Space (3-D)

If angles with some of the axes are given,



$$F_y = F \cos \theta_y$$

$$F_{xz} = F \sin \theta_y$$



$$F_y = F \cos \theta_y$$

$$F_x = F \sin \theta_y \cos \phi$$

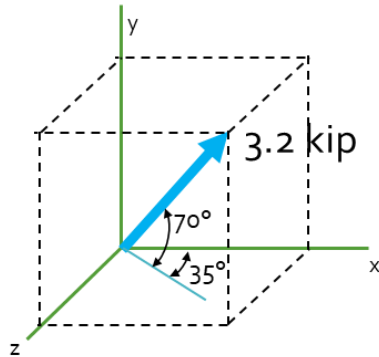
$$F_z = F \sin \theta_y \sin \phi$$

Magnitude of a
force in space

$$\vec{F} = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

Example 10 - Again

- Determine the component forces for the vector shown and place them in Cartesian Vector form.



Rectangular Components in Space (3-D)

If direction angles are given,

Scalar components of a force F

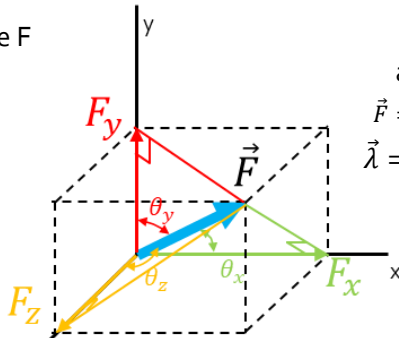
$$F_x = F \cos \theta_x$$

$$F_y = F \cos \theta_y$$

$$F_z = F \cos \theta_z$$

Vector expression
of a force F

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$



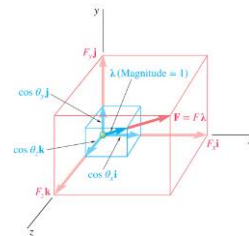
Unit vector

along the line of action of F

$$\vec{F} = F \cos \theta_x \hat{i} + F \cos \theta_y \hat{j} + F \cos \theta_z \hat{k}$$

$$\vec{\lambda} = \cos \theta_x \hat{i} + \cos \theta_y \hat{j} + \cos \theta_z \hat{k}$$

$$\vec{F} = F \vec{\lambda}$$



Example 11

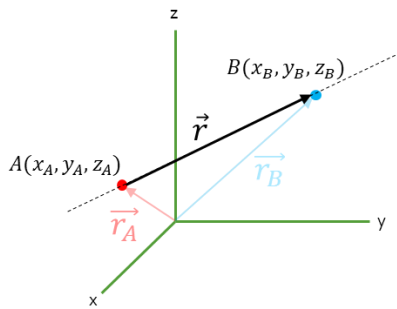
- A force of 500 N forms angles of 60° , 45° , and 120° , respectively, with the x , y , and z axes. Find the components F_x , F_y , and F_z of the force and express the force in terms of unit vectors.

Example 12

- A force has the components $F_x=20$ lb, $F_y=-30$ lb, and $F_z=60$ lb. Determine its magnitude F and the angles θ_x , θ_y , and θ_z it forms with the coordinate axes.

Unit Vector in Any Direction

- The positions of A and B are known with respect to the origin of the coordinate system.



$$\vec{r} = \vec{r}_B - \vec{r}_A$$

$$\vec{r} = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$$

$$\vec{r} = d_x\hat{i} + d_y\hat{j} + d_z\hat{k}$$

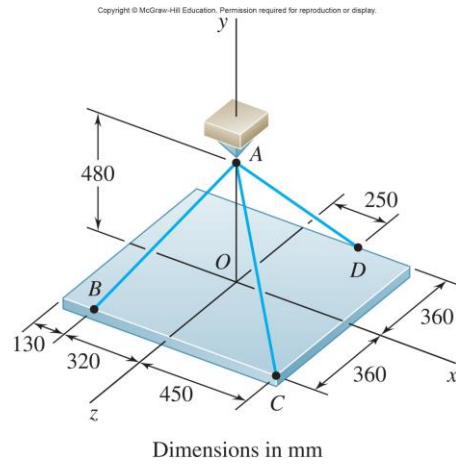
$$|\vec{r}| = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

$$\vec{\lambda} = \frac{\vec{r}}{|\vec{r}|} = \frac{d_x\hat{i} + d_y\hat{j} + d_z\hat{k}}{\sqrt{d_x^2 + d_y^2 + d_z^2}}$$

$$\vec{F} = F\vec{\lambda}$$

Example 13

- A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AD is 429 N, determine the components of the force exerted on the plate at D.



Adding Concurrent Forces in Space

- When adding concurrent vectors in space, add like components of vectors in Cartesian Vector form.

$$\left. \begin{aligned} \vec{A} &= A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \\ \vec{B} &= B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \end{aligned} \right\} \vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

- To determine the resultant, take the square root of the sum of the squares of the components.

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

- Use the cosine function to determine the direction angles of the resultant vector.

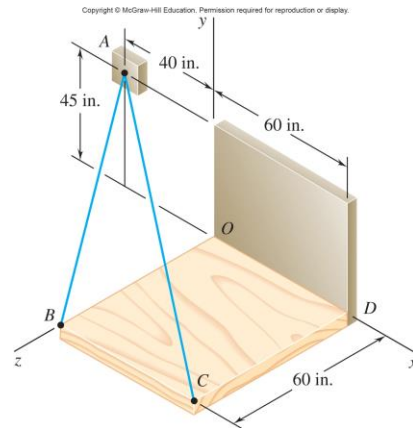
$$\cos\theta_x = \frac{R_x}{R}$$

$$\cos\theta_y = \frac{R_y}{R}$$

$$\cos\theta_z = \frac{R_z}{R}$$

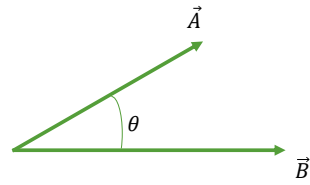
Example 14

- Knowing that the tension is 425 lb in cable AB and 510 lb in AC, determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.



PROJECTION OF A VECTOR

Scalar (Dot) Product



- Use the dot product to find:
 - Angle formed by two given vectors or intersecting lines.
 - The components of a vector parallel and perpendicular to a given axis or line.
 - Also called the projection of a vector
- By definition:

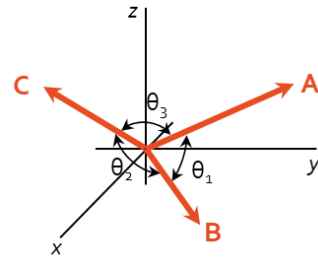
$$\vec{A} \cdot \vec{B} = AB \cos \theta = A_x B_x + A_y B_y + A_z B_z$$

- The dot product is commutative and distributive

$$\vec{P} \cdot \vec{Q} = \vec{Q} \cdot \vec{P} \qquad \vec{P} \cdot (\vec{Q}_1 + \vec{Q}_2) = \vec{P} \cdot \vec{Q}_1 + \vec{P} \cdot \vec{Q}_2$$

Example 15

- If $\vec{A} = 20\hat{i} + 30\hat{j} + 10\hat{k}$, $\vec{B} = 4\hat{i} + 6\hat{j} - 5\hat{k}$, and $\vec{C} = 7\hat{i} - 15\hat{j} + 12\hat{k}$, calculate the angle between the following vectors:
 - \vec{A} and \vec{B}
 - \vec{C} and \vec{B}
 - \vec{A} and \vec{C}



Projection of a Vector on an Axis

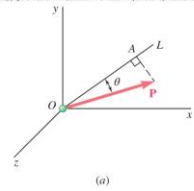
1. Find the unit vector of **P** in the direction of the axis or directed line
2. Calculate the dot product between the vector **P** and unit vector λ to find the component of **P** on **OL**.

$$P_{OL} = \vec{P} \cdot \vec{\lambda}_P$$

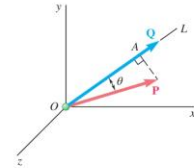
3. In vector cartesian form, write the vector of **P** along **OL**.

$$\vec{P}_{OL} = P_{OL} \vec{\lambda}_{OL} = \underbrace{(\vec{P} \cdot \vec{\lambda}_P)}_{\text{Magnitude}} \vec{\lambda}_{OL} \quad \text{Direction}$$

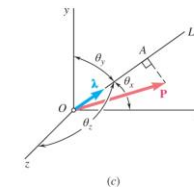
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(a)



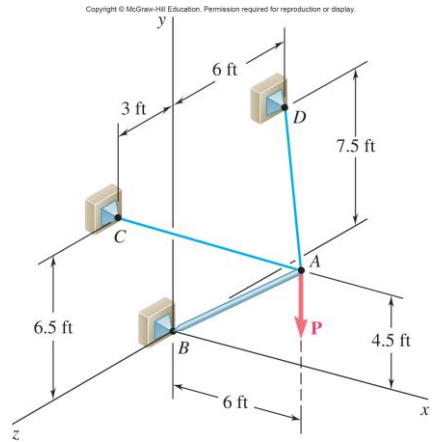
(b)



(c)

Example 16

- Knowing that the tension in cable AC is 280 lb, determine:
 - The angle between cable AC and the boom AB
 - The projection on AB of the force exerted by cable AC at point A



EQUILIBRIUM OF A PARTICLE

Equilibrium of a Particle

- When the resultant of all the forces acting on a particle is zero, the particle is in equilibrium.

$$\vec{R} = \Sigma \vec{F} = 0$$

- Scalar form for equilibrium of a particle

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma F_z = 0$$

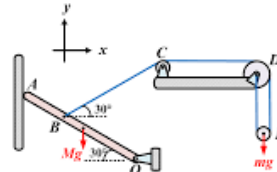
- Newton's First Law

- If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

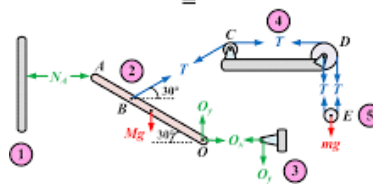


Free-Body Diagrams (FBD)

- Space Diagram
 - A sketch showing the physical conditions of the problem



- Free-Body Diagram
 - A sketch showing a particle (or component) and all the forces acting on it
 - Free from all other bodies in the actual situation



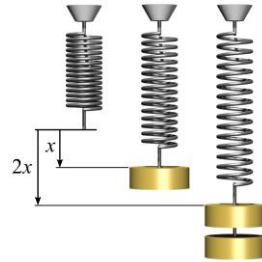
https://encrypted-tbn0.gstatic.com/images?q=tbn:ANdYGR5ozH-YnzIAUCMyoWTMqJNix_IgmuTNLyyA&usqp=CAU

Common Connections for Particles

- Springs

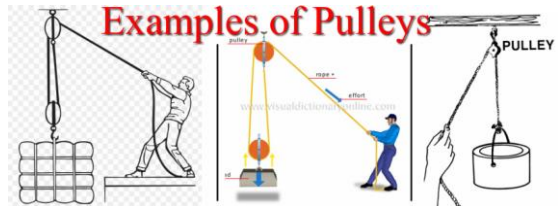
$$F = ks$$

- s is determined from the difference in the spring's deformed length and its undeformed length



- Cables and Pulleys

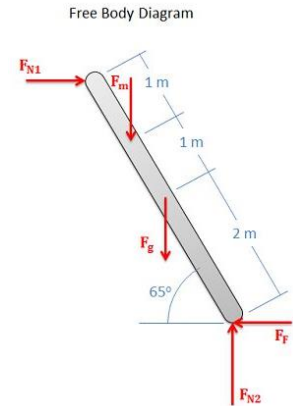
- Assume cables (or cords) have negligible weight and cannot stretch
- A cable can support only tension which always acts in the direction of the cable.



<https://upload.wikimedia.org/wikipedia/commons/ff/cf/Hookes-law-springs.png>

How to Construct a Free-Body Diagram

1. Decide which body (or combination of bodies) is to be isolated and analyzed (shown in the free-body diagram)
2. Draw a figure of the particle(s) isolated from its "environment"
3. Replace all physical contacts between the particle and the environment with forces (of assumed direction)
 - Contact forces occur from bodies, connections, friction in physical contact with the particle
 - Body forces is a force that acts throughout the volume of a body
 - i.e. the earth-pull on (or weight of) a body
4. Choose the set of coordinate axes to be used in solving the problem and indicate their directions on the free-body diagram.
 - Place any dimensions required for solution of the problem on the diagram.



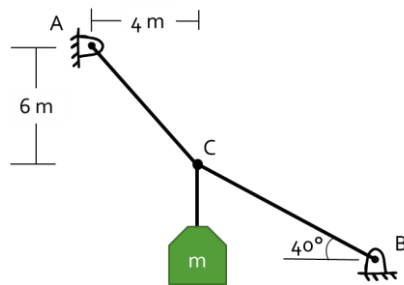
<https://qph.cf2.quoracdn.net/main-qimg-c3cb61215d758ba783fdde67e9fd783-pj1>

Tips for Drawing a Correct FBD

- **Always** establish the x , y , and z axes
- **Always** draw your FBD first
- Label **ALL** known and unknown force magnitudes and directions on the diagram
- Label **ALL** dimensions
- **Calculate** important angles
- The sense of a force having an unknown magnitude can be assumed
 - If the solution is negative, then the assumed direction is incorrect (opposite of what you assumed)

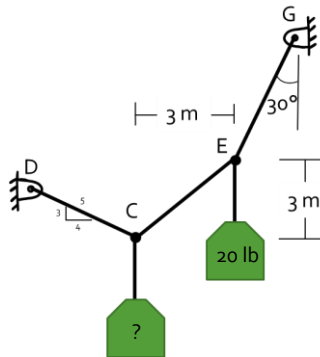
Example 17

- Two cables are tied together at C and are loaded with a mass of 120 kg. Determine the tension in cable AC and cable BC.



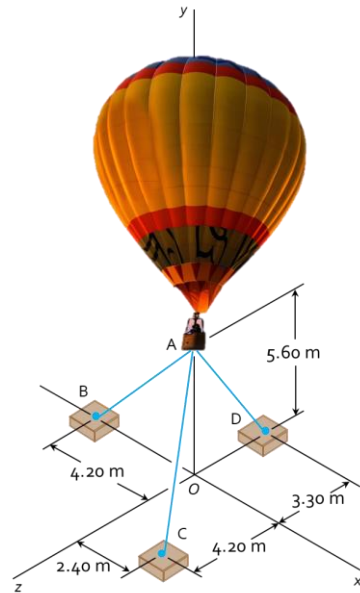
Example 18

- Determine the weight, in lb, attached at C for the system to be in static equilibrium.



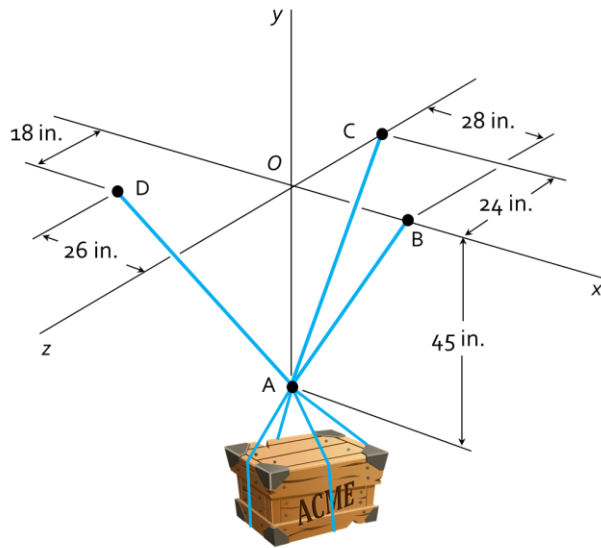
Example 19

- Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800 N vertical force at A, determine the tension in each wire.



Example 20

- A crate is supported by three cables as shown. Determine the weight, W , of the crate knowing that the tension in cable AD is 924 lb.



https://gamepedia.cursecdn.com/worldofmayhem_gamepedia_en/5/5u/Free_Crate.png