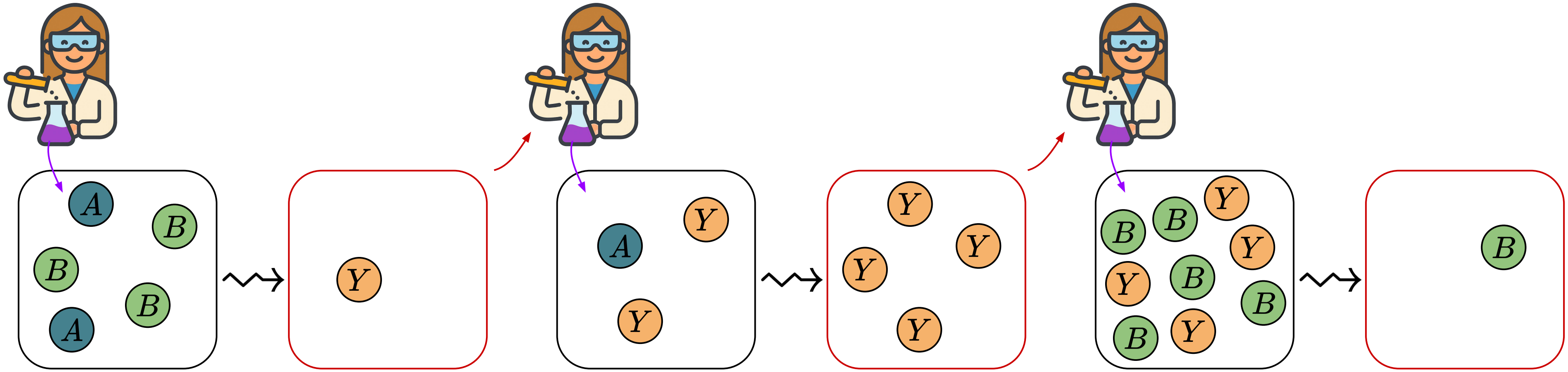
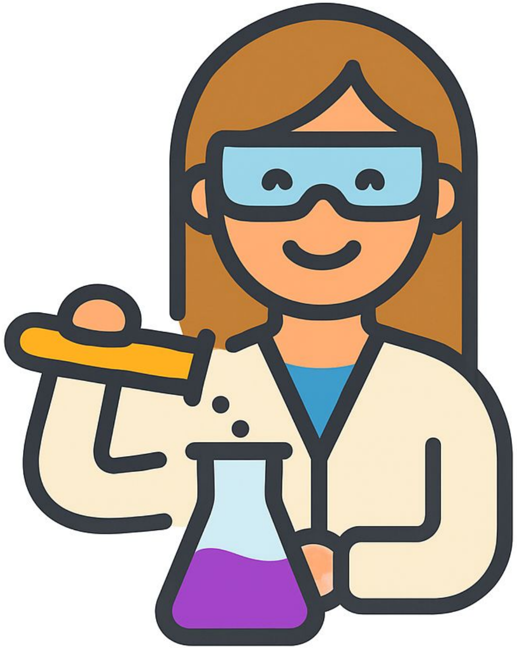
# Polynomial Simulation of CRN Models with Trimolecular Void Step-Cycle CRNs

The 22nd International Conference on Unconventional Computation and Natural Computation
Sept 1-5, 2025 | Nice, France




**Austin Luchsinger**, Aiden Massie, Robert Schweller, Evan Tomai, Tim Wylie

# (Stochastic) Chemical Reaction Networks



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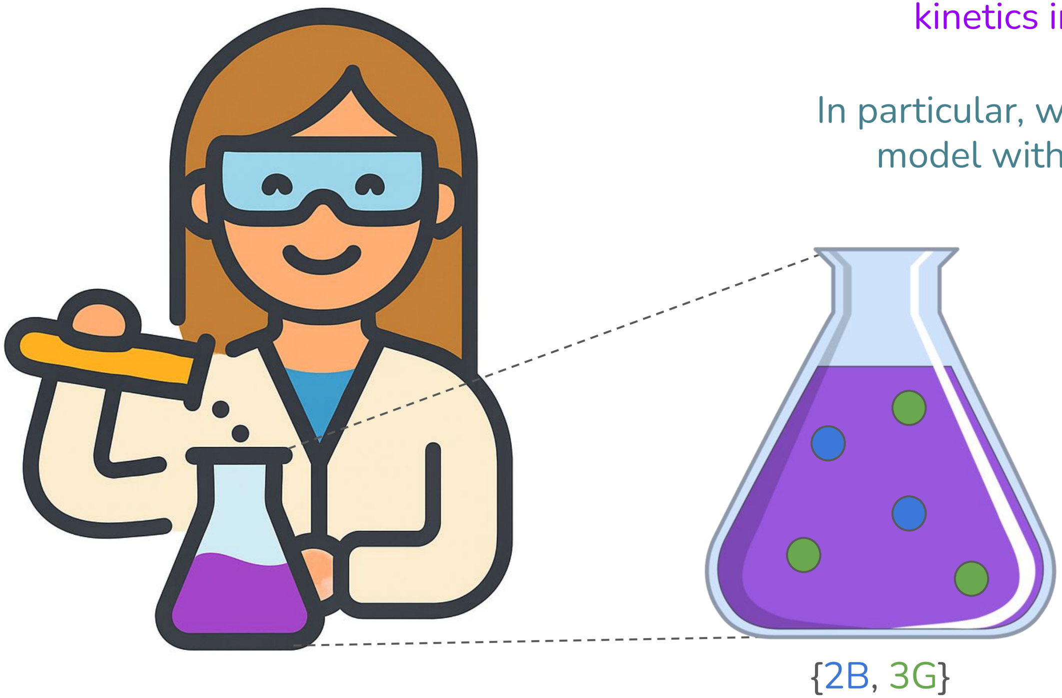

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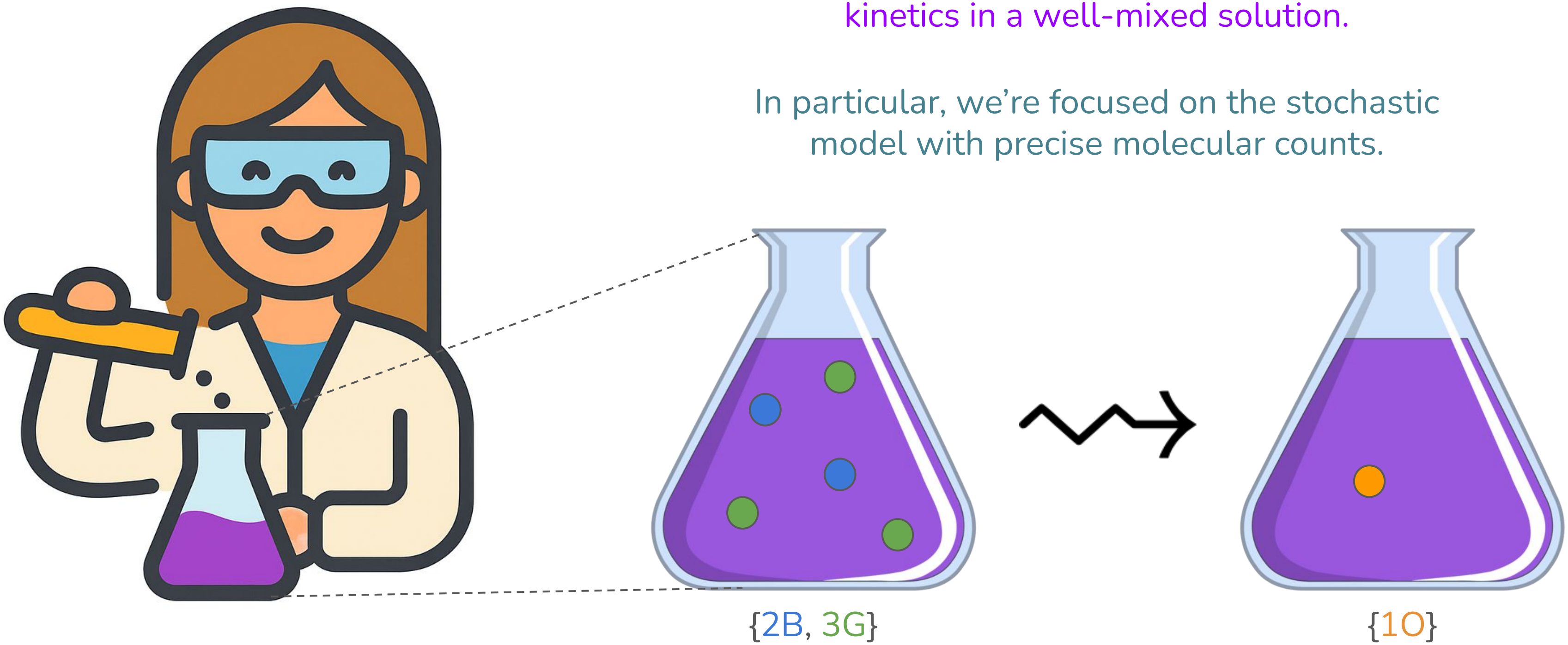

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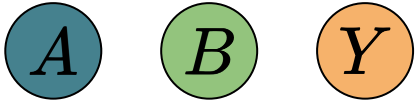

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<u>Species</u>

$A \quad B \quad Y$

<u>Reactions</u>

$$A \rightarrow Y + Y$$

$$B + Y \rightarrow \emptyset$$

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Species

$A$ $B$ $Y$

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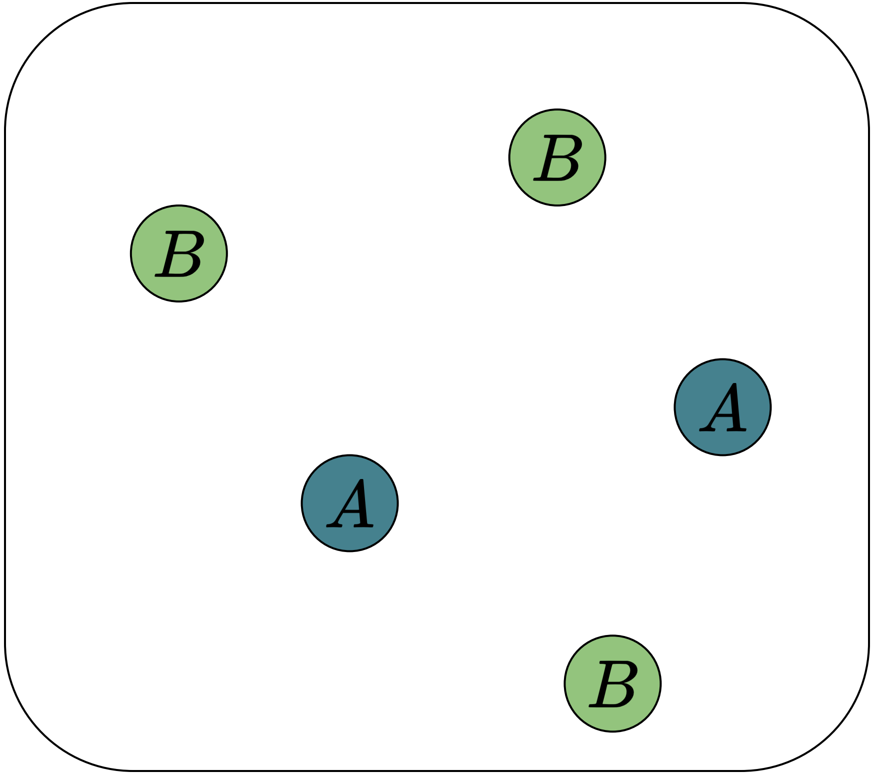
reactants

products

# CRN Configurations

$$(a, b, y) \in \mathbb{Z}_{\geq 0}^{\Lambda}$$

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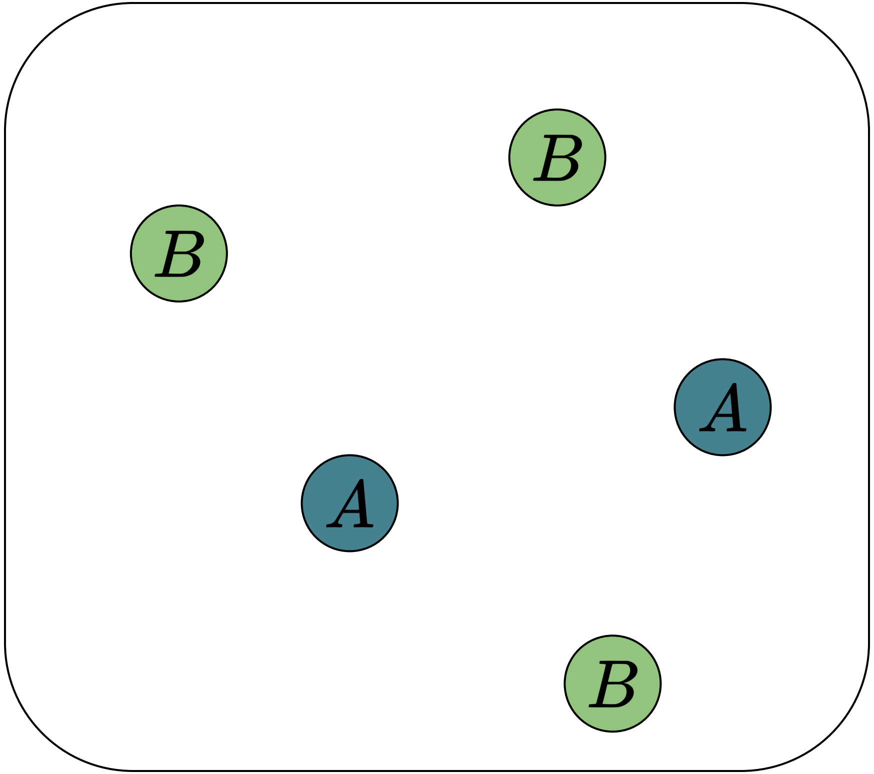


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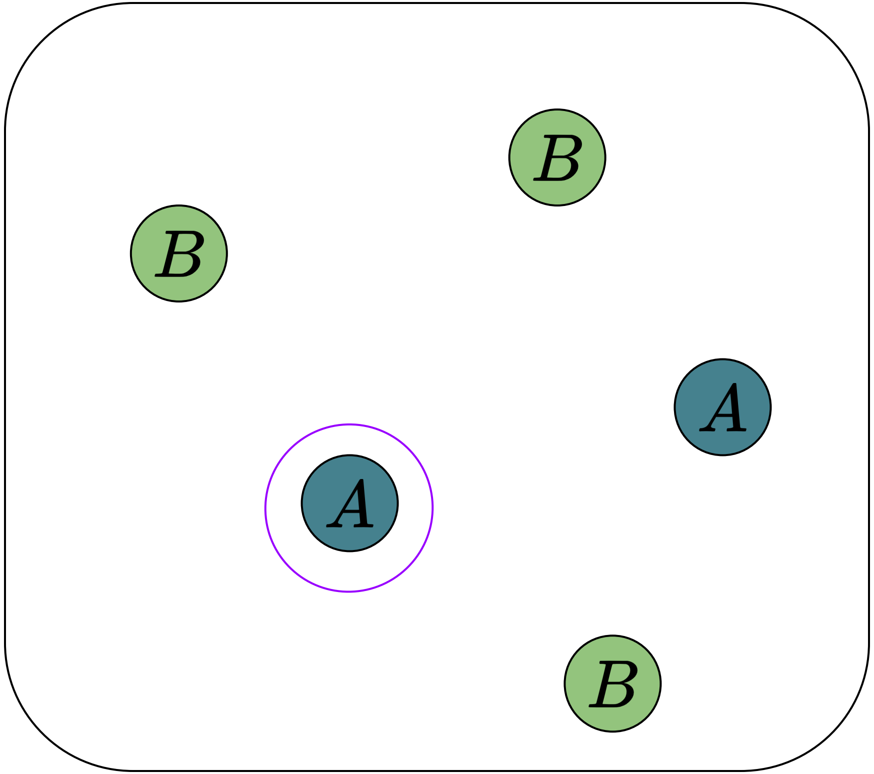


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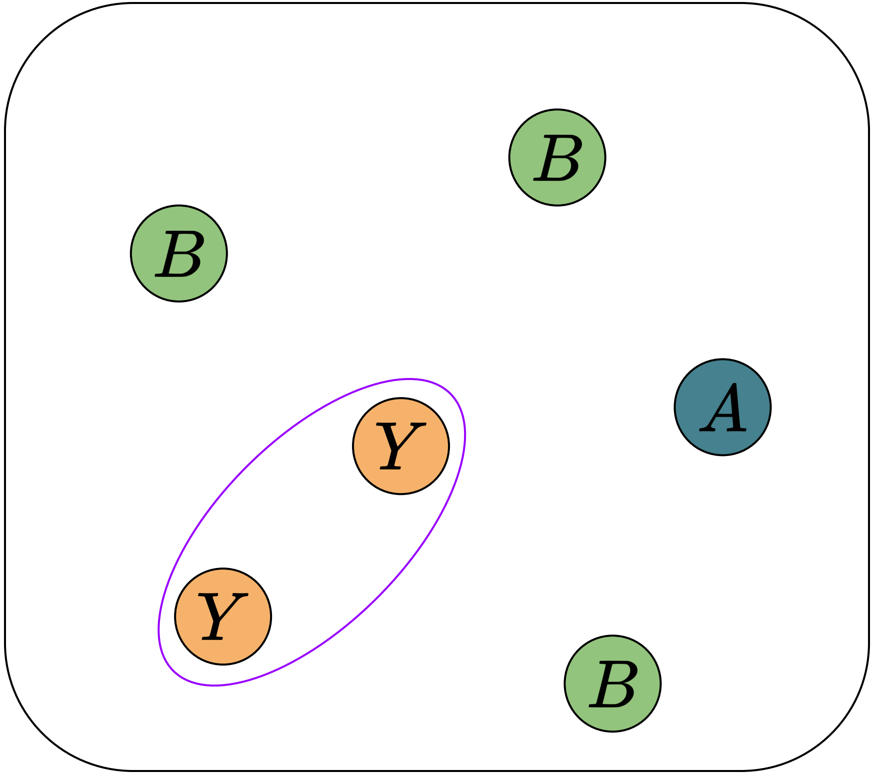


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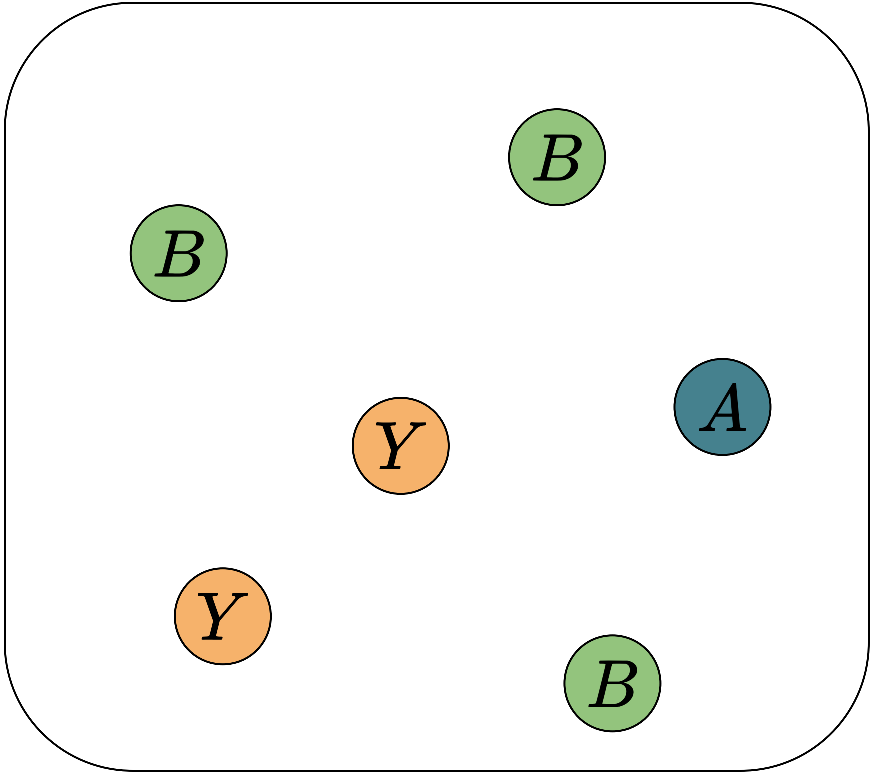


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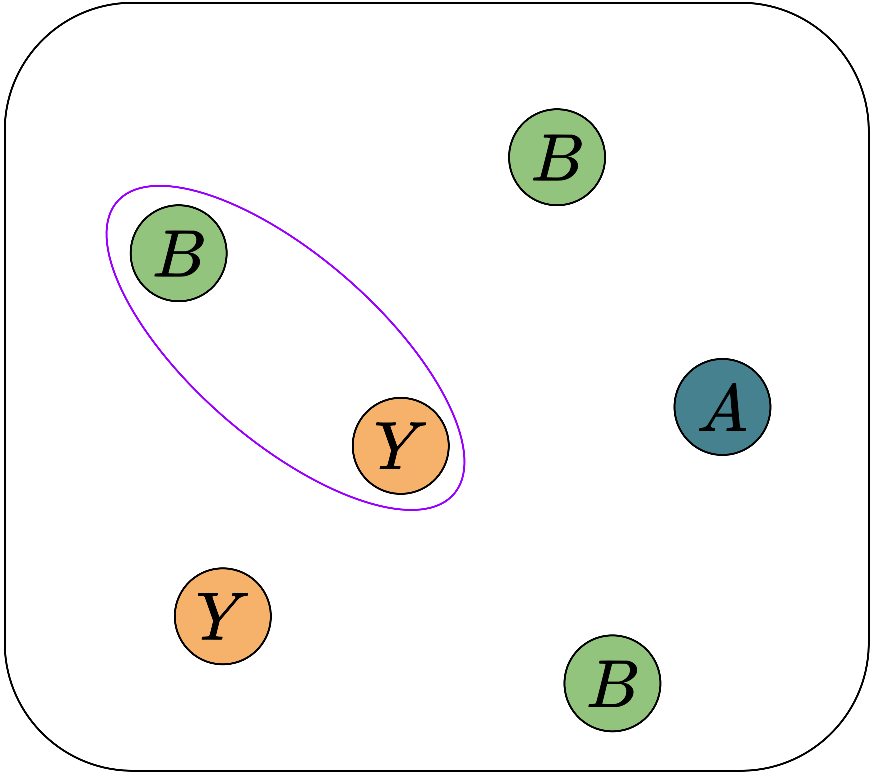


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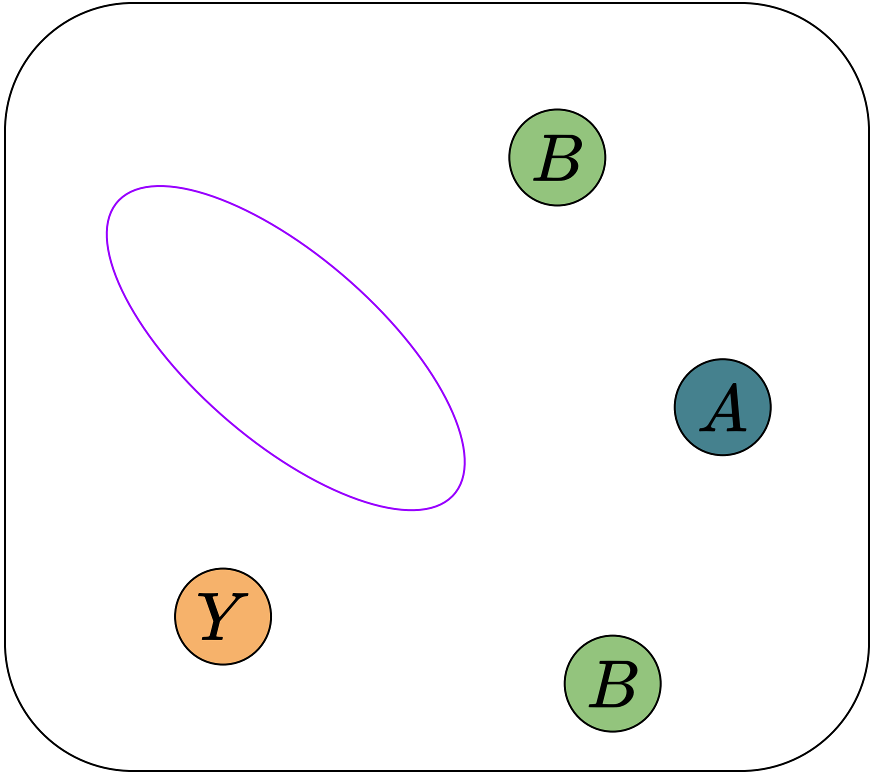


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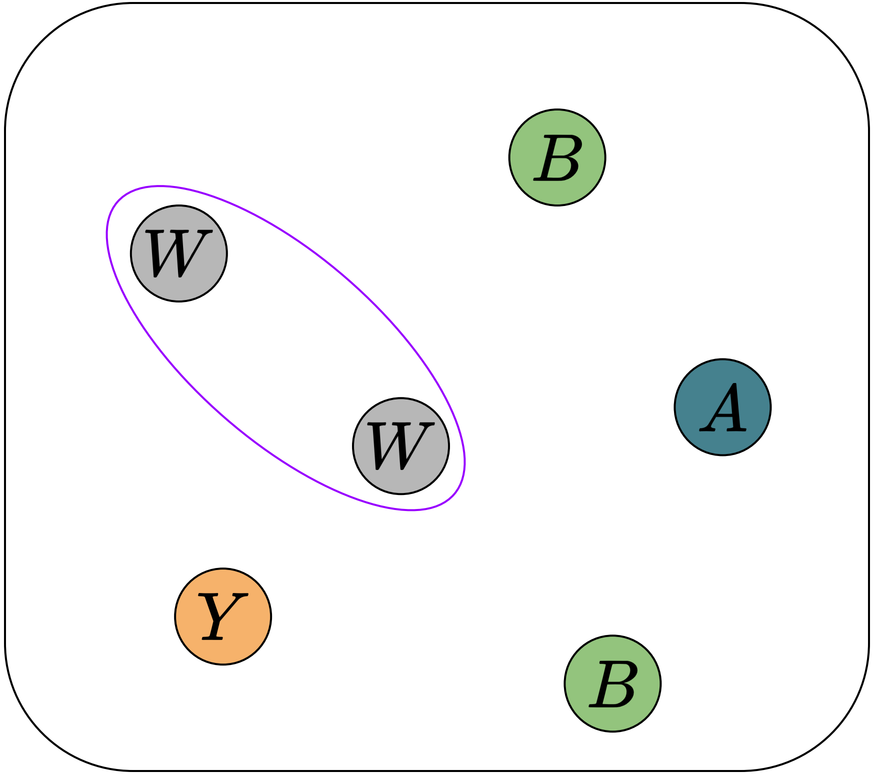


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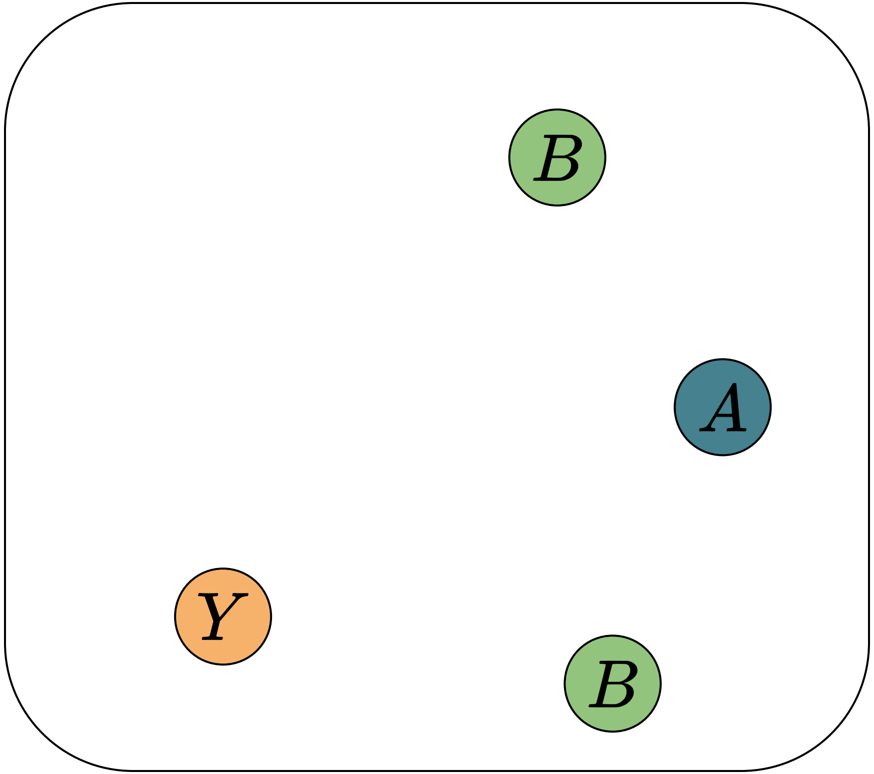


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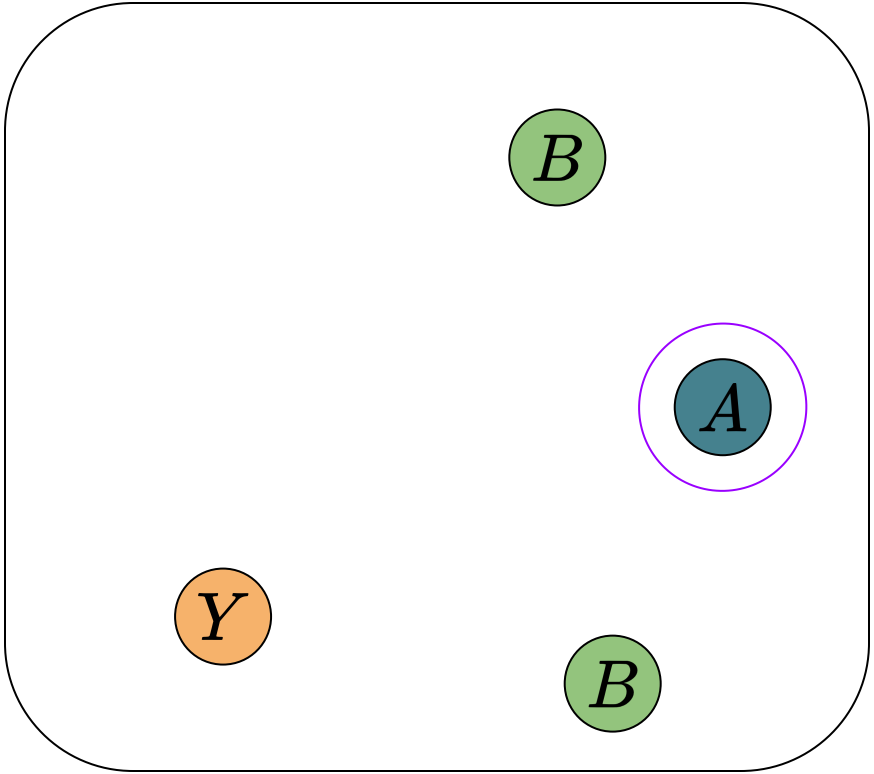


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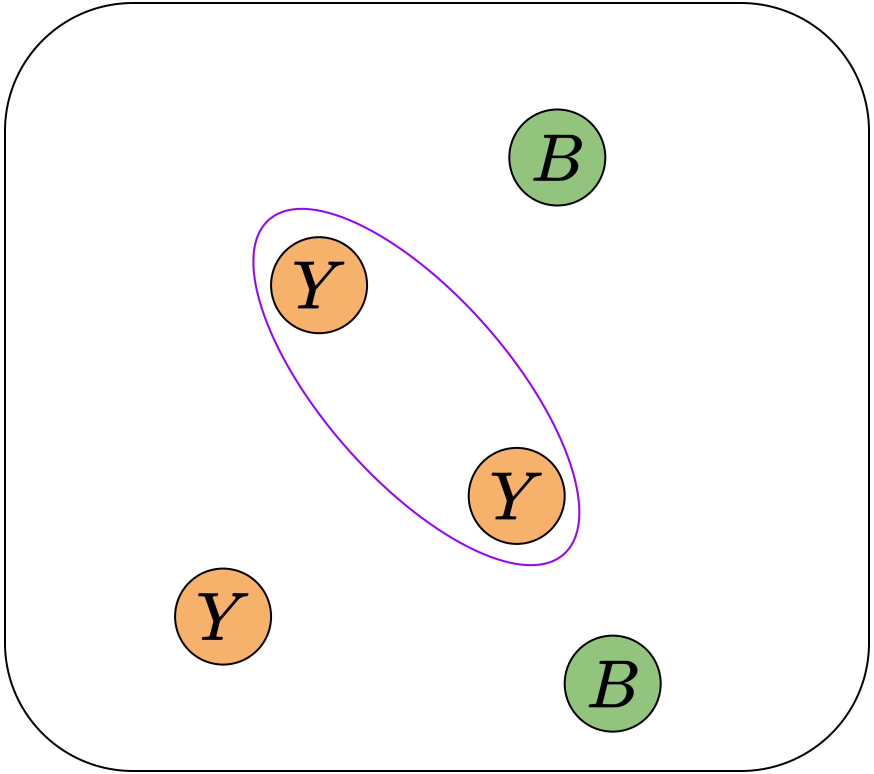


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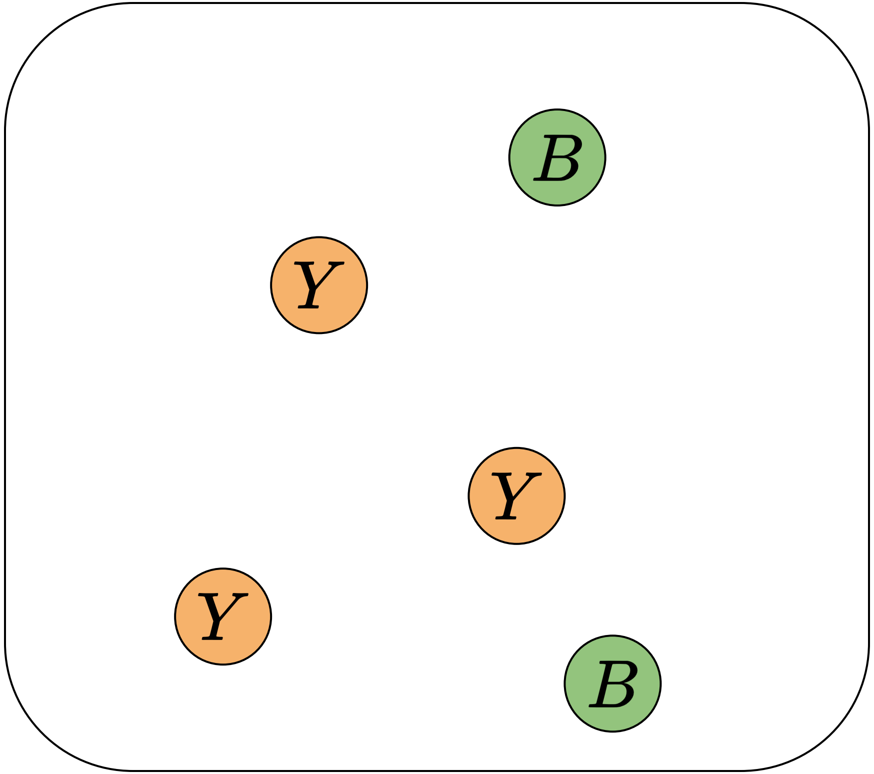


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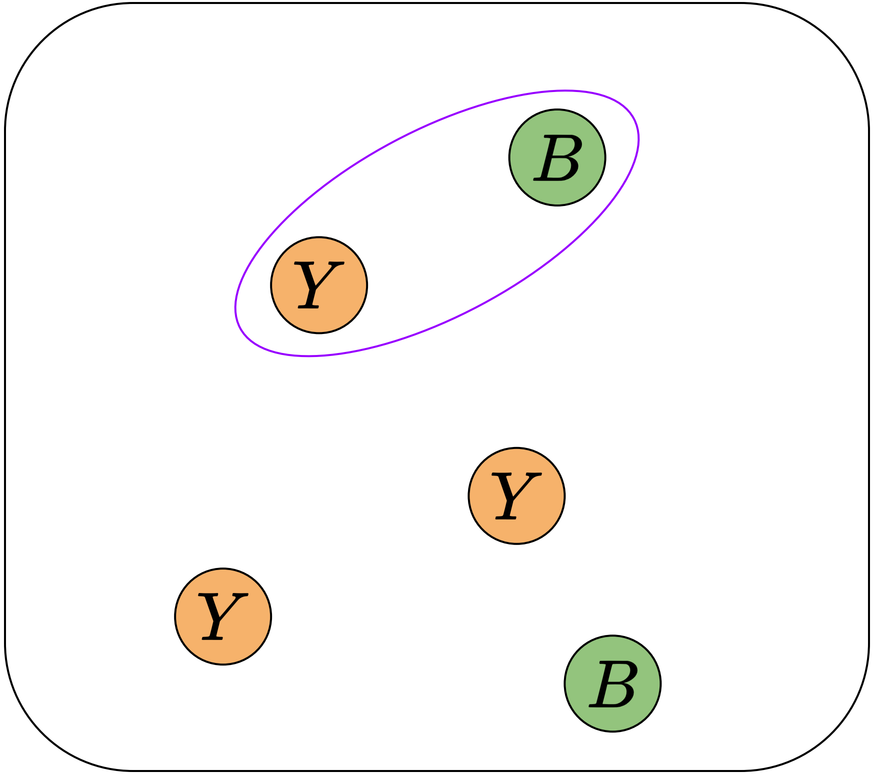


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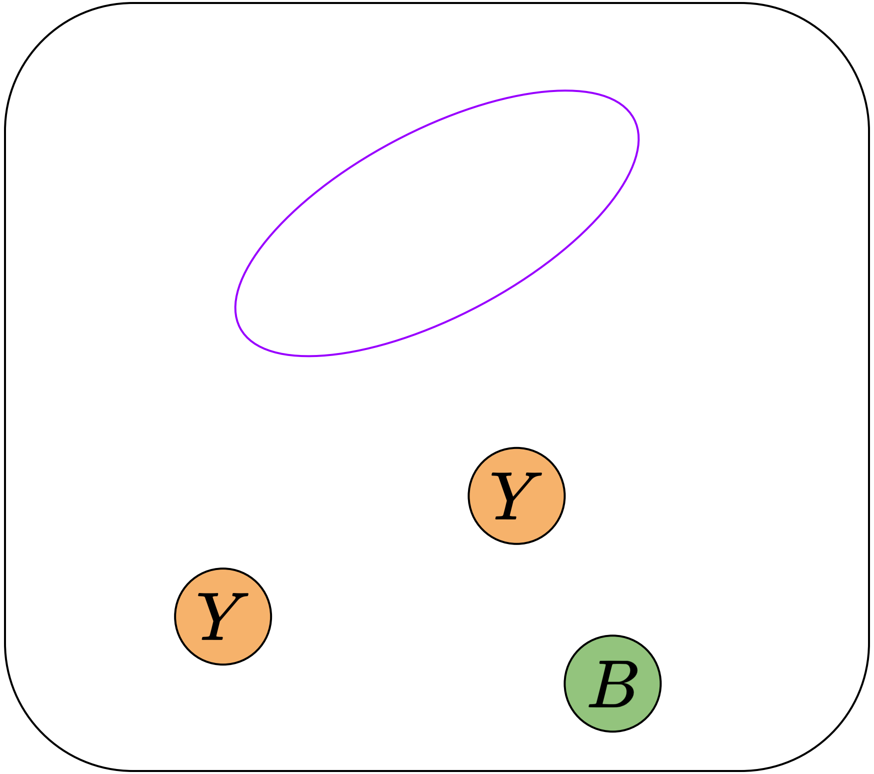


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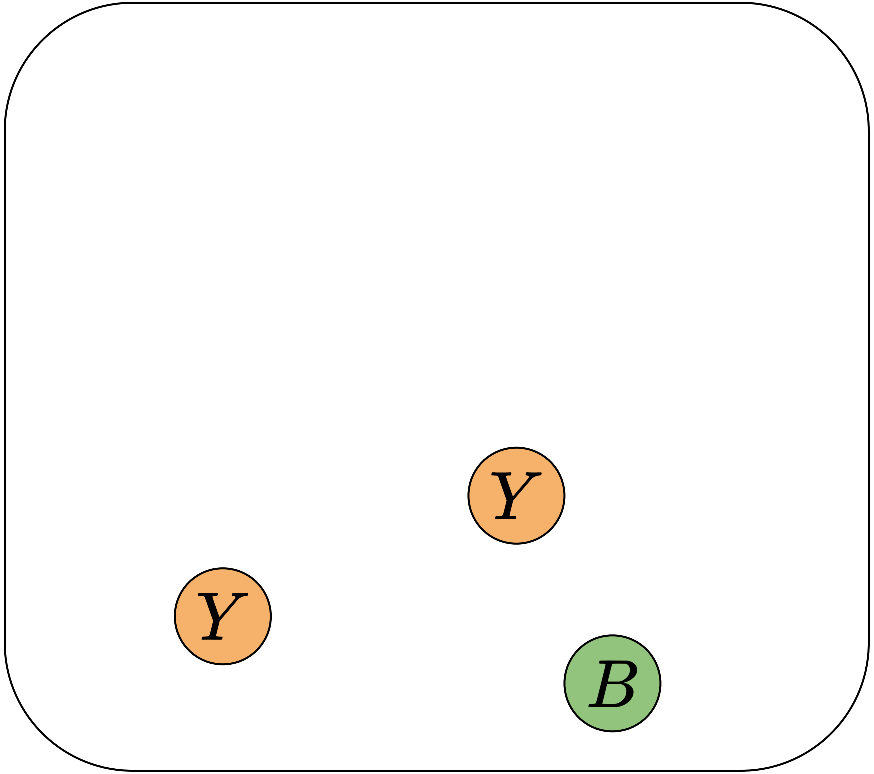


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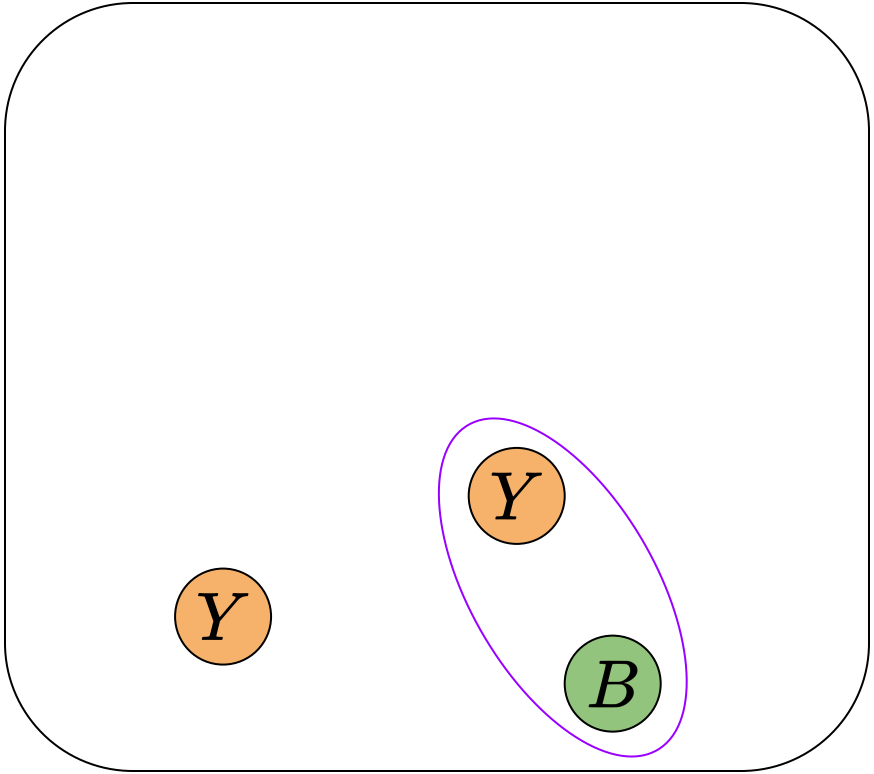


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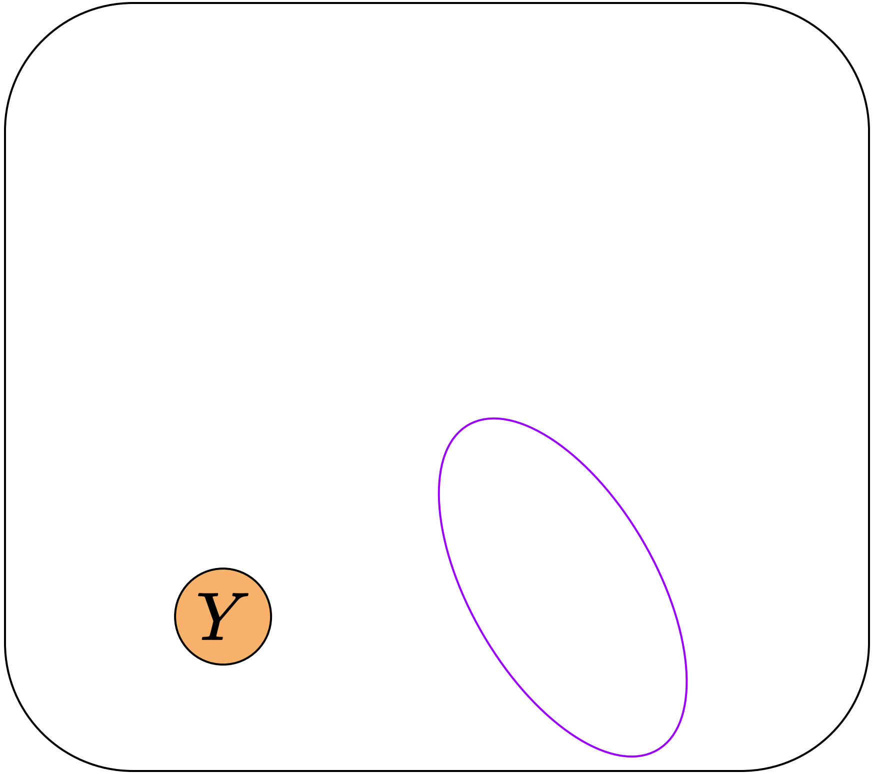


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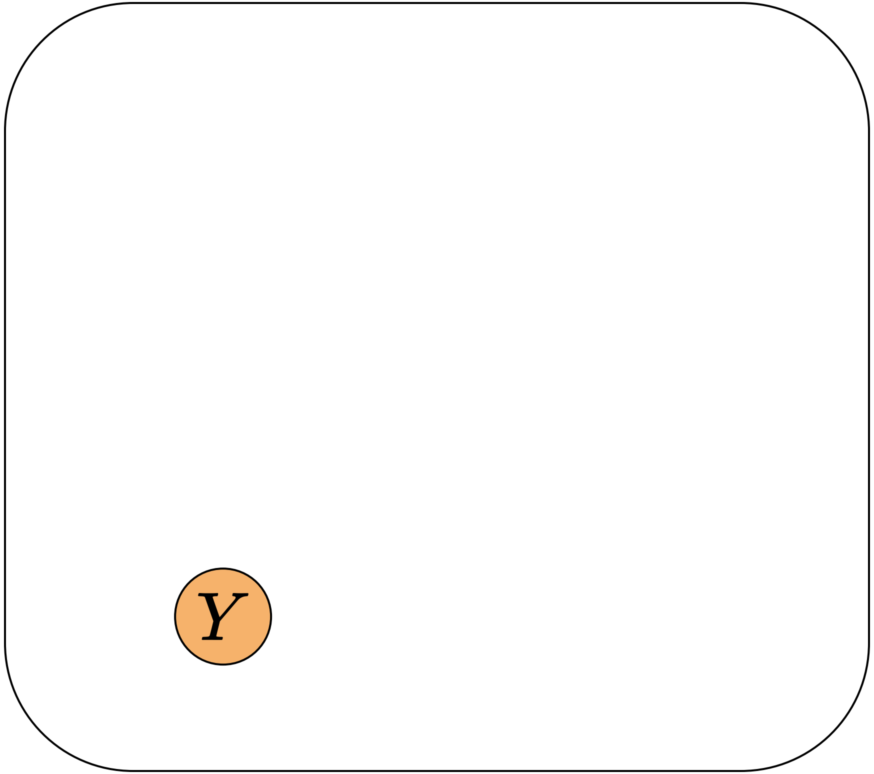


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# CRN Configurations & Dynamics

$\{1Y\}$ is **terminal** since no reactions can happen (are applicable).

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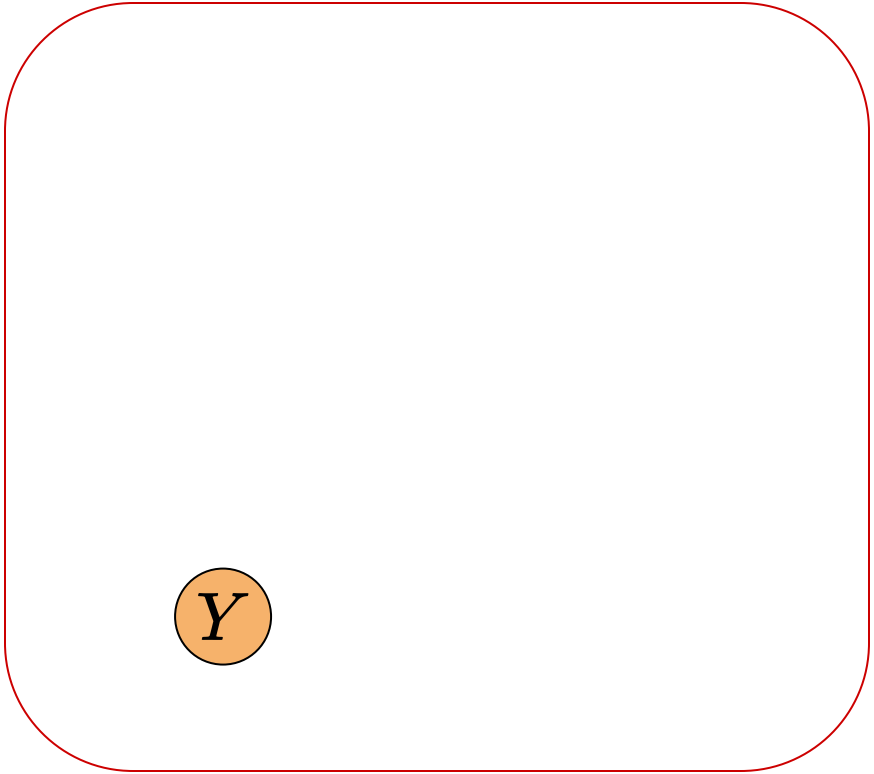


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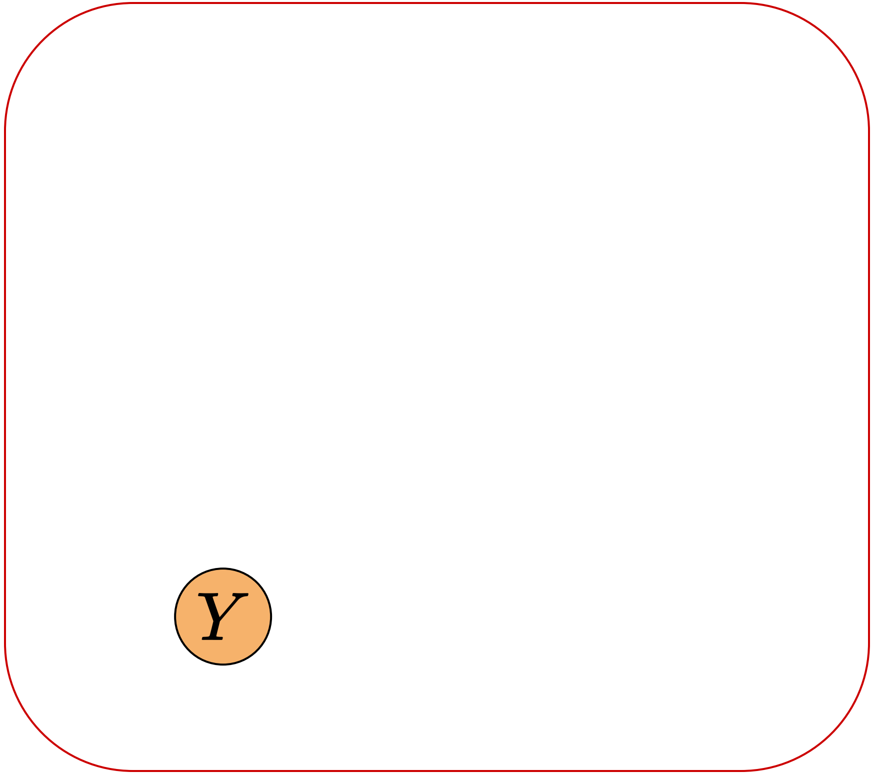


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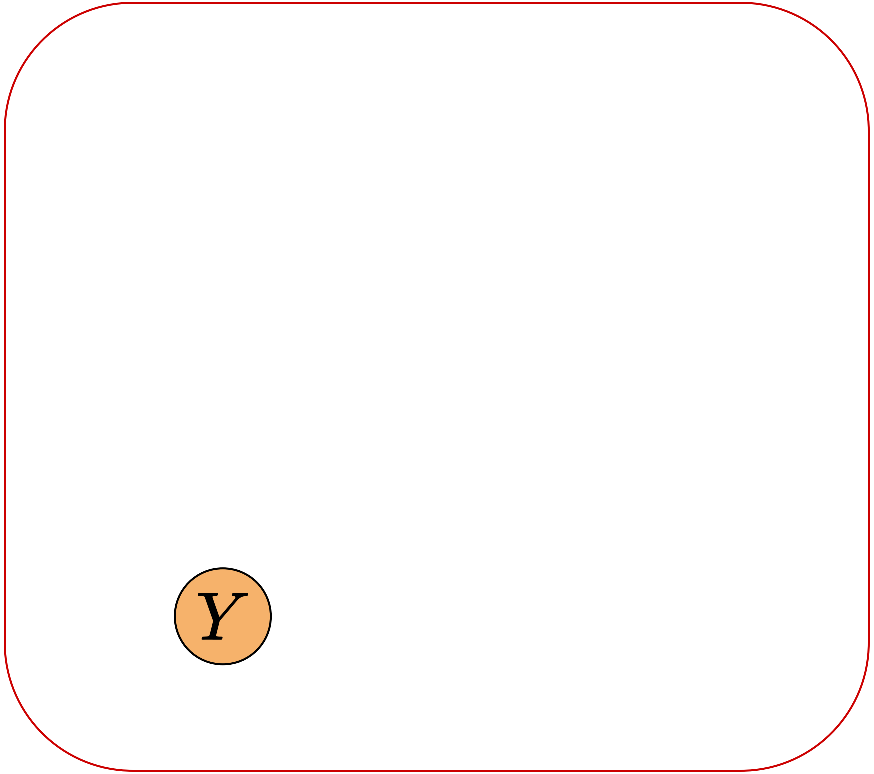
$$\{2A, 3B\} \rightsquigarrow \{1Y\}$$

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# Step CRN Configurations & Dynamics

<u>Reactions</u>

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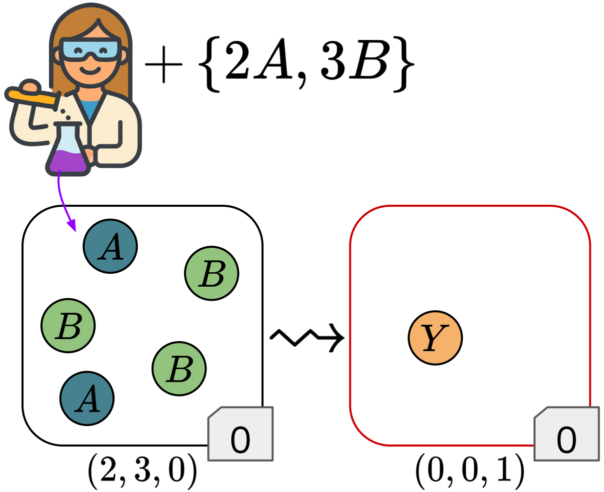

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Step CRNs also specify a sequence of k steps that each add a collection of species to terminal configurations.

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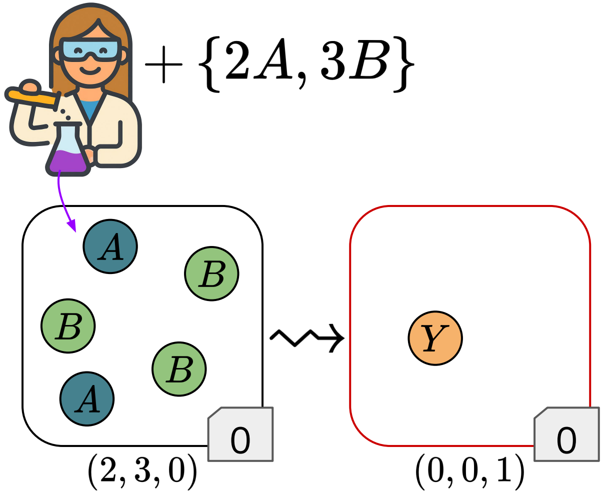

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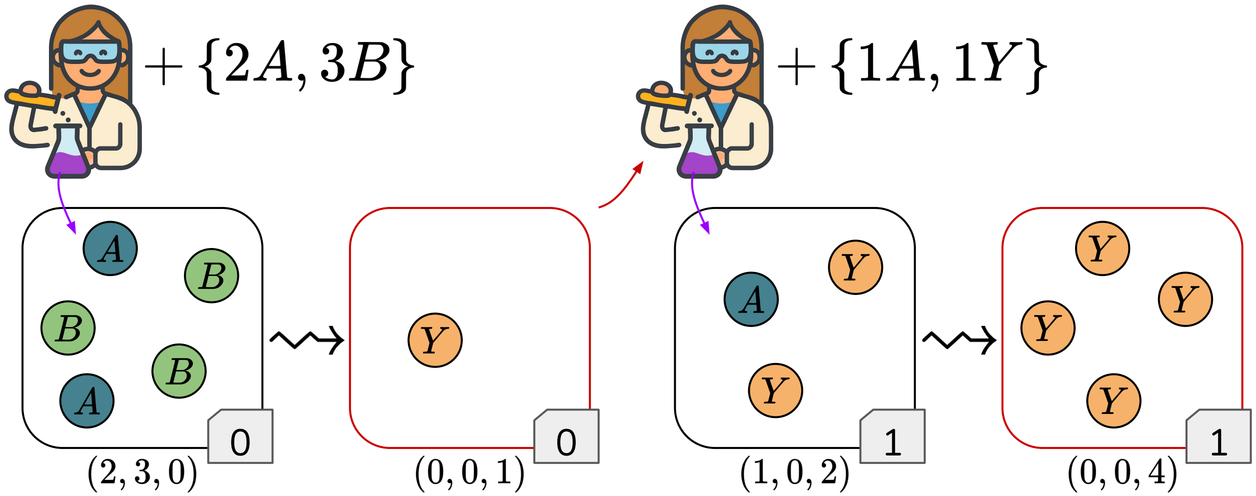

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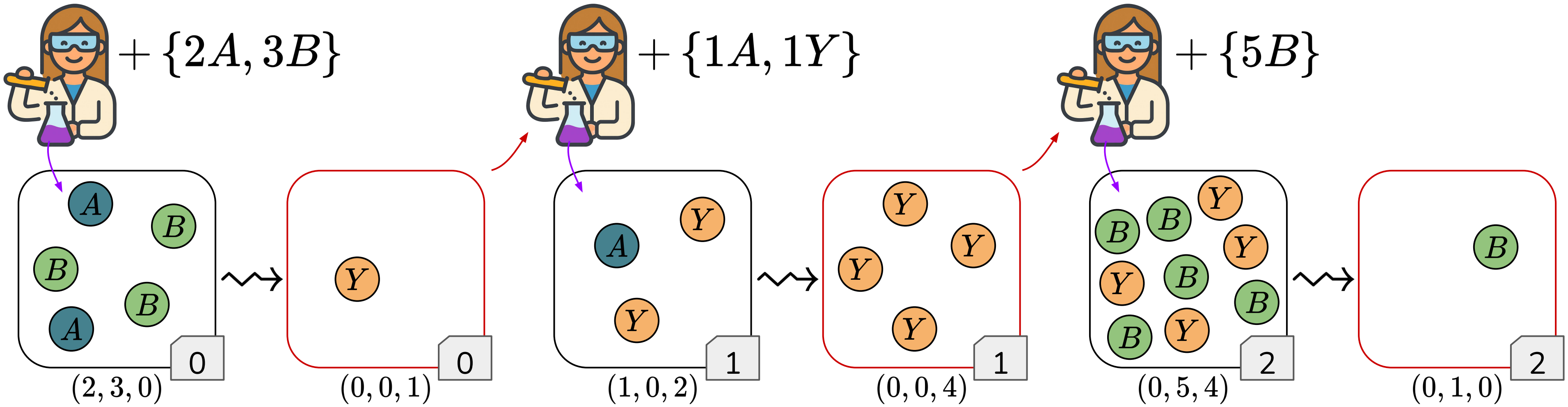

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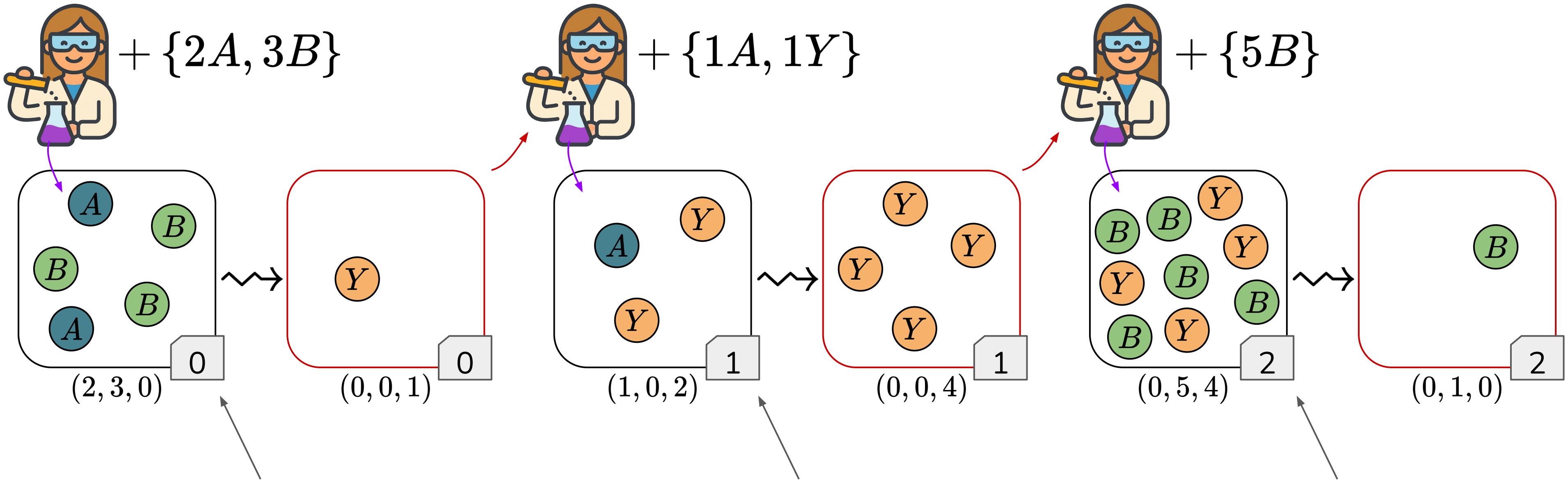


Step CRN Configurations also include an index to indicate the current step.

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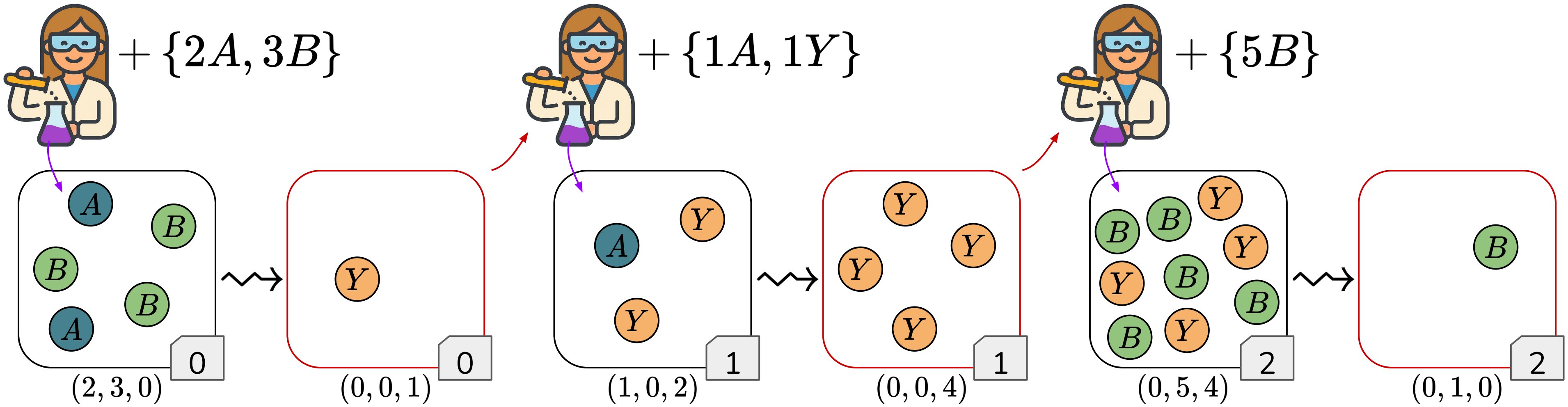

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Step-Cycle

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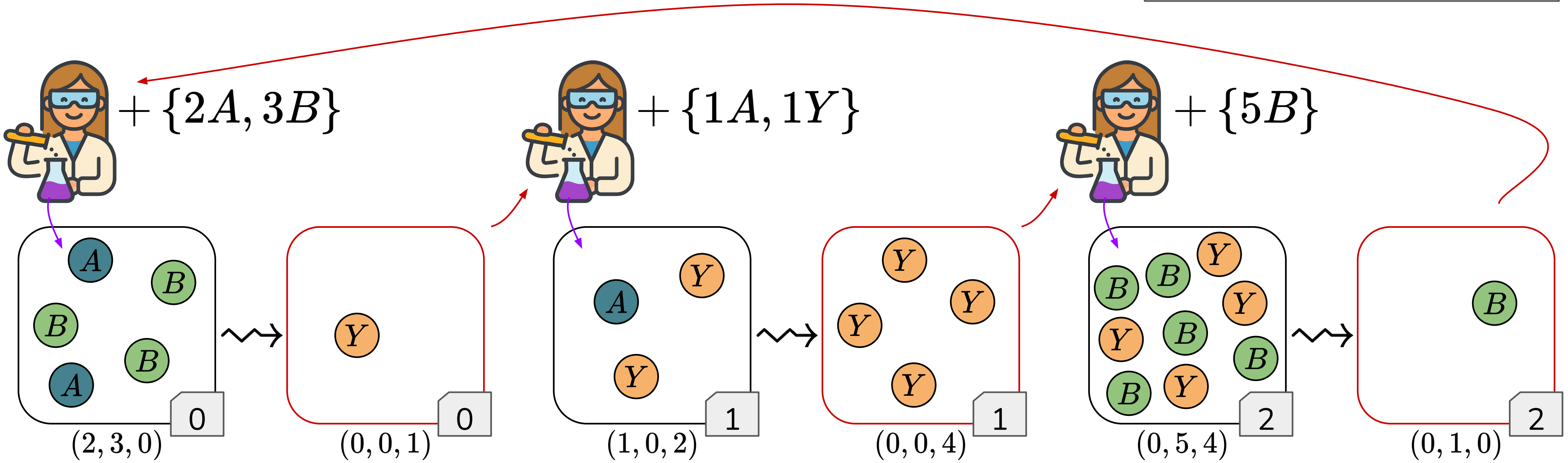

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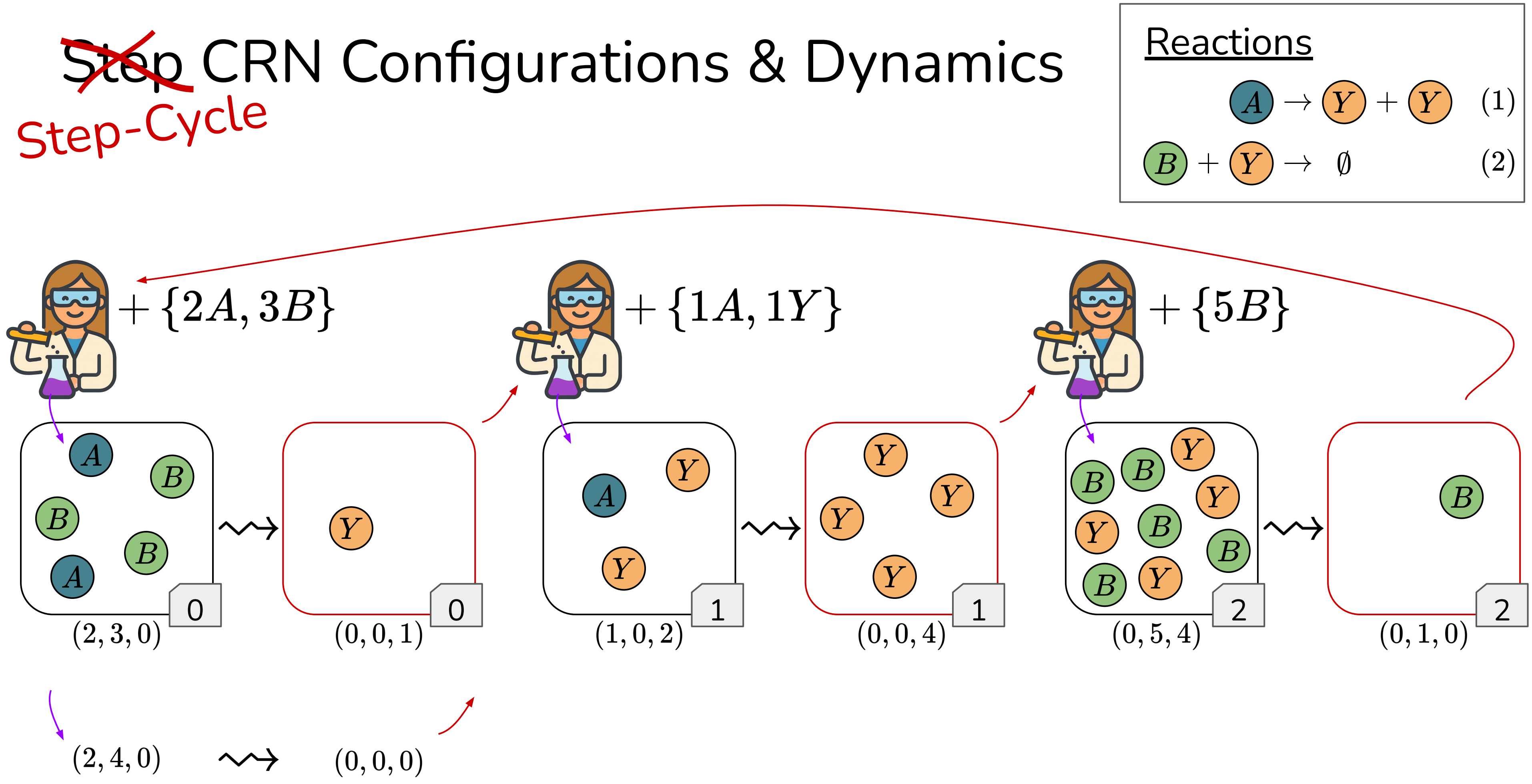

# ~~Step~~ Step-Cycle CRN Configurations & Dynamics

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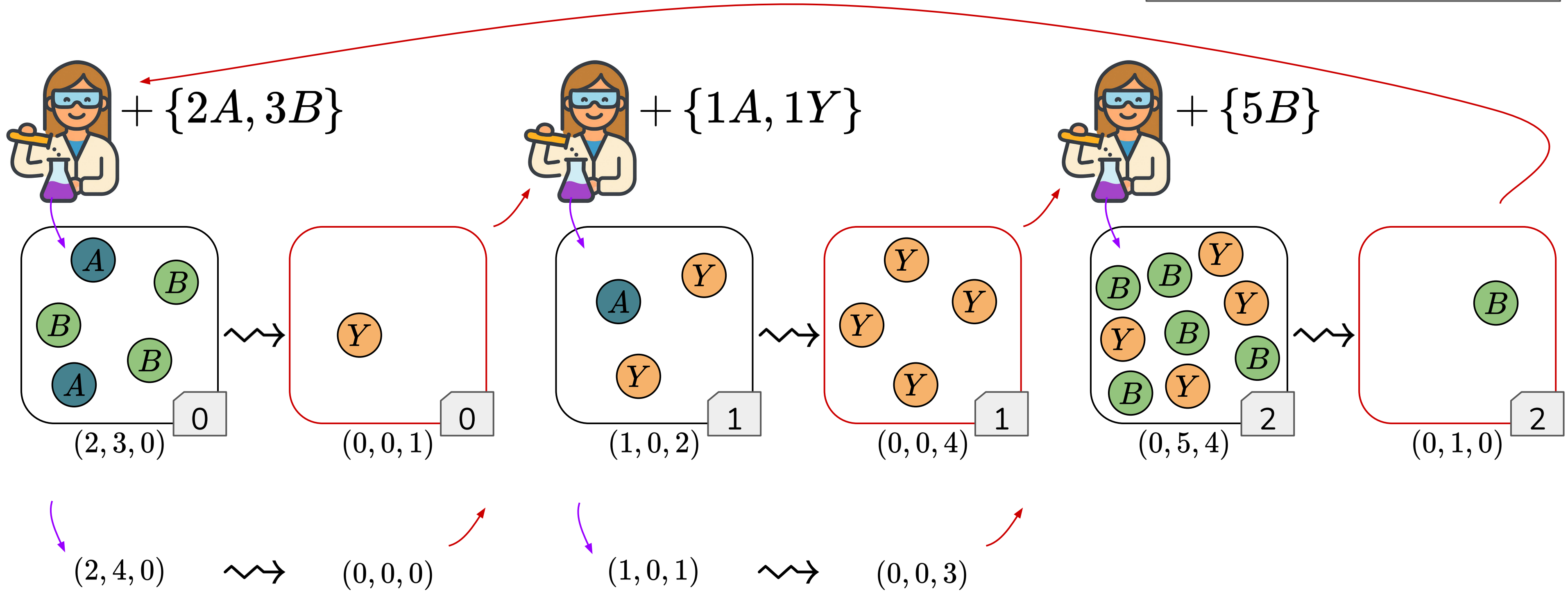

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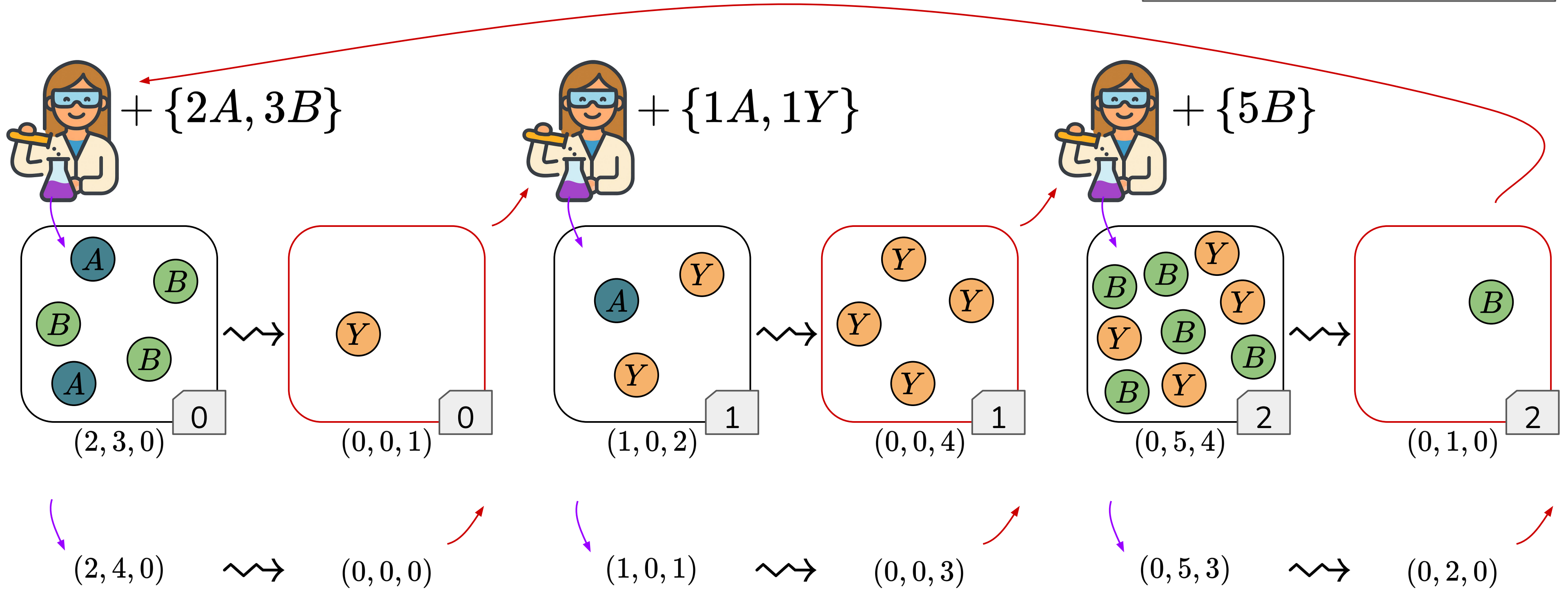

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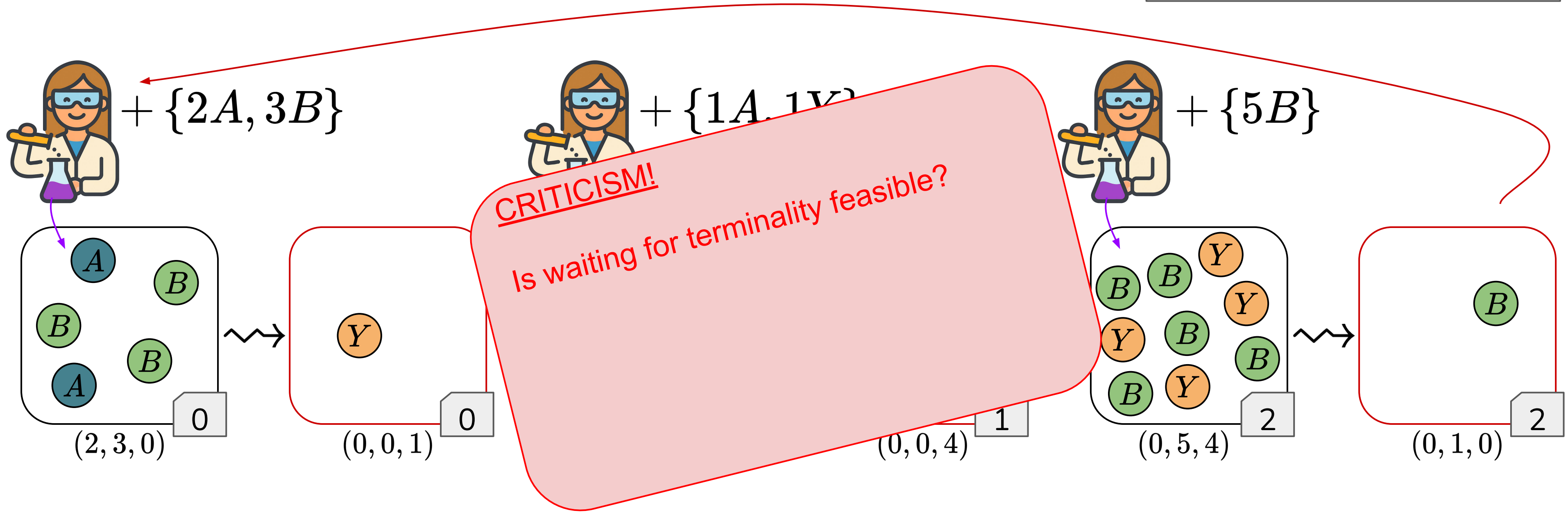

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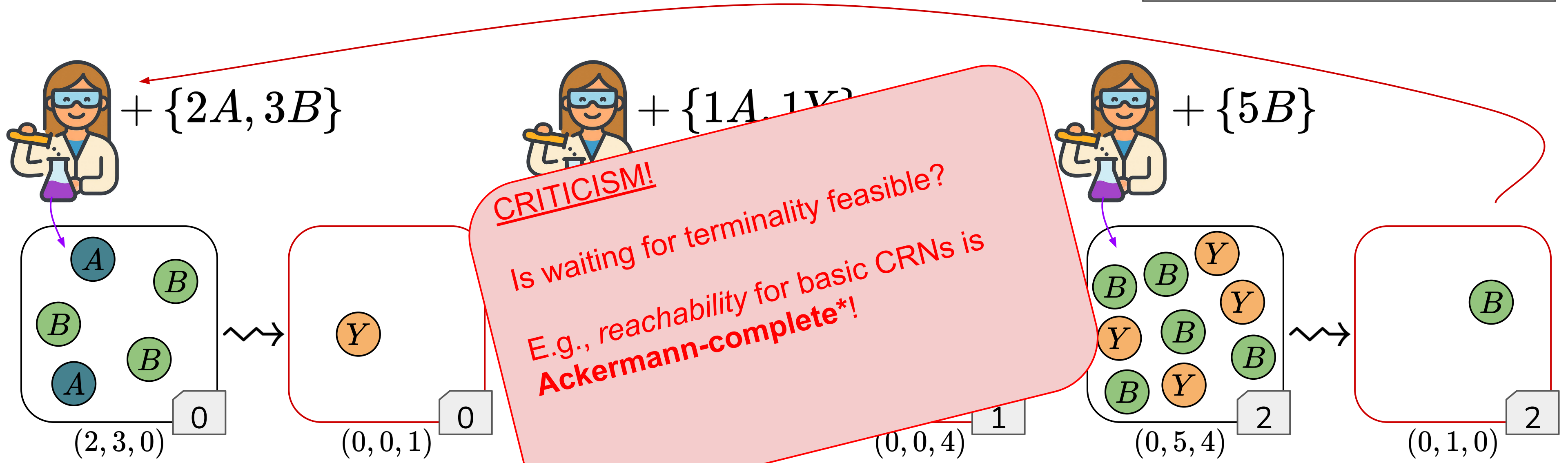


CRITICISM!

Is waiting for terminality feasible?

E.g., *reachability* for basic CRNs is **Ackermann-complete***!

*[Wojciech Czerwiński and Łukasz Orlikowski, FOCS'21]
*[Jérôme Leroux, FOCS'21]

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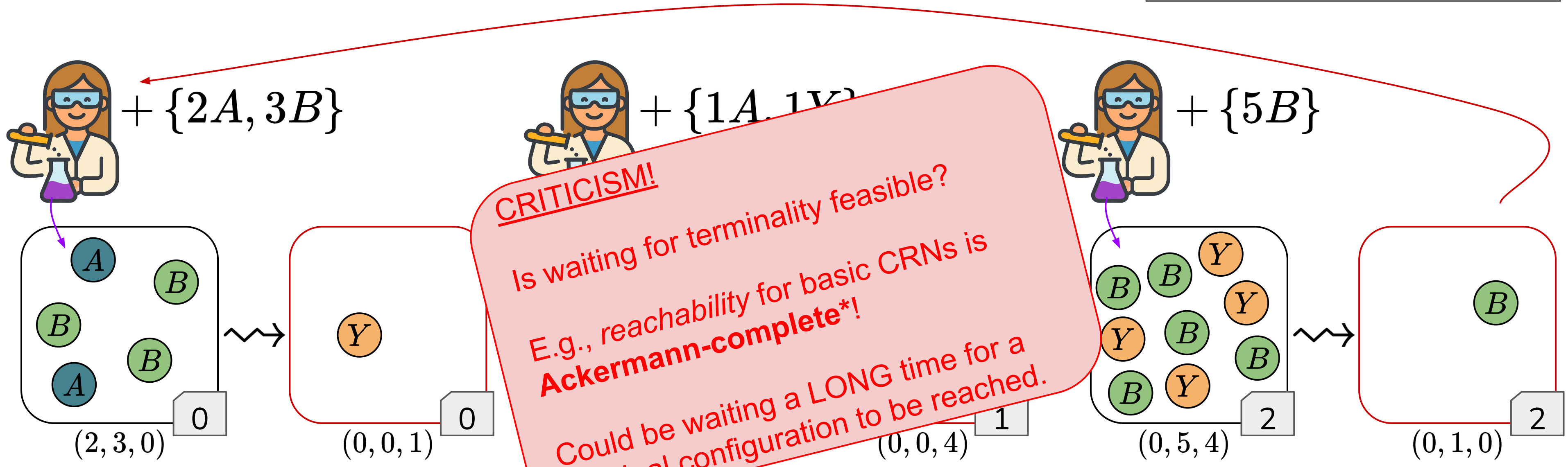


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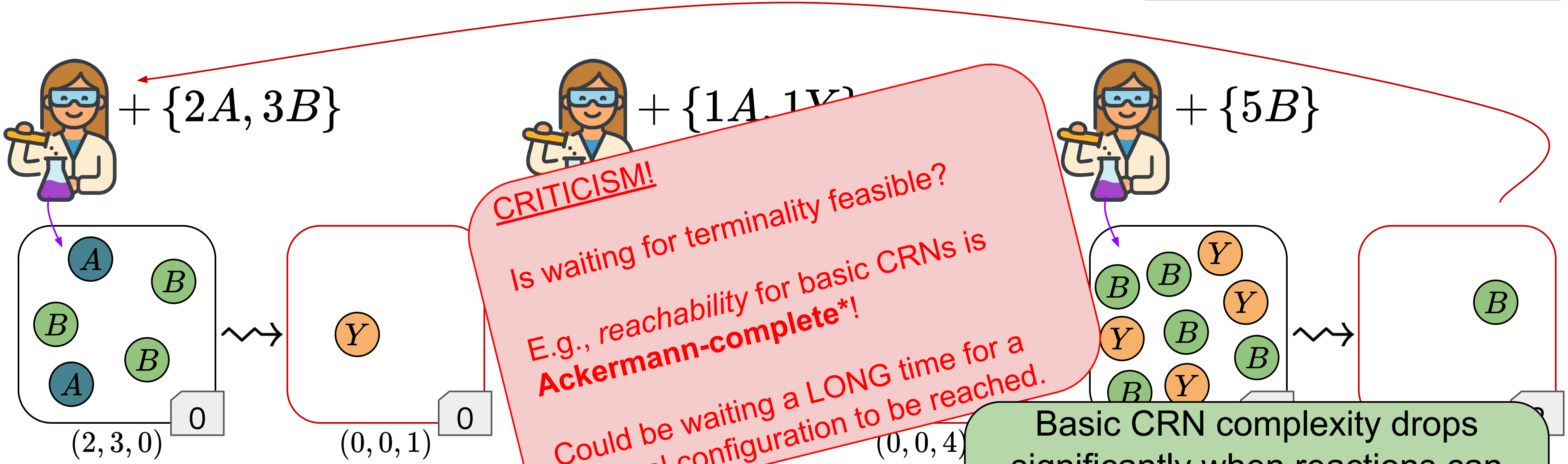


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Basic CRN complexity drops significantly when reactions can only **add** or **delete** species.

E.g., *reachability* falls within NP!

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# Deletion-Only Reactions (Void Rules)

<u>Rule Size:</u>

(2,0) $A + B \rightarrow \emptyset$

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} Catalytic Reactions

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Catalytic Reactions

We consider Step-Cycle CRNs that use only (3,1) void rules.

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Catalytic Reactions

We consider Step-Cycle CRNs that use only (3,1) void rules.

Despite the significant loss in power for basic CRNs, (3,1) Void Step-Cycle CRNs are ***equivalent*** to general Step-Cycle CRNs!

# Not All Equivalencies are Equivalent

Register Machine

Turing Machine

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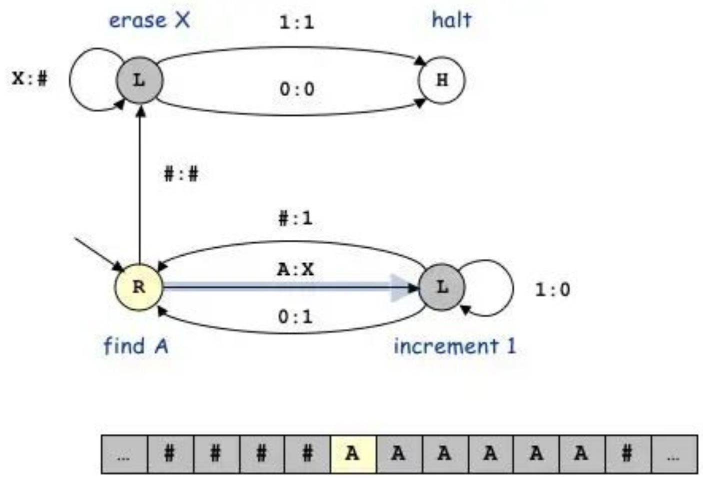

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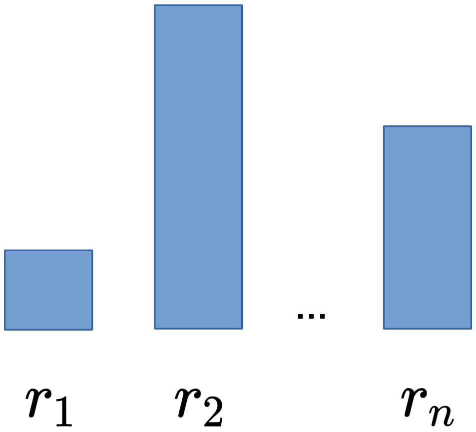
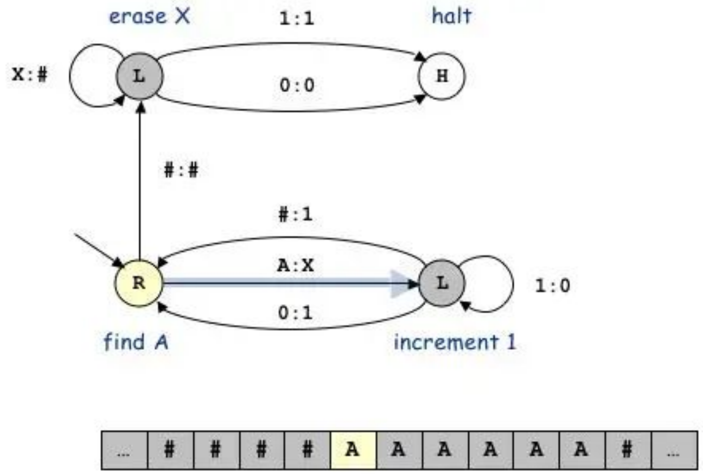



$$inc(r_i, s_j)$$
$$dec(r_i, s_j, s_k)$$

## Turing Machine



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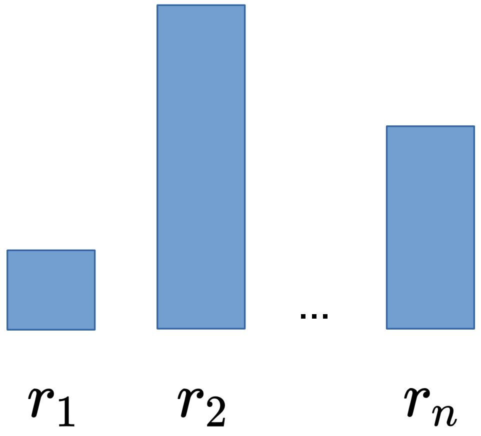


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Computationally Equivalent

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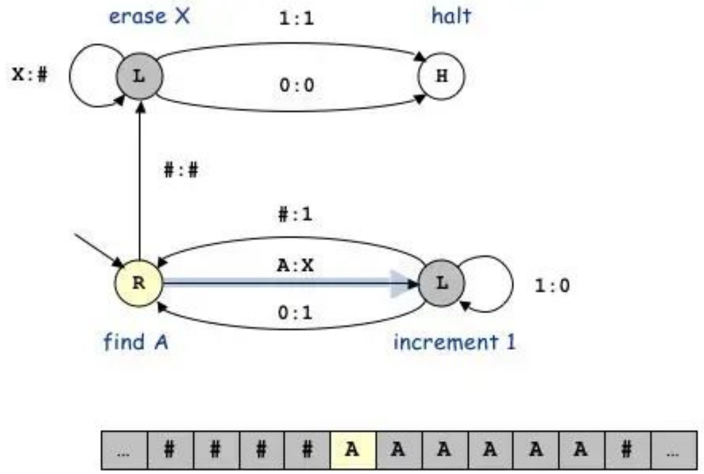

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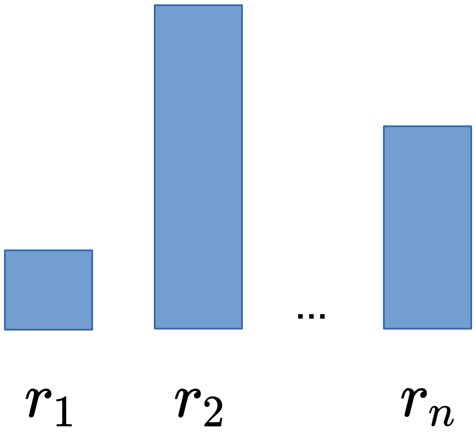


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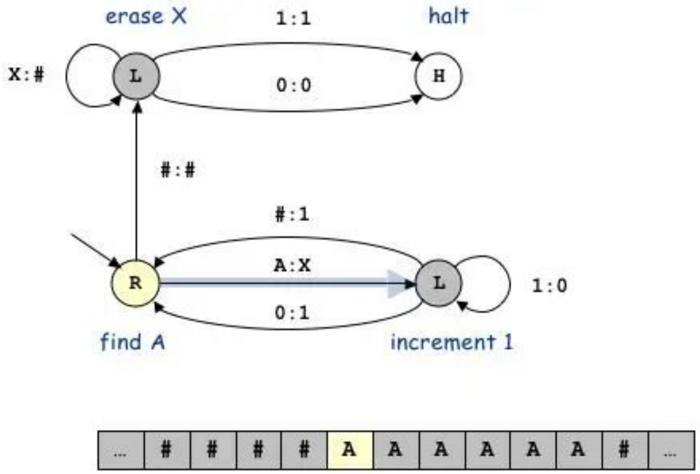
Computationally Equivalent

Exponentially Slower!

## Turing Machine



# Configuration Mapping

(3,1) Void Step-Cycle CRN $C'$

$$A + B + Y \rightarrow B$$

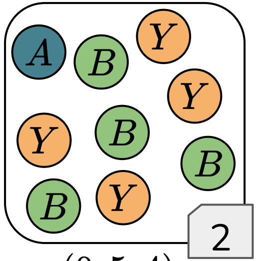
General Step-Cycle CRN $C$

$$D + A \rightarrow B + Y + C$$

# Configuration Mapping

## (3,1) Void Step-Cycle CRN $C'$

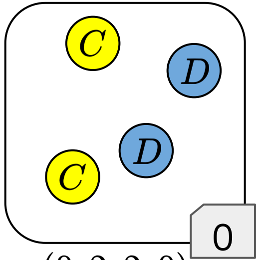
$$A + B + Y \rightarrow B$$




$(0, 5, 4)$

## General Step-Cycle CRN $C$

$$D + A \rightarrow B + Y + C$$

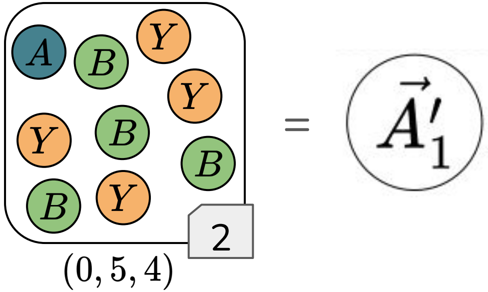



$(0, 2, 2, 0)$

# Configuration Mapping

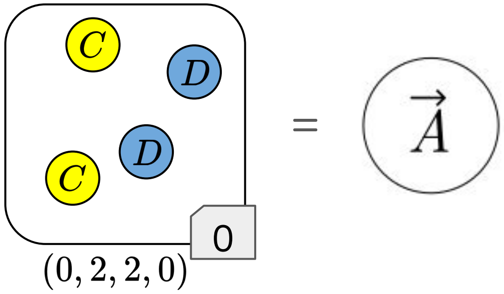
## (3,1) Void Step-Cycle CRN $C'$

$$A + B + Y \rightarrow B$$




## General Step-Cycle CRN $C$

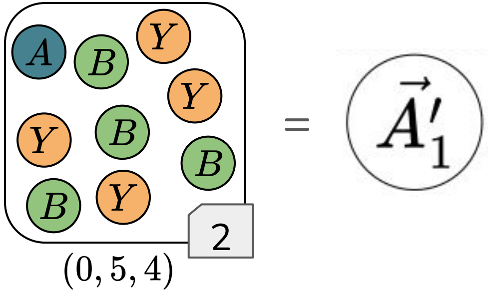
$$D + A \rightarrow B + Y + C$$



# Configuration Mapping

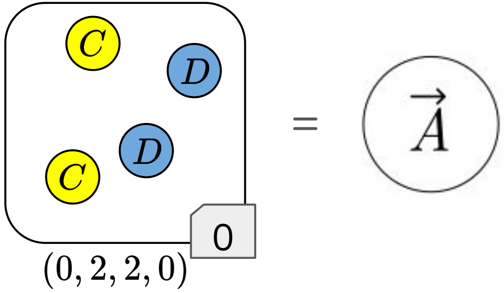
(3,1) Void Step-Cycle CRN $C'$

$$A + B + Y \rightarrow B$$




General Step-Cycle CRN $C$

$$D + A \rightarrow B + Y + C$$




*Polynomial-time* computable function $M : \text{configs}_{C'} \rightarrow \text{configs}_{C}$

# Configuration Mapping

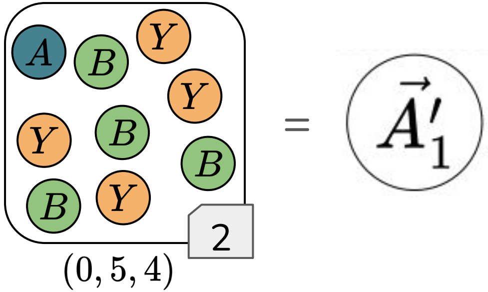
(3,1) Void Step-Cycle CRN $C'$

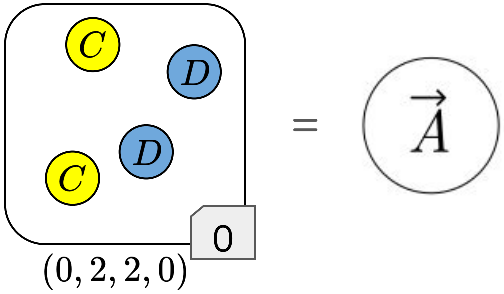
$A + B + Y \to B$

General Step-Cycle CRN $C$

$D + A \to B + Y + C$

$$M\left(\overrightarrow{A'_1}\right) = \vec{A}$$

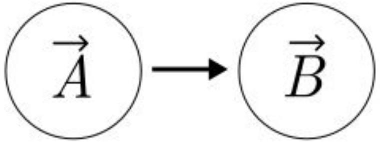



*Polynomial-time* computable function $M : \text{configs}_{C'} \to \text{configs}_C$

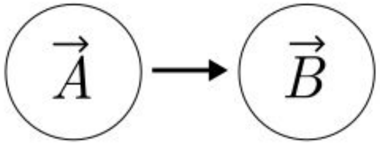
# Simulation

Configuration Space
of Original System

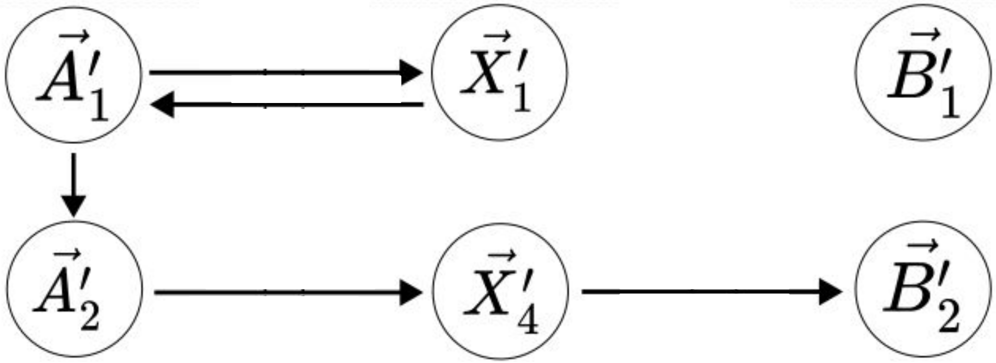


# Simulation

Configuration Space
of Original System

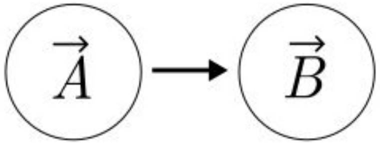



Configuration Space
of Simulating System

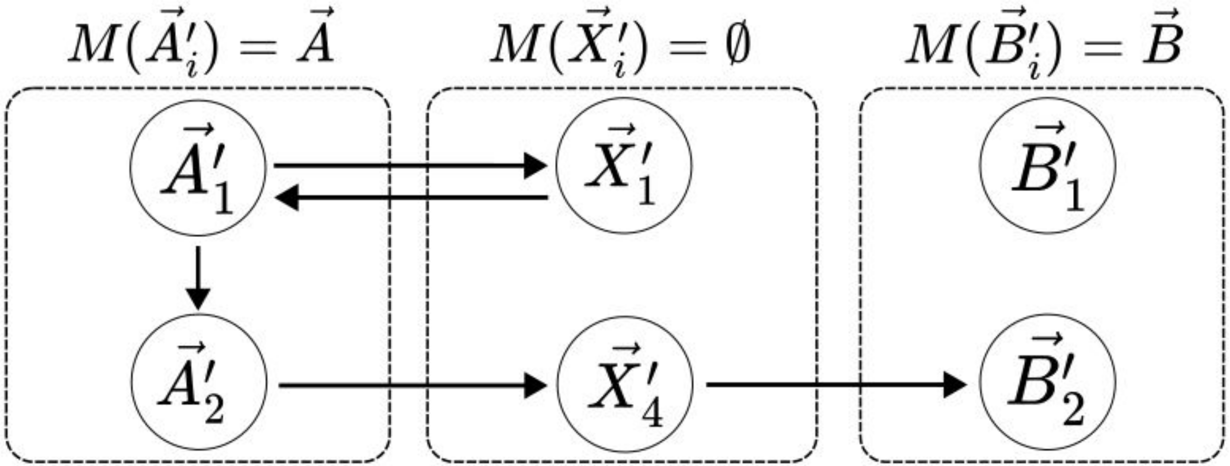


# Simulation

Configuration Space
of Original System



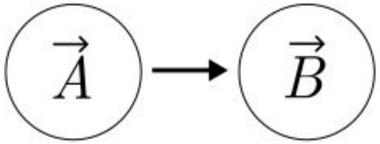

Configuration Space
of Simulating System



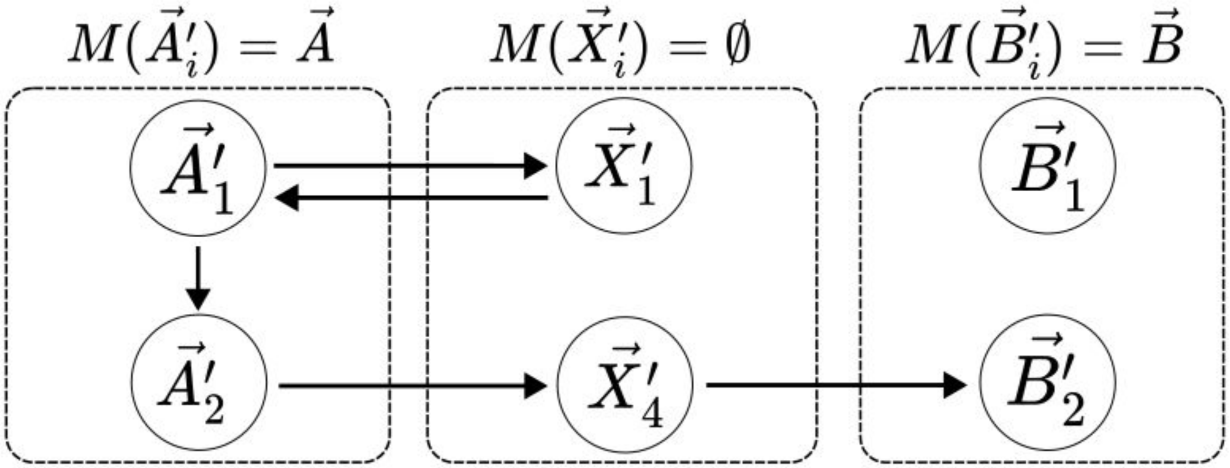

$$M : \text{configs}_{C'} \rightarrow \text{configs}_C$$

# Simulation

Configuration Space
of Original System




Configuration Space
of Simulating System



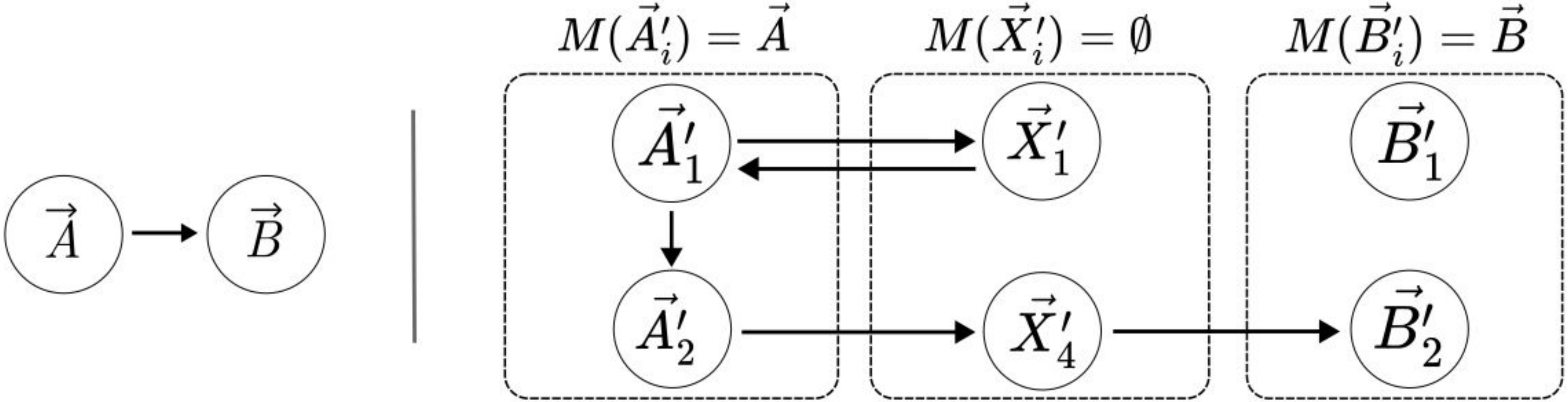

$$M : \text{configs}_{C'} \to \text{configs}_C$$

**Definition 7 (It Follows).** *We say system $T$ follows system $T'$ if whenever $\overrightarrow{A'} \Rightarrow_{T'} \overrightarrow{B'}$ and $M(\overrightarrow{A'}) \neq M(\overrightarrow{B'})$, then $M(\overrightarrow{A'}) \rightarrow_T M(\overrightarrow{B'})$.*

# Simulation

Configuration Space
<u>of Original System</u>

Configuration Space
<u>of Simulating System</u>

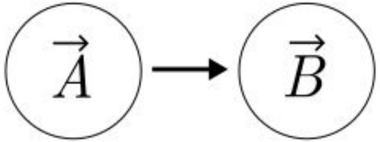



$$M : \text{configs}_{C'} \rightarrow \text{configs}_C$$

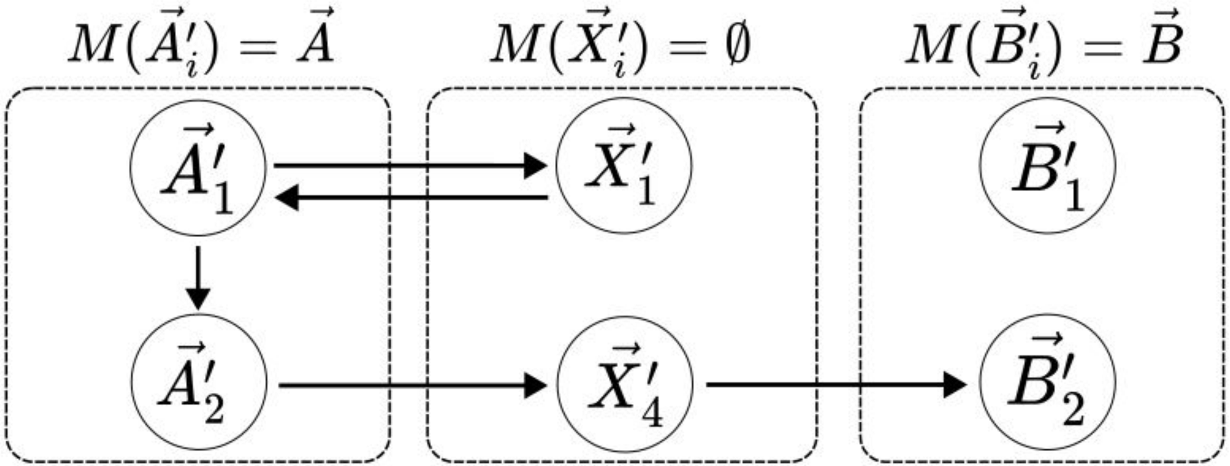
**Definition 8 (Models).** *We say system $T'$ models system $T$ if $\overrightarrow{A} \rightarrow_T \overrightarrow{B}$ implies that $\forall \overrightarrow{A'} \in [\![ \overrightarrow{A} ]\!], \exists \overrightarrow{B'} \in [\![ \overrightarrow{B} ]\!]$ such that $\overrightarrow{A'} \Rightarrow_{T'} \overrightarrow{B'}$.*

# Simulation

Configuration Space
of Original System




Configuration Space
of Simulating System




$$M : \text{configs}_{C'} \rightarrow \text{configs}_C$$

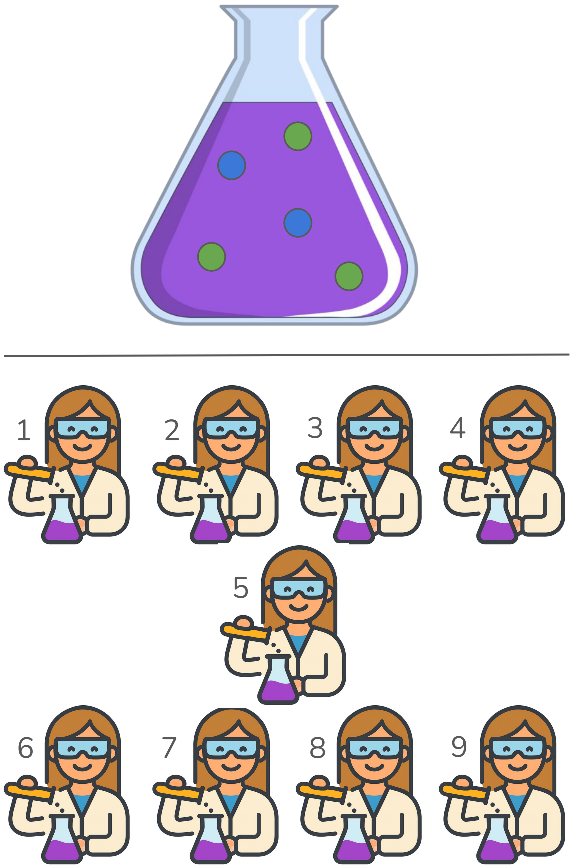
Polynomial Simulation:

- Polynomial System Size
- (Expected) Polynomial System Transitions

# (3,1) Void Step-Cycle Simulates **Basic** CRNs

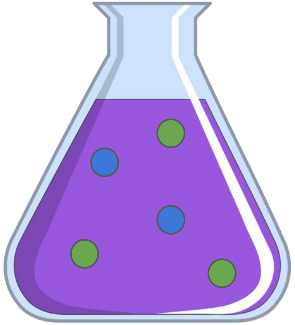
| Step | Additions | Relevant Rules |
|---|---|---|
| 0 | $\vec{G}$ | $\forall \gamma_i \in \Gamma :$<br>1. $\vec{G} + \vec{G}_i + \vec{g}_i \to \emptyset$<br>24. $\vec{z} + \vec{G} \to \vec{z}$ |
| 1 | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i :$ $\vec{E}_i^j$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i :$<br>2. $\vec{G}_i + \vec{E}_i^j \to \vec{G}_i$ |
| 2 | $\vec{w}$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i :$<br>3. $\vec{w} + \vec{g}_i \to \vec{w}$<br>4. $\vec{w} + \vec{r}_j + \vec{E}_i^j \to \vec{w}$ |
| 3 | $\vec{a}_y$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i :$<br>5. $\vec{E}_i^j + \vec{a}_y + \vec{w} \to \vec{E}_i^j$<br>25. $\vec{a}_y + \vec{w} + \vec{z} \to \emptyset$ |
| 4 | $\vec{a}_n$ | 6. $\vec{a}_y + \vec{a}_n + \vec{w} \to \vec{a}_y$ |
| 5 | $\forall \gamma_i \in \Gamma :$<br>$\forall r_j \in \mathcal{R}_i :$ $\vec{r}_j, \vec{R}_i^j$<br>$\forall p_j \in \mathcal{P}_i :$ $\vec{p}_j, \vec{P}_i^j$ | $\forall \gamma_i \in \Gamma :$<br>$\forall r_j \in \mathcal{R}_i :$<br>7. $\vec{G}_i + \vec{r}_j + \vec{R}_i^j \to \vec{G}_i$<br>8. $\vec{E}_i^j + \vec{r}_j + \vec{R}_i^j \to \emptyset$<br>9. $\vec{a}_y + \vec{r}_j + \vec{R}_i^j \to \vec{a}_y$<br>$\forall p_j \in \mathcal{P}_i :$<br>10. $\vec{G}_i + \vec{p}_j + \vec{P}_i^j \to \vec{G}_i$<br>11. $\vec{a}_n + \vec{p}_j + \vec{P}_i^j \to \vec{a}_n$ |
| 6 | $\forall \gamma_i \in \Gamma :$ $\vec{x}, \vec{G}'_i$ | $\forall \gamma_i \in \Gamma :$<br>12. $\vec{x} + \vec{G}'_i + \vec{G}_i \to \emptyset$ |
| 7 | $\forall \gamma_i \in \Gamma :$ $\vec{c}$ | $\forall \gamma_i \in \Gamma :$<br>13. $\vec{a}_y + \vec{c} + \vec{G}'_i \to \vec{a}_y$ |
| 8 | $\forall \gamma_i \in \Gamma :$ $\vec{g_i}$<br>$\vec{y}_1$ | $\forall \gamma_i \in \Gamma :$<br>$\forall r_j \in \mathcal{R}_i :$<br>14. $\vec{y}_1 + \vec{E}_i^j \to \vec{y}_1$<br>15. $\vec{y}_1 + \vec{R}_i^j \to \vec{y}_1$<br>$\forall p_j \in \mathcal{P}_i :$<br>16. $\vec{y}_1 + \vec{P}_i^j \to \vec{y}_1$<br>17. $\vec{G}'_i + \vec{g}_i \to \vec{G}'_i$<br>18. $\vec{y}_1 + \vec{x} \to \vec{y}_1$<br>19. $\vec{y}_1 + \vec{a}_y \to \vec{y}_1$<br>20. $\vec{y}_1 + \vec{a}_n \to \vec{y}_1$<br>21. $\vec{y}_1 + \vec{c} \to \vec{y}_1$ |
| 9 | $\vec{y}_2, \vec{z}$<br>$\forall \gamma_i \in \Gamma :$ $\vec{G}_i$ | 22. $\vec{y}_1 + \vec{y}_2 \to \emptyset$<br>$\forall \gamma_i \in \Gamma :$<br>23. $\vec{z} + \vec{g}_i \to \vec{g}_i$ |
| Return to Step 0. | | |

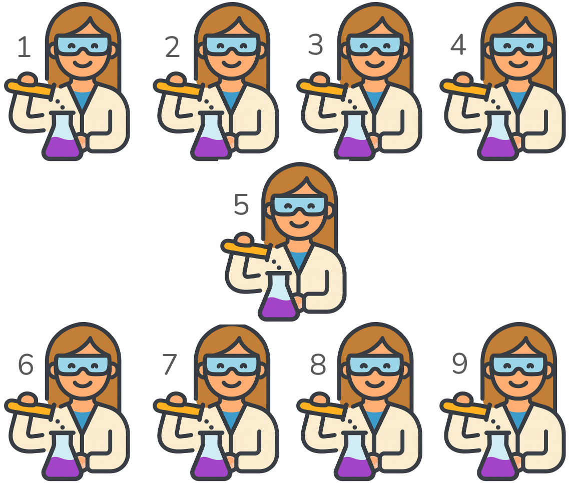
# (3,1) Void Step-Cycle Simulates **Basic** CRNs




| Step | Additions | Relevant Rules |
|---|---|---|
| 0 | $\vec{G}$ | $\forall\gamma_i \in \Gamma$ : 1. $\vec{G} + \vec{G}_i + \vec{g}_i \to \emptyset$<br>24. $\vec{z} + \vec{G} \to \vec{z}$ |
| 1 | $\forall\gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i$ : $\vec{E}_i^j$ | $\forall\gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i$ : 2. $\vec{G}_i + \vec{E}_i^j \to \vec{G}_i$ |
| 2 | $\vec{w}$ | $\forall\gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i$ : 3. $\vec{w} + \vec{g}_i \to \vec{w}$<br>4. $\vec{w} + \vec{r}_j + \vec{E}_i^j \to \vec{w}$ |
| 3 | $\vec{d}_y$ | $\forall\gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i$ : 5. $\vec{E}_i^j + \vec{d}_y + \vec{w} \to \vec{E}_i^j$<br>25. $\vec{d}_y + \vec{w} + \vec{z} \to \emptyset$ |
| 4 | $\vec{d}_n$ | 6. $\vec{d}_y + \vec{d}_n + \vec{w} \to \vec{d}_y$ |
| 5 | $\forall\gamma_i \in \Gamma$ :<br>$\forall r_j \in \mathcal{R}_i$ : $\vec{r}_j$, $\vec{R}_i^j$<br>$\forall p_j \in \mathcal{P}_i$ : $\vec{p}_j$, $\vec{P}_i^j$ | $\forall\gamma_i \in \Gamma$ :<br>7. $\vec{G}_i + \vec{r}_j + \vec{R}_i^j \to \vec{G}_i$<br>$\forall r_j \in \mathcal{R}_i$ : 8. $\vec{E}_i^j + \vec{r}_j + \vec{R}_i^j \to \emptyset$<br>9. $\vec{d}_y + \vec{r}_j + \vec{R}_i^j \to \vec{d}_y$<br>10. $\vec{G}_i + \vec{p}_j + \vec{P}_i^j \to \vec{G}_i$<br>$\forall p_j \in \mathcal{P}_i$ : 11. $\vec{d}_n + \vec{p}_j + \vec{P}_i^j \to \vec{d}_n$ |
| 6 | $\forall\gamma_i \in \Gamma$ : $\vec{x}$, $\vec{G}'_i$ | $\forall\gamma_i \in \Gamma$ : 12. $\vec{x} + \vec{G}'_i + \vec{G}_i \to \emptyset$ |
| 7 | $\forall\gamma_i \in \Gamma$ : $\vec{c}$ | $\forall\gamma_i \in \Gamma$ : 13. $\vec{d}_y + \vec{c} + \vec{G}'_i \to \vec{d}_y$ |
| 8 | $\forall\gamma_i \in \Gamma$ : $\vec{g}_i$<br>$\vec{y}_1$ | $\forall\gamma_i \in \Gamma$ :<br>$\forall r_j \in \mathcal{R}_i$ : 14. $\vec{y}_1 + \vec{E}_i^j \to \vec{y}_1$<br>15. $\vec{y}_1 + \vec{R}_i^j \to \vec{y}_1$<br>$\forall p_j \in \mathcal{P}_i$ : 16. $\vec{y}_1 + \vec{P}_i^j \to \vec{y}_1$<br>17. $\vec{G}'_i + \vec{g}_i \to \vec{G}'_i$<br>18. $\vec{y}_1 + \vec{x} \to \vec{y}_1$<br>19. $\vec{y}_1 + \vec{d}_y \to \vec{y}_1$<br>20. $\vec{y}_1 + \vec{d}_n \to \vec{y}_1$<br>21. $\vec{y}_1 + \vec{c} \to \vec{y}_1$ |
| 9 | $\vec{y}_2$, $\vec{z}$<br>$\forall\gamma_i \in \Gamma$ : $\vec{G}_i$ | 22. $\vec{y}_1 + \vec{y}_2 \to \emptyset$<br>$\forall\gamma_i \in \Gamma$ : 23. $\vec{z} + \vec{g}_i \to \vec{g}_i$ |
| Return to Step 0. | | |

# (3,1) Void Step-Cycle Simulates **Basic** CRNs





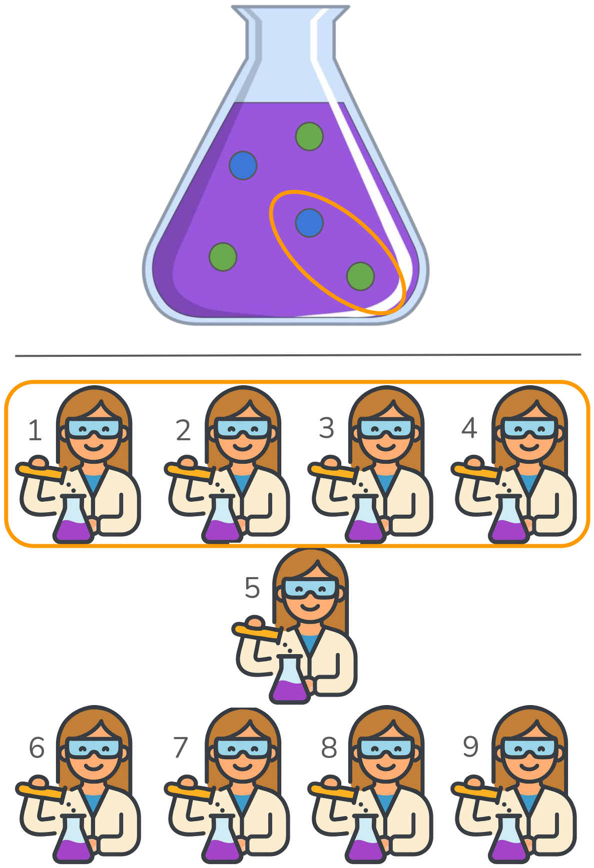

Selection

Execution

Clean Up

| Step | Additions | Relevant Rules |
|---|---|---|
| 0 | $\overrightarrow{G}$ | $\forall \gamma_i \in \Gamma:$ 1. $\overrightarrow{G} + \overrightarrow{G}_i + \overrightarrow{g}_i \rightarrow \emptyset$ <br> 24. $\overrightarrow{z} + \overrightarrow{G} \rightarrow \overrightarrow{z}$ |
| 1 | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i:$ $\overrightarrow{E}_i^j$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i:$ 2. $\overrightarrow{G}_i + \overrightarrow{E}_i^j \rightarrow \overrightarrow{G}_i$ |
| 2 | $\overrightarrow{w}$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i:$ 3. $\overrightarrow{w} + \overrightarrow{g}_i \rightarrow \overrightarrow{w}$ <br> 4. $\overrightarrow{w} + \overrightarrow{r}_j + \overrightarrow{E}_i^j \rightarrow \overrightarrow{w}$ |
| 3 | $\overrightarrow{a}_y$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i:$ 5. $\overrightarrow{E}_i^j + \overrightarrow{a}_y + \overrightarrow{w} \rightarrow \overrightarrow{E}_i^j$ <br> 25. $\overrightarrow{a}_y + \overrightarrow{w} + \overrightarrow{z} \rightarrow \emptyset$ |
| 4 | $\overrightarrow{a}_n$ | 6. $\overrightarrow{a}_y + \overrightarrow{a}_n + \overrightarrow{w} \rightarrow \overrightarrow{a}_y$ |
| 5 | $\forall \gamma_i \in \Gamma:$ <br> $\forall r_j \in \mathcal{R}_i:$ $\overrightarrow{r}_j, \overrightarrow{R}_i^j$ <br> $\forall p_j \in \mathcal{P}_i:$ $\overrightarrow{p}_j, \overrightarrow{P}_i^j$ | $\forall \gamma_i \in \Gamma:$ <br> 7. $\overrightarrow{G}_i + \overrightarrow{r}_j + \overrightarrow{R}_i^j \rightarrow \overrightarrow{G}_i$ <br> $\forall r_j \in \mathcal{R}_i:$ 8. $\overrightarrow{E}_i^j + \overrightarrow{r}_j + \overrightarrow{R}_i^j \rightarrow \emptyset$ <br> 9. $\overrightarrow{a}_y + \overrightarrow{r}_j + \overrightarrow{R}_i^j \rightarrow \overrightarrow{a}_y$ <br> 10. $\overrightarrow{G}_i + \overrightarrow{p}_j + \overrightarrow{P}_i^j \rightarrow \overrightarrow{G}_i$ <br> $\forall p_j \in \mathcal{P}_i:$ 11. $\overrightarrow{a}_n + \overrightarrow{p}_j + \overrightarrow{P}_i^j \rightarrow \overrightarrow{a}_n$ |
| 6 | $\forall \gamma_i \in \Gamma:$ $\overrightarrow{x}, \overrightarrow{G}_i'$ | $\forall \gamma_i \in \Gamma:$ 12. $\overrightarrow{x} + \overrightarrow{G}_i' + \overrightarrow{G}_i \rightarrow \emptyset$ |
| 7 | $\forall \gamma_i \in \Gamma:$ $\overrightarrow{c}$ | $\forall \gamma_i \in \Gamma:$ 13. $\overrightarrow{a}_y + \overrightarrow{c} + \overrightarrow{G}_i' \rightarrow \overrightarrow{a}_y$ |
| 8 | $\forall \gamma_i \in \Gamma:$ $\overrightarrow{g_i}$ <br> $\overrightarrow{y}_1$ | $\forall \gamma_i \in \Gamma:$ <br> $\forall r_j \in \mathcal{R}_i:$ 14. $\overrightarrow{y}_1 + \overrightarrow{E}_i^j \rightarrow \overrightarrow{y}_1$ <br> 15. $\overrightarrow{y}_1 + \overrightarrow{R}_i^j \rightarrow \overrightarrow{y}_1$ <br> $\forall p_j \in \mathcal{P}_i:$ 16. $\overrightarrow{y}_1 + \overrightarrow{P}_i^j \rightarrow \overrightarrow{y}_1$ <br> 17. $\overrightarrow{G}_i' + \overrightarrow{g}_i \rightarrow \overrightarrow{G}_i'$ <br> 18. $\overrightarrow{y}_1 + \overrightarrow{x} \rightarrow \overrightarrow{y}_1$ <br> 19. $\overrightarrow{y}_1 + \overrightarrow{a}_y \rightarrow \overrightarrow{y}_1$ <br> 20. $\overrightarrow{y}_1 + \overrightarrow{a}_n \rightarrow \overrightarrow{y}_1$ <br> 21. $\overrightarrow{y}_1 + \overrightarrow{c} \rightarrow \overrightarrow{y}_1$ |
| 9 | $\forall \gamma_i \in \Gamma:$ $\overrightarrow{y}_2, \overrightarrow{z}$ <br> $\overrightarrow{G}_i$ | $\forall \gamma_i \in \Gamma:$ 22. $\overrightarrow{y}_1 + \overrightarrow{y}_2 \rightarrow \emptyset$ <br> 23. $\overrightarrow{z} + \overrightarrow{g}_i \rightarrow \overrightarrow{g}_i$ |
| Return to Step 0. | | |

# (3,1) Void Step-Cycle Simulates **Basic** CRNs



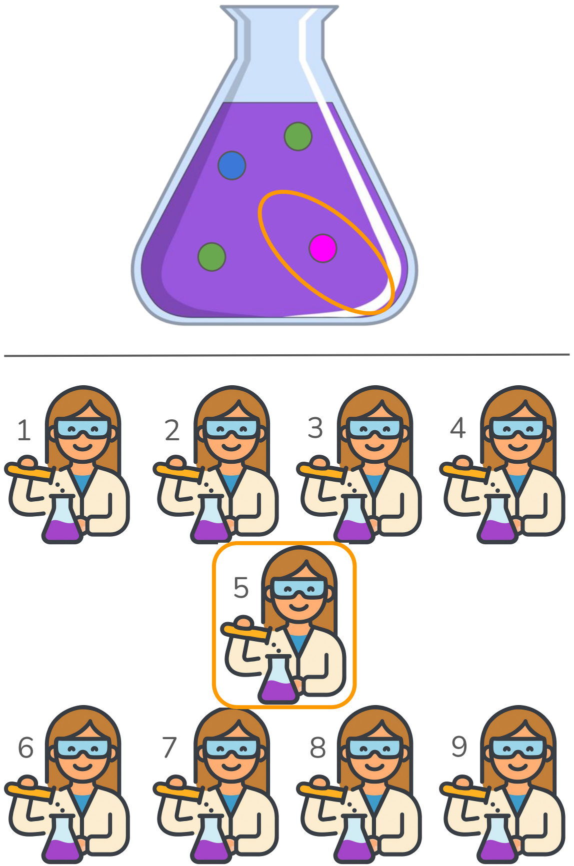

Selection (Steps 0–4)

Execution (Step 5)

Clean Up (Steps 6–9)

| Step | Additions | | Relevant Rules | |
|---|---|---|---|---|
| 0 | | $\vec{G}$ | $\forall \gamma_i \in \Gamma :$ | 1. $\vec{G} + \vec{G}_i + \vec{g}_i \rightarrow \emptyset$<br>24. $\vec{z} + \vec{G} \rightarrow \vec{z}$ |
| 1 | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i :$ | $\vec{E}_i^j$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i :$ | 2. $\vec{G}_i + \vec{E}_i^j \rightarrow \vec{G}_i$ |
| 2 | | $\vec{w}$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i :$ | 3. $\vec{w} + \vec{g}_i \rightarrow \vec{w}$<br>4. $\vec{w} + \vec{r}_j + \vec{E}_i^j \rightarrow \vec{w}$ |
| 3 | | $\vec{a}_y$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i :$ | 5. $\vec{E}_i^j + \vec{a}_y + \vec{w} \rightarrow \vec{E}_i^j$<br>25. $\vec{a}_y + \vec{w} + \vec{z} \rightarrow \emptyset$ |
| 4 | | $\vec{a}_n$ | | 6. $\vec{a}_y + \vec{a}_n + \vec{w} \rightarrow \vec{a}_y$ |
| 5 | $\forall \gamma_i \in \Gamma :$ | $\forall r_j \in \mathcal{R}_i : \vec{r}_j, \vec{R}_i^j$<br>$\forall p_j \in \mathcal{P}_i : \vec{p}_j, \vec{P}_i^j$ | $\forall \gamma_i \in \Gamma :$<br>$\forall r_j \in \mathcal{R}_i :$<br>$\forall p_j \in \mathcal{P}_i :$ | 7. $\vec{G}_i + \vec{r}_j + \vec{R}_i^j \rightarrow \vec{G}_i$<br>8. $\vec{E}_i^j + \vec{r}_j + \vec{R}_i^j \rightarrow \emptyset$<br>9. $\vec{a}_y + \vec{r}_j + \vec{R}_i^j \rightarrow \vec{a}_y$<br>10. $\vec{G}_i + \vec{p}_j + \vec{P}_i^j \rightarrow \vec{G}_i$<br>11. $\vec{a}_n + \vec{p}_j + \vec{P}_i^j \rightarrow \vec{a}_n$ |
| 6 | $\forall \gamma_i \in \Gamma :$ | $\vec{x}, \vec{G}'_i$ | $\forall \gamma_i \in \Gamma :$ | 12. $\vec{x} + \vec{G}'_i + \vec{G}_i \rightarrow \emptyset$ |
| 7 | $\forall \gamma_i \in \Gamma :$ | $\vec{c}$ | $\forall \gamma_i \in \Gamma :$ | 13. $\vec{a}_y + \vec{c} + \vec{G}'_i \rightarrow \vec{a}_y$ |
| 8 | $\forall \gamma_i \in \Gamma :$ | $\vec{g_i}$<br>$\vec{y}_1$ | $\forall \gamma_i \in \Gamma :$<br>$\forall r_j \in \mathcal{R}_i :$<br>$\forall p_j \in \mathcal{P}_i :$ | 14. $\vec{y}_1 + \vec{E}_i^j \rightarrow \vec{y}_1$<br>15. $\vec{y}_1 + \vec{R}_i^j \rightarrow \vec{y}_1$<br>16. $\vec{y}_1 + \vec{P}_i^j \rightarrow \vec{y}_1$<br>17. $\vec{G}'_i + \vec{g}_i \rightarrow \vec{G}'_i$<br>18. $\vec{y}_1 + \vec{x} \rightarrow \vec{y}_1$<br>19. $\vec{y}_1 + \vec{a}_y \rightarrow \vec{y}_1$<br>20. $\vec{y}_1 + \vec{a}_n \rightarrow \vec{y}_1$<br>21. $\vec{y}_1 + \vec{c} \rightarrow \vec{y}_1$ |
| 9 | $\forall \gamma_i \in \Gamma :$ | $\vec{y}_2, \vec{z}$<br>$\vec{G}_i$ | $\forall \gamma_i \in \Gamma :$ | 22. $\vec{y}_1 + \vec{y}_2 \rightarrow \emptyset$<br>23. $\vec{z} + \vec{g}_i \rightarrow \vec{g}_i$ |
| Return to Step 0. | | | | |

# (3,1) Void Step-Cycle Simulates **Basic** CRNs



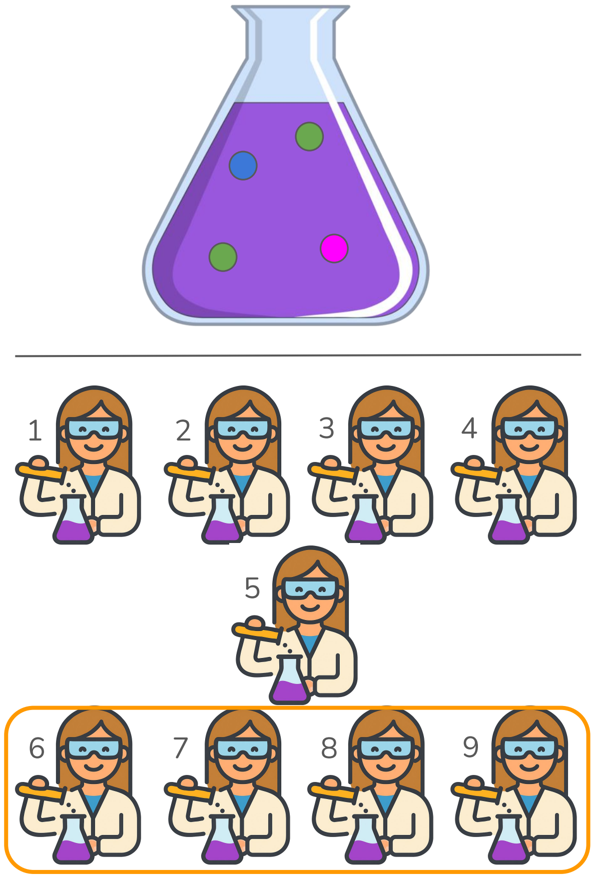

Selection

Execution

Clean Up

| Step | Additions | Relevant Rules |
|---|---|---|
| 0 | $\overrightarrow{G}$ | $\forall \gamma_i \in \Gamma$ : 1. $\overrightarrow{G} + \overrightarrow{G}_i + \overrightarrow{g}_i \rightarrow \emptyset$<br>24. $\overrightarrow{z} + \overrightarrow{G} \rightarrow \overrightarrow{z}$ |
| 1 | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i$ : $\overrightarrow{E}_i^j$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i$ : 2. $\overrightarrow{G}_i + \overrightarrow{E}_i^j \rightarrow \overrightarrow{G}_i$ |
| 2 | $\overrightarrow{w}$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i$ : 3. $\overrightarrow{w} + \overrightarrow{g}_i \rightarrow \overrightarrow{w}$<br>4. $\overrightarrow{w} + \overrightarrow{r}_j + \overrightarrow{E}_i^j \rightarrow \overrightarrow{w}$ |
| 3 | $\overrightarrow{a}_y$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i$ : 5. $\overrightarrow{E}_i^j + \overrightarrow{a}_y + \overrightarrow{w} \rightarrow \overrightarrow{E}_i^j$<br>25. $\overrightarrow{a}_y + \overrightarrow{w} + \overrightarrow{z} \rightarrow \emptyset$ |
| 4 | $\overrightarrow{a}_n$ | 6. $\overrightarrow{a}_y + \overrightarrow{a}_n + \overrightarrow{w} \rightarrow \overrightarrow{a}_y$ |
| 5 | $\forall \gamma_i \in \Gamma$ :<br>$\forall r_j \in \mathcal{R}_i$ : $\overrightarrow{r}_j, \overrightarrow{R}_i^j$<br>$\forall p_j \in \mathcal{P}_i$ : $\overrightarrow{p}_j, \overrightarrow{P}_i^j$ | $\forall \gamma_i \in \Gamma$ :<br>$\forall r_j \in \mathcal{R}_i$ : 7. $\overrightarrow{G}_i + \overrightarrow{r}_j + \overrightarrow{R}_i^j \rightarrow \overrightarrow{G}_i$<br>8. $\overrightarrow{E}_i^j + \overrightarrow{r}_j + \overrightarrow{R}_i^j \rightarrow \emptyset$<br>9. $\overrightarrow{a}_y + \overrightarrow{r}_j + \overrightarrow{R}_i^j \rightarrow \overrightarrow{a}_y$<br>$\forall p_j \in \mathcal{P}_i$ : 10. $\overrightarrow{G}_i + \overrightarrow{p}_j + \overrightarrow{P}_i^j \rightarrow \overrightarrow{G}_i$<br>11. $\overrightarrow{a}_n + \overrightarrow{p}_j + \overrightarrow{P}_i^j \rightarrow \overrightarrow{a}_n$ |
| 6 | $\forall \gamma_i \in \Gamma$ : $\overrightarrow{x}, \overrightarrow{G}'_i$ | $\forall \gamma_i \in \Gamma$ : 12. $\overrightarrow{x} + \overrightarrow{G}'_i + \overrightarrow{G}_i \rightarrow \emptyset$ |
| 7 | $\forall \gamma_i \in \Gamma$ : $\overrightarrow{c}$ | $\forall \gamma_i \in \Gamma$ : 13. $\overrightarrow{a}_y + \overrightarrow{c} + \overrightarrow{G}'_i \rightarrow \overrightarrow{a}_y$ |
| 8 | $\forall \gamma_i \in \Gamma$ : $\overrightarrow{g_i}$<br>$\overrightarrow{y}_1$ | $\forall \gamma_i \in \Gamma$ :<br>$\forall r_j \in \mathcal{R}_i$ : 14. $\overrightarrow{y}_1 + \overrightarrow{E}_i^j \rightarrow \overrightarrow{y}_1$<br>15. $\overrightarrow{y}_1 + \overrightarrow{R}_i^j \rightarrow \overrightarrow{y}_1$<br>$\forall p_j \in \mathcal{P}_i$ : 16. $\overrightarrow{y}_1 + \overrightarrow{P}_i^j \rightarrow \overrightarrow{y}_1$<br>17. $\overrightarrow{G}'_i + \overrightarrow{g}_i \rightarrow \overrightarrow{G}'_i$<br>18. $\overrightarrow{y}_1 + \overrightarrow{x} \rightarrow \overrightarrow{y}_1$<br>19. $\overrightarrow{y}_1 + \overrightarrow{a}_y \rightarrow \overrightarrow{y}_1$<br>20. $\overrightarrow{y}_1 + \overrightarrow{a}_n \rightarrow \overrightarrow{y}_1$<br>21. $\overrightarrow{y}_1 + \overrightarrow{c} \rightarrow \overrightarrow{y}_1$ |
| 9 | $\overrightarrow{y}_2, \overrightarrow{z}$<br>$\forall \gamma_i \in \Gamma$ : $\overrightarrow{G}_i$ | 22. $\overrightarrow{y}_1 + \overrightarrow{y}_2 \rightarrow \emptyset$<br>$\forall \gamma_i \in \Gamma$ : 23. $\overrightarrow{z} + \overrightarrow{g}_i \rightarrow \overrightarrow{g}_i$ |
| Return to Step 0. | | |

# (3,1) Void Step-Cycle Simulates **Basic** CRNs



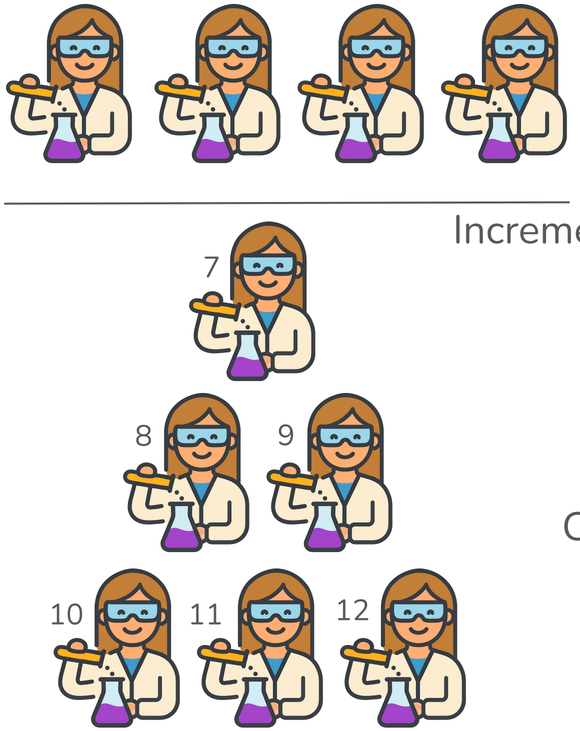

Selection

Execution

Clean Up

| Step | Additions | | Relevant Rules | |
|---|---|---|---|---|
| 0 | | $\vec{G}$ | $\forall \gamma_i \in \Gamma:$ | 1. $\vec{G} + \vec{G}_i + \vec{g}_i \to \emptyset$<br>24. $\vec{z} + \vec{G} \to \vec{z}$ |
| 1 | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i:$ | $\vec{E}_i^j$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i:$ | 2. $\vec{G}_i + \vec{E}_i^j \to \vec{G}_i$ |
| 2 | | $\vec{w}$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i:$ | 3. $\vec{w} + \vec{g}_i \to \vec{w}$<br>4. $\vec{w} + \vec{r}_j + \vec{E}_i^j \to \vec{w}$ |
| 3 | | $\vec{a}_y$ | $\forall \gamma_i \in \Gamma, \forall r_j \in \mathcal{R}_i:$ | 5. $\vec{E}_i^j + \vec{a}_y + \vec{w} \to \vec{E}_i^j$<br>25. $\vec{a}_y + \vec{w} + \vec{z} \to \emptyset$ |
| 4 | | $\vec{a}_n$ | | 6. $\vec{a}_y + \vec{a}_n + \vec{w} \to \vec{a}_y$ |
| 5 | $\forall \gamma_i \in \Gamma:$ | $\forall r_j \in \mathcal{R}_i:$ $\vec{r}_j, \vec{R}_i^j$<br>$\forall p_j \in \mathcal{P}_i:$ $\vec{p}_j, \vec{P}_i^j$ | $\forall \gamma_i \in \Gamma:$<br>$\forall r_j \in \mathcal{R}_i:$<br>$\forall p_j \in \mathcal{P}_i:$ | 7. $\vec{G}_i + \vec{r}_j + \vec{R}_i^j \to \vec{G}_i$<br>8. $\vec{E}_i^j + \vec{r}_j + \vec{R}_i^j \to \emptyset$<br>9. $\vec{a}_y + \vec{r}_j + \vec{R}_i^j \to \vec{a}_y$<br>10. $\vec{G}_i + \vec{p}_j + \vec{P}_i^j \to \vec{G}_i$<br>11. $\vec{a}_n + \vec{p}_j + \vec{P}_i^j \to \vec{a}_n$ |
| 6 | $\forall \gamma_i \in \Gamma:$ | $\vec{x}, \vec{G}'_i$ | $\forall \gamma_i \in \Gamma:$ | 12. $\vec{x} + \vec{G}'_i + \vec{G}_i \to \emptyset$ |
| 7 | $\forall \gamma_i \in \Gamma:$ | $\vec{c}$ | $\forall \gamma_i \in \Gamma:$ | 13. $\vec{a}_y + \vec{c} + \vec{G}'_i \to \vec{a}_y$ |
| 8 | $\forall \gamma_i \in \Gamma:$ | $\vec{g_i}$<br>$\vec{y}_1$ | $\forall \gamma_i \in \Gamma:$<br>$\forall r_j \in \mathcal{R}_i:$<br>$\forall p_j \in \mathcal{P}_i:$ | 14. $\vec{y}_1 + \vec{E}_i^j \to \vec{y}_1$<br>15. $\vec{y}_1 + \vec{R}_i^j \to \vec{y}_1$<br>16. $\vec{y}_1 + \vec{P}_i^j \to \vec{y}_1$<br>17. $\vec{G}'_i + \vec{g}_i \to \vec{G}'_i$<br>18. $\vec{y}_1 + \vec{x} \to \vec{y}_1$<br>19. $\vec{y}_1 + \vec{a}_y \to \vec{y}_1$<br>20. $\vec{y}_1 + \vec{a}_n \to \vec{y}_1$<br>21. $\vec{y}_1 + \vec{c} \to \vec{y}_1$ |
| 9 | $\forall \gamma_i \in \Gamma:$ | $\vec{y}_2, \vec{z}$<br>$\vec{G}_i$ | $\forall \gamma_i \in \Gamma:$ | 22. $\vec{y}_1 + \vec{y}_2 \to \emptyset$<br>23. $\vec{z} + \vec{g}_i \to \vec{g}_i$ |
| Return to Step 0. | | | | |

# (3,1) Void Step-Cycle Simulates **Step** CRNs



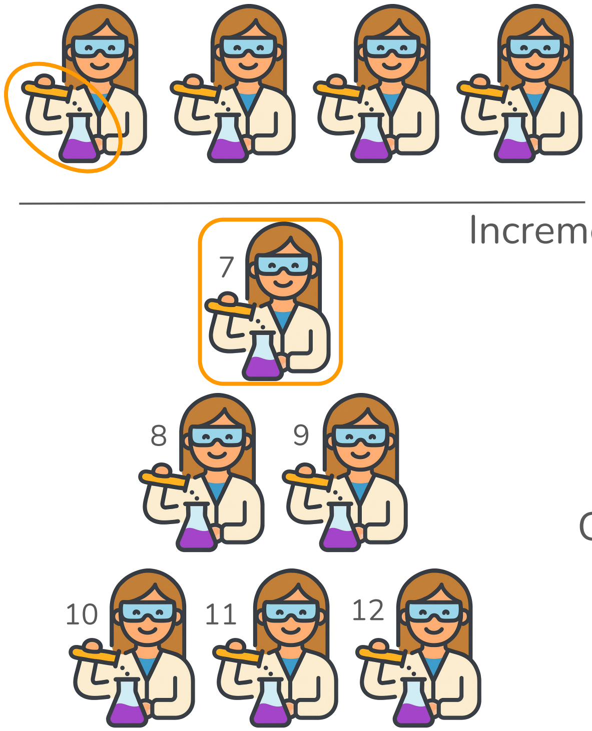

Add Step Species

Increment Step

Clean Up

| Step | Additions | | Relevant Rules | |
|---|---|---|---|---|
| | Steps 0-6 follow from Table 1 | | | |
| 7 | $\forall \gamma_i \in \Gamma :$<br>$\forall \vec{S}_l \in \mathcal{C}_S : \forall s_j \in \vec{S}_l :$ | $\vec{c}$<br>$\vec{s}_j, \vec{S}_l^j, \vec{E}_{l+1}, \vec{b}$ | $\forall \gamma_i \in \Gamma :$<br>$\forall \vec{S}_l \in \mathcal{C}_S :$ | 13. $\vec{d}_y + \vec{c} + \vec{G}'_i \to \vec{d}_y$<br>14. $\vec{S}_l + \vec{s}_j + \vec{S}_l^j \to \vec{S}_l$<br>15. $\vec{S}_l + \vec{E}_{l+1} + \vec{b} \to \vec{S}_l$<br>16. $\vec{x} + \vec{s}_j + \vec{S}_l^j \to \vec{x}$<br>17. $\vec{x} + \vec{E}_l + \vec{b} \to \vec{x}$ |
| 8 | $\forall \gamma_i \in \Gamma :$ | $\vec{y}$<br>$\vec{g}_i$ | $\forall \gamma_i \in \Gamma :$ $\forall r_j \in \mathcal{R}_i :$<br>$\forall p_j \in \mathcal{P}_i :$<br>$\forall \vec{S}_l \in \mathcal{C}_S :$ | 18. $\vec{y}_1 + \vec{E}_i^j \to \vec{y}$<br>19. $\vec{y}_1 + \vec{R}_i^j \to \vec{y}_1$<br>20. $\vec{y}_1 + \vec{P}_i^j \to \vec{y}_1$<br>21. $\vec{G}'_i + \vec{g}_i \to \vec{G}'_i$<br>22. $\vec{y}_1 + \vec{d}_y \to \vec{y}_1$<br>23. $\vec{y}_1 + \vec{d}_n \to \vec{y}_1$<br>36. $\vec{y}_1 + \vec{b} \to \vec{y}_1$<br>37. $\vec{y}_1 + \vec{c} \to \vec{y}_1$<br>26. $\vec{y}_1 + \vec{S}_l^j \to \vec{y}_1$ |
| 9 | $\forall \vec{S}_l \in \mathcal{C}_S :$ | $\vec{S}_l$ | $\forall \vec{S}_l \in \mathcal{C}_S :$ | 27. $\vec{S}_l + \vec{S}_l + \vec{E}_l \to \emptyset$ |
| 10 | $\forall \vec{S}_l \in \mathcal{C}_S :$ | $\vec{d}_l$ | $\forall \vec{S}_l \in \mathcal{C}_S$ | 28. $\vec{S}_l + \vec{S}_l + \vec{d}_l \to \vec{S}_l$<br>29. $\vec{S}_l + \vec{d}_l + \vec{x} \to \vec{x}$ |
| 11 | $\forall \gamma_i \in \Gamma :$ | $\vec{y}_2$<br>$\vec{g}_i$ | $\forall \vec{S}_l \in \mathcal{C}_S :$<br>$\forall \gamma_i \in \Gamma :$ | 30. $\vec{y}_2 + \vec{d}_l \to \vec{y}_2$<br>31. $\vec{y}_2 + \vec{x} \to \vec{y}_2$<br>32. $\vec{y}_2 + \vec{y}_1 \to \vec{y}_2$<br>33. $\vec{G}'_i + \vec{g}_i \to \vec{G}'_i$ |
| 12 | $\forall \gamma_i \in \Gamma :$ | $\vec{y}_2, \vec{z}$<br>$\vec{G}_i$ | | 34. $\vec{y}_2 + \vec{y}_2 \to \emptyset$<br>35. $\vec{z} + \vec{g}_i \to \vec{g}_i$ |
| | Return to Step 0. | | | |

# (3,1) Void Step-Cycle Simulates **Step** CRNs



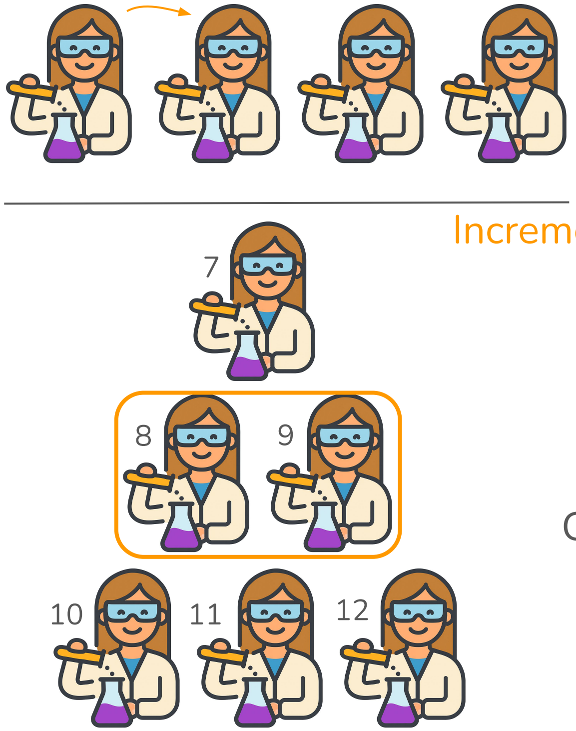

Add Step Species

Increment Step

Clean Up

| Step | Additions | | Relevant Rules | |
|---|---|---|---|---|
| Steps 0-6 follow from Table 1 | | | | |
| 7 | $\forall \gamma_i \in \Gamma$ :<br>$\forall \vec{S}_l \in \mathcal{C}_S : \forall s_j \in \vec{S}_l$ : | $\vec{c}$<br>$\vec{s}_j, \vec{S}^j_l, \vec{E}_{l+1}, \vec{b}$ | $\forall \gamma_i \in \Gamma$ :<br>$\forall \vec{S}_l \in \mathcal{C}_S$ : | 13. $\vec{a}_y + \vec{c} + \vec{G}'_i \to \vec{a}_y$<br>14. $\vec{S}_l + \vec{s}_j + \vec{S}^j_l \to \vec{S}_l$<br>15. $\vec{S}_l + \vec{E}_{l+1} + \vec{b} \to \vec{S}_l$<br>16. $\vec{x} + \vec{s}_j + \vec{S}^j_l \to \vec{x}$<br>17. $\vec{x} + \vec{E}_l + \vec{b} \to \vec{x}$ |
| 8 | $\forall \gamma_i \in \Gamma$ : | $\vec{y}$<br>$\vec{g}_i$ | $\forall \gamma_i \in \Gamma$ : $\forall r_j \in \mathcal{R}_i$ :<br>$\forall p_j \in \mathcal{P}_i$ :<br>$\forall \vec{S}_l \in \mathcal{C}_S$ : | 18. $\vec{y}_1 + \vec{E}^j_i \to \vec{y}$<br>19. $\vec{y}_1 + \vec{R}^j_i \to \vec{y}_1$<br>20. $\vec{y}_1 + \vec{P}^j_i \to \vec{y}_1$<br>21. $\vec{G}'_i + \vec{g}_i \to \vec{G}'_i$<br>22. $\vec{y}_1 + \vec{a}_y \to \vec{y}_1$<br>23. $\vec{y}_1 + \vec{a}_n \to \vec{y}_1$<br>36. $\vec{y}_1 + \vec{b} \to \vec{y}_1$<br>37. $\vec{y}_1 + \vec{c} \to \vec{y}_1$<br>26. $\vec{y}_1 + \vec{S}^j_l \to \vec{y}_1$ |
| 9 | $\forall \vec{S}_l \in \mathcal{C}_S$ : | $\vec{S}_l$ | $\forall \vec{S}_l \in \mathcal{C}_S$ : | 27. $\vec{S}_l + \vec{S}_l + \vec{E}_l \to \emptyset$ |
| 10 | $\forall \vec{S}_l \in \mathcal{C}_S$ : | $\vec{d}_l$ | $\forall \vec{S}_l \in \mathcal{C}_S$ | 28. $\vec{S}_l + \vec{S}_l + \vec{d}_l \to \vec{S}_l$<br>29. $\vec{S}_l + \vec{d}_l + \vec{x} \to \vec{x}$ |
| 11 | $\forall \gamma_i \in \Gamma$ : | $\vec{y}_2$<br>$\vec{g}_i$ | $\forall \vec{S}_l \in \mathcal{C}_S$ :<br>$\forall \gamma_i \in \Gamma$ : | 30. $\vec{y}_2 + \vec{d}_l \to \vec{y}_2$<br>31. $\vec{y}_2 + \vec{x} \to \vec{y}_2$<br>32. $\vec{y}_2 + \vec{y}_1 \to \vec{y}_2$<br>33. $\vec{G}'_i + \vec{g}_i \to \vec{G}'_i$ |
| 12 | $\forall \gamma_i \in \Gamma$ : | $\vec{y}_2, \vec{z}$<br>$\vec{G}_i$ | | 34. $\vec{y}_2 + \vec{y}_2 \to \emptyset$<br>35. $\vec{z} + \vec{g}_i \to \vec{g}_i$ |
| Return to Step 0. | | | | |

# (3,1) Void Step-Cycle Simulates **Step** CRNs



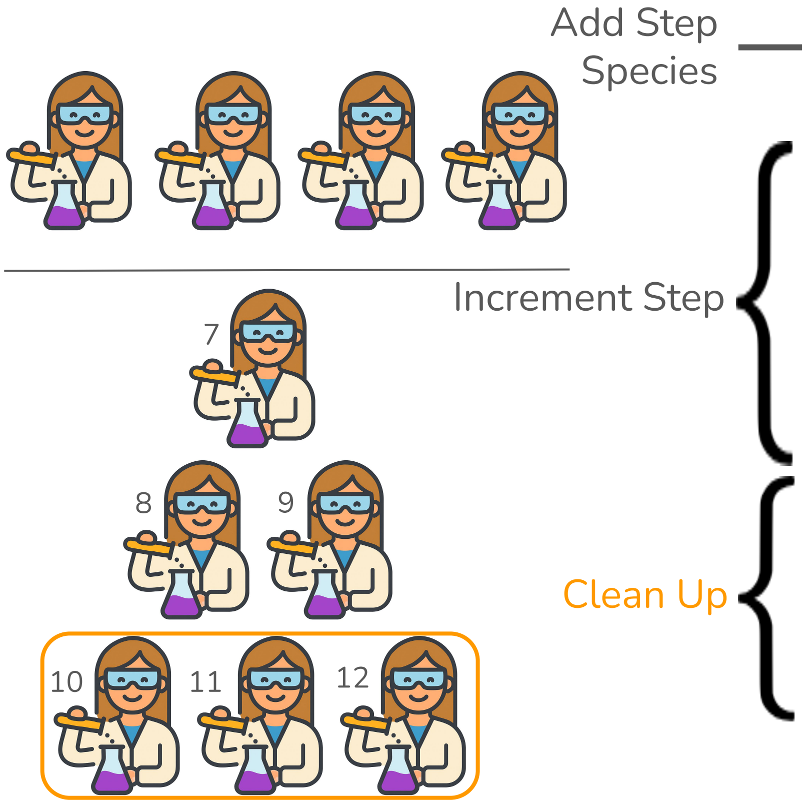

Add Step Species

Increment Step

Clean Up

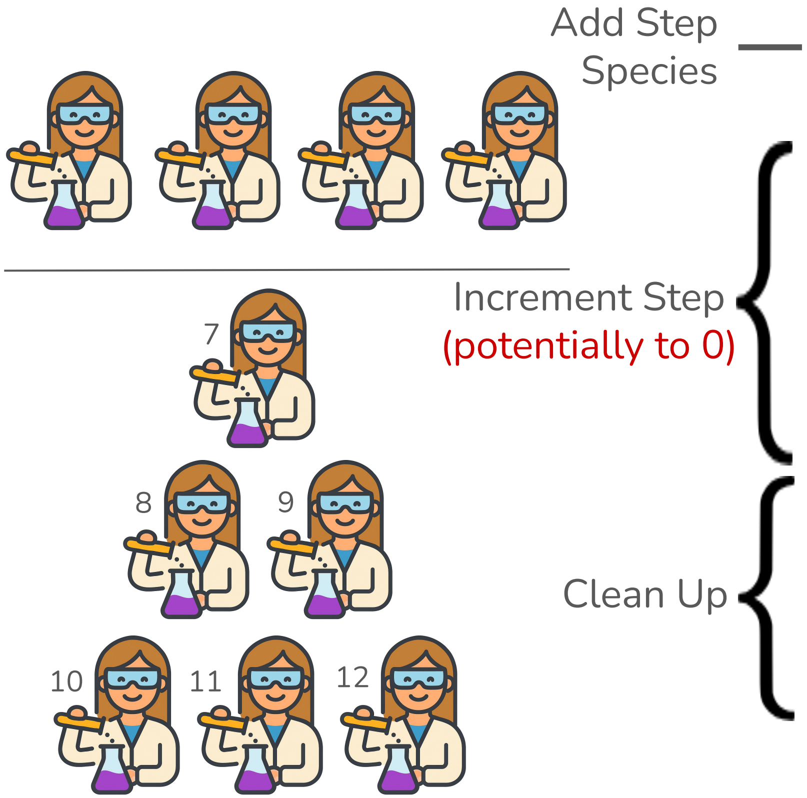
| Step | Additions | | Relevant Rules | |
|---|---|---|---|---|
| Steps 0-6 follow from Table 1 | | | | |
| 7 | $\forall \gamma_i \in \Gamma$ : | $\vec{c}$ | $\forall \gamma_i \in \Gamma$ : | 13. $\vec{d}_y + \vec{c} + \vec{G}'_i \rightarrow \vec{d}_y$ |
| | $\forall \vec{S}_l \in \mathcal{C}_S : \forall s_j \in \vec{S}_l$ : | $\vec{s}_j,\ \vec{S}^j_l,\ \vec{E}_{l+1},\ \vec{b}$ | $\forall \vec{S}_l \in \mathcal{C}_S$ : | 14. $\vec{S}_l + \vec{s}_j + \vec{S}^j_l \rightarrow \vec{S}_l$<br>15. $\vec{S}_l + \vec{E}_{l+1} + \vec{b} \rightarrow \vec{S}_l$<br>16. $\vec{x} + \vec{s}_j + \vec{S}^j_l \rightarrow \vec{x}$<br>17. $\vec{x} + \vec{E}_l + \vec{b} \rightarrow \vec{x}$ |
| 8 | $\forall \gamma_i \in \Gamma$ : | $\vec{y}$<br>$\vec{g}_i$ | $\forall \gamma_i \in \Gamma$ : $\forall r_j \in \mathcal{R}_i$ : | 18. $\vec{y}_1 + \vec{E}^j_i \rightarrow \vec{y}$<br>19. $\vec{y}_1 + \vec{R}^j_i \rightarrow \vec{y}_1$ |
| | | | $\forall p_j \in \mathcal{P}_i$ : | 20. $\vec{y}_1 + \vec{P}^j_i \rightarrow \vec{y}_1$ |
| | | | | 21. $\vec{G}'_i + \vec{g}_i \rightarrow \vec{G}'_i$<br>22. $\vec{y}_1 + \vec{d}_y \rightarrow \vec{y}_1$<br>23. $\vec{y}_1 + \vec{d}_n \rightarrow \vec{y}_1$<br>36. $\vec{y}_1 + \vec{b} \rightarrow \vec{y}_1$<br>37. $\vec{y}_1 + \vec{c} \rightarrow \vec{y}_1$ |
| | | | $\forall \vec{S}_l \in \mathcal{C}_S$ : | 26. $\vec{y}_1 + \vec{S}^j_l \rightarrow \vec{y}_1$ |
| 9 | $\forall \vec{S}_l \in \mathcal{C}_S$ : | $\vec{S}_l$ | $\forall \vec{S}_l \in \mathcal{C}_S$ : | 27. $\vec{S}_l + \vec{S}_l + \vec{E}_l \rightarrow \emptyset$ |
| 10 | $\forall \vec{S}_l \in \mathcal{C}_S$ : | $\vec{d}_l$ | $\forall \vec{S}_l \in \mathcal{C}_S$ | 28. $\vec{S}_l + \vec{S}_l + \vec{d}_l \rightarrow \vec{S}_l$<br>29. $\vec{S}_l + \vec{d}_l + \vec{x} \rightarrow \vec{x}$ |
| 11 | $\forall \gamma_i \in \Gamma$ : | $\vec{y}_2$<br>$\vec{g}_i$ | $\forall \vec{S}_l \in \mathcal{C}_S$ : | 30. $\vec{y}_2 + \vec{d}_l \rightarrow \vec{y}_2$<br>31. $\vec{y}_2 + \vec{x} \rightarrow \vec{y}_2$<br>32. $\vec{y}_2 + \vec{y}_1 \rightarrow \vec{y}_2$ |
| | | | $\forall \gamma_i \in \Gamma$ : | 33. $\vec{G}'_i + \vec{g}_i \rightarrow \vec{G}'_i$ |
| 12 | $\forall \gamma_i \in \Gamma$ : | $\vec{y}_2,\ \vec{z}$<br>$\vec{G}_i$ | | 34. $\vec{y}_2 + \vec{y}_2 \rightarrow \emptyset$<br>35. $\vec{z} + \vec{g}_i \rightarrow \vec{g}_i$ |
| Return to Step 0. | | | | |

# (3,1) Void Step-Cycle Simulates **Step** CRNs




| Step | Additions | | Relevant Rules | |
|---|---|---|---|---|
| Steps 0-6 follow from Table 1 | | | | |
| 7 | $\forall \gamma_i \in \Gamma:$ | $\vec{c}$ | $\forall \gamma_i \in \Gamma:$ | 13. $\vec{a}_y + \vec{c} + \vec{G}'_i \to \vec{a}_y$ |
| | $\forall \vec{S}_l \in \mathcal{C}_S : \forall s_j \in \vec{S}_l :$ | $\vec{s}_j, \vec{S}^j_l, \vec{E}_{l+1}, \vec{b}$ | $\forall \vec{S}_l \in \mathcal{C}_S :$ | 14. $\vec{S}_l + \vec{s}_j + \vec{S}^j_l \to \vec{S}_l$<br>15. $\vec{S}_l + \vec{E}_{l+1} + \vec{b} \to \vec{S}_l$<br>16. $\vec{x} + \vec{s}_j + \vec{S}^j_l \to \vec{x}$<br>17. $\vec{x} + \vec{E}_l + \vec{b} \to \vec{x}$ |
| 8 | $\forall \gamma_i \in \Gamma:$ | $\vec{y}$<br>$\vec{g}_i$ | $\forall \gamma_i \in \Gamma:$ $\forall r_j \in \mathcal{R}_i :$ | 18. $\vec{y}_1 + \vec{E}^j_i \to \vec{y}$<br>19. $\vec{y}_1 + \vec{R}^j_i \to \vec{y}_1$ |
| | | | $\forall p_j \in \mathcal{P}_i :$ | 20. $\vec{y}_1 + \vec{P}^j_i \to \vec{y}_1$<br>21. $\vec{G}'_i + \vec{g}_i \to \vec{G}'_i$<br>22. $\vec{y}_1 + \vec{a}_y \to \vec{y}_1$<br>23. $\vec{y}_1 + \vec{a}_n \to \vec{y}_1$<br>36. $\vec{y}_1 + \vec{b} \to \vec{y}_1$<br>37. $\vec{y}_1 + \vec{c} \to \vec{y}_1$ |
| | | | $\forall \vec{S}_l \in \mathcal{C}_S :$ | 26. $\vec{y}_1 + \vec{S}^j_l \to \vec{y}_1$ |
| 9 | $\forall \vec{S}_l \in \mathcal{C}_S :$ | $\vec{S}_l$ | $\forall \vec{S}_l \in \mathcal{C}_S :$ | 27. $\vec{S}_l + \vec{S}_l + \vec{E}_l \to \emptyset$ |
| 10 | $\forall \vec{S}_l \in \mathcal{C}_S :$ | $\vec{d}_l$ | $\forall \vec{S}_l \in \mathcal{C}_S$ | 28. $\vec{S}_l + \vec{S}_l + \vec{d}_l \to \vec{S}_l$<br>29. $\vec{S}_l + \vec{d}_l + \vec{x} \to \vec{x}$ |
| 11 | $\forall \gamma_i \in \Gamma:$ | $\vec{y}_2$<br>$\vec{g}_i$ | $\forall \vec{S}_l \in \mathcal{C}_S :$ | 30. $\vec{y}_2 + \vec{d}_l \to \vec{y}_2$<br>31. $\vec{y}_2 + \vec{x} \to \vec{y}_2$<br>32. $\vec{y}_2 + \vec{y}_1 \to \vec{y}_2$ |
| | | | $\forall \gamma_i \in \Gamma:$ | 33. $\vec{G}'_i + \vec{g}_i \to \vec{G}'_i$ |
| 12 | $\forall \gamma_i \in \Gamma:$ | $\vec{y}_2, \vec{z}$<br>$\vec{G}_i$ | | 34. $\vec{y}_2 + \vec{y}_2 \to \emptyset$<br>35. $\vec{z} + \vec{g}_i \to \vec{g}_i$ |
| Return to Step 0. | | | | |

# (3,1) Void Step-Cycle Simulates ~~Step~~ Step-Cycle CRNs




| Step | Additions | | Relevant Rules | |
|---|---|---|---|---|
| Steps 0-6 follow from Table 1 | | | | |
| 7 | $\forall \gamma_i \in \Gamma$ :<br>$\forall \vec{S}_l \in \mathcal{C}_S : \forall s_j \in \vec{S}_l$ : | $\vec{c}$<br>$\vec{s}_j, \vec{S}_l^j, \vec{E}_{l+1}, \vec{b}$<br>$\vec{E}_0$ | $\forall \gamma_i \in \Gamma$ :<br>$\forall \vec{S}_l \in \mathcal{C}_S$ : | 13. $\vec{a}_y + \vec{c} + \vec{G}'_i \to \vec{a}_y$<br>14. $\vec{S}_l + \vec{s}_j + \vec{S}_l^j \to \vec{S}_l$<br>15. $\vec{S}_l + \vec{E}_{l+1} + \vec{b} \to \vec{S}_l$<br>16. $\vec{x} + \vec{s}_j + \vec{S}_l^j \to \vec{x}$<br>17. $\vec{x} + \vec{E}_l + \vec{b} \to \vec{x}$ |
| 8 | $\forall \gamma_i \in \Gamma$ : | $\vec{y}$<br>$\vec{g}_i$ | $\forall \gamma_i \in \Gamma$ : $\forall r_j \in \mathcal{R}_i$ :<br>$\forall p_j \in \mathcal{P}_i$ :<br>$\forall \vec{S}_l \in \mathcal{C}_S$ : | 18. $\vec{y}_1 + \vec{E}_i^j \to \vec{y}$<br>19. $\vec{y}_1 + \vec{R}_i^j \to \vec{y}_1$<br>20. $\vec{y}_1 + \vec{P}_i^j \to \vec{y}_1$<br>21. $\vec{G}'_i + \vec{g}_i \to \vec{G}'_i$<br>22. $\vec{y}_1 + \vec{a}_y \to \vec{y}_1$<br>23. $\vec{y}_1 + \vec{a}_n \to \vec{y}_1$<br>36. $\vec{y}_1 + \vec{b} \to \vec{y}_1$<br>37. $\vec{y}_1 + \vec{c} \to \vec{y}_1$<br>26. $\vec{y}_1 + \vec{S}_l^j \to \vec{y}_1$ |
| 9 | $\forall \vec{S}_l \in \mathcal{C}_S$ : | $\vec{S}_l$ | $\forall \vec{S}_l \in \mathcal{C}_S$ : | 27. $\vec{S}_l + \vec{S}_l + \vec{E}_l \to \emptyset$ |
| 10 | $\forall \vec{S}_l \in \mathcal{C}_S$ : | $\vec{d}_l$ | $\forall \vec{S}_l \in \mathcal{C}_S$ | 28. $\vec{S}_l + \vec{S}_l + \vec{d}_l \to \vec{S}_l$<br>29. $\vec{S}_l + \vec{d}_l + \vec{x} \to \vec{x}$ |
| 11 | $\forall \gamma_i \in \Gamma$ : | $\vec{y}_2$<br>$\vec{g}_i$ | $\forall \vec{S}_l \in \mathcal{C}_S$ :<br>$\forall \gamma_i \in \Gamma$ : | 30. $\vec{y}_2 + \vec{d}_l \to \vec{y}_2$<br>31. $\vec{y}_2 + \vec{x} \to \vec{y}_2$<br>32. $\vec{y}_2 + \vec{y}_1 \to \vec{y}_2$<br>33. $\vec{G}'_i + \vec{g}_i \to \vec{G}'_i$ |
| 12 | $\forall \gamma_i \in \Gamma$ : | $\vec{y}_2, \vec{z}$<br>$\vec{G}_i$ | | 34. $\vec{y}_2 + \vec{y}_2 \to \emptyset$<br>35. $\vec{z} + \vec{g}_i \to \vec{g}_i$ |
| Return to Step 0. | | | | |

# (3,1) Step-Cycle CRNs is Turing Universal

(3,1) Void Step-Cycle CRN $C'$

$$A + B + Y \rightarrow B$$

General Step-Cycle CRN $C$

$$D + A \rightarrow B + Y + C$$

equivalent under
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*[Bajaj et al., ISAAC 2025 (to appear)]

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(3,1) Void CRN

General CRN

Reachability is
NP-complete

=/=

Reachability is
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# Future Work

Our notion of polynomial simulation actually captures “expected” polynomial-ness. Can we achieve a tighter connection between the transition probabilities for both the original and simulating systems?

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# Questions?