1 The Tile Complexity Gap Between Determinism and Nondeterminism

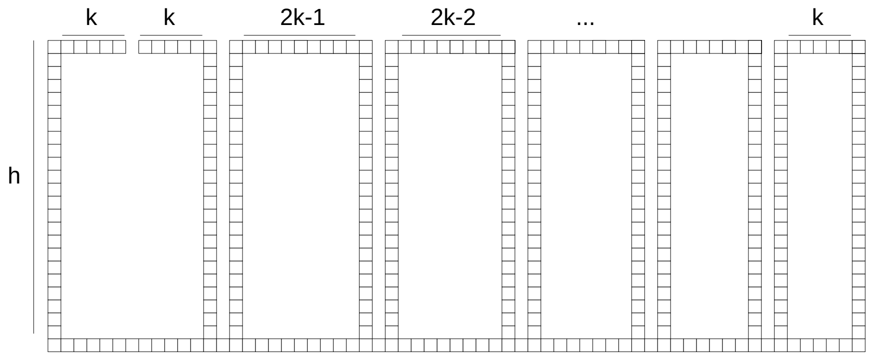

Figure 1: Here's a shape!

Conjecture 1.1. For a given h and k, the deterministic tile complexity of the shape from Figure 1 is at least $\Omega(kh + k^2)$.

Observation 1.2. For a given h and k, The nondeterministic tile complexity of the shape from Figure 1 is at most $O(h + k^2)$.

Observation 1.3. For a given h and k, the size of the shape from Figure 1 is $\Theta(kh + k^2)$.

Theorem 1.4. There exists a class of shapes such that the smallest tileset to deterministically assemble a given shape is a factor $\Theta(n^{1/3})$ larger than the smallest tileset to nondeterministically assembly the given shape, where n is the size of the shape.

Proof. For the class of shapes described in Figure 1, consider the subclass consisting of the shapes in which the value $k = h^{1/2}$. With this value of k, we know from Conjecture 1.1 that the deterministic tile complexity for the assembly of each shape is $\Omega(h^{3/2})$. From Observation 1.2, we know the nondeterministic tile complexity is $O(h)$. Therefore, the ratio of the deterministic complexity over the nondeterministic complexity is at least $\Omega(h^{1/2})$. Finally, we know from Observation 1.3 that the shape has size $n = \Theta(h^{3/2})$. Thus, the $\Omega(h^{1/2})$ factor gap between deterministic and nondeterministic tile complexity is at least $\Theta(n^{1/3})$. □