How efficiently can you build an $n \times n$ square?
How efficiently can you build an n x n square?

Tile Complexity: 2n
How efficiently can you build an $n \times n$ square?

-Use $\log n$ tile types to seed counter:
How efficiently can you build an n x n square?

- Use $\log n$ tile types capable of binary counting:

- Use 8 additional tile types capable of binary counting:
How efficiently can you build an $n \times n$ square?

- Use $\log n$ tile types capable of binary counting:

- Use 8 additional tile types capable of binary counting:

<table>
<thead>
<tr>
<th>1</th>
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<tr>
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<td>0 0 0</td>
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<td>0 0 0 1</td>
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<td>0 0 0 0</td>
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</tbody>
</table>

$log n$
How efficiently can you build an n x n square?

- Use $\log n$ tile types capable of Binary counting:

- Use 8 additional tile types capable of binary counting:

```
1 1 1 1
1 1 1 0
1 1 0 1
1 1 0 0
1 0 1 1
1 0 1 0
1 0 0 1
1 0 0 0
0 1 1 1
0 1 1 0
0 1 0 1
0 1 0 0
0 0 1 1
0 0 1 0
0 0 0 1
0 0 0 0
```
How efficiently can you build an $n \times n$ square?

- Use $\log n$ tile types capable of Binary counting:

- Use 8 additional tile types capable of binary counting:
How efficiently can you build an $n \times n$ square?

Tile Complexity:
$O(\log n)$
(Rothemund, Winfree 2000)
Binary Counter
(Rothemund, Winfree 2000)

Tile set:

Assembly:

8 tile types

log n tile types

n x log n rectangle: $O(\log n)$ tile complexity
How efficiently can you build an $n \times n$ square?

Tile Complexity: $O(\log n)$
(Rothemund, Winfree 2000)

Can this be beat?

What is a lower bound?
How efficiently can you build an $n \times n$ square?

Tile Complexity: $O(\log n)$
(Rothemund, Winfree 2000)

Can this be beat?

What is a lower bound?

Challenge:
Derive information theoretic lower bound
Wow. Can we meet that?

**Lower Bound:**
(almost all $n$)

$$\Omega\left(\frac{\log n}{\log \log n}\right)$$
(Rothemund, Winfree 2000)

**Current Upper Bound:**

$O(\log n)$
(Rothemund, Winfree 2000)
Build a 2 x 16 rectangle: $t = 2$

2 x $n$ lines

<table>
<thead>
<tr>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega(\sqrt{n})$</td>
<td>$O(\sqrt{n})$</td>
</tr>
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</table>
Building $k \times n$ Rectangles

$k$-digit, base $n^{(1/k)}$ counter:
Building $k \times n$ Rectangles

$k$-digit, base $n^{(1/k)}$ counter:

<p>| | | | | | | | | |</p>
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<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>S0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Tile Complexity: $O(k + n^{1/k})$
Building $k \times n$ Rectangles

$k$-digit, base $n^{(1/k)}$ counter:

\[
\begin{array}{cccc}
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 1 \\
 S0 & 1 & 2 & 0 \\
\end{array}
\quad \rightarrow \quad
\begin{array}{ccc}
 2 & 2 & 2 \\
 2 & 2 & 2 \\
 2 & 2 & 2 \\
 0 & 1 & 2 \\
\end{array}
\]

$k \times n$ rectangles

Lower Bound \quad Upper Bound

? \quad \mathcal{O}(k + n^{1/k})
Outline

• Background, Motivation
• Model
• Rectangle and Squares
How efficiently can you build an n x n square?
How efficiently can you build an $n \times n$ square?
How efficiently can you build an \( n \times n \) square?

Tile Complexity: \( 2n \)
How efficiently can you build an $n \times n$ square?

Tile Complexity:

$O(k + n^{1/k})$
How efficiently can you build an n x n square?

Tile Complexity:

\[ O(k + n^{1/k}) \]

Tile Complexity:

\[ O\left(\frac{\log n}{\log \log n}\right) \]

(Adleman, Cheng, Goel, Huang STOC 2001)
How efficiently can you build an $n \times n$ square?

**Tile Complexity:**

$$O(k + n^{1/k})$$

**Tile Complexity:**

$$O\left(\frac{\log n}{\log \log n}\right)$$

(Adleman, Cheng, Goel, Huang STOC 2001)

**Lower Bound:**

$$\Omega\left(\frac{\log n}{\log \log n}\right)$$

(Rothemund, Winfree 2000)