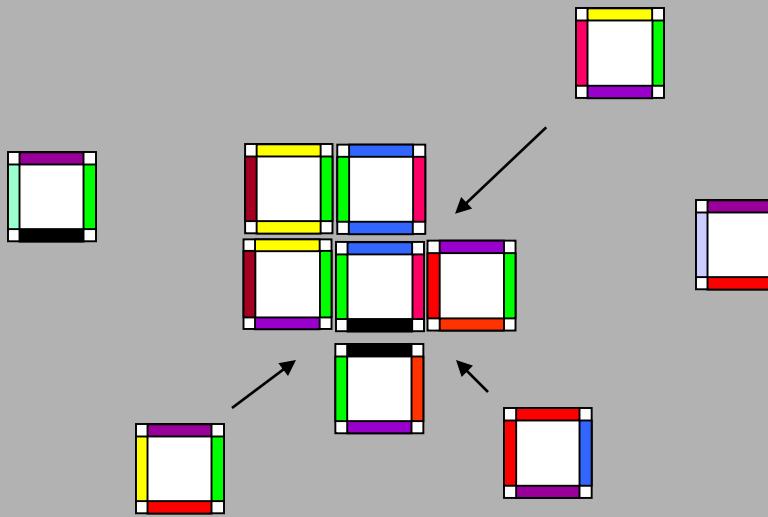
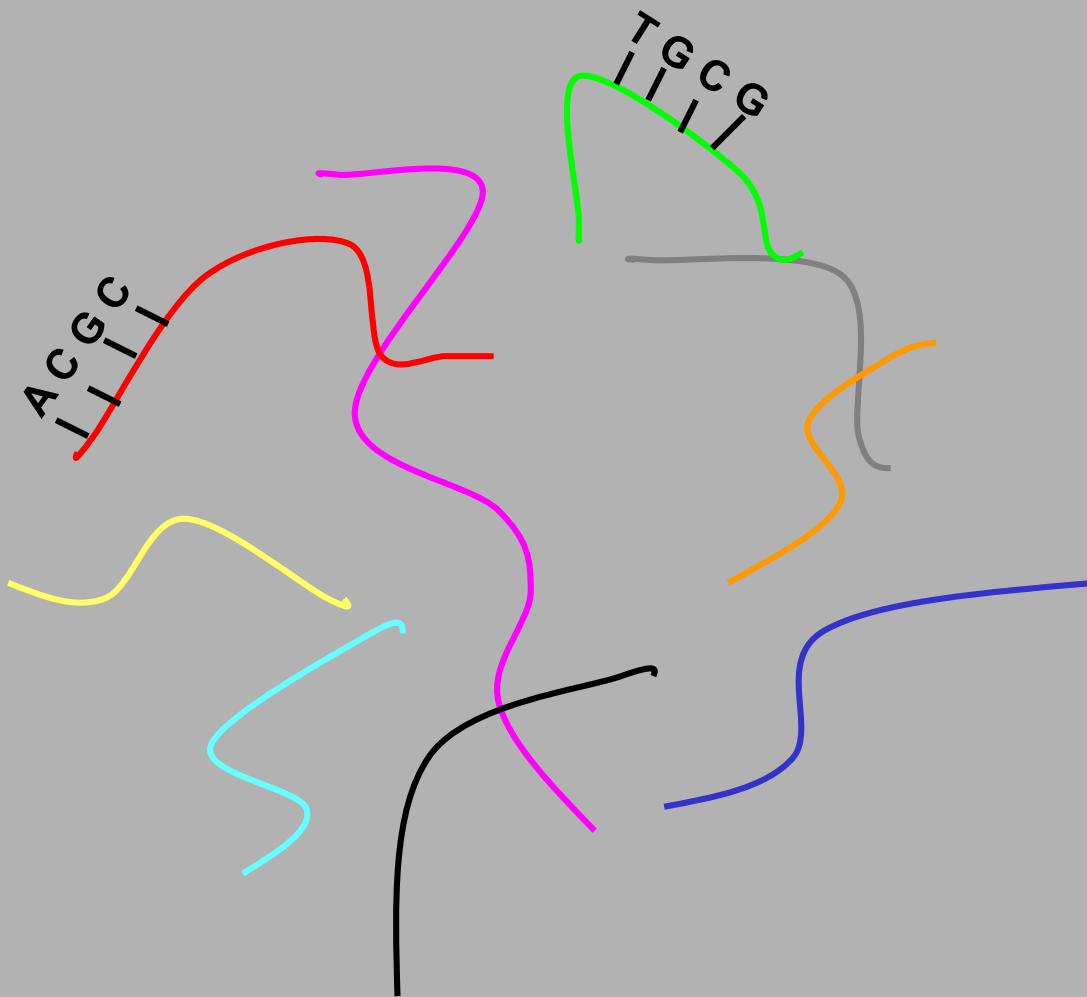


Abstract Tile Self-Assembly Model and the Complexity of Self-Assembled Rectangles

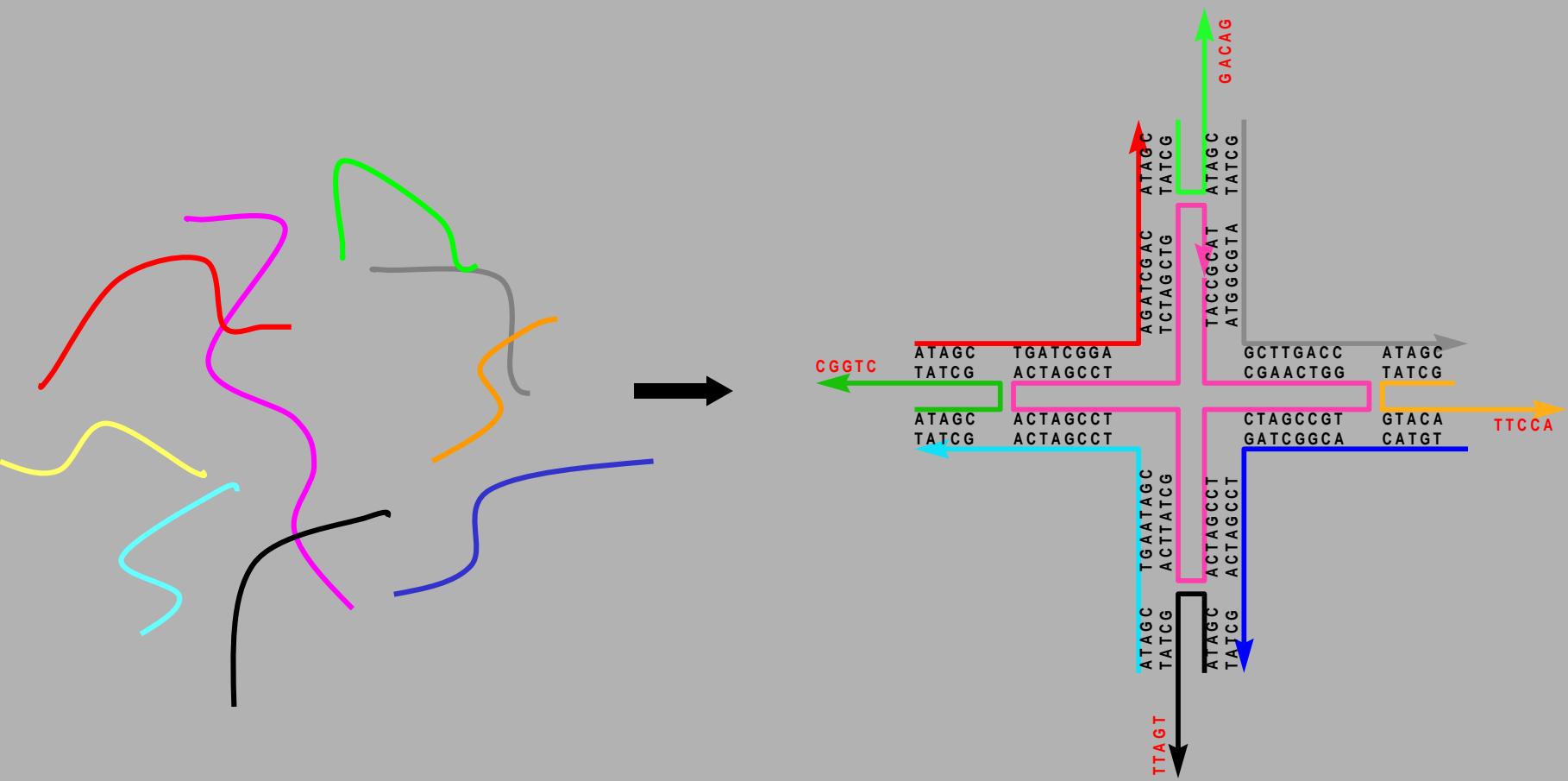


Molecular Building Blocks

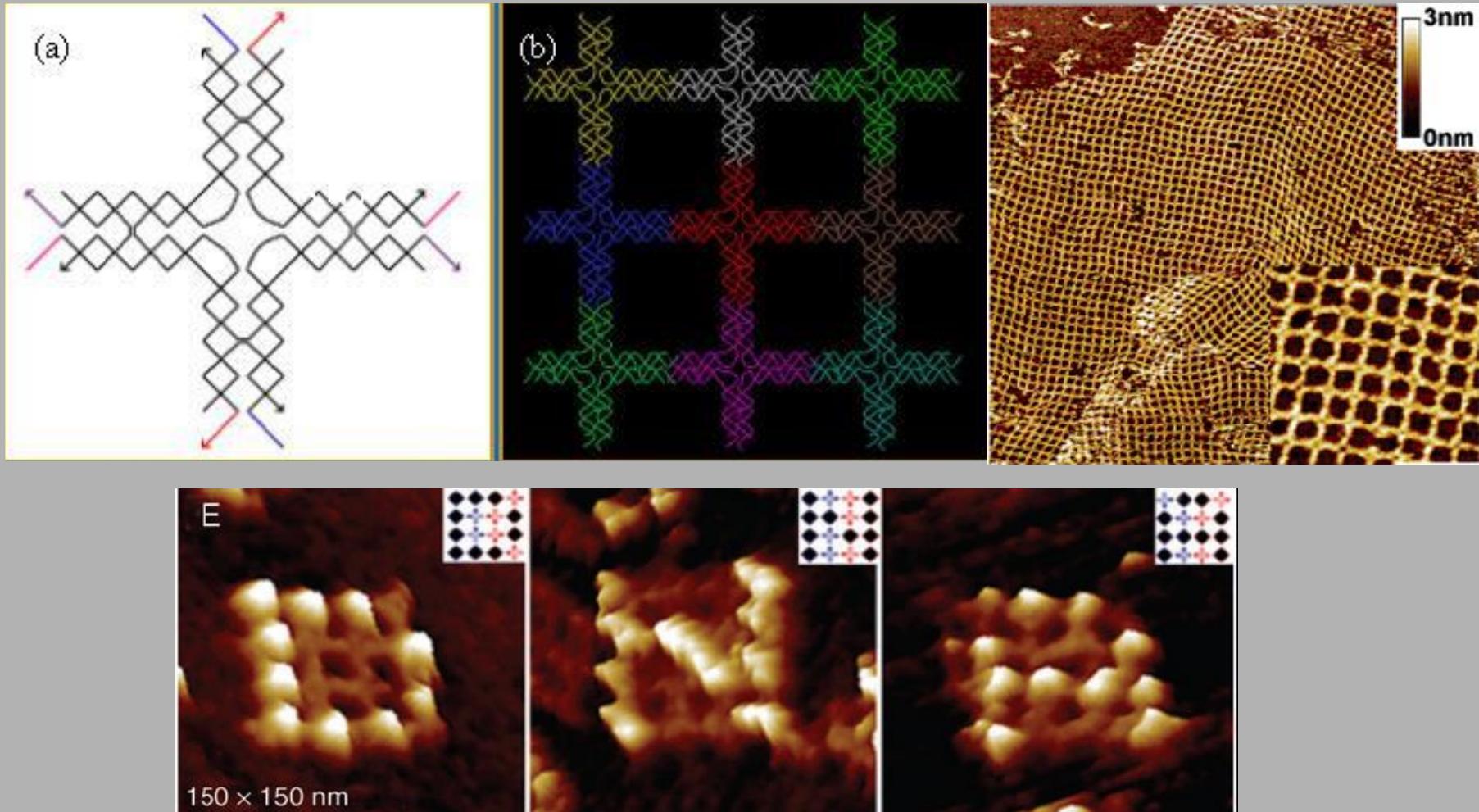


Molecular Building Blocks

[Reif's Group, Duke University]



DNA Scaffolding

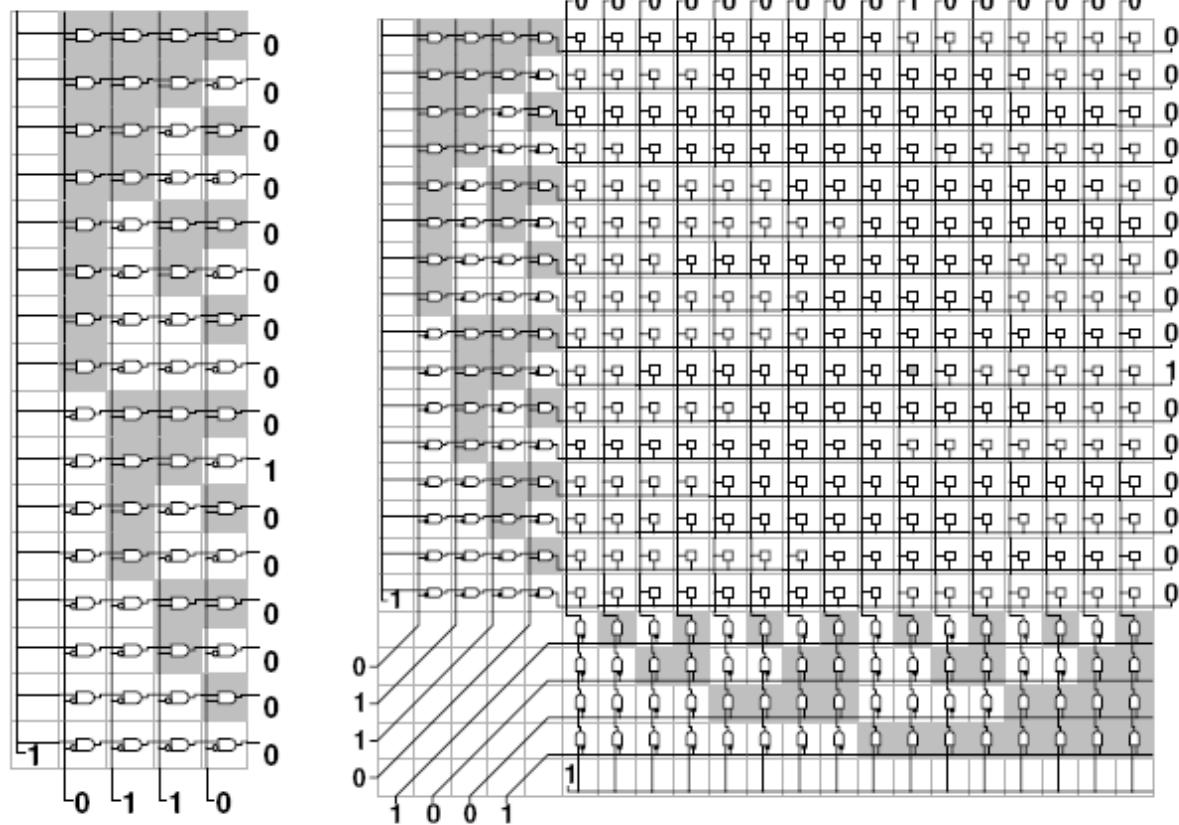


[Sung Ha Park, Constantin Pistol, Sang Jung Ahn, John H. Reif,
Alvin R. Lebeck, Chris Dwyer, and Thomas H. LaBean, 2006]

Self-Assembly for Circuit Patterns

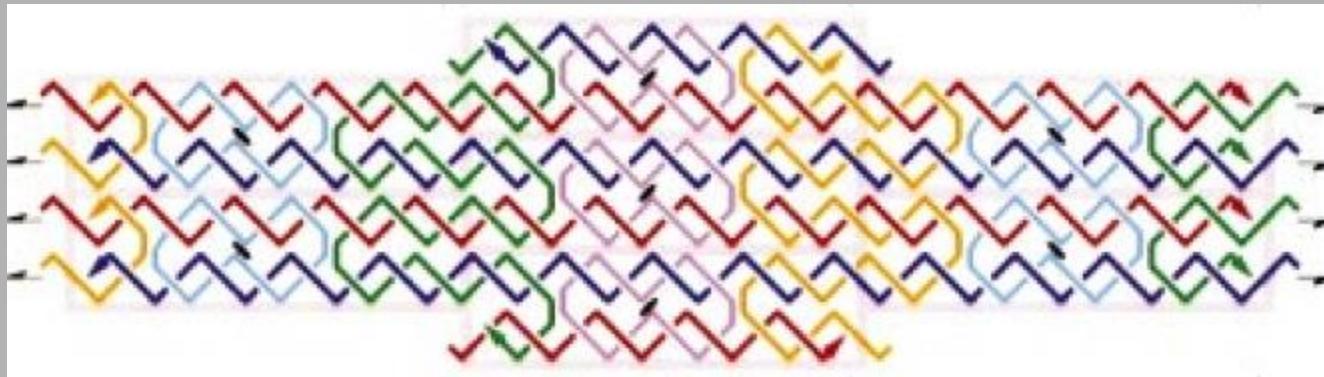
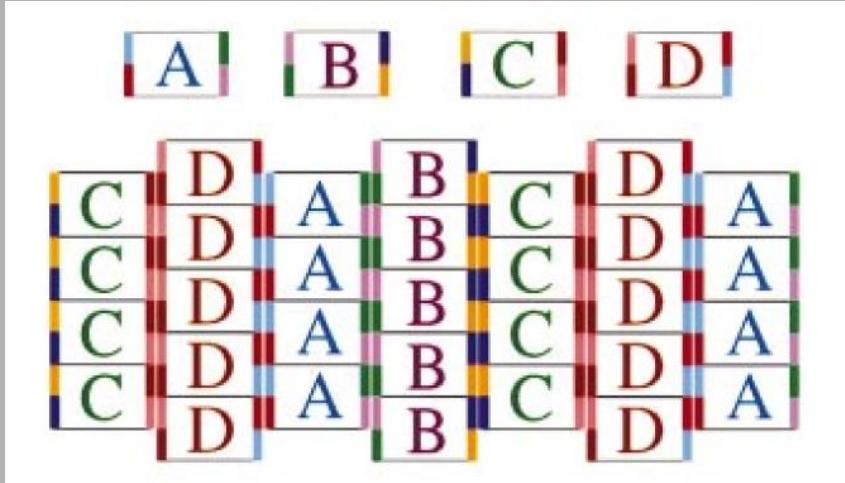
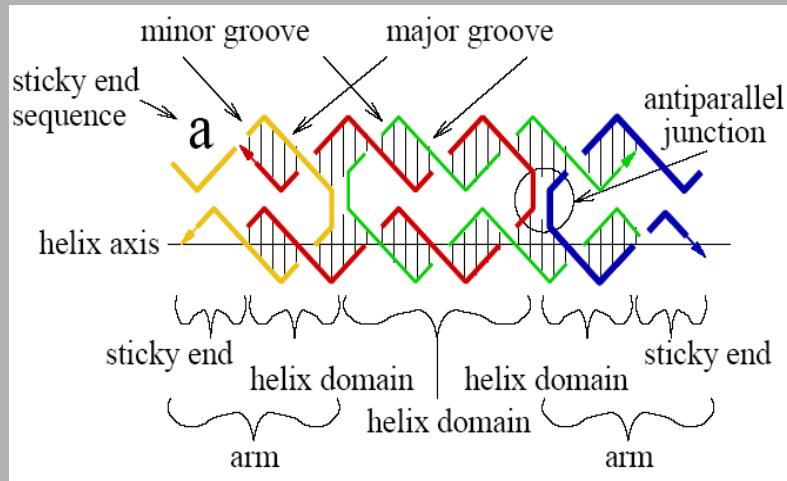
[Cook, Rothemund, and Winfree, 2003]

seed tile		:	
WIRE		:	
input tiles		:	
AND-NOT		:	
rule tiles		:	
AND		:	
AND-NOT		:	
WIRE		:	



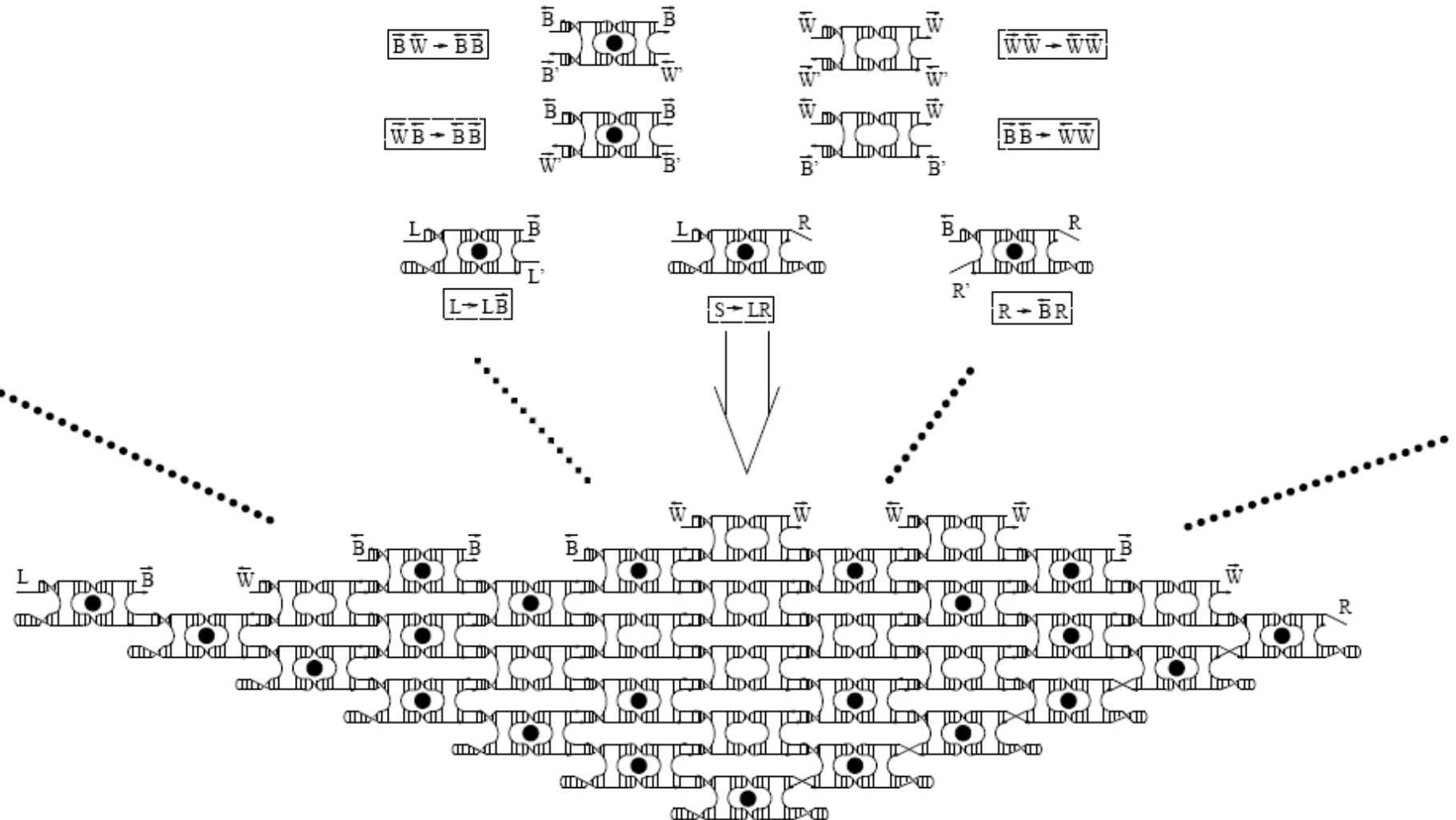
More Examples of DNA Tiles

[Winfree's Group, Cal Tech]



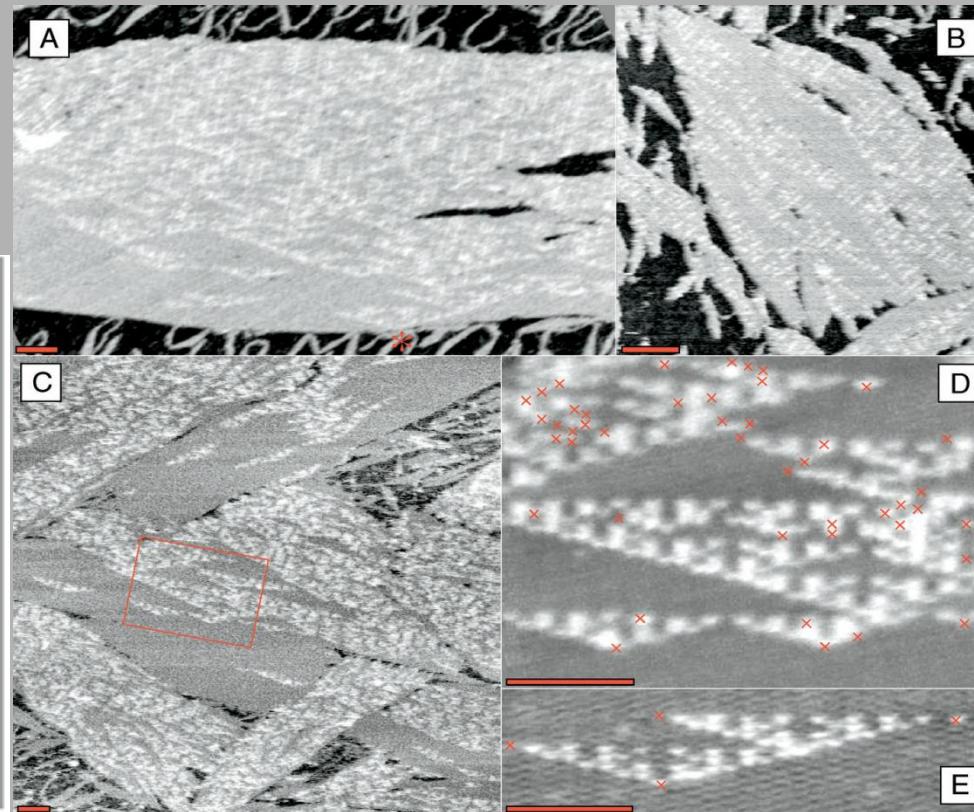
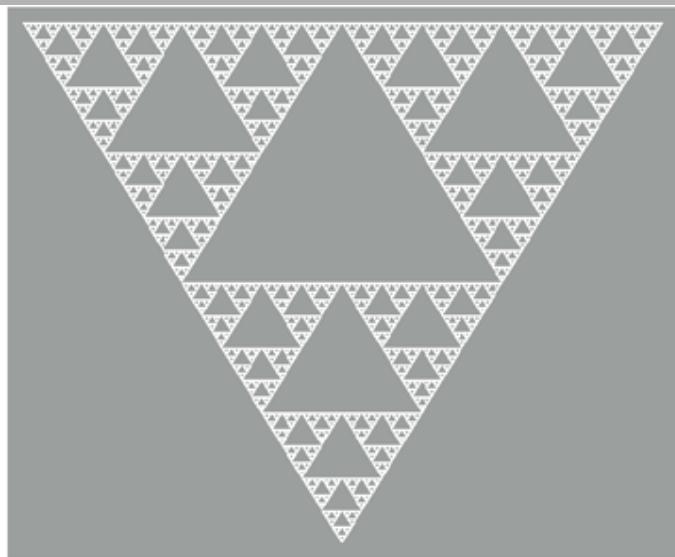
2D Self-Assembly for Turing Machines

[Winfree, Yang, and Seeman, 1998]



Simulation of Cellular Automata

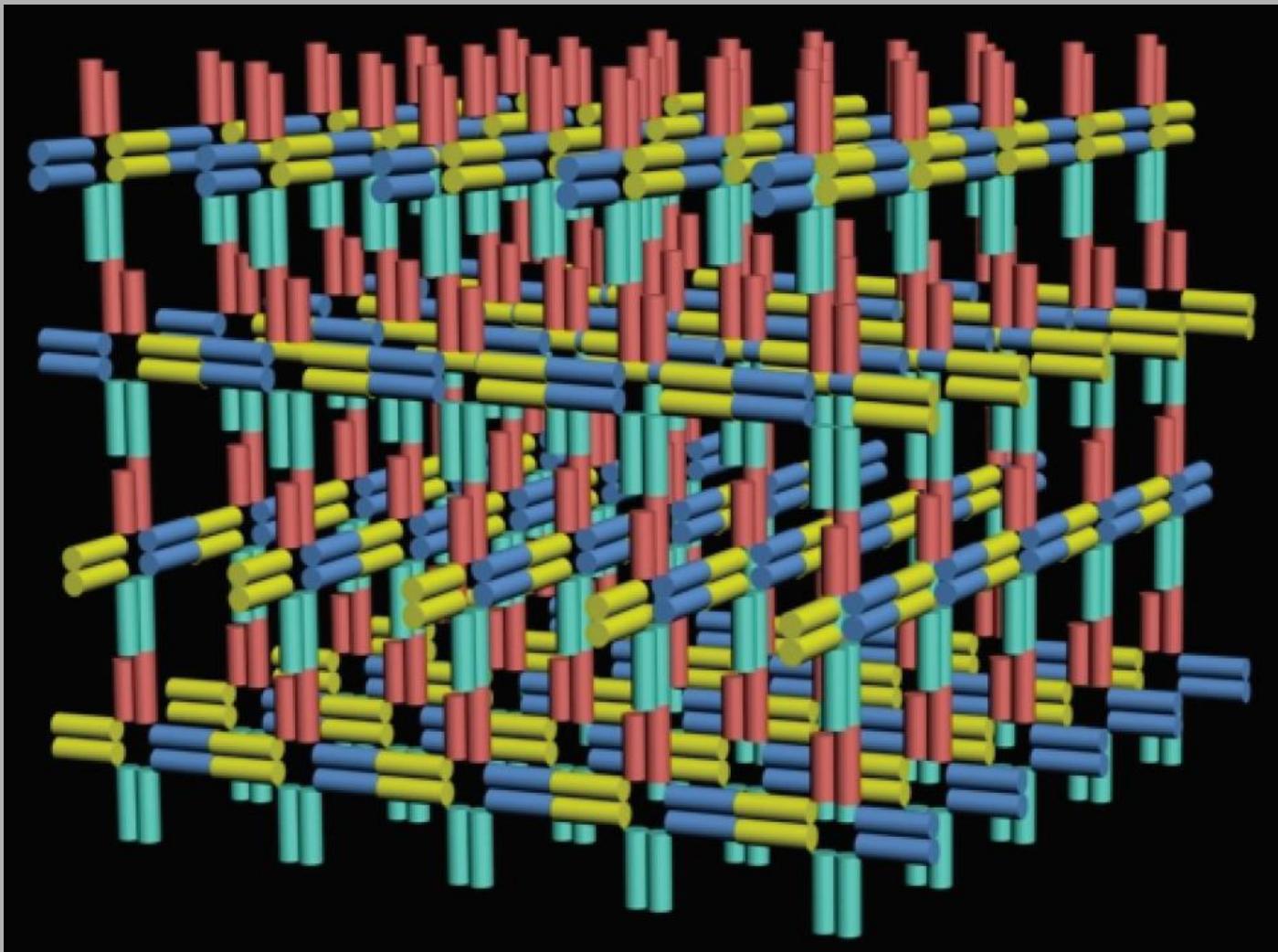
Paul Rothemund, Nick
Papadakis, Erik Winfree,
PLoS Biology 2: e424
(2004)



340nm

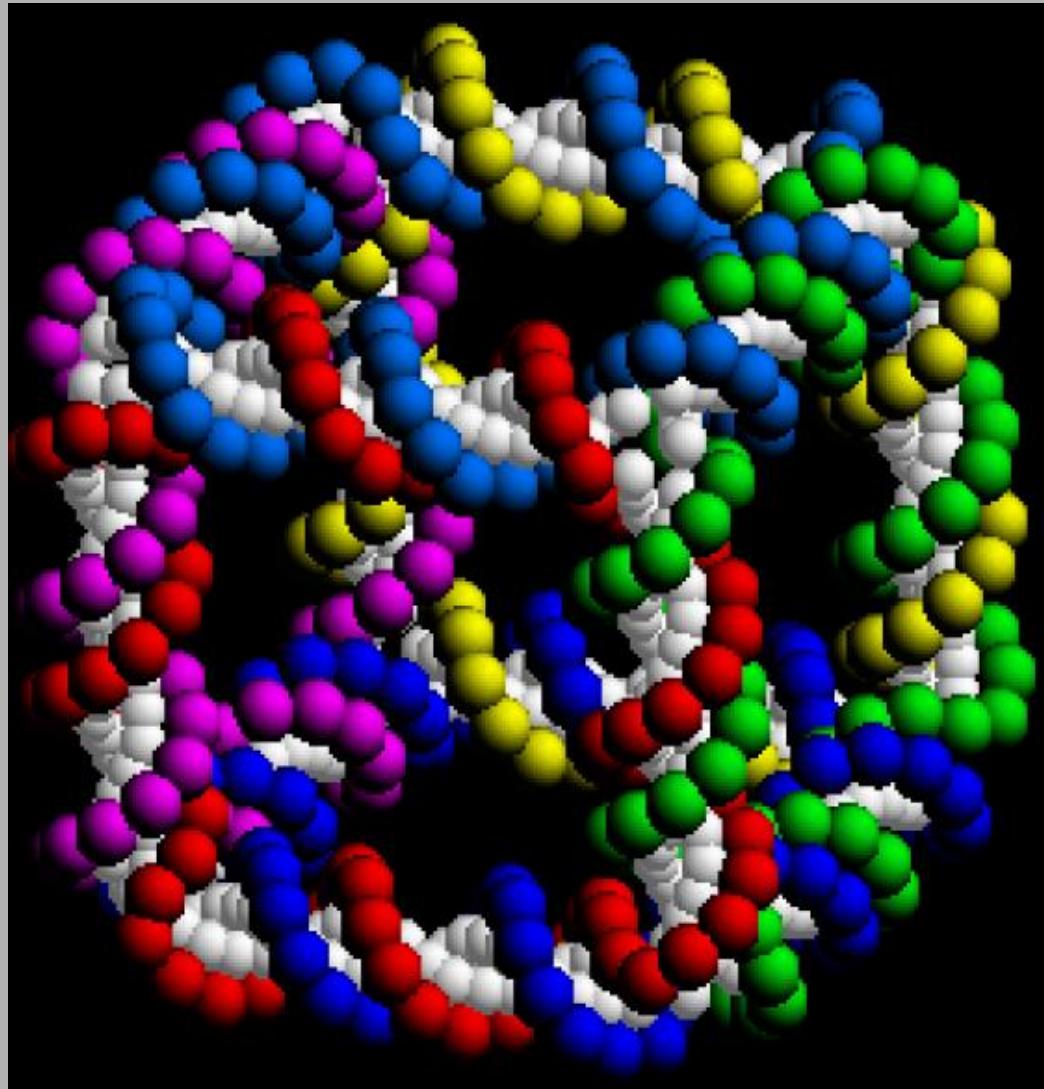
Example of 3D Self-Assembly

[Shaw, University of Southern California]



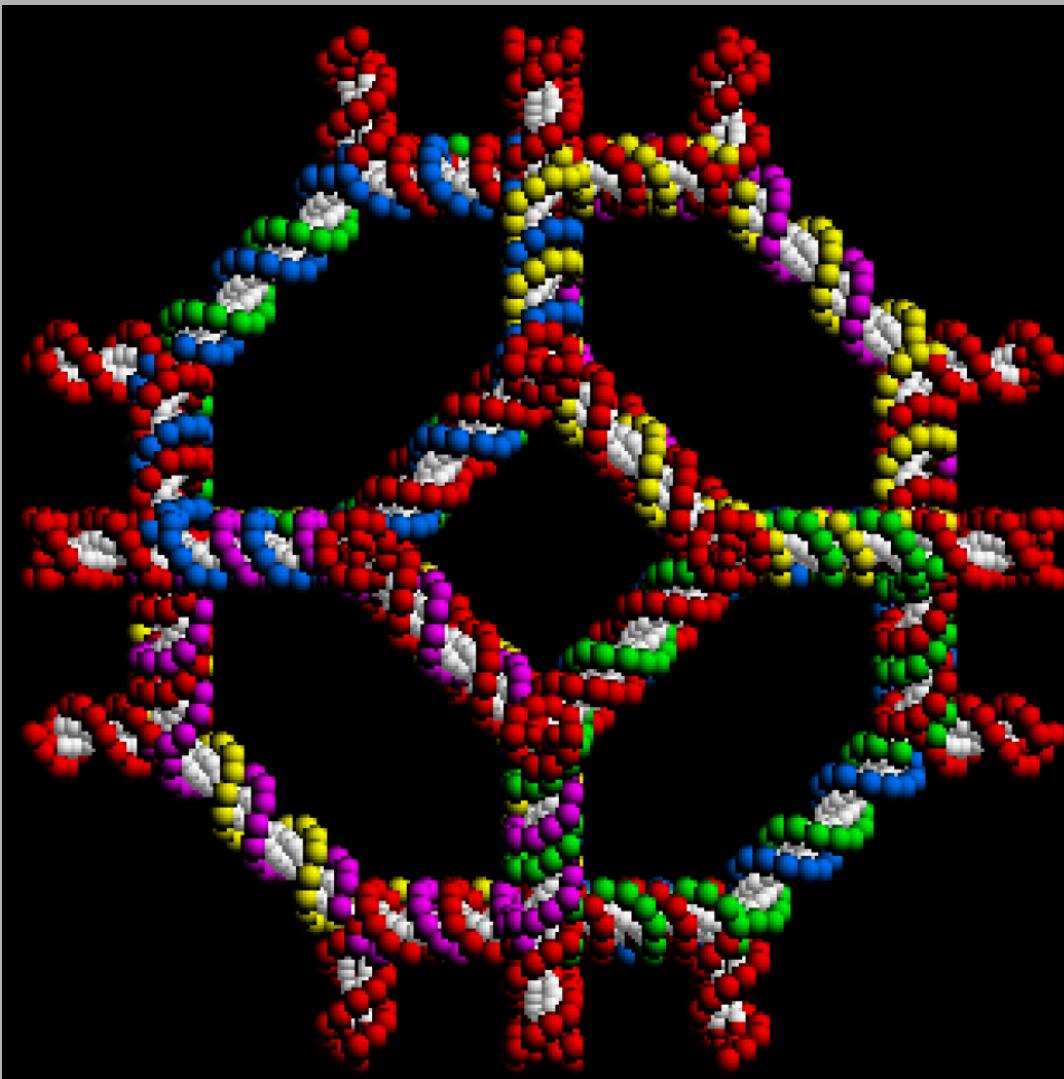
3D DNA Cube

[Seeman, New York University]



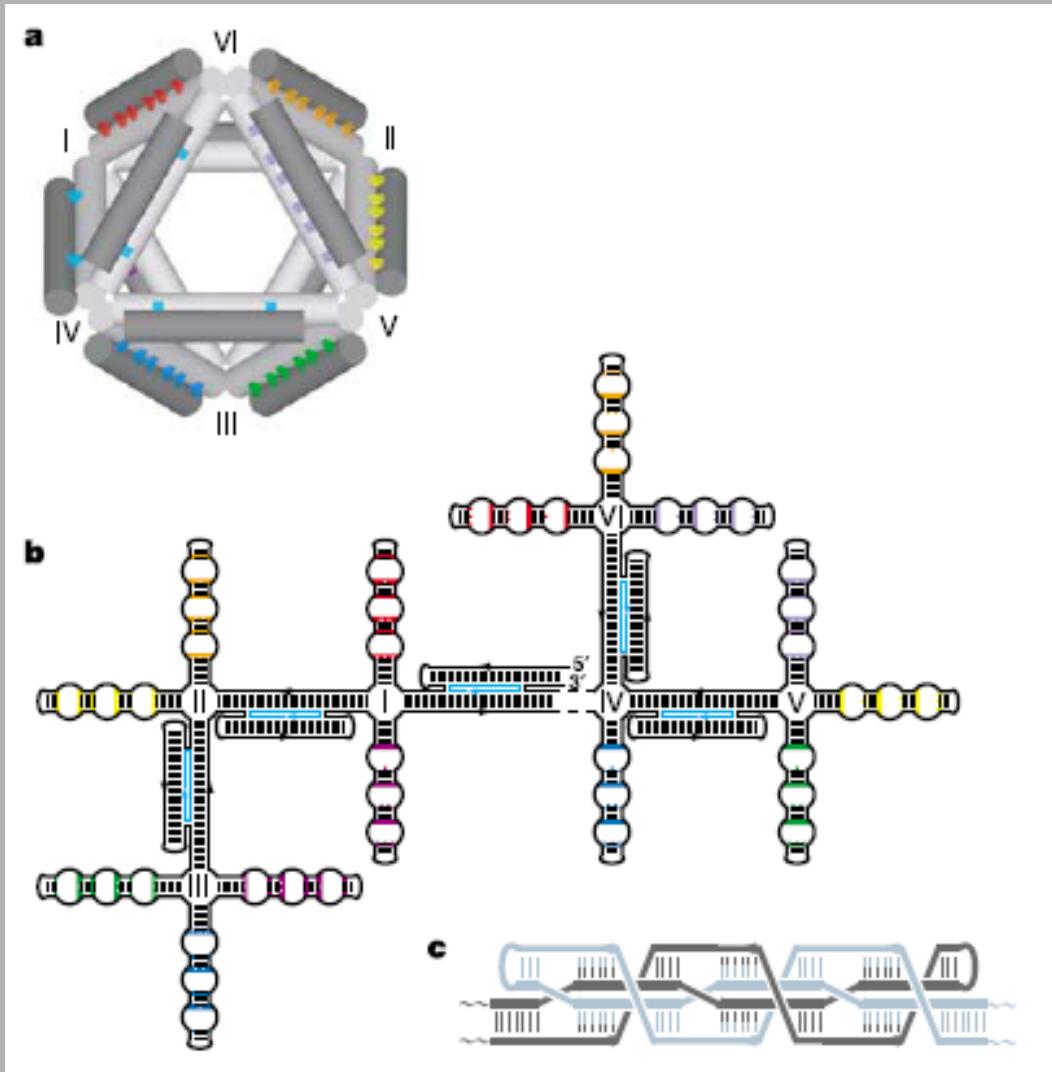
3D DNA Truncated Octahedron

[Seeman, New York University]



Clonable DNA Octahedron

[Shih, Quispe, Joyce, 2004]



Outline

- Background, Motivation
- Model
- Rectangle

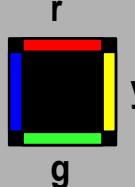
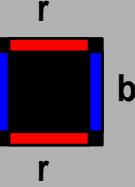
Tile Model of Self-Assembly

(Rothenmund, Winfree STOC 2000)

Tile System: $\{t, G, T, s\}$

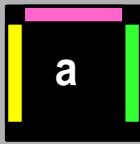
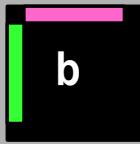
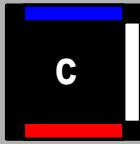
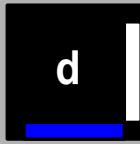
t : temperature, positive integer

G: glue function $G : \Sigma \times \Sigma \rightarrow \{0, 1, \dots, t\}$

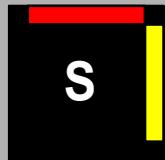
T: tileset $\{$  ,  ,  , ... $\}$

s: seed tile

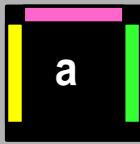
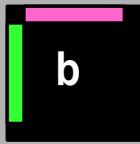
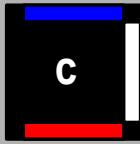
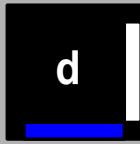
How a tile system self assembles

$T =$				$G(y) = 2$
				$G(g) = 2$
				$G(r) = 2$
				$G(b) = 2$
				$G(p) = 1$
				$G(w) = 1$

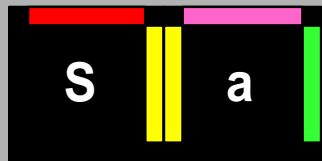
$t = 2$



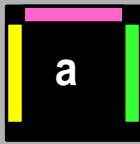
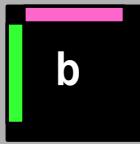
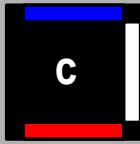
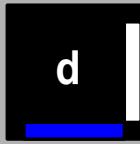
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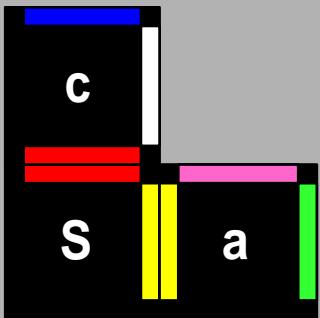
$t = 2$



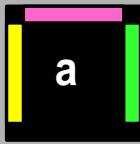
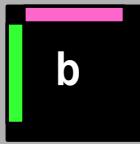
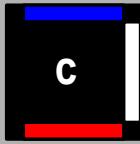
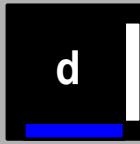
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				$G(w) = 1$

$t = 2$

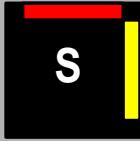
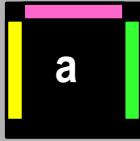
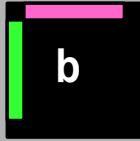
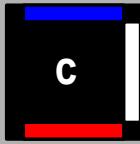
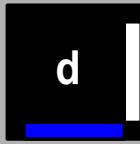


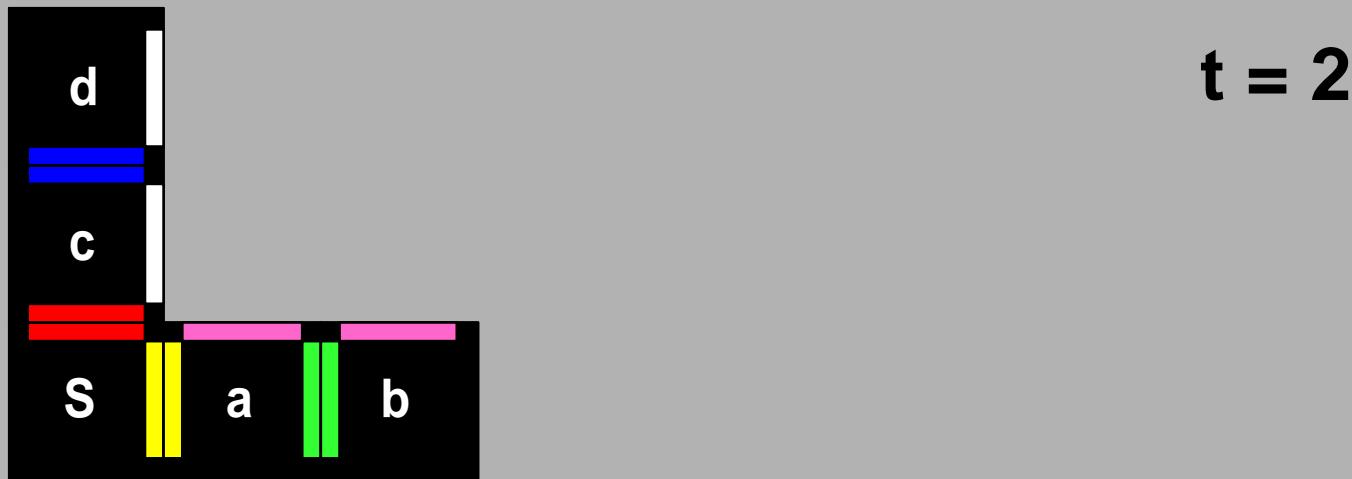
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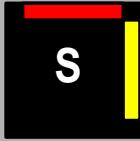
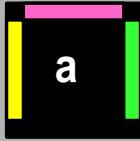
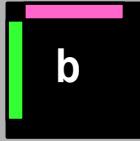
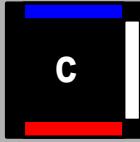
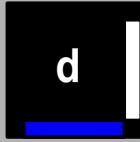


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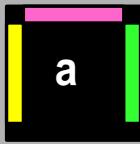
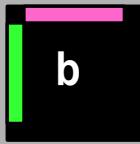
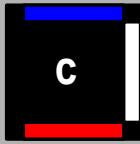
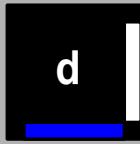


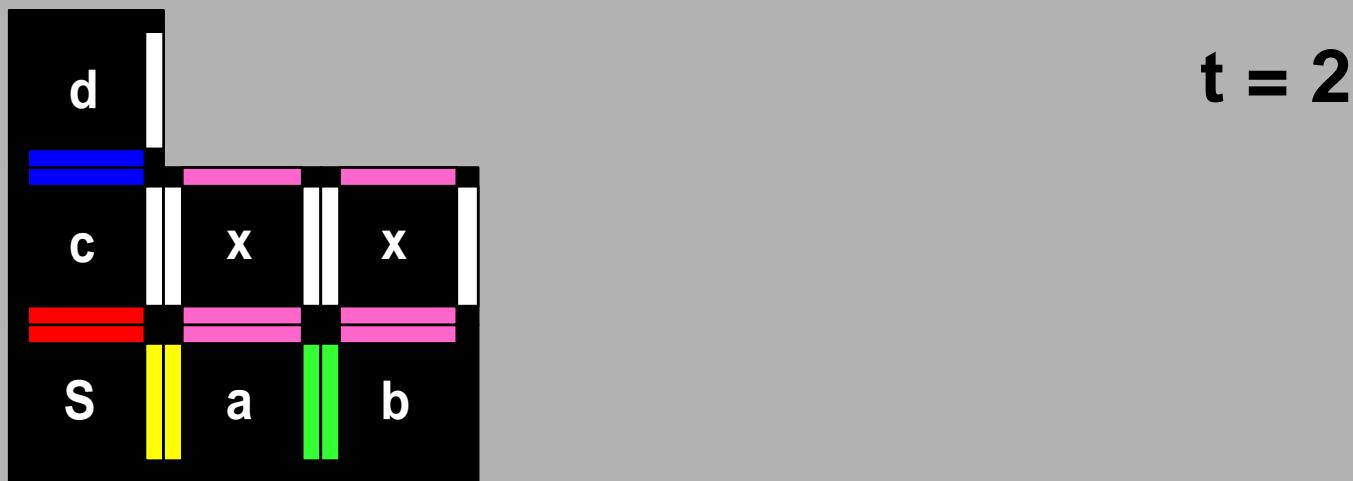
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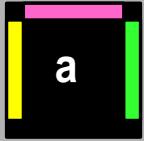
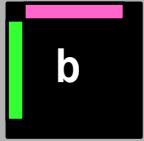
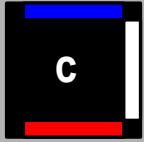
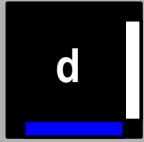


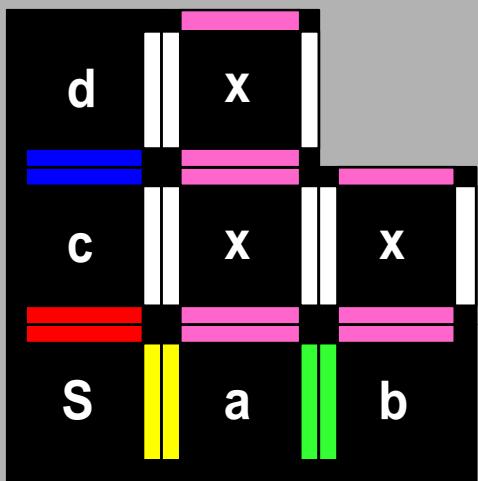
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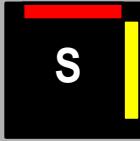
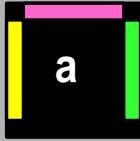
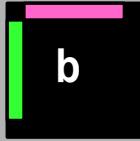
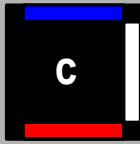
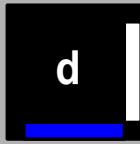
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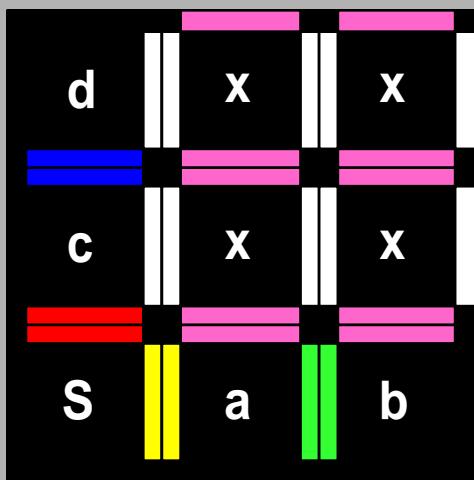
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				$G(w) = 1$



$t = 2$

How a tile system self assembles

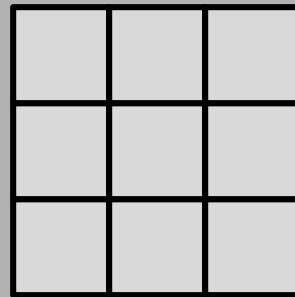
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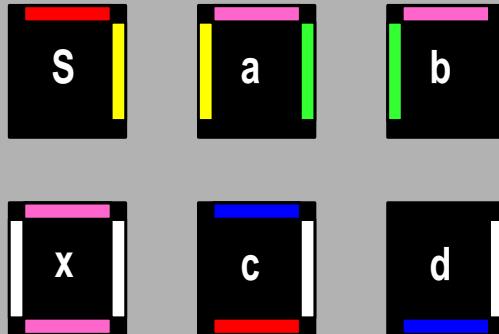
Efficient Assembly of Shapes

- Given a Shape:

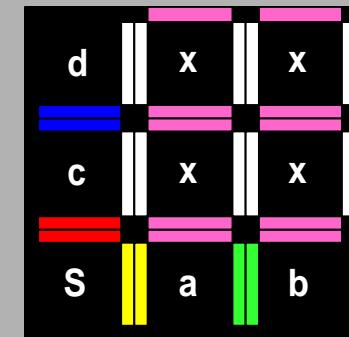


- Design an **efficient** tile system that uniquely builds the shape:

$T =$



$$\begin{aligned}G(y) &= 2 \\G(g) &= 2 \\G(r) &= 2 \\G(b) &= 2 \\G(p) &= 1 \\G(w) &= 1\end{aligned}$$



Alphabet of Shapes, Built with DNA Tiles

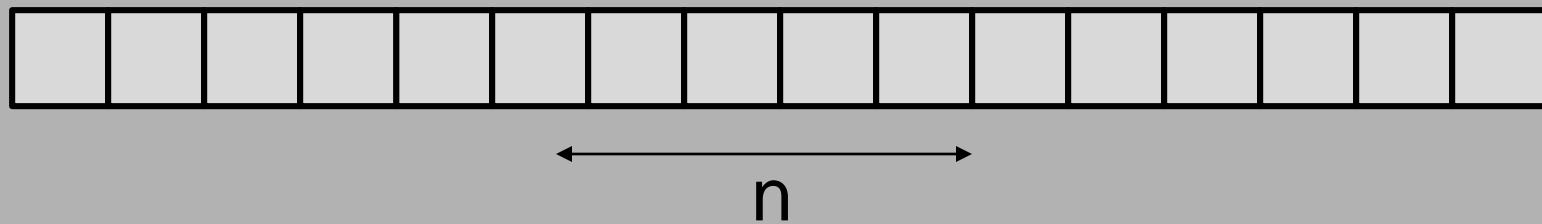


[Bryan Wei, Mingjie Dai, Peng Yin, Nature 2012]

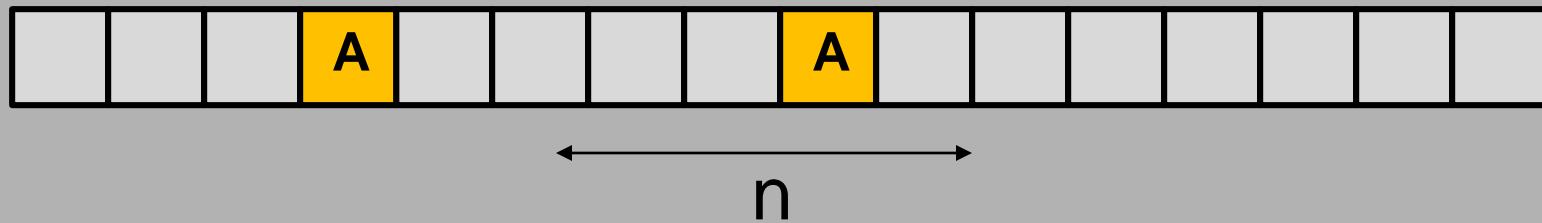
Outline

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- Rectangles

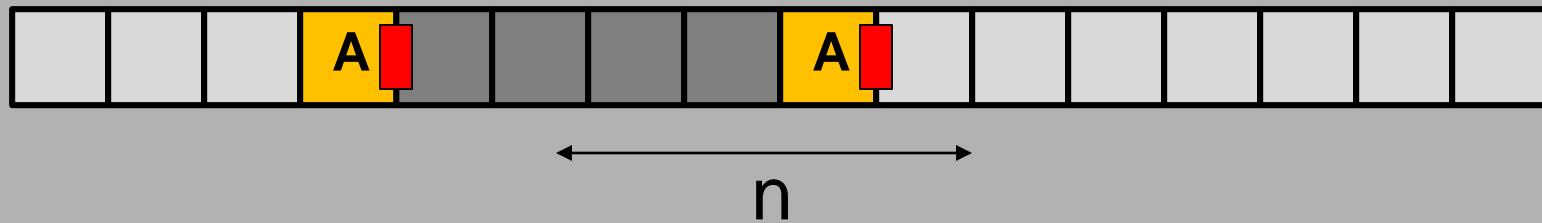
Building 1xn Lines



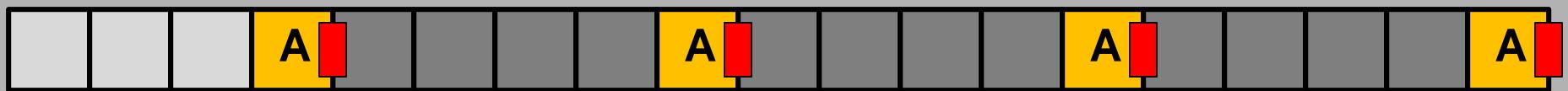
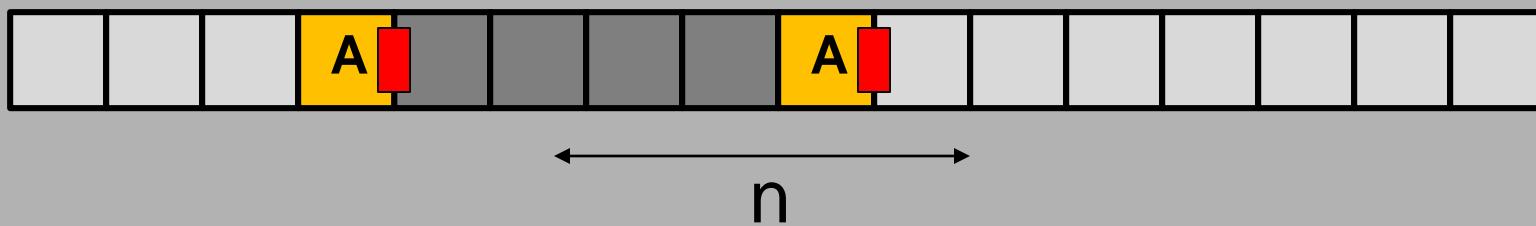
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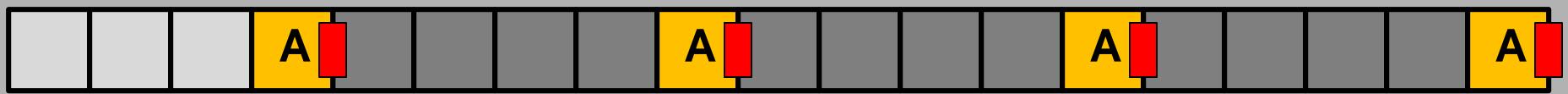
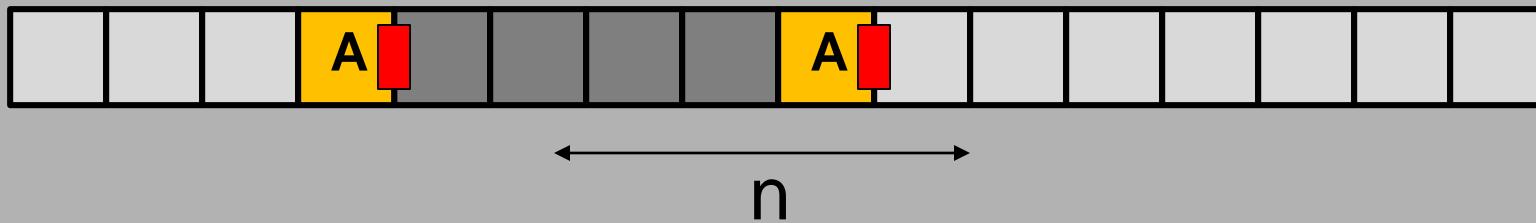
Building 1xn Lines



Building 1xn Lines

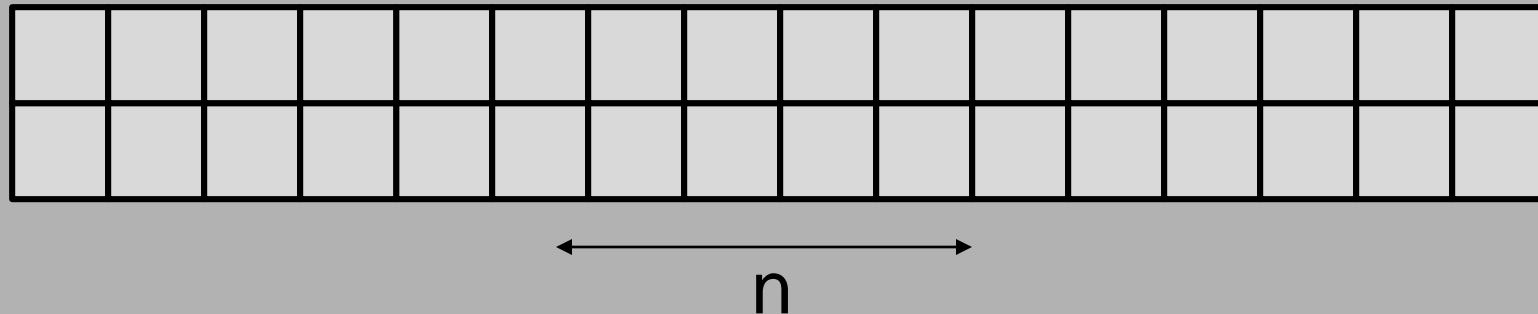


Building 1xn Lines

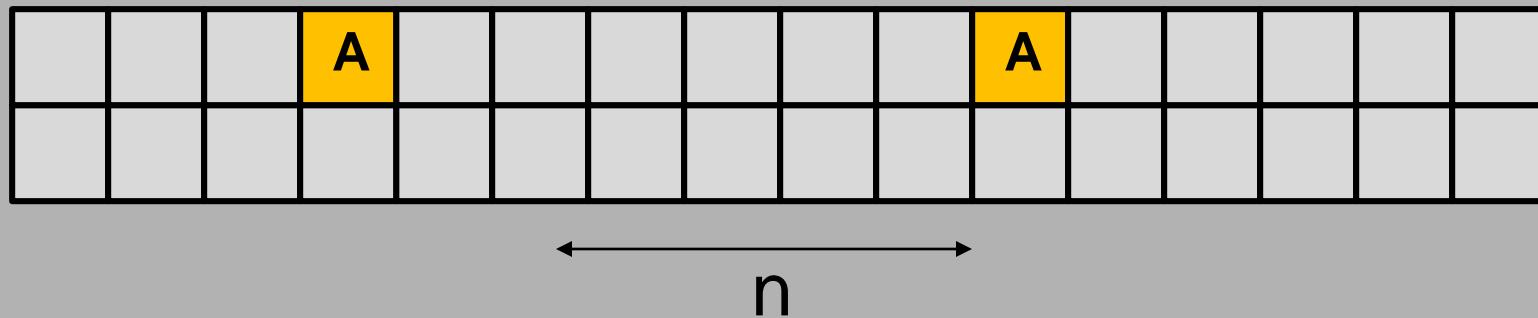


Tile Complexity: n

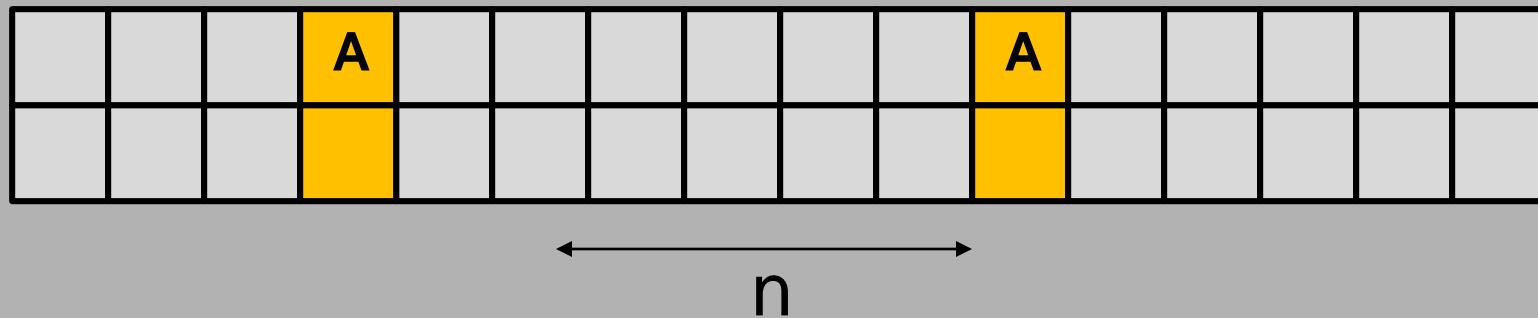
Building $2 \times n$ Rectangles



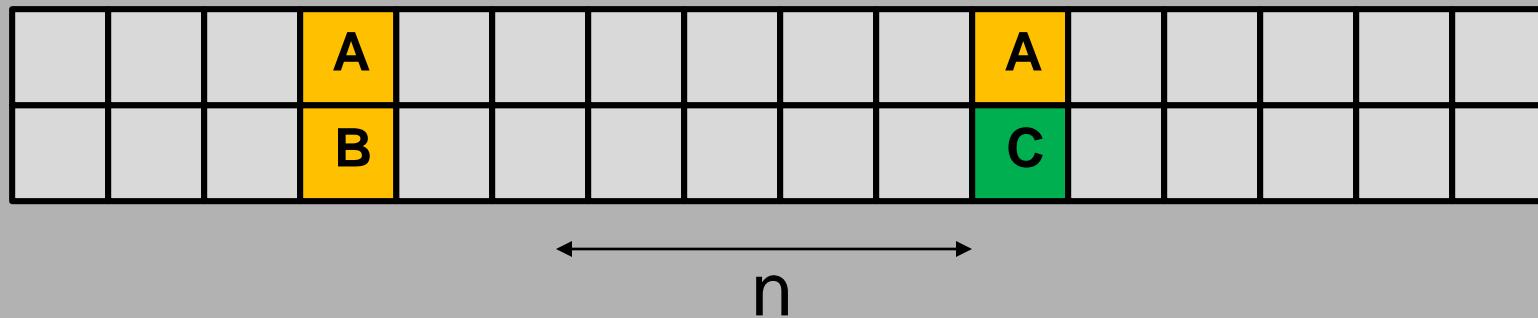
Building $2 \times n$ Rectangles



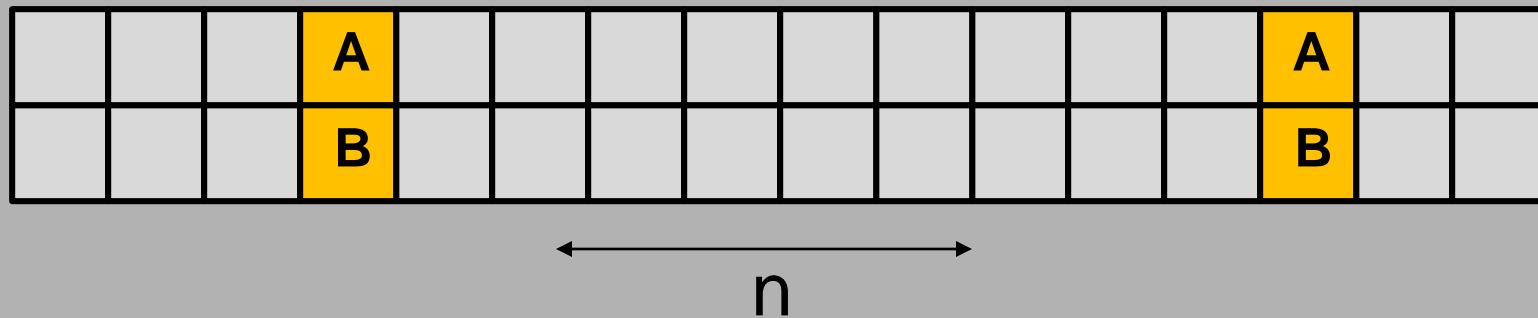
Building $2 \times n$ Rectangles



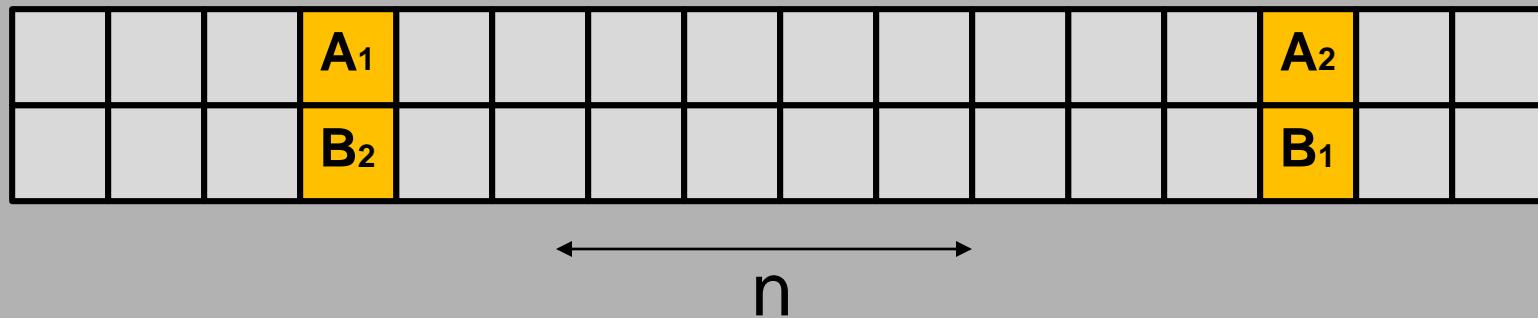
Building $2 \times n$ Rectangles



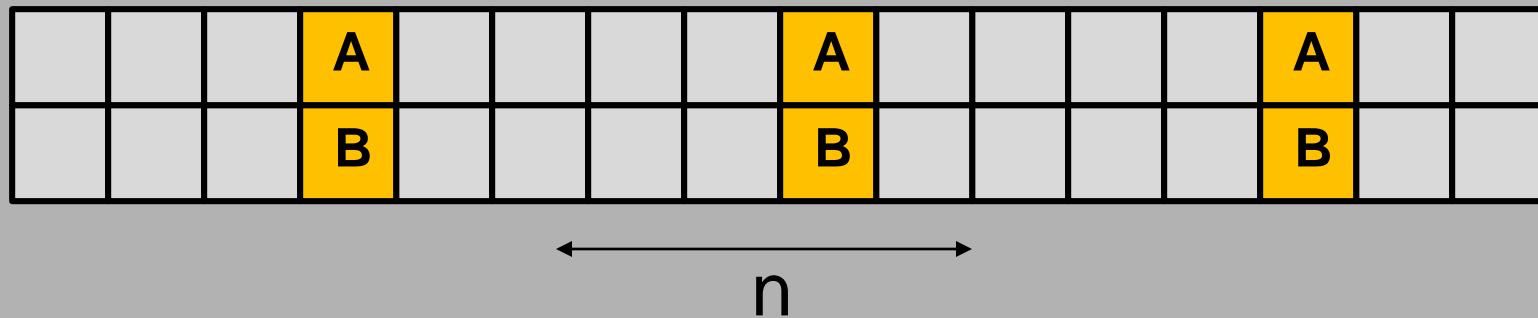
Building $2 \times n$ Rectangles



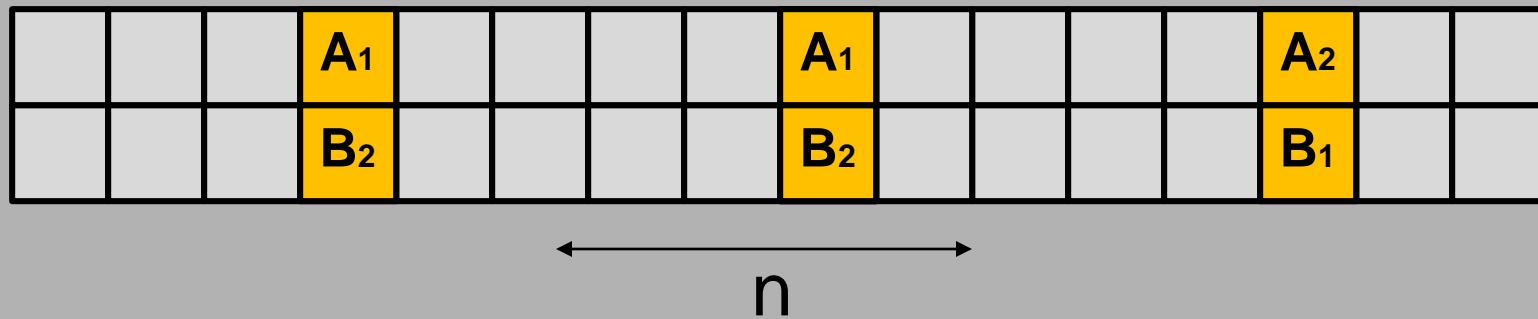
Building $2 \times n$ Rectangles



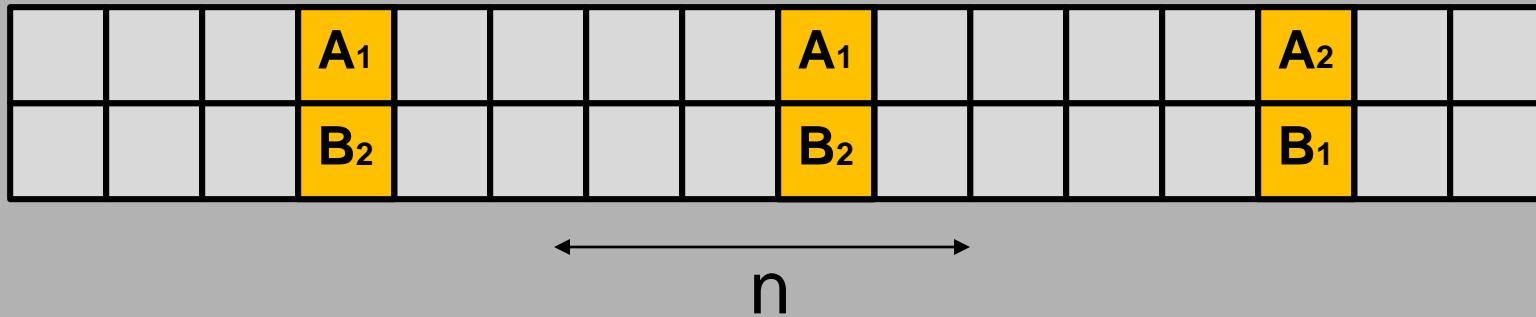
Building $2 \times n$ Rectangles



Building $2 \times n$ Rectangles

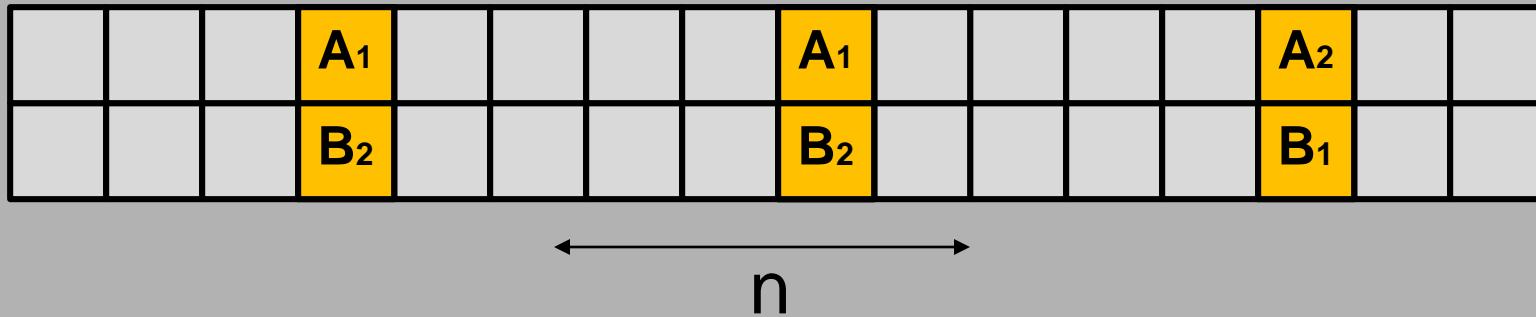


Building $2 \times n$ Rectangles



Lower Bound: $\sqrt{\frac{n}{2}} = \Omega(\sqrt{n})$

Building $2 \times n$ Rectangles



$2 \times n$ lines

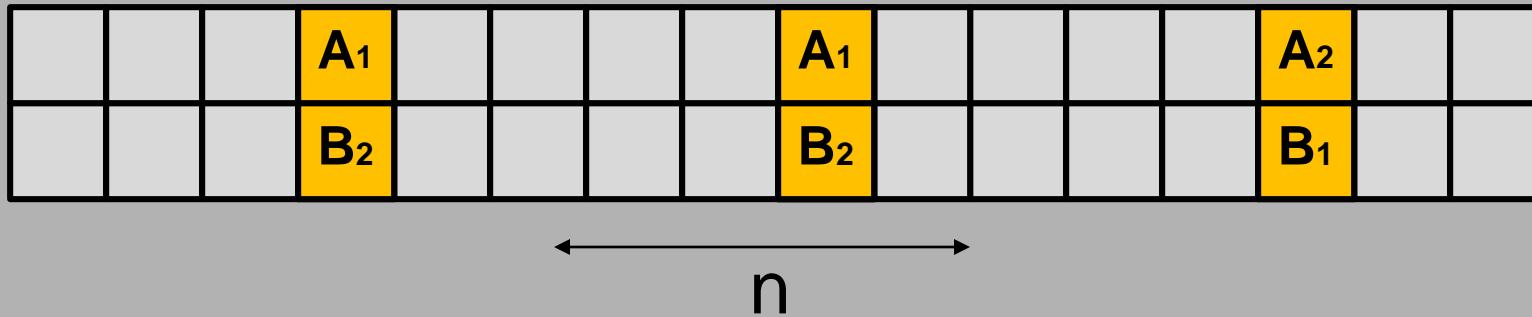
Lower Bound

$$\Omega(\sqrt{n})$$

Upper Bound

$$O(n)$$

Building $2 \times n$ Rectangles



$2 \times n$ lines

Lower Bound

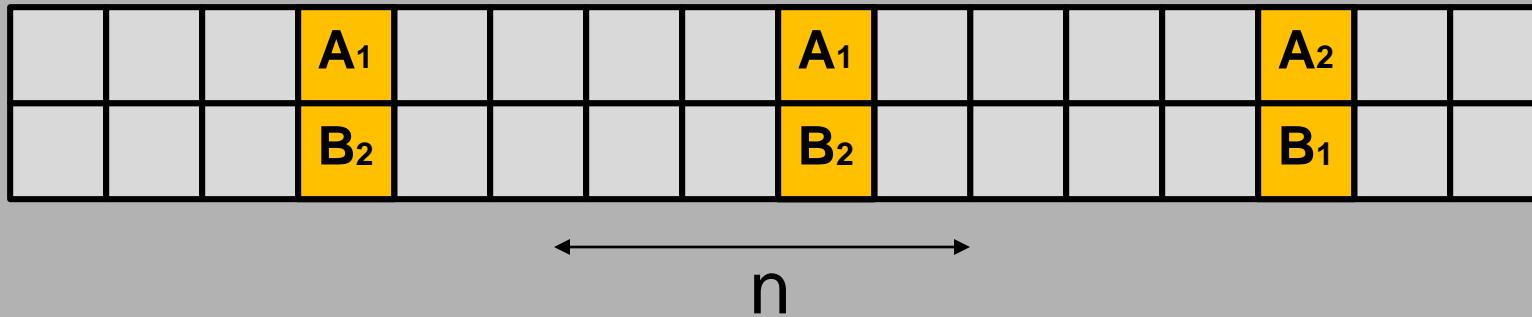
$$\Omega(\sqrt{n})$$

Upper Bound

$$O(n)$$

Can we do better than $O(n)$?

Building $2 \times n$ Rectangles



$2 \times n$ lines

Lower Bound

$$\Omega(\sqrt{n})$$

Upper Bound

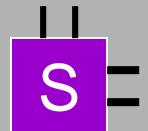
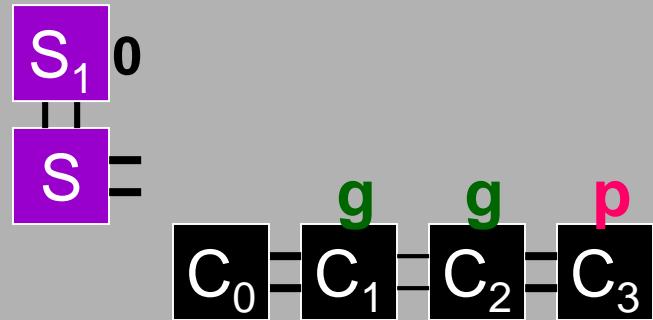
~~$O(n)$~~

$O(\sqrt{n})$

Can we do better than $O(n)$?
-YES

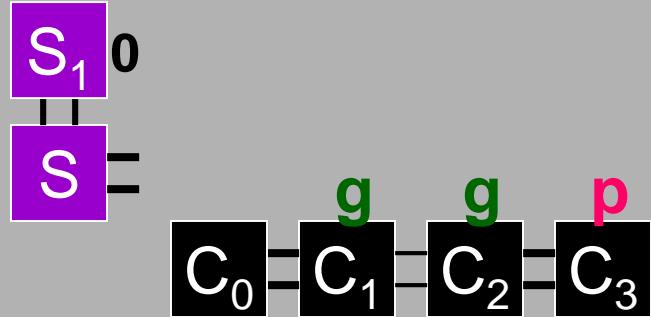
Build a 2×16 rectangle:

$t = 2$



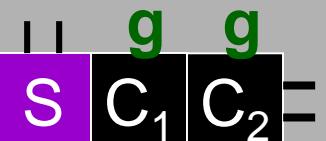
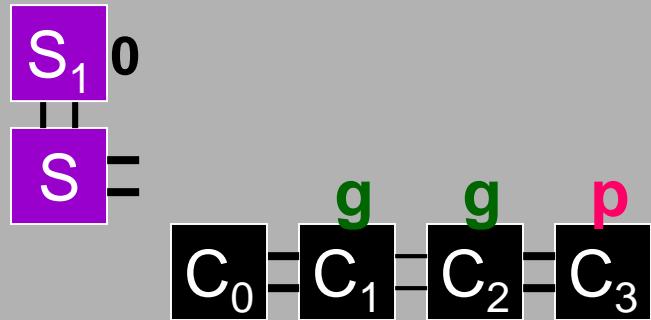
Build a 2×16 rectangle:

$t = 2$



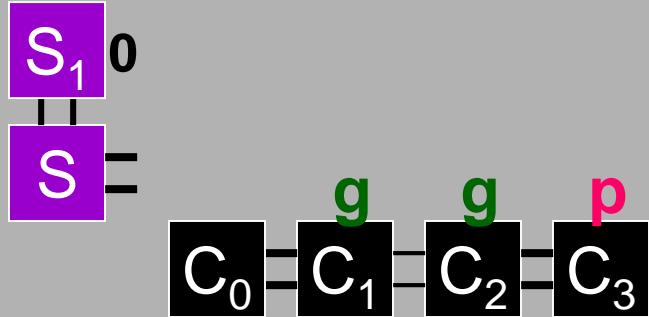
Build a 2×16 rectangle:

$t = 2$



Build a 2×16 rectangle:

$t = 2$



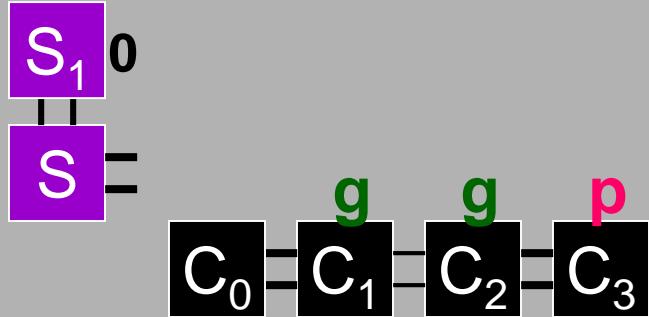
g g p

C_0 $=$ C_1 $=$ C_2 $=$ C_3

S C_1 C_2 C_3

Build a 2×16 rectangle:

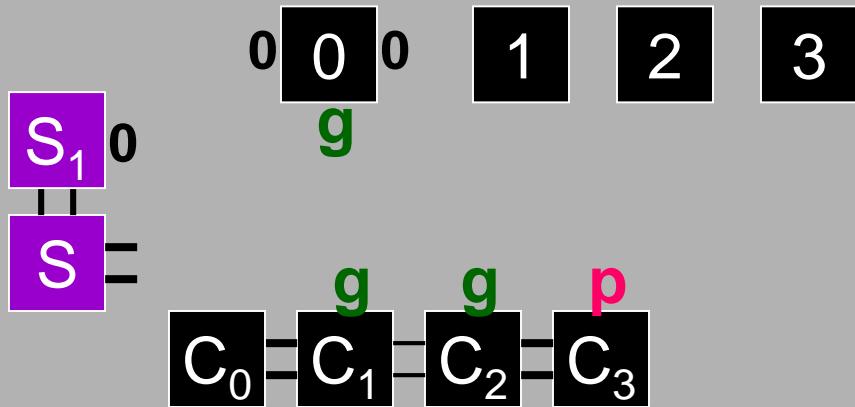
$t = 2$



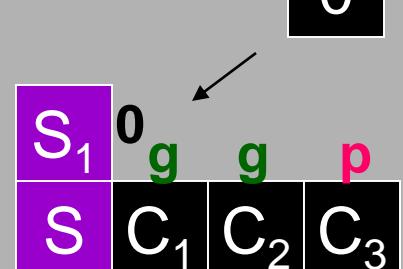
S_1	0	g	g	p
S	C_1	C_2	C_3	

Build a 2×16 rectangle:

$t = 2$

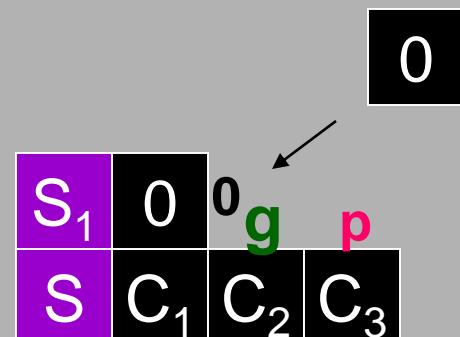
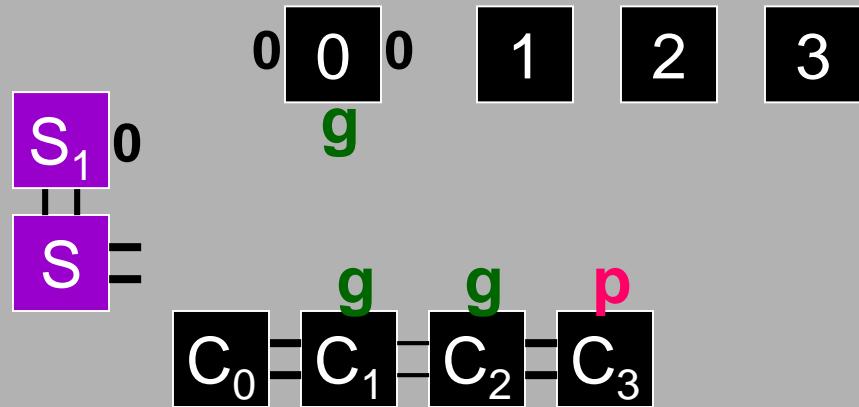


0



Build a 2×16 rectangle:

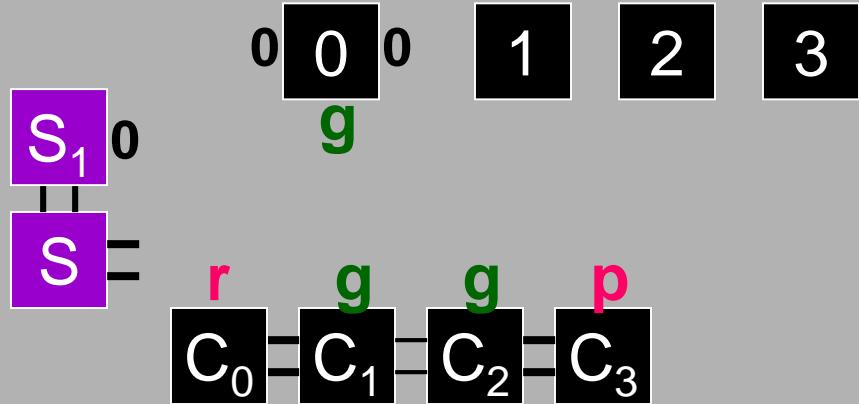
$t = 2$



Build a 2×16 rectangle:

$t = 2$

$$\begin{matrix} & g & g \\ 0 & 0 & = & 1 & 1 \\ p & r & & & \end{matrix}$$

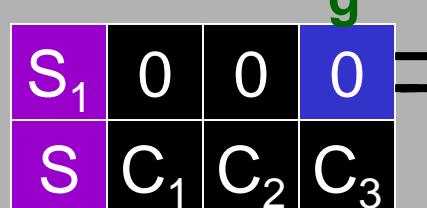
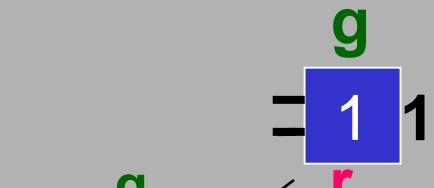
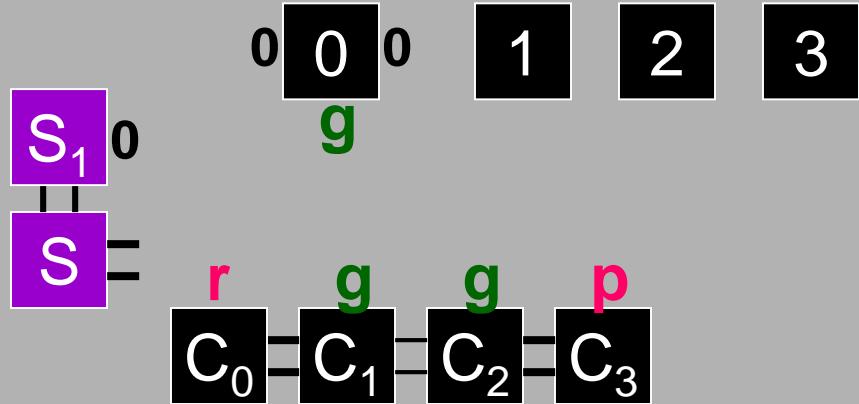
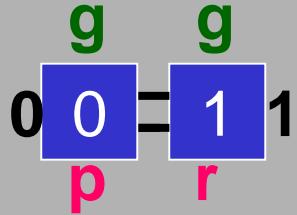


$$\begin{matrix} & g \\ 0 & 0 & = \\ p & & \end{matrix}$$

S ₁	0	0	0	p
S	C ₁	C ₂	C ₃	

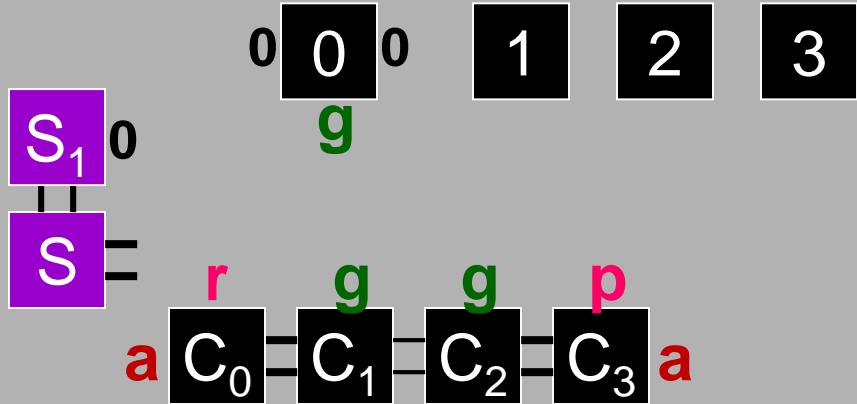
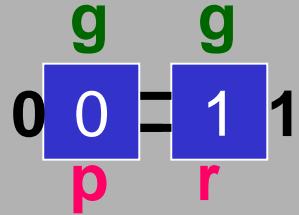
Build a 2×16 rectangle:

$t = 2$



Build a 2×16 rectangle:

$t = 2$

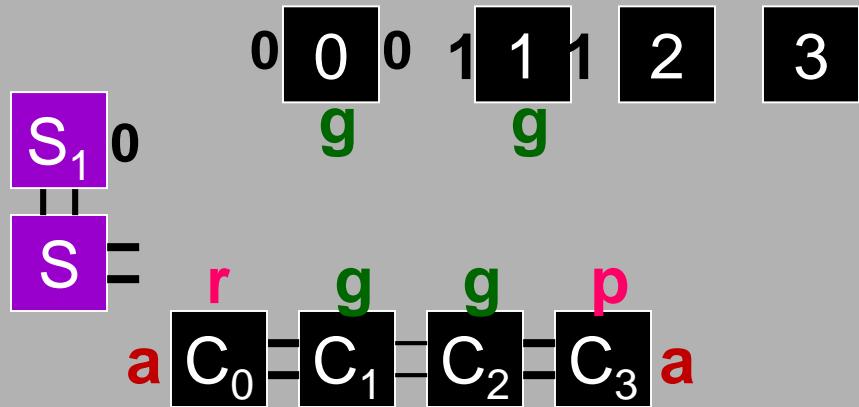


S_1	0	0	0	1	1
S	C_1	C_2	C_3	a	r

Build a 2×16 rectangle:

$t = 2$

$$\begin{matrix} & g & g \\ 0 & 0 & = & 1 & 1 \\ p & r \end{matrix}$$



S_1	0	0	0	1	1	g	g	p
S	C_1	C_2	C_3	C_0	C_1	C_2	C_3	

Build a 2×16 rectangle:

$t = 2$

$$\begin{matrix} g & g \\ 0 & 0 \\ p & r \end{matrix} = \begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}$$

$0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 2 \quad 3$

$S_1 \quad 0$

$S \quad =$

$r \quad g \quad g \quad p$

$a \quad C_0 = C_1 = C_2 = C_3 \quad a$

$$1 = 2$$

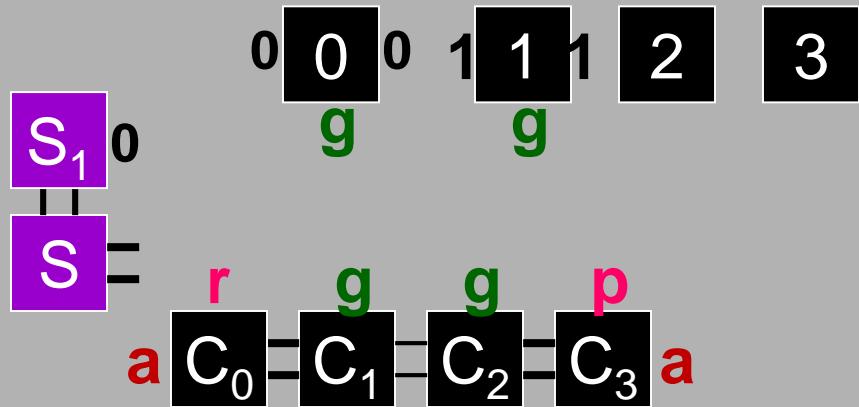
$$2 = 3$$

S_1	0	0	0	1	1	1	1
S	C_1	C_2	C_3	C_0	C_1	C_2	C_3

Build a 2×16 rectangle:

$t = 2$

$$\begin{matrix} g & g \\ 0 & 0 \\ p & r \end{matrix} = \begin{matrix} 1 & 1 \end{matrix}$$



$3 \quad X \quad p$

$$\begin{matrix} 1 & = & 2 \\ 2 & = & 3 \end{matrix}$$

S_1	0	0	0	1	1	1	1	2	2	2	2	3	3	3	3	p
S	C_1	C_2	C_3	C_0	C_1	C_2	C_3	C_0	C_1	C_2	C_3	C_0	C_1	C_2	C_3	

Build a 2×16 rectangle:

$t = 2$

$$\begin{matrix} g & g \\ 0 & 0 \\ p & r \end{matrix} = \begin{matrix} 1 & 1 \end{matrix}$$

$\begin{matrix} 0 & 0 & 0 & 1 & 1 & 1 & 2 & 3 \end{matrix}$

S_1

S

$a \quad C_0 = C_1 = C_2 = C_3 \quad a$

$3 \quad X \quad p$

$$1 = 2$$

$$2 = 3$$

S_1	0	0	0	1	1	1	1	2	2	2	2	3	3	3	P
S	C ₁	C ₂	C ₃	C ₀	C ₁	C ₂	C ₃	C ₀	C ₁	C ₂	C ₃	C ₀	C ₁	C ₂	C ₃

Build a 2×16 rectangle:

$t = 2$

S_1	0	0	0	1	1	1	1	2	2	2	2	3	3	3	P
S	C_1	C_2	C_3	C_0	C_1	C_2	C_3	C_0	C_1	C_2	C_3	C_0	C_1	C_2	C_3

$2 \times n$ lines

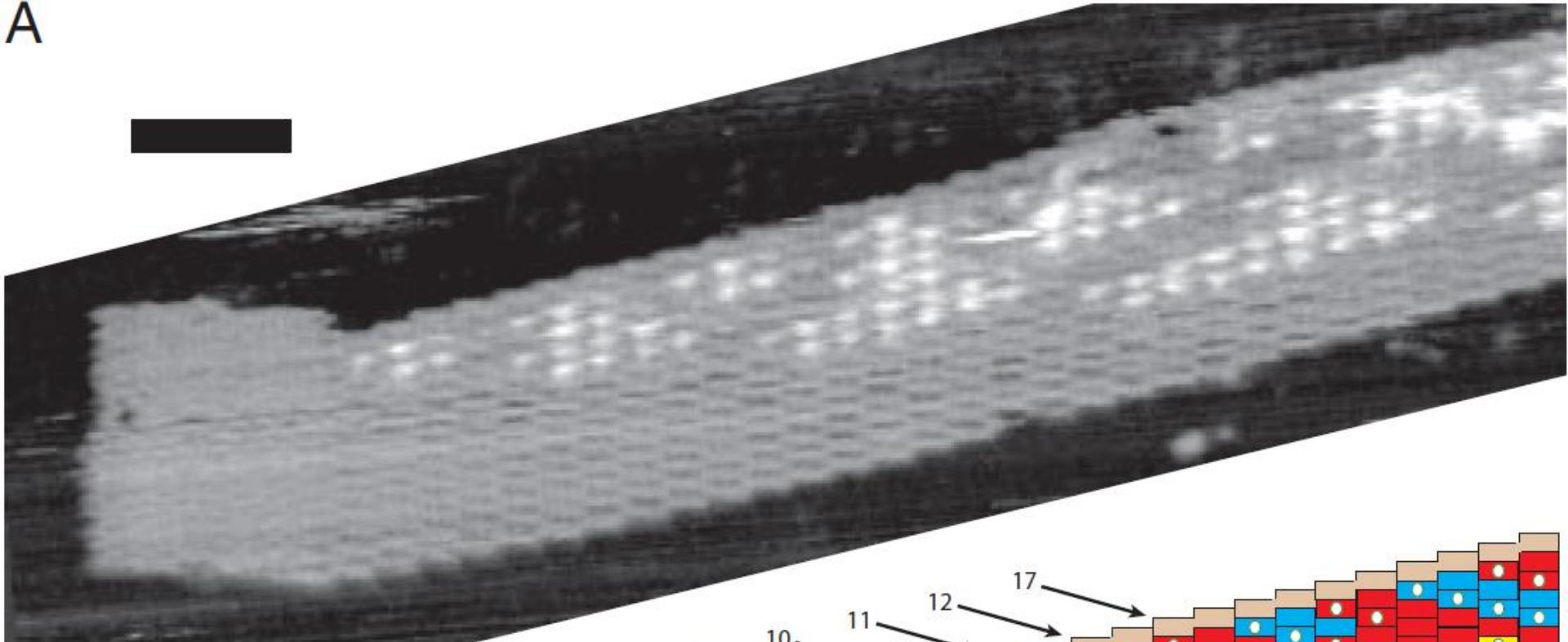
Lower Bound

$$\Omega(\sqrt{n})$$

Upper Bound

$$O(\sqrt{n})$$

A



B

