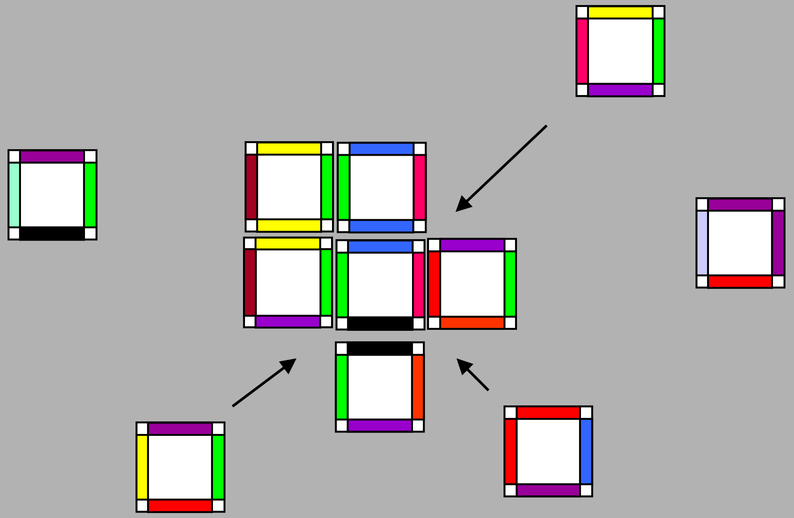
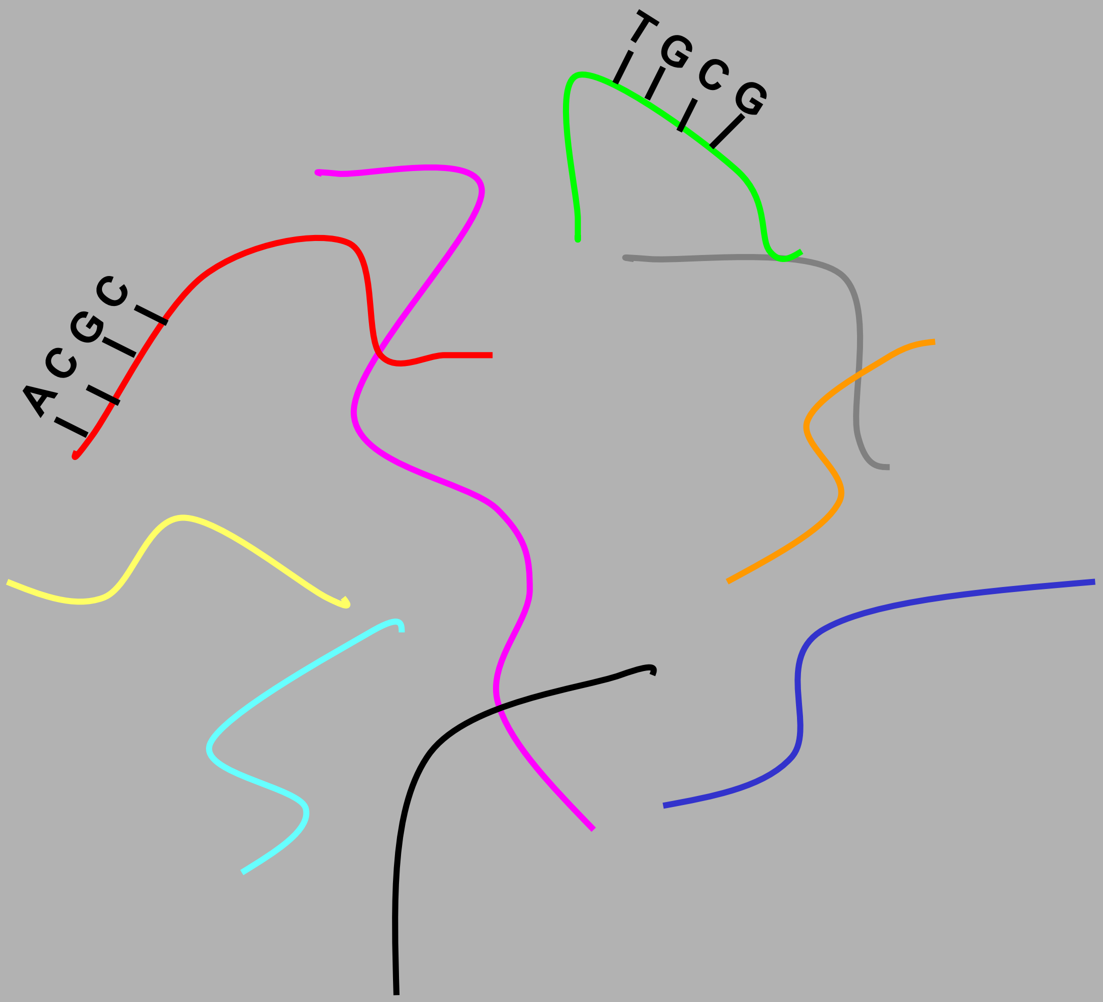
Abstract Tile Self-Assembly Model and the Complexity of Self-Assembled Rectangles

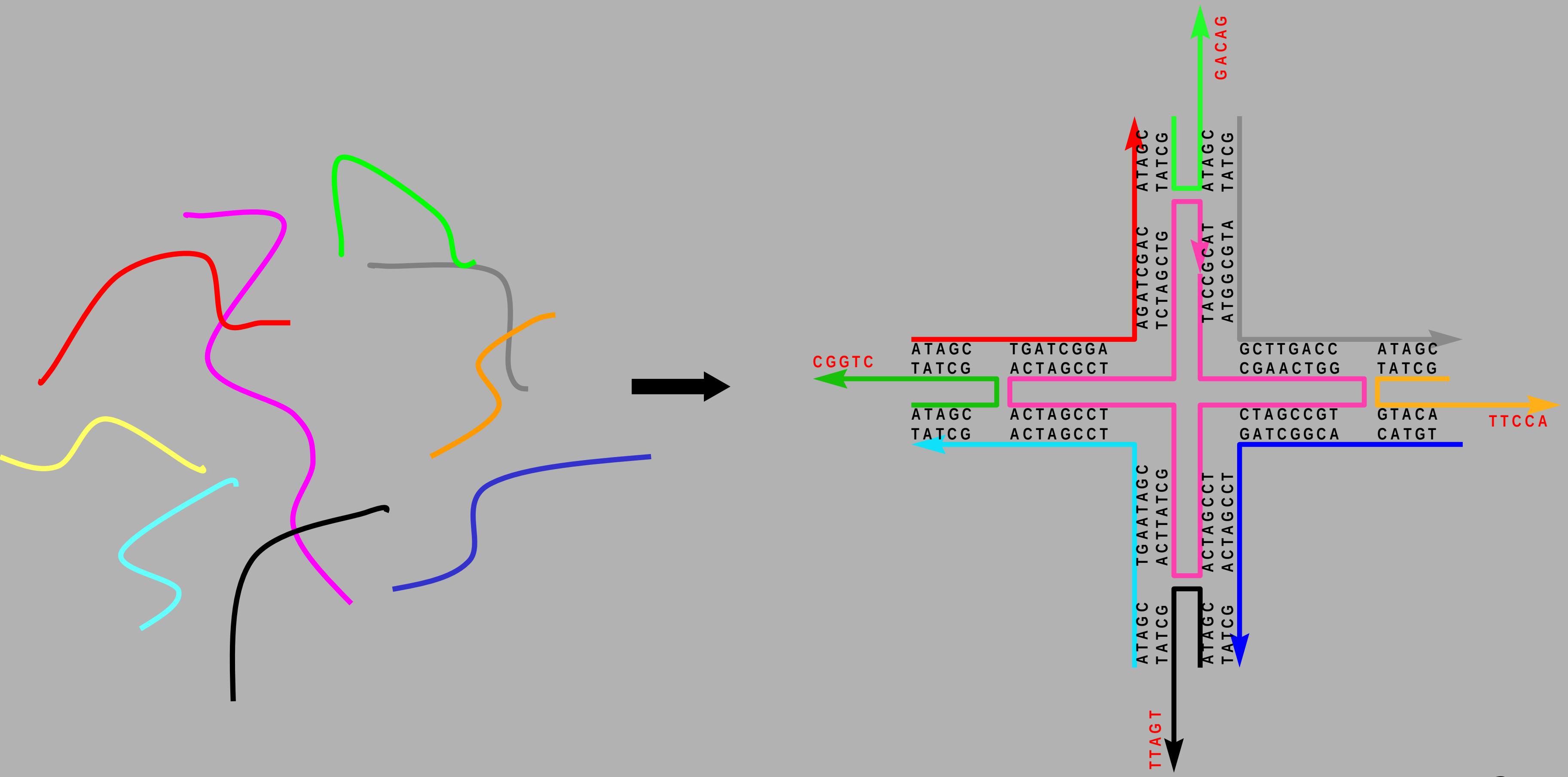


Molecular Building Blocks

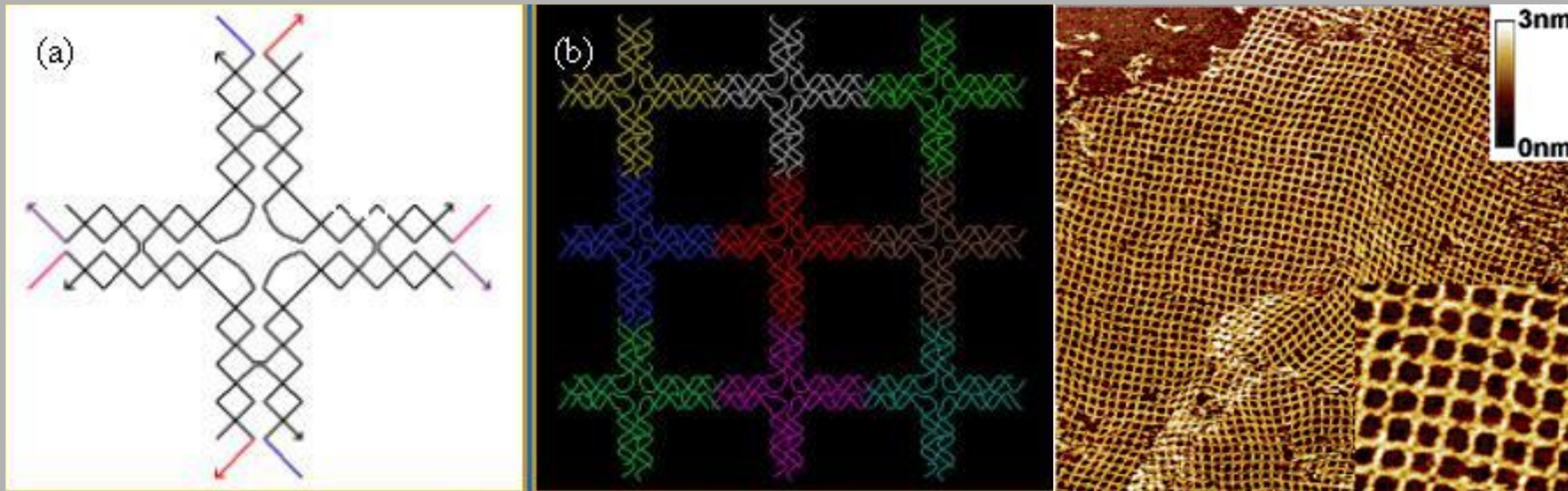


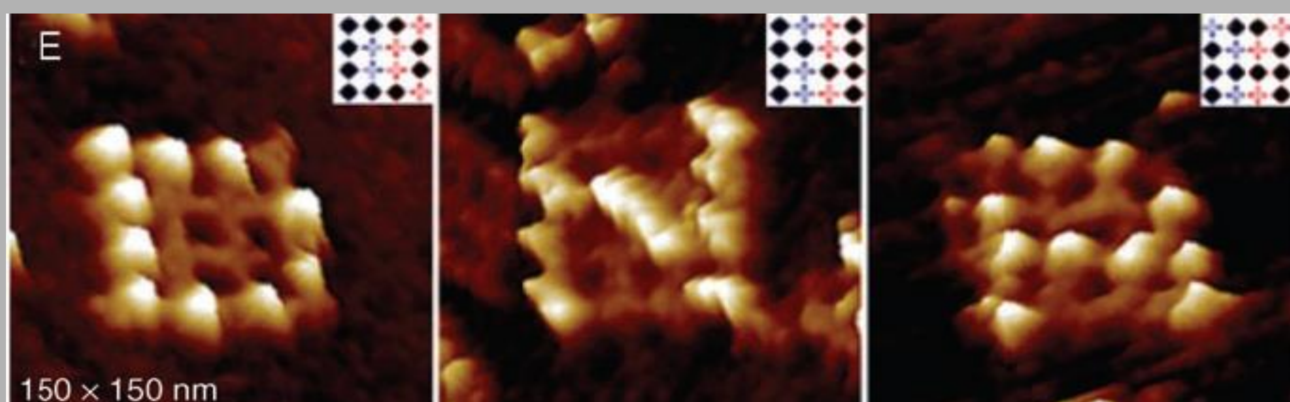
Molecular Building Blocks

[Reif’s Group, Duke University]



DNA Scaffolding

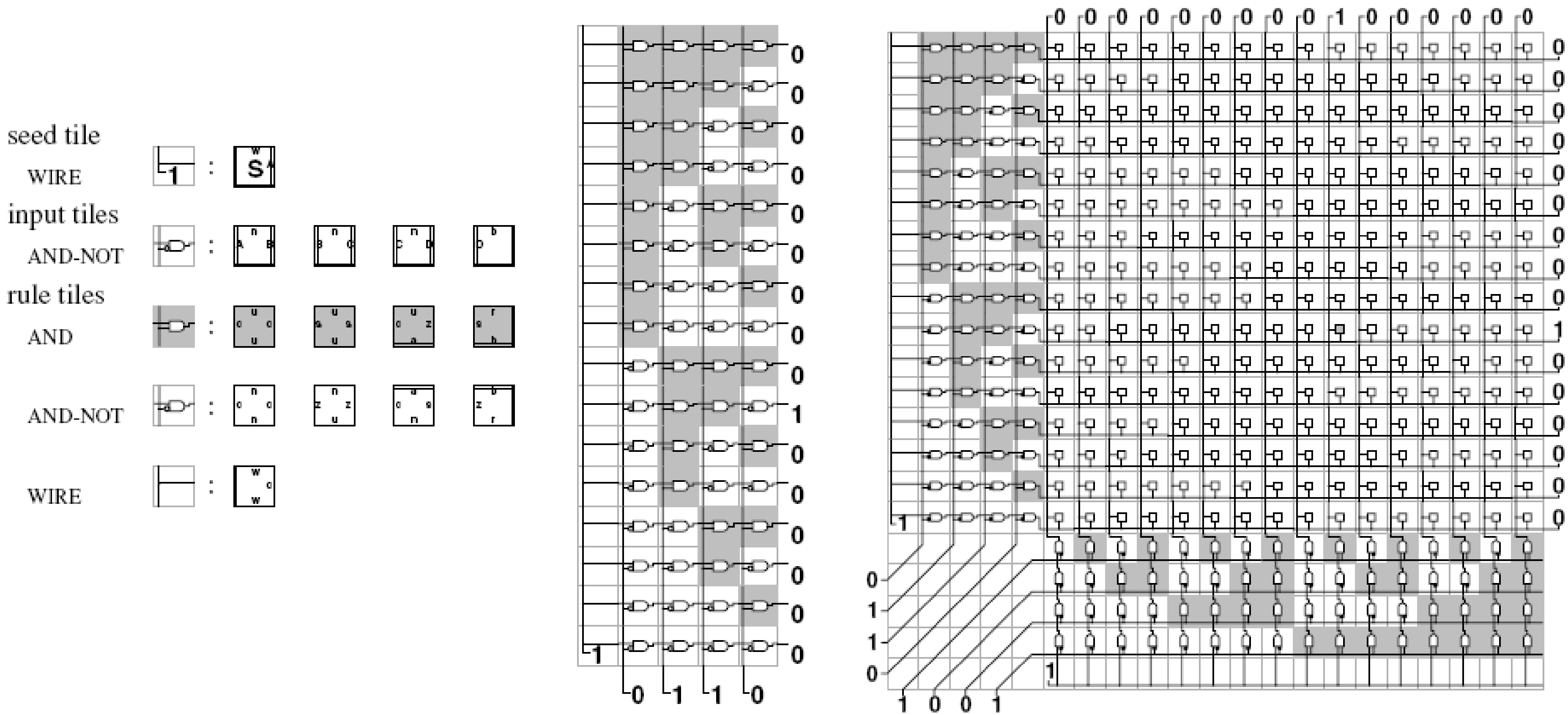




[Sung Ha Park, Constantin Pistol, Sang Jung Ahn, John H. Reif, Alvin R. Lebeck, Chris Dwyer, and Thomas H. LaBean, 2006]

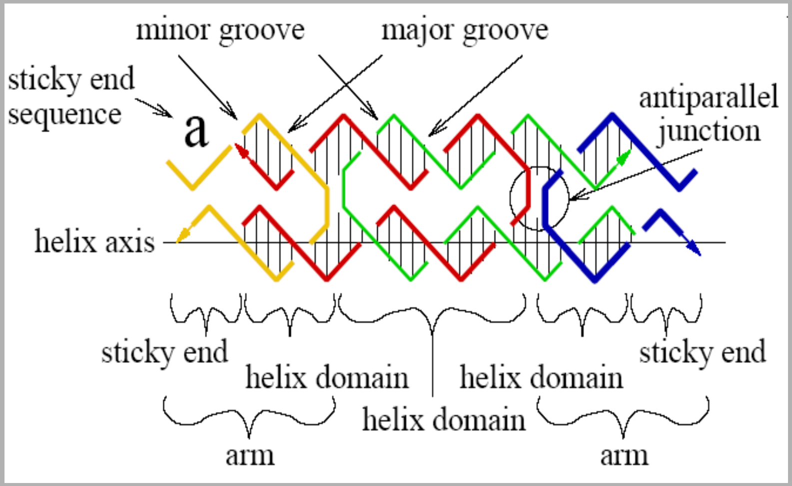
Self-Assembly for Circuit Patterns

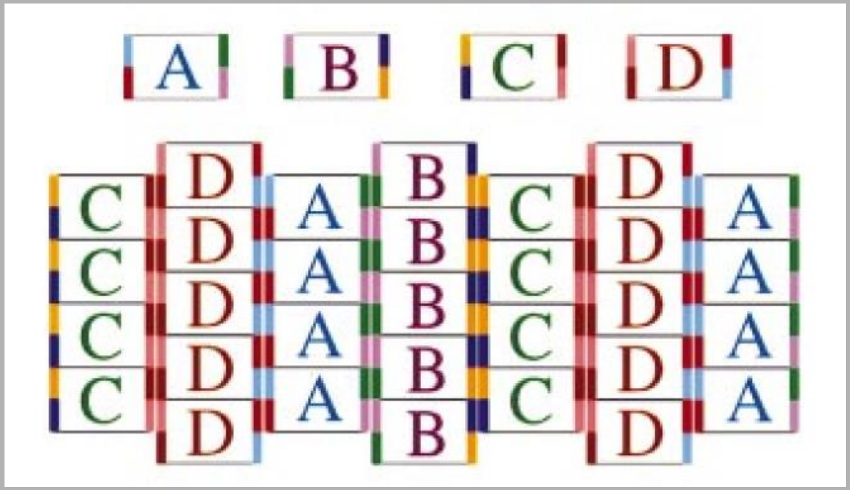
[Cook, Rothemund, and Winfree, 2003]

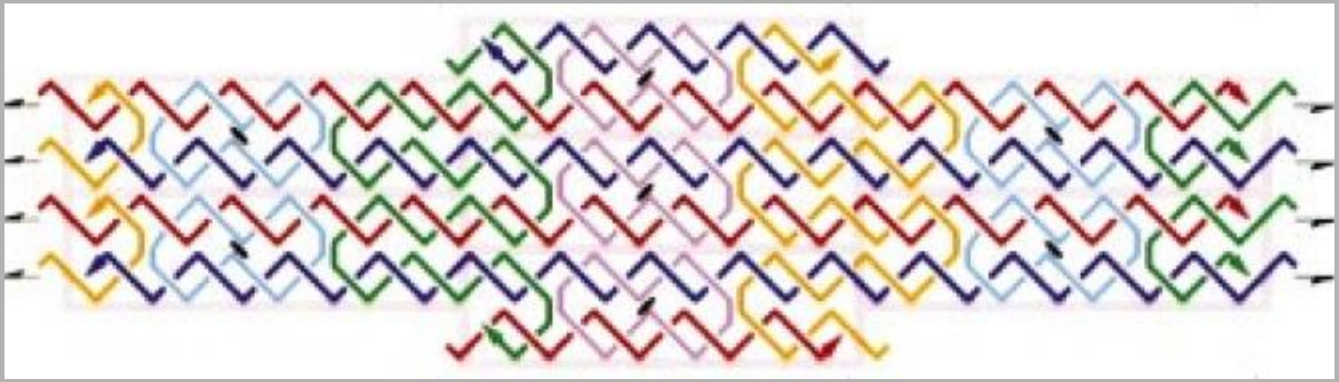


More Examples of DNA Tiles

[Winfree's Group, Cal Tech]

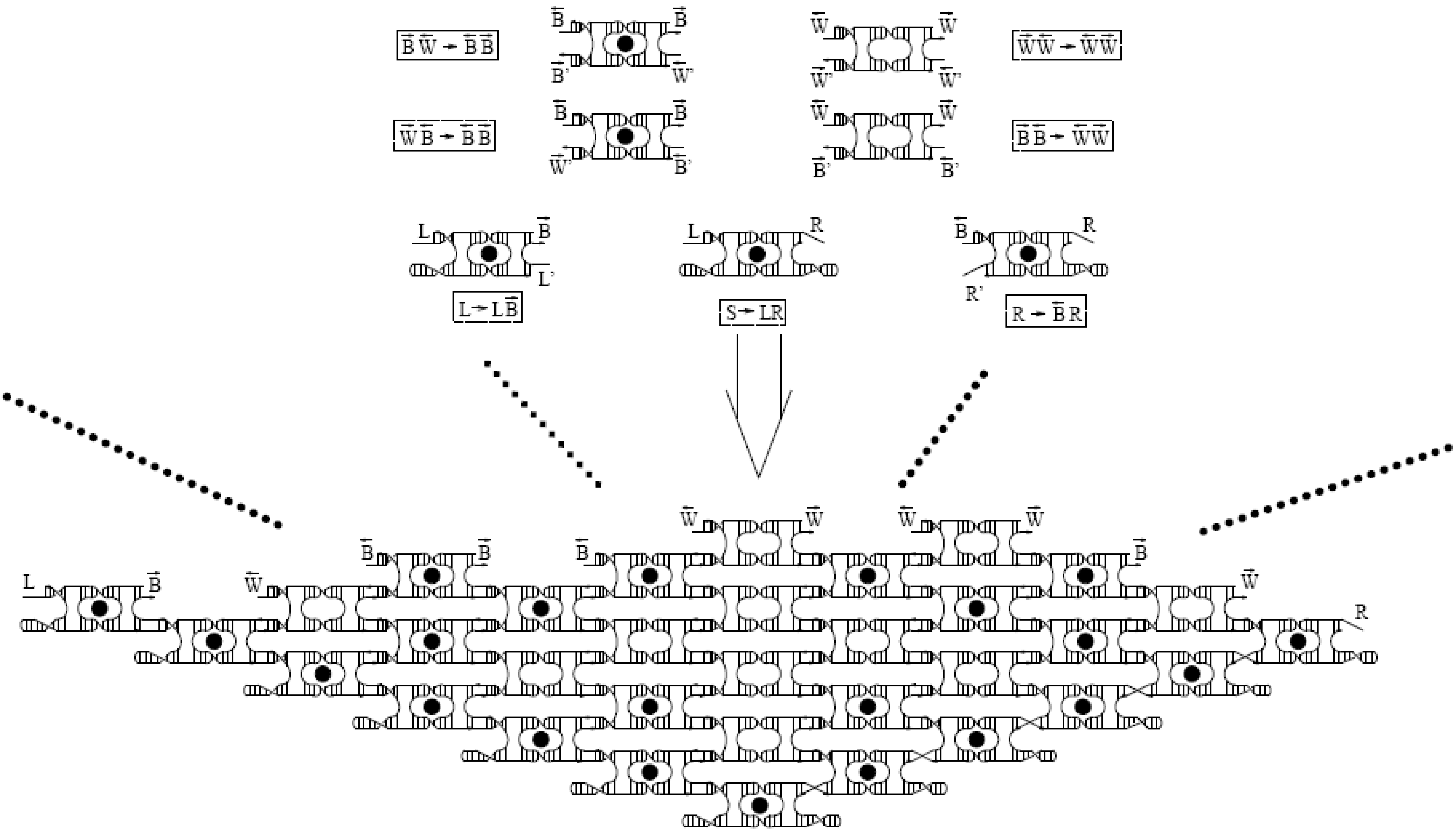






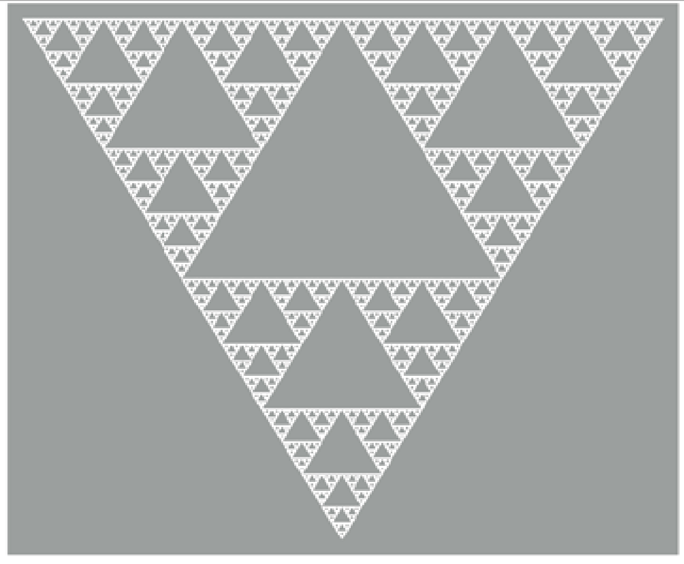
2D Self-Assembly for Turing Machines

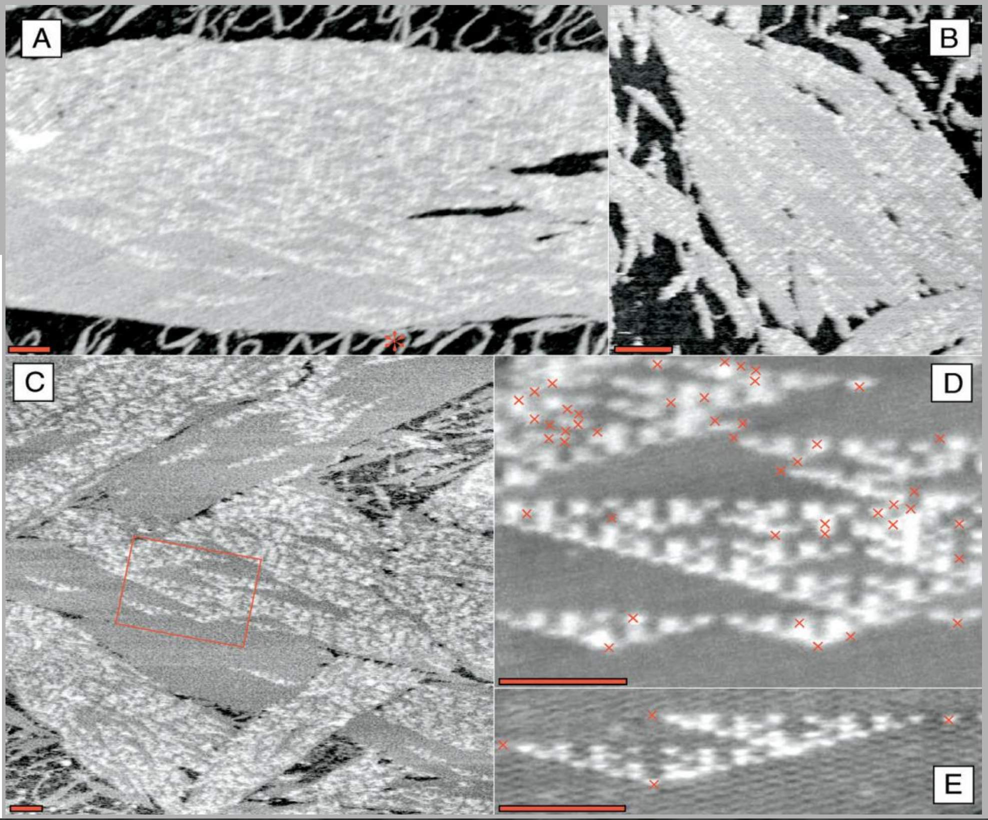
[Winfree, Yang, and Seeman, 1998]



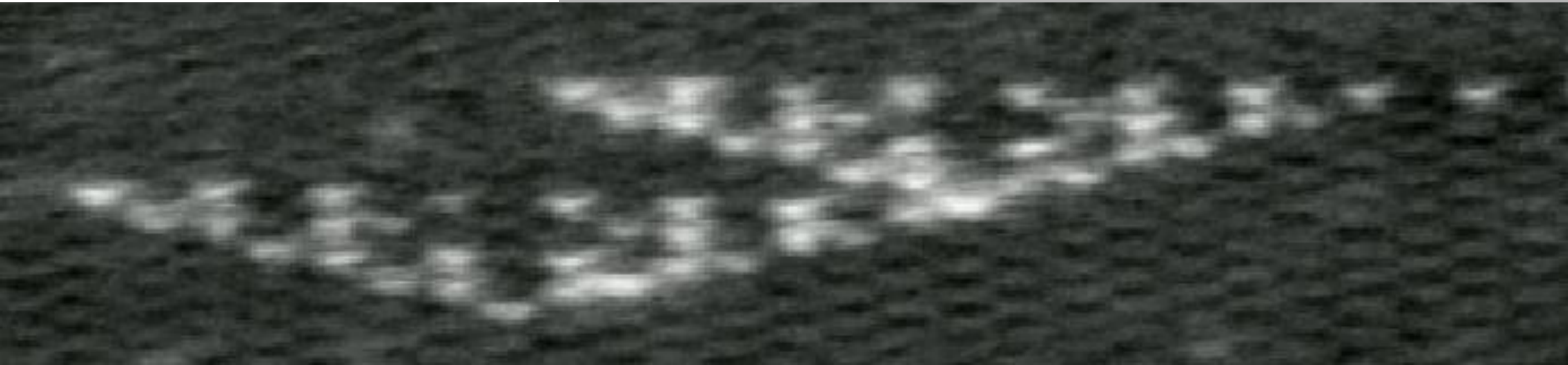
Simulation of Cellular Automata

Paul Rothemund, Nick Papadakis, Erik Winfree, PLoS Biology 2: e424 (2004)



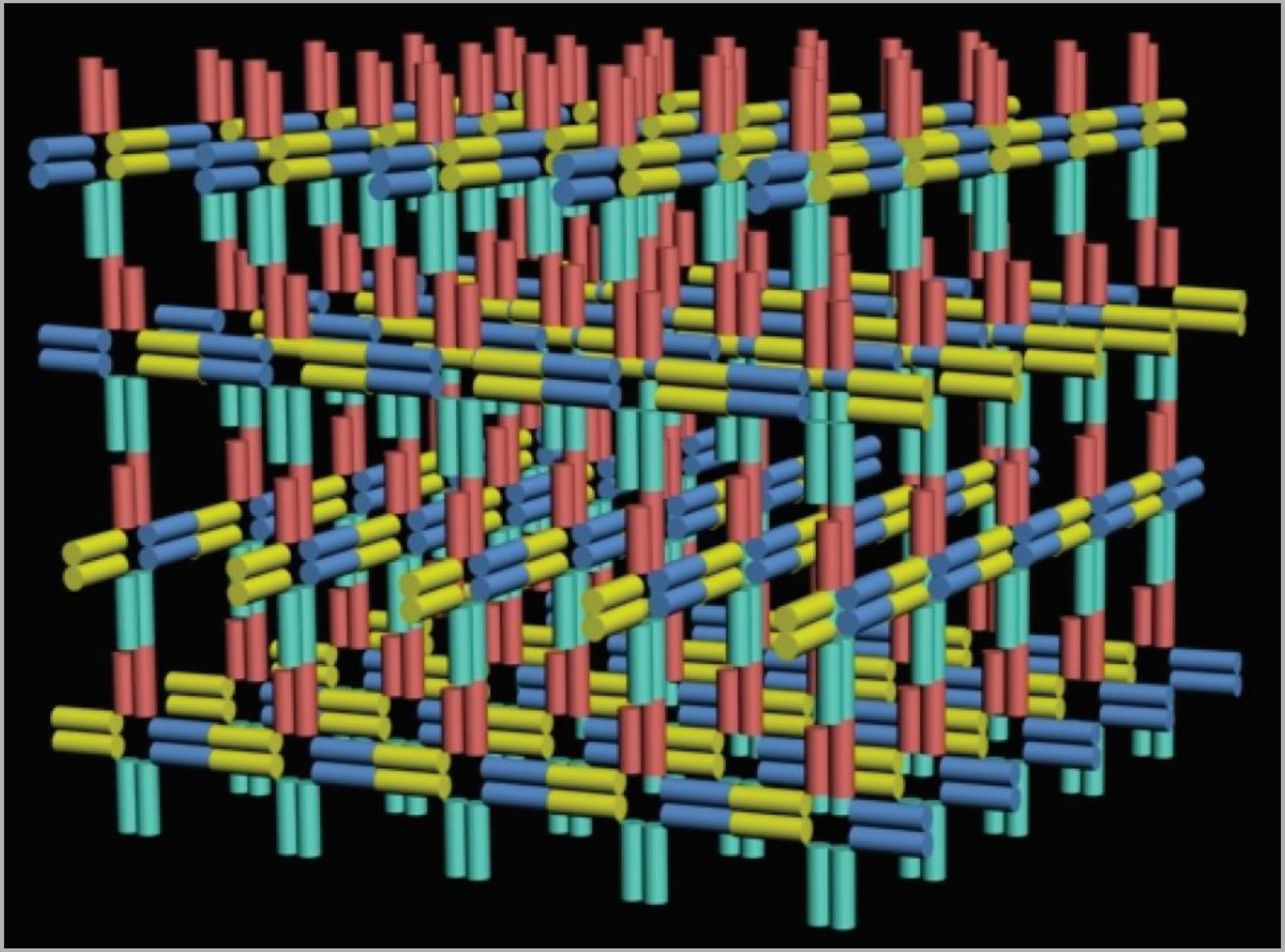


340nm



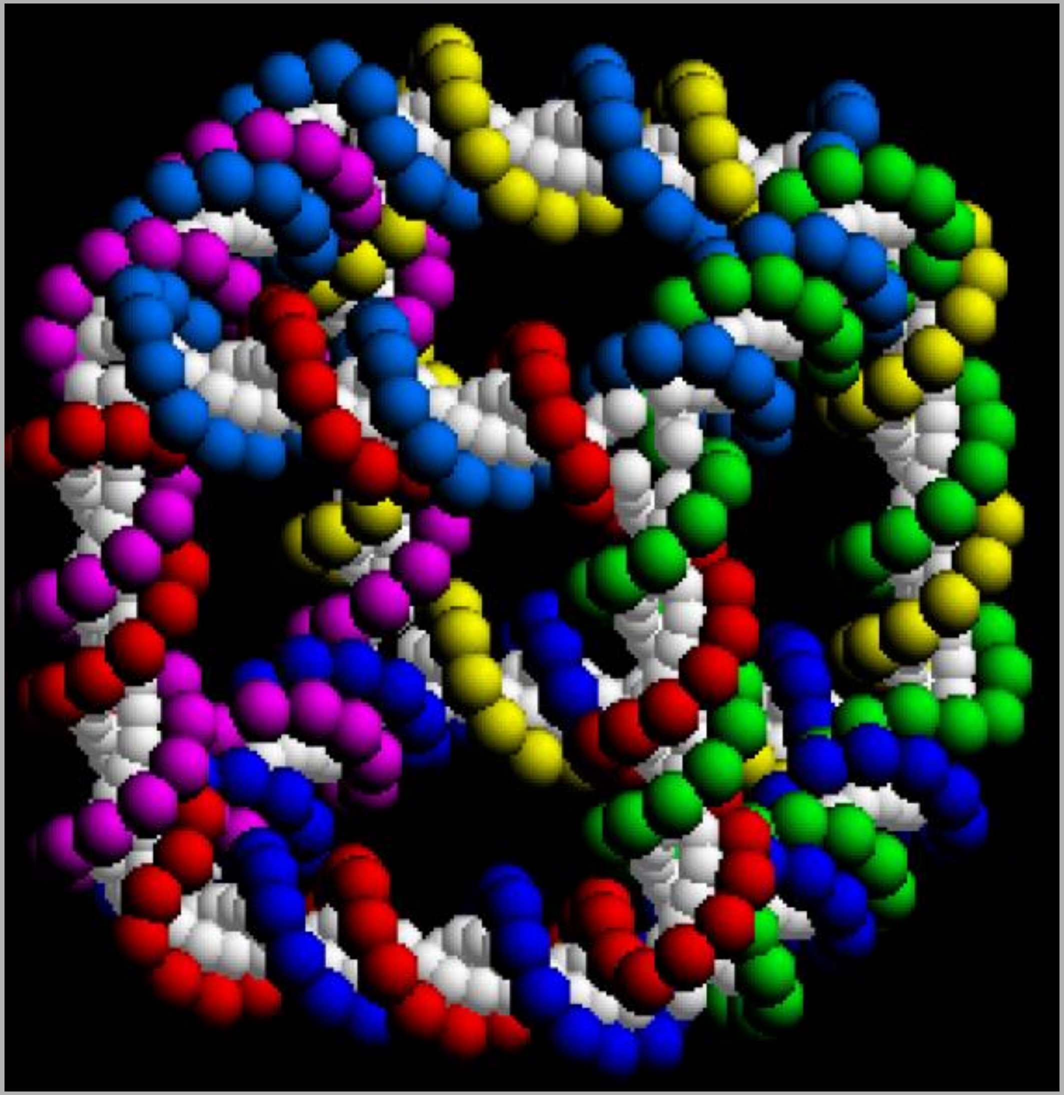
Example of 3D Self-Assembly

[Shaw, University of Southern California]



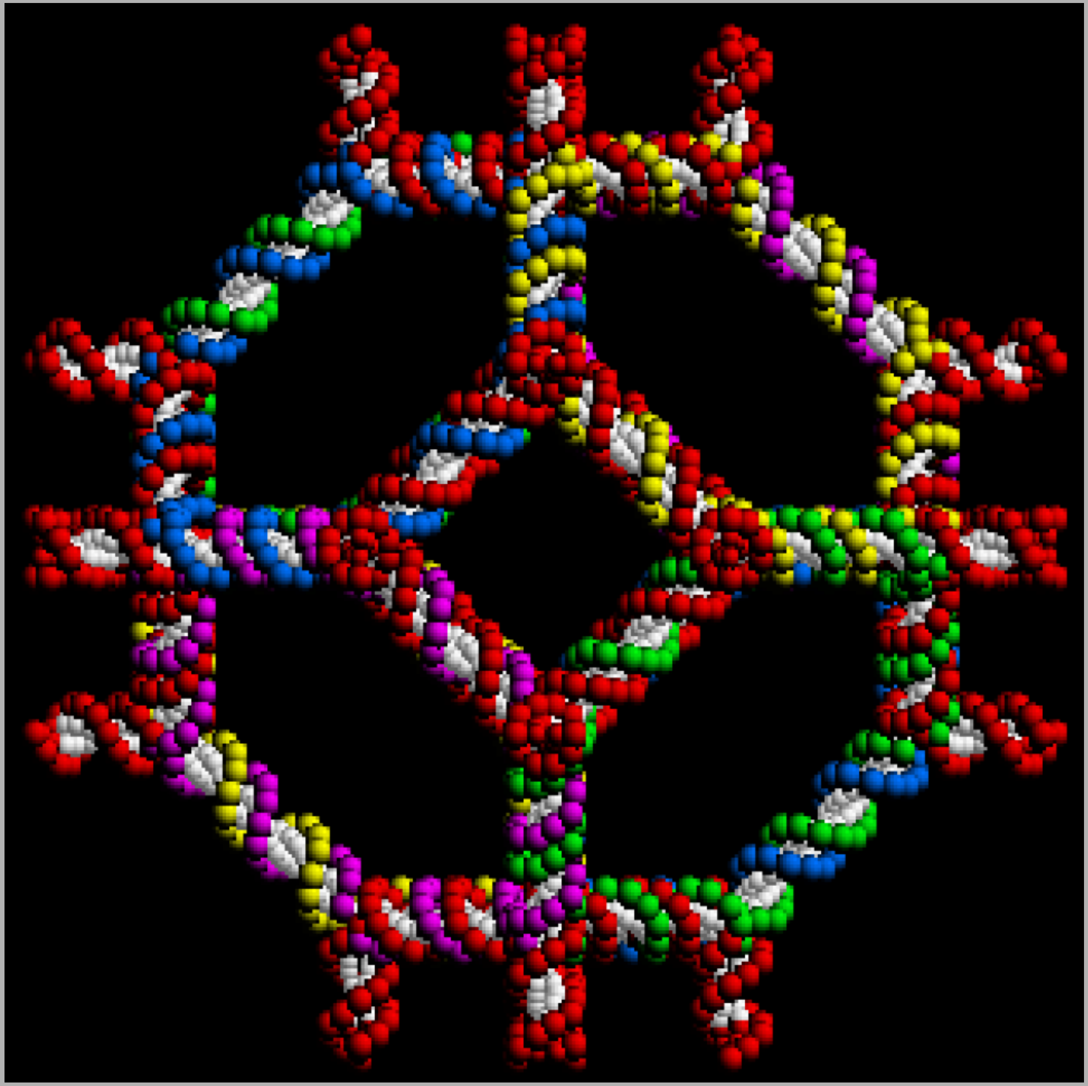
3D DNA Cube

[Seeman, New York University]



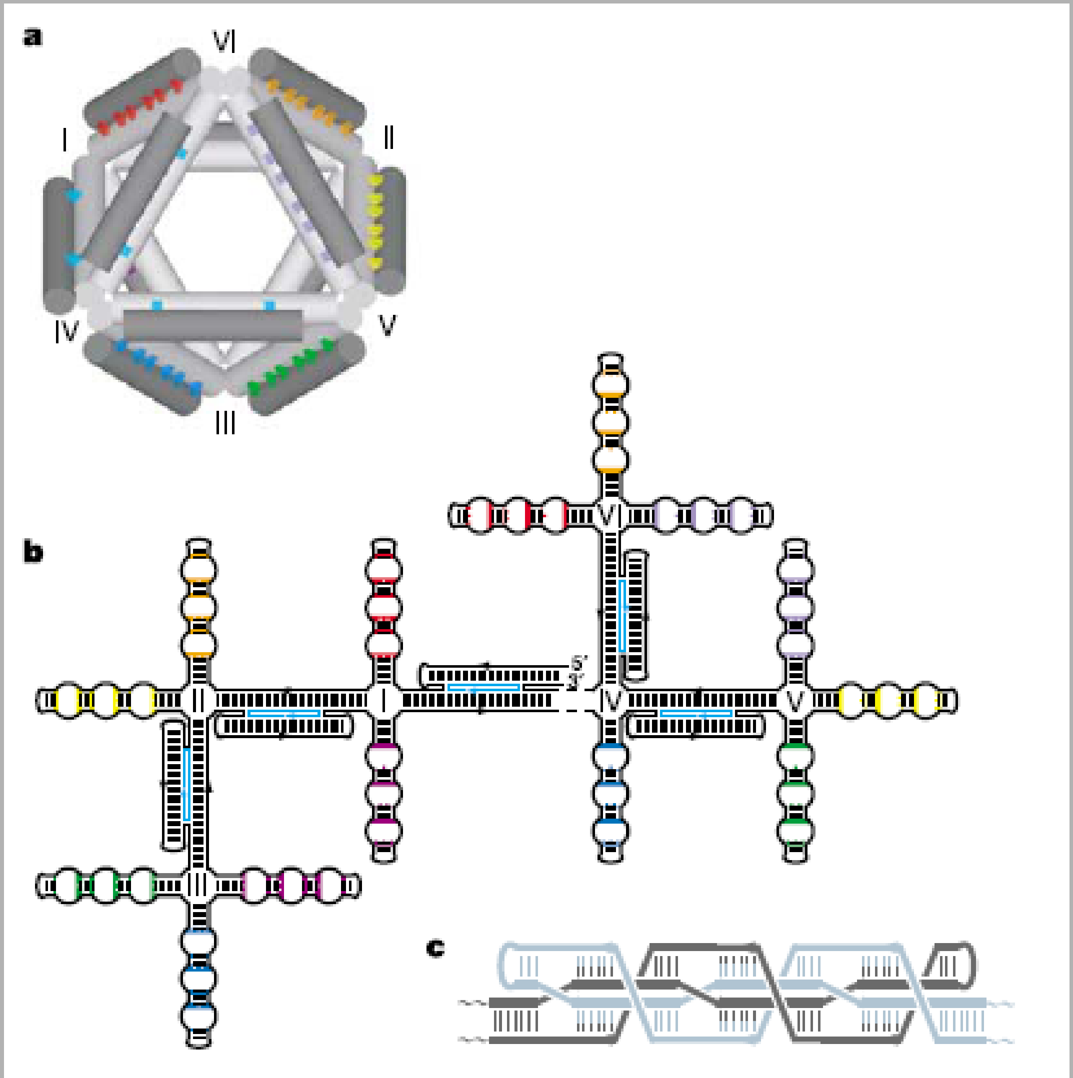
3D DNA Truncated Octahedron

[Seeman, New York University]



Clonable DNA Octahedron

[Shih, Quispe, Joyce, 2004]



Outline

- Background, Motivation
- Model
- Rectangle

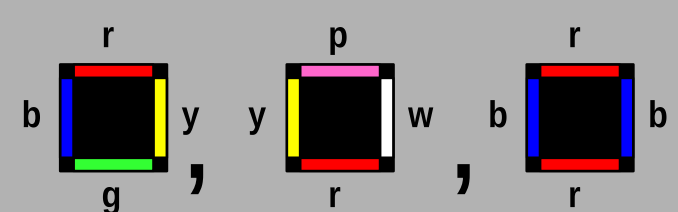
Tile Model of Self-Assembly

(Rothemund, Winfree STOC 2000)

Tile System: $\{t, G, T, s\}$

t : temperature, positive integer

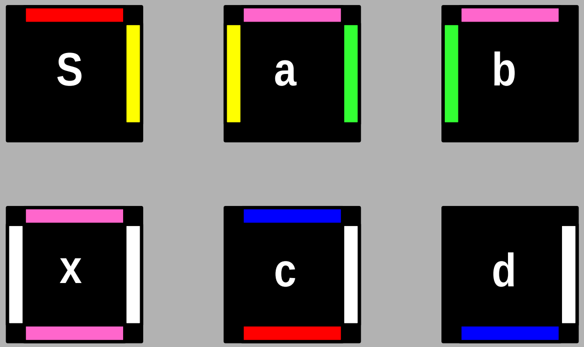
G: glue function $G : \sum \times \sum \rightarrow \{0,1,\ldots,t\}$

T: tileset $\{$ 

$, \ldots \}$

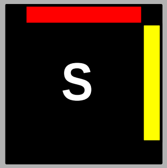
s: seed tile

How a tile system self assembles

T = 

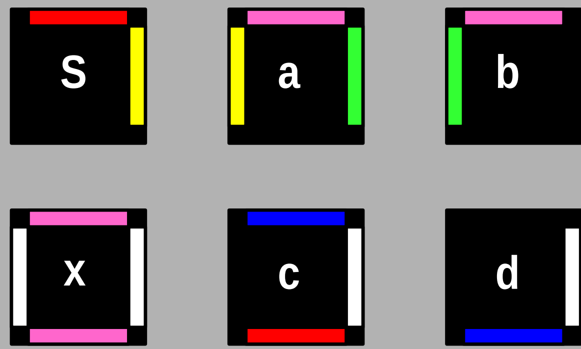
G(y) = 2
G(g) = 2
G(r) = 2
G(b) = 2
G(p) = 1
G(w) = 1

t = 2



How a tile system self assembles

T =



G(y) = 2

G(g) = 2

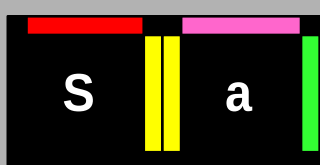
G(r) = 2

G(b) = 2

G(p) = 1

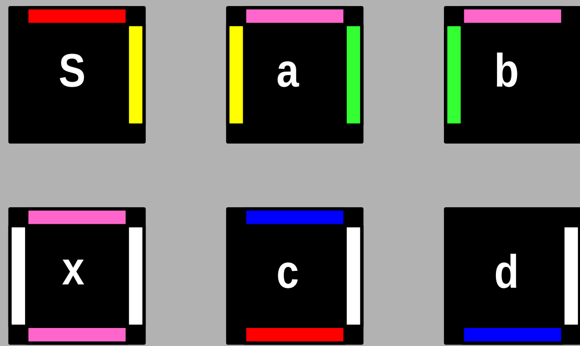
G(w) = 1

t = 2



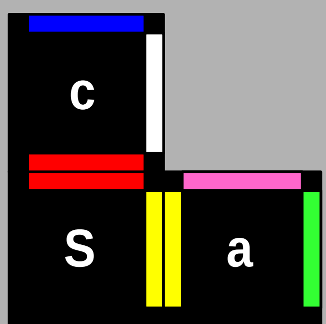
How a tile system self assembles

T =



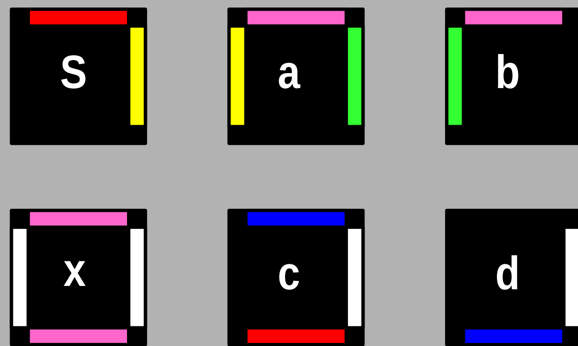
G(y) = 2
G(g) = 2
G(r) = 2
G(b) = 2
G(p) = 1
G(w) = 1

t = 2



How a tile system self assembles

T =



G(y) = 2

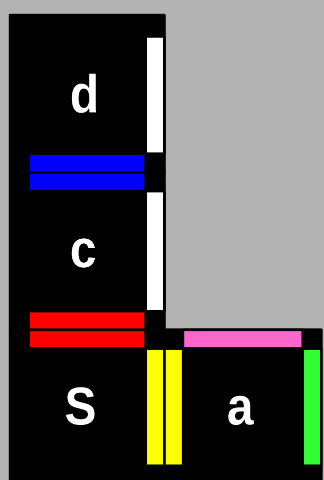
G(g) = 2

G(r) = 2

G(b) = 2

G(p) = 1

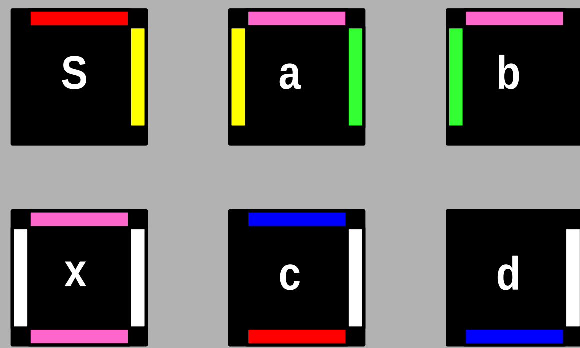
G(w) = 1



t = 2

How a tile system self assembles

T =



G(y) = 2

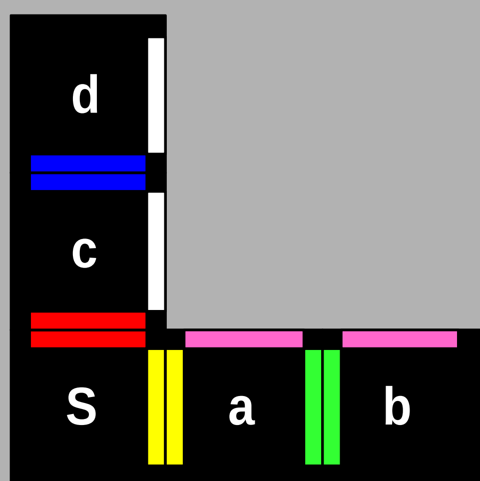
G(g) = 2

G(r) = 2

G(b) = 2

G(p) = 1

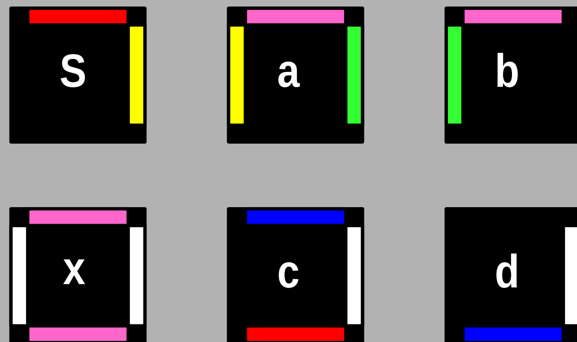
G(w) = 1



t = 2

How a tile system self assembles

T =



G(y) = 2

G(g) = 2

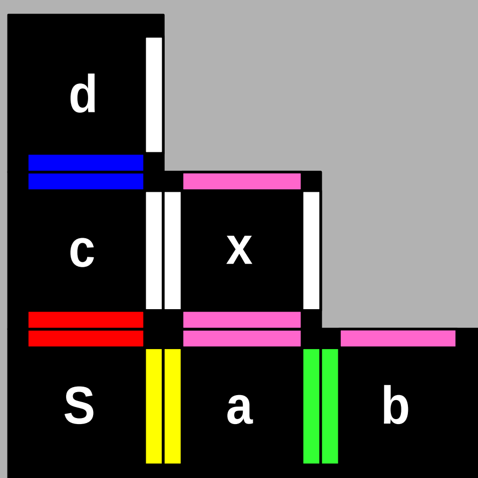
G(r) = 2

G(b) = 2

G(p) = 1

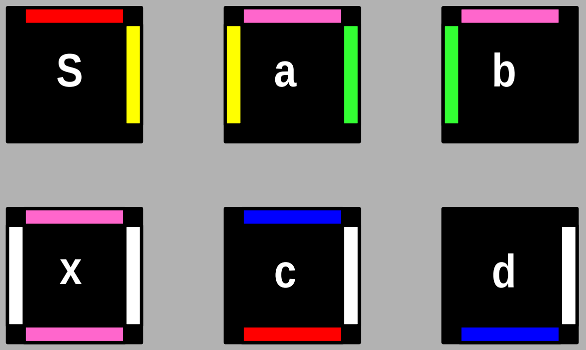
G(w) = 1

t = 2



How a tile system self assembles

T =



G(y) = 2

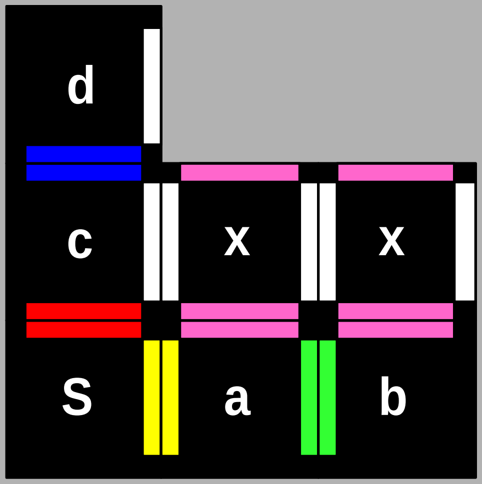
G(g) = 2

G(r) = 2

G(b) = 2

G(p) = 1

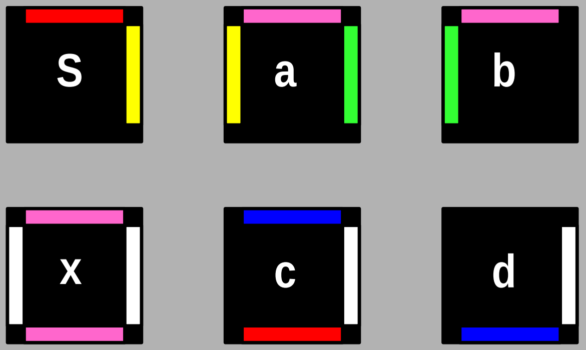
G(w) = 1



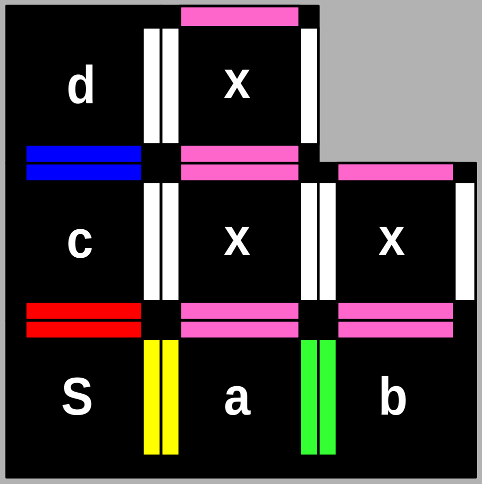
t = 2

How a tile system self assembles

T =



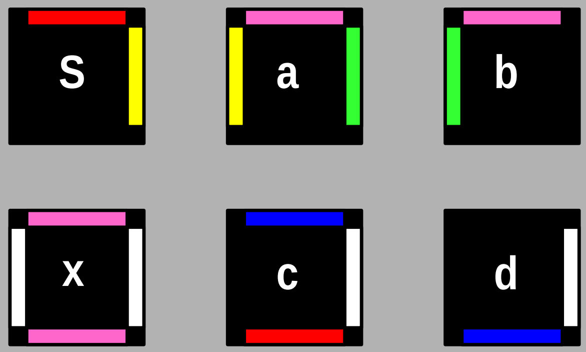
G(y) = 2
G(g) = 2
G(r) = 2
G(b) = 2
G(p) = 1
G(w) = 1



t = 2

How a tile system self assembles

T =



G(y) = 2

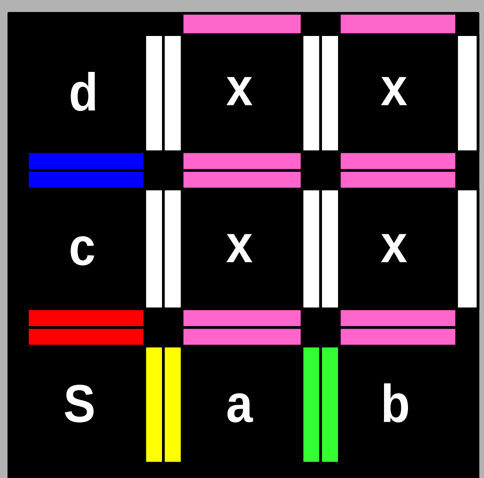
G(g) = 2

G(r) = 2

G(b) = 2

G(p) = 1

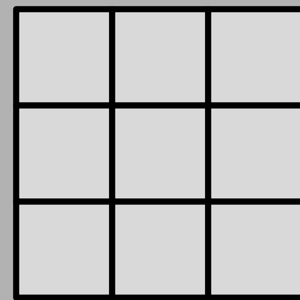
G(w) = 1



t = 2

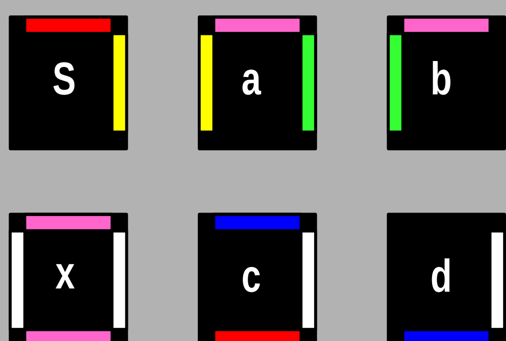
Efficient Assembly of Shapes

- Given a Shape:

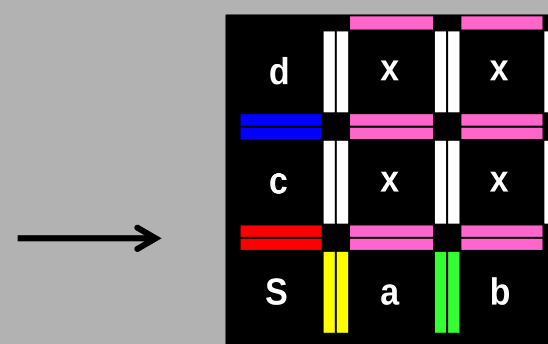


- Design an efficient tile system that uniquely builds the shape:

T =



G(y) = 2
G(g) = 2
G(r) = 2
G(b) = 2
G(p) = 1
G(w) = 1



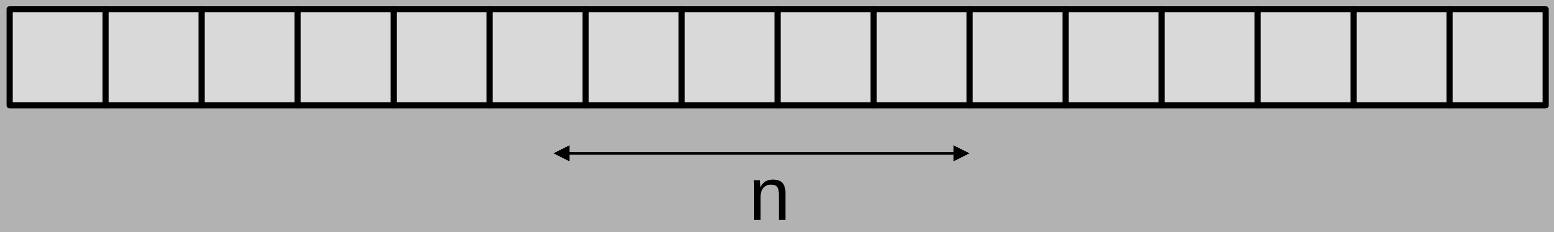
Alphabet of Shapes, Built with DNA Tiles

[Bryan Wei, Mingjie Dai, Peng Yin, Nature 2012]

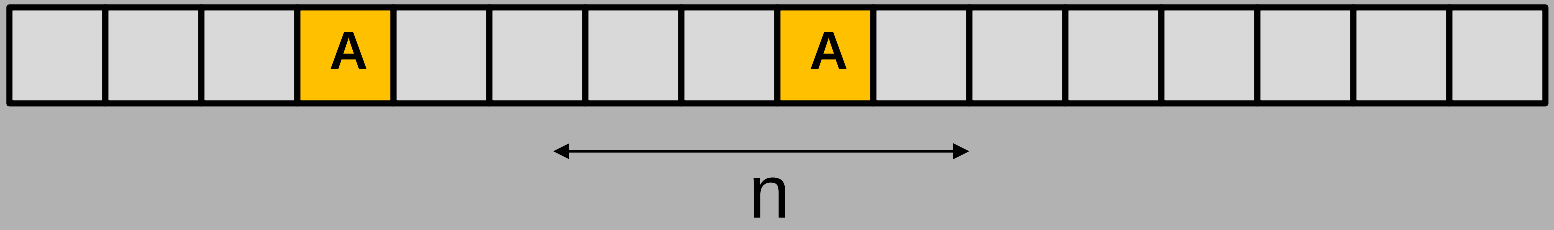
Outline

- Background, Motivation
- Model
- Rectangles

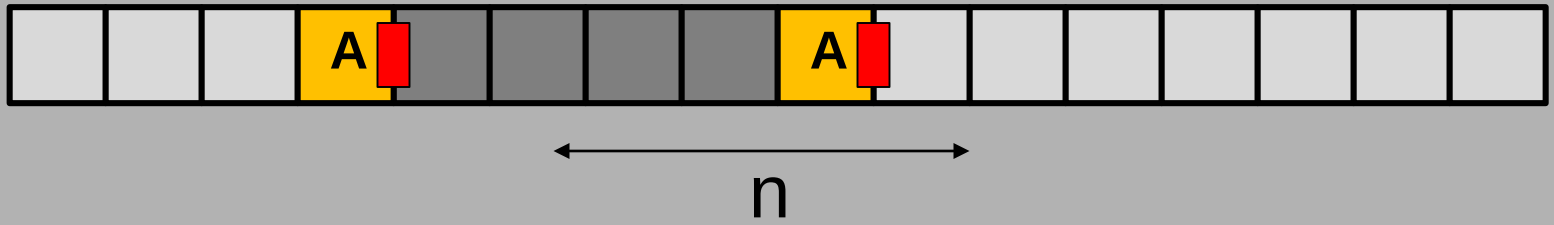
Building 1xn Lines



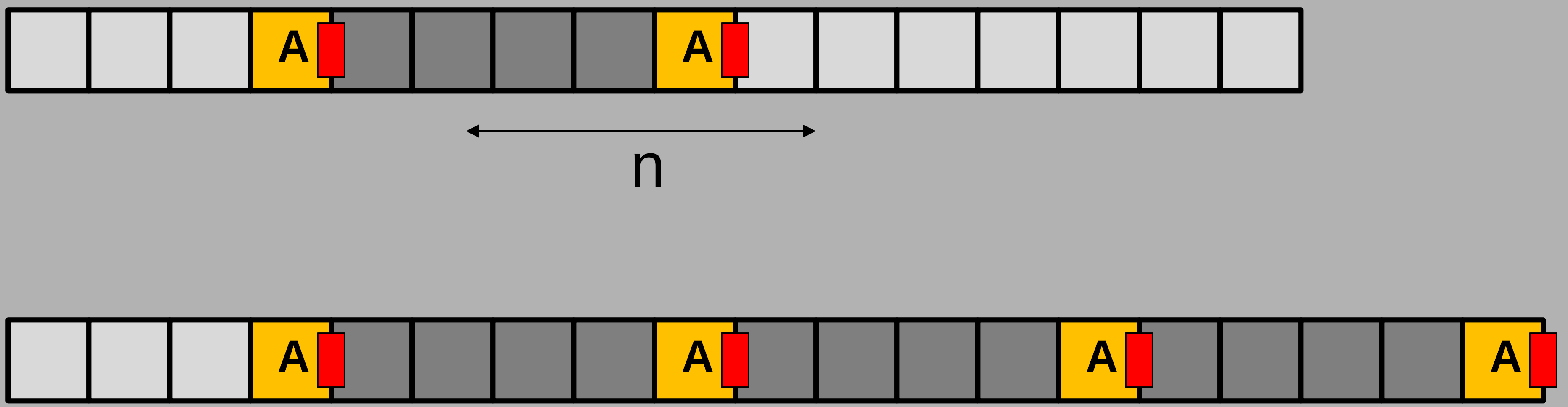
Building 1xn Lines



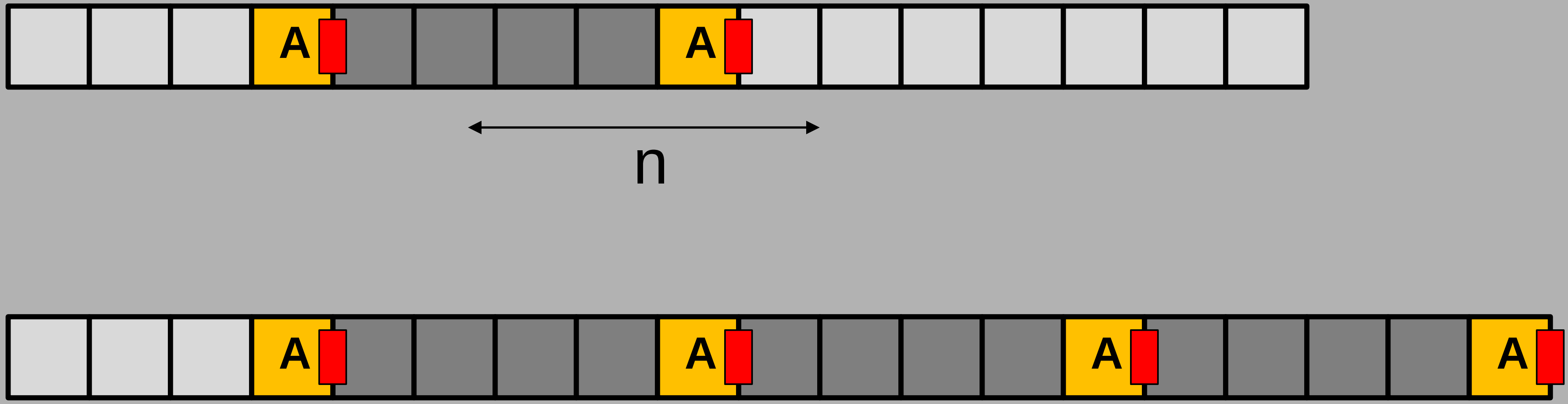
Building 1xn Lines



Building 1xn Lines

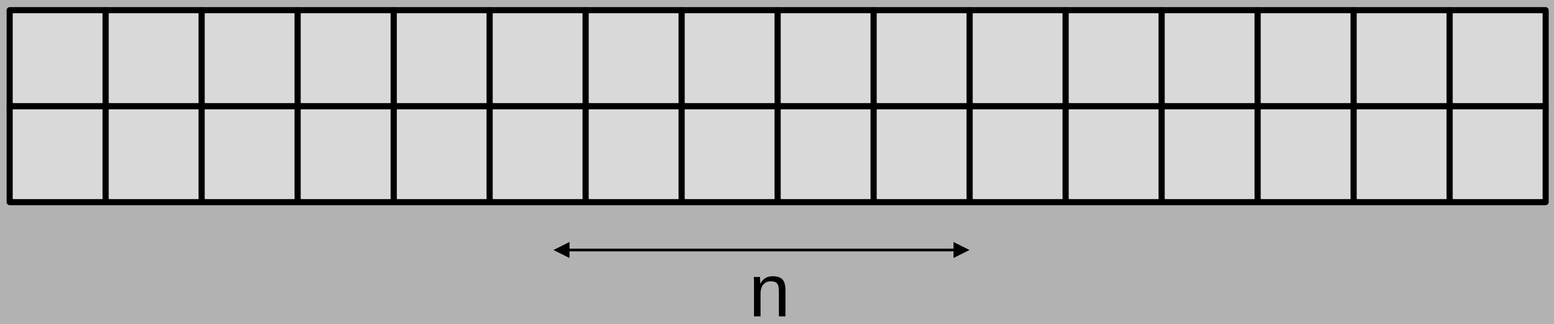


Building 1xn Lines

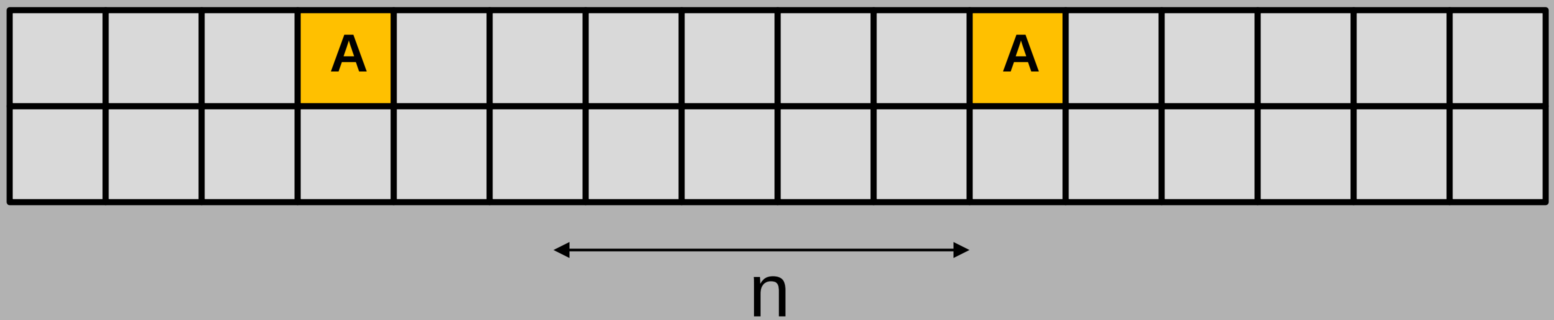


Tile Complexity: n

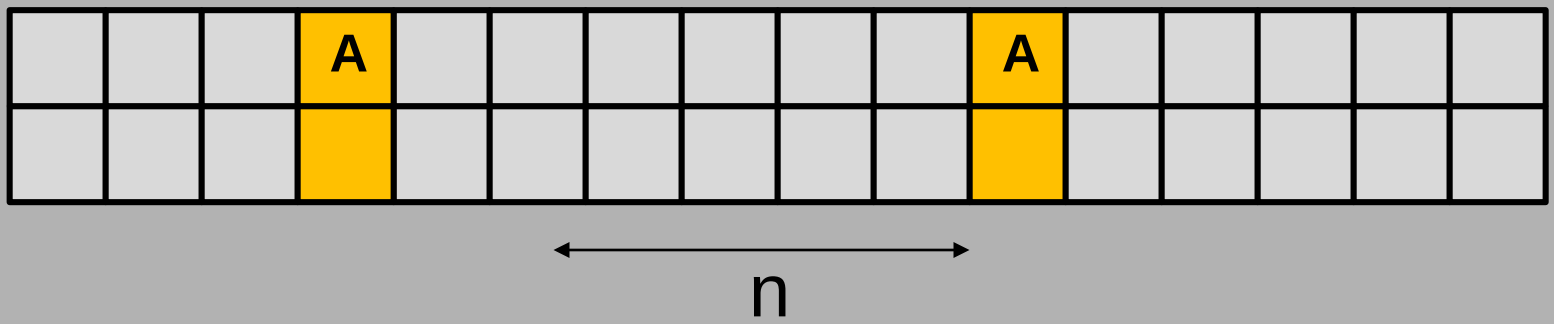
Building 2xn Rectanlges



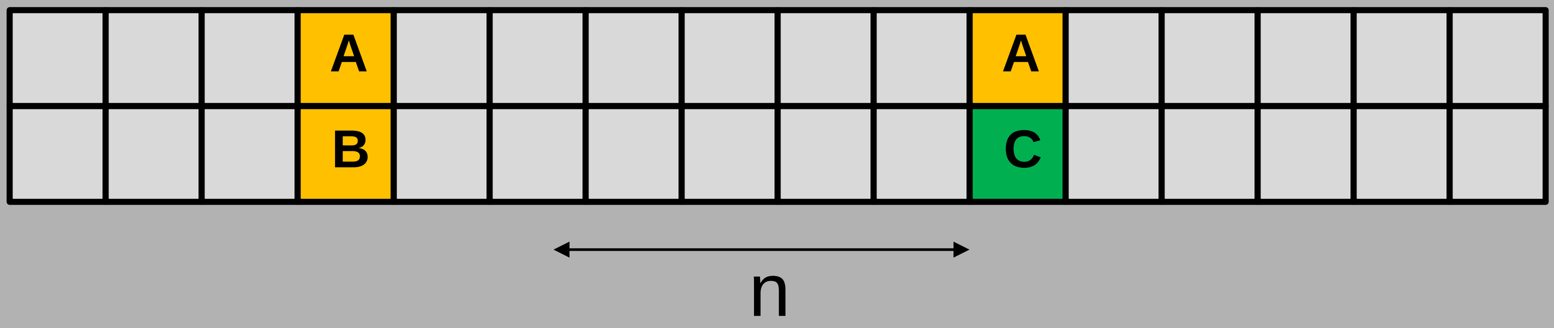
Building 2xn Rectangles



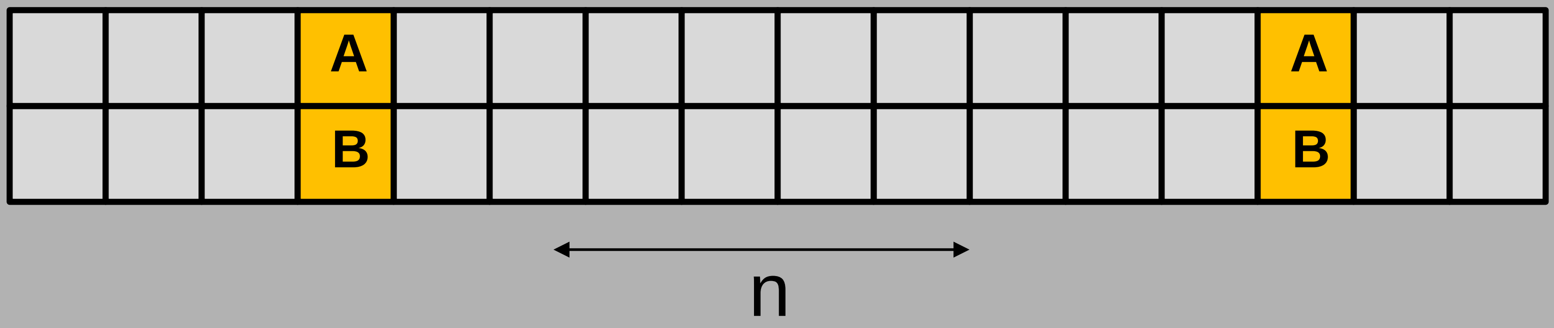
Building 2xn Rectangles



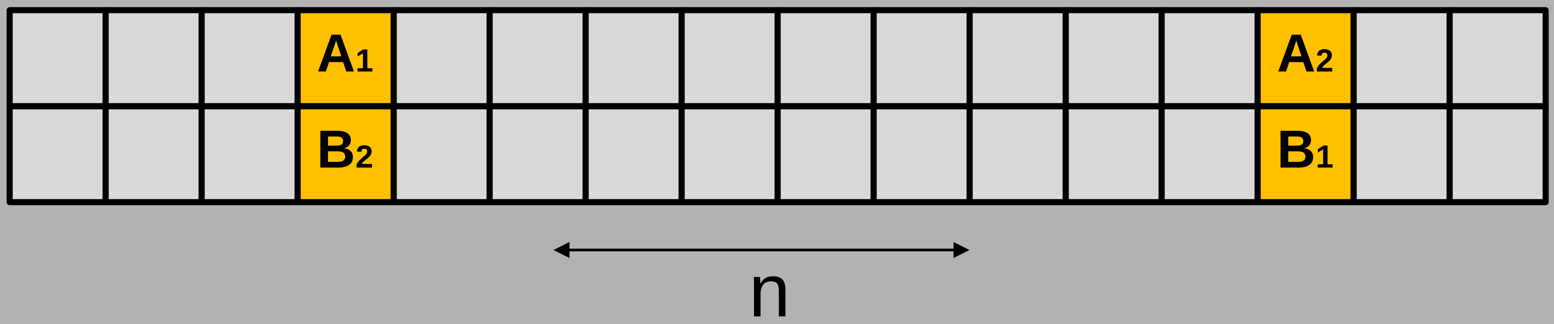
Building 2xn Rectangles



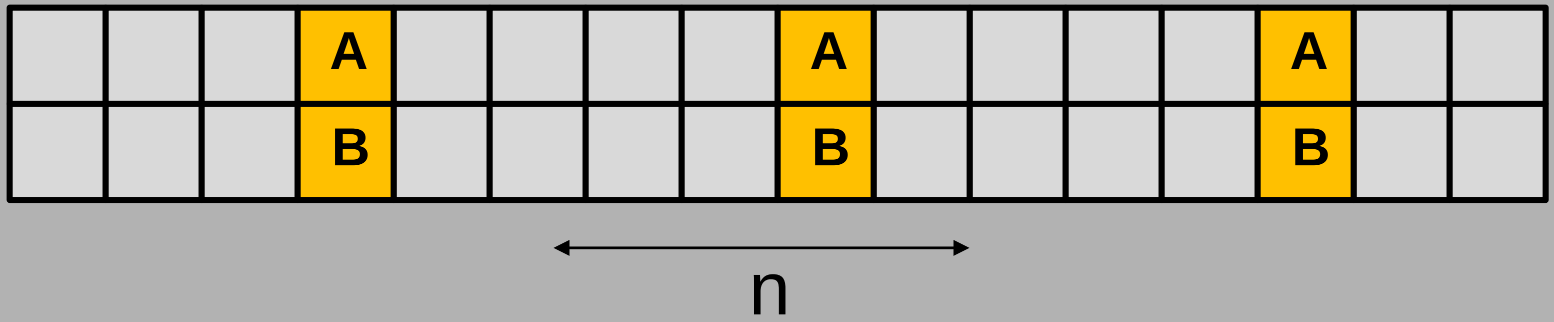
Building 2xn Rectangles



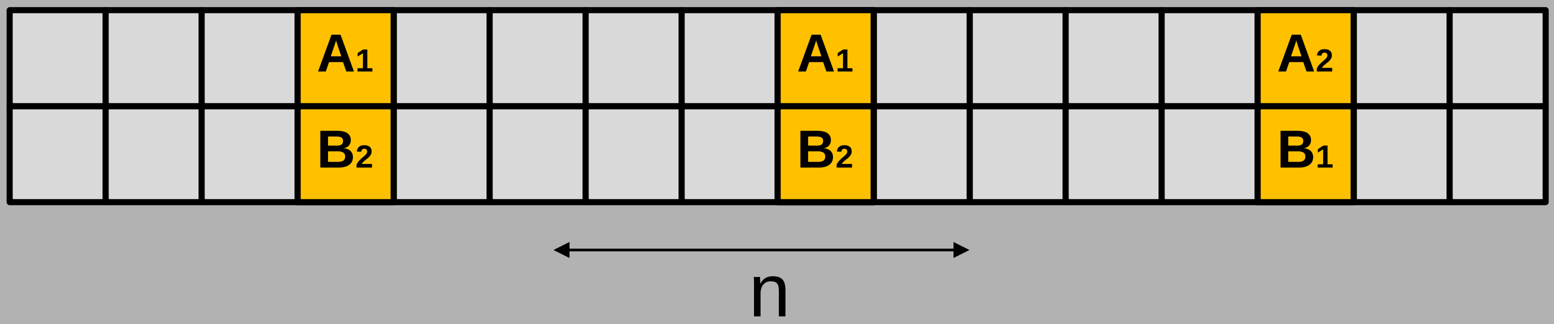
Building 2xn Rectangles



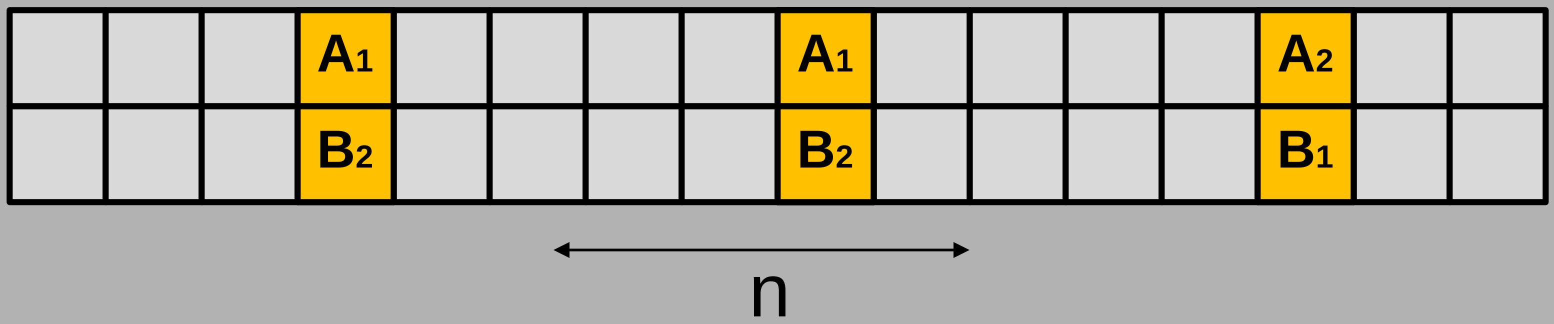
Building 2xn Rectangles



Building 2xn Rectangles

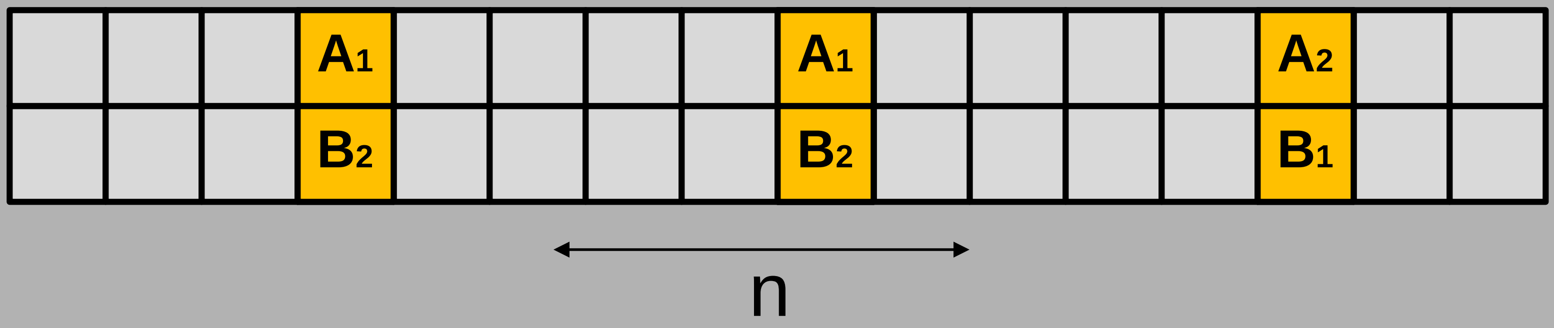


Building 2xn Rectangles



Lower Bound: $$\sqrt{\frac{n}{2}} = \Omega(\sqrt{n})$$

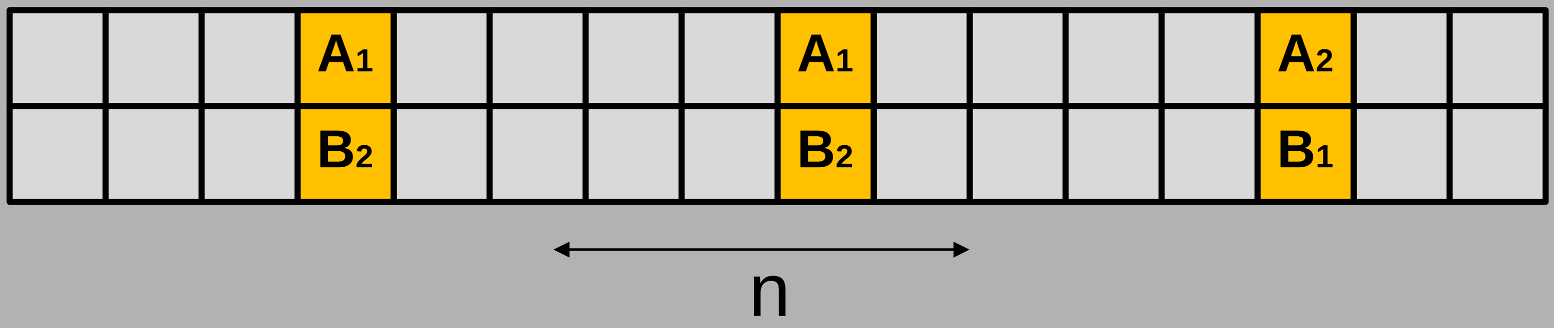
Building 2xn Rectangles



2 x n lines

Lower Bound	Upper Bound
$\Omega(\sqrt{n})$	$O(n)$

Building 2xn Rectangles

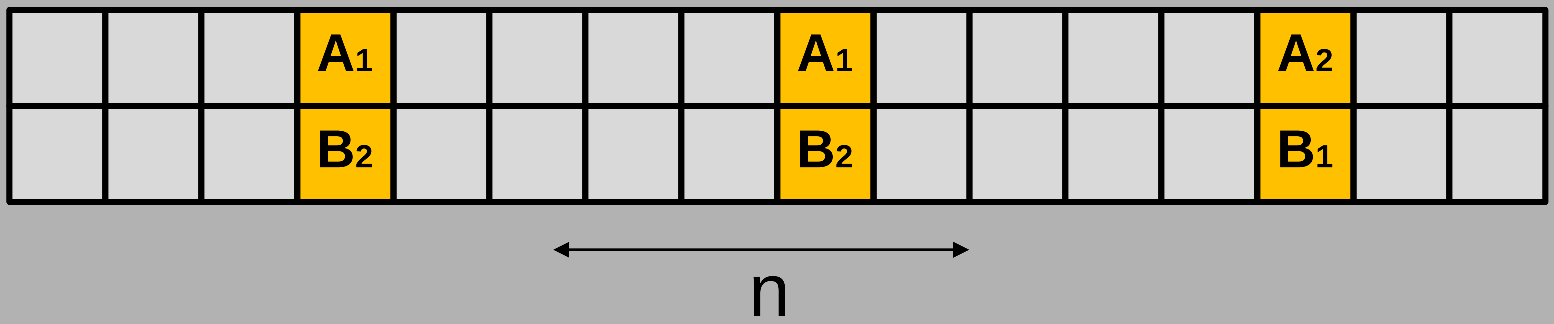


2 x n lines

Lower Bound	Upper Bound
$\Omega(\sqrt{n})$	$O(n)$

Can we do better than O(n)?

Building 2xn Rectangles



2 x n lines

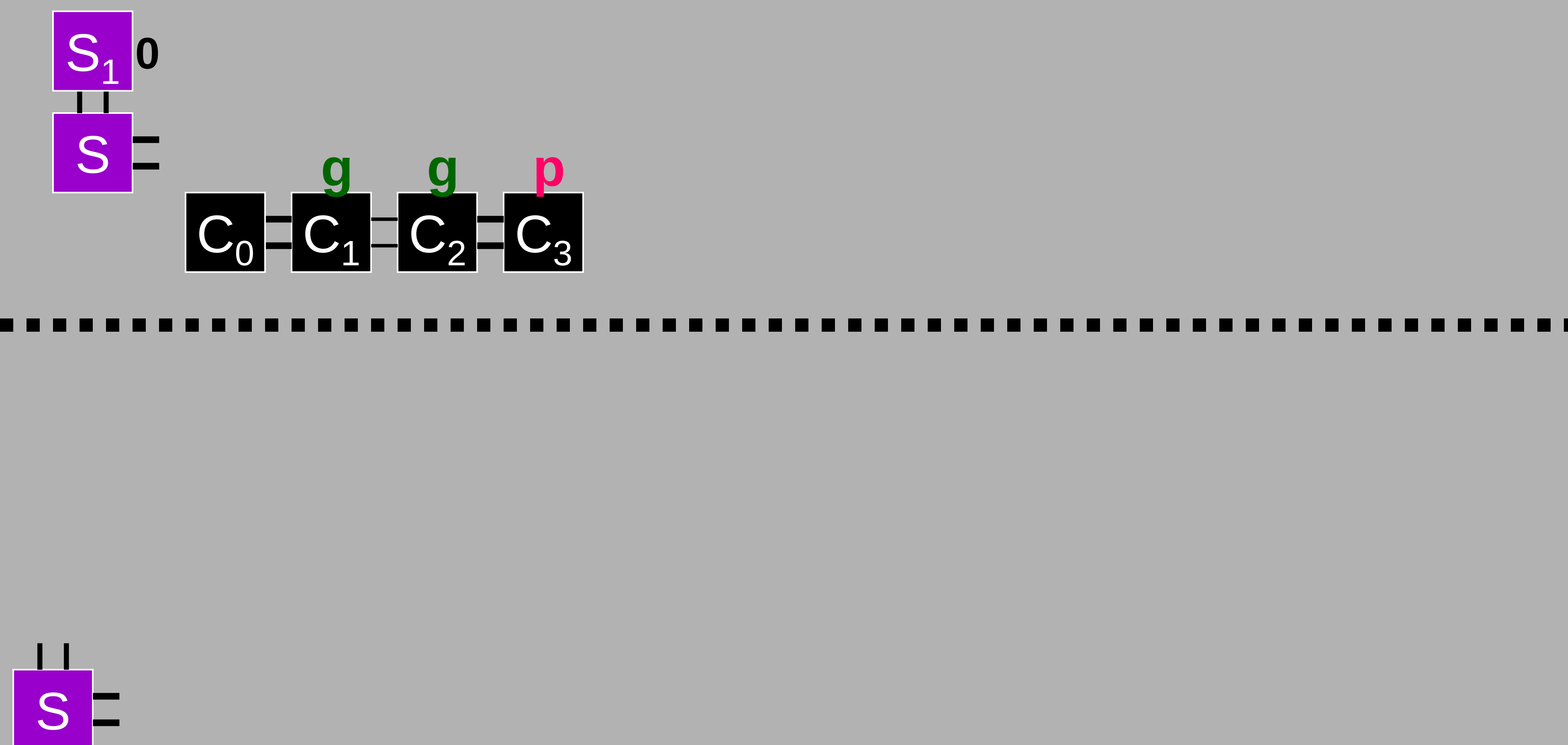
Lower Bound	Upper Bound
$\Omega(\sqrt{n})$	$\cancel{O(n)}$ $O(\sqrt{n})$

Can we do better than O(n)?

-YES

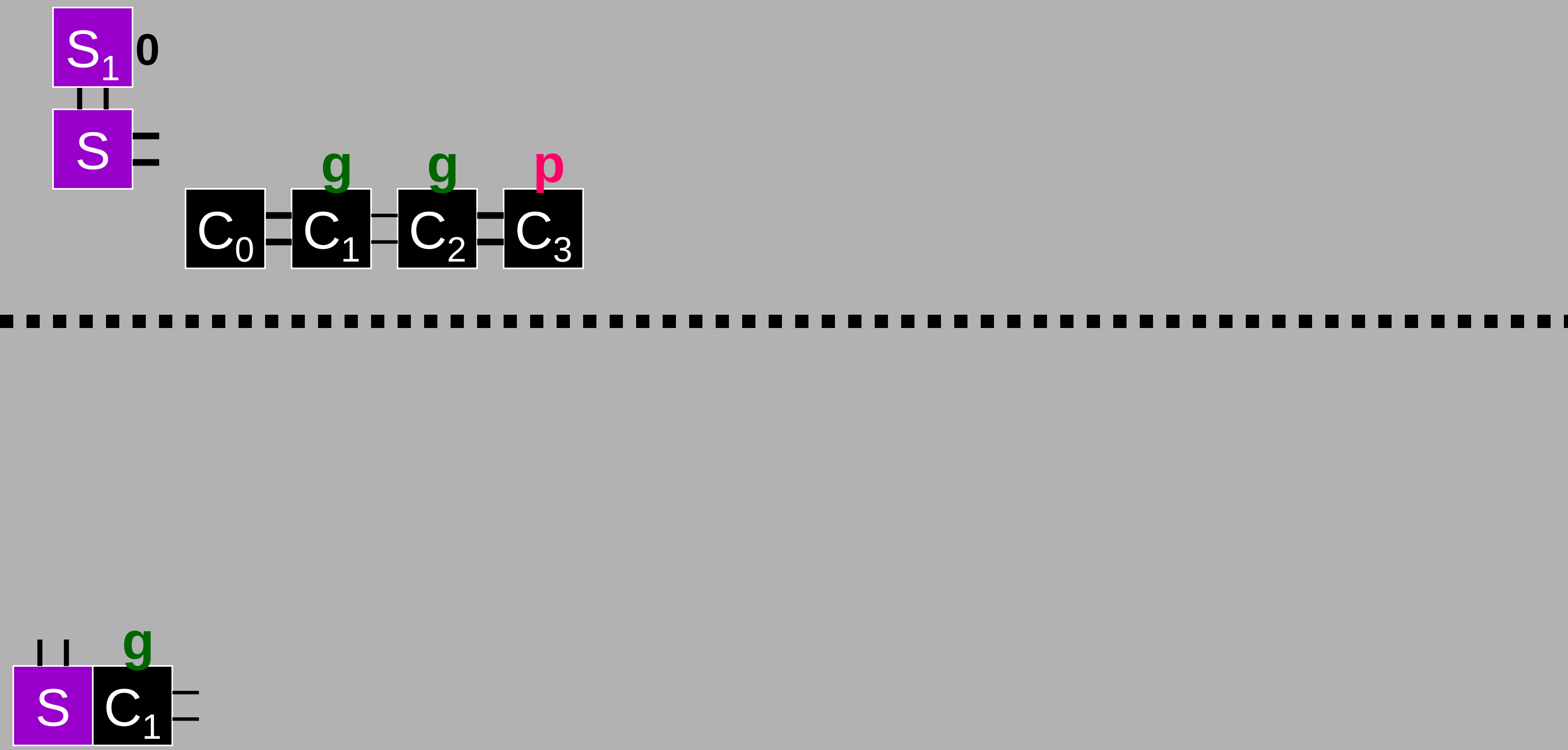
Build a 2 x 16 rectangle:

t = 2



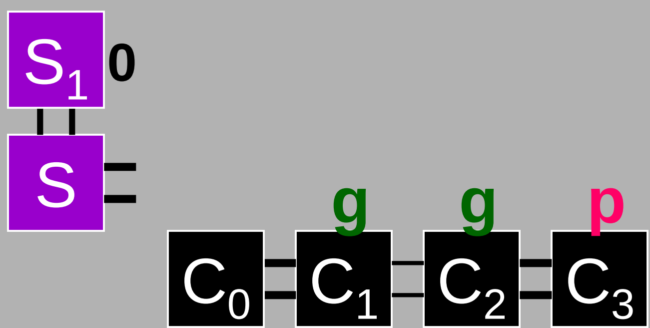
Build a 2 x 16 rectangle:

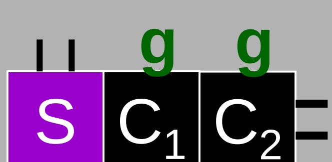
t = 2



Build a 2 x 16 rectangle:

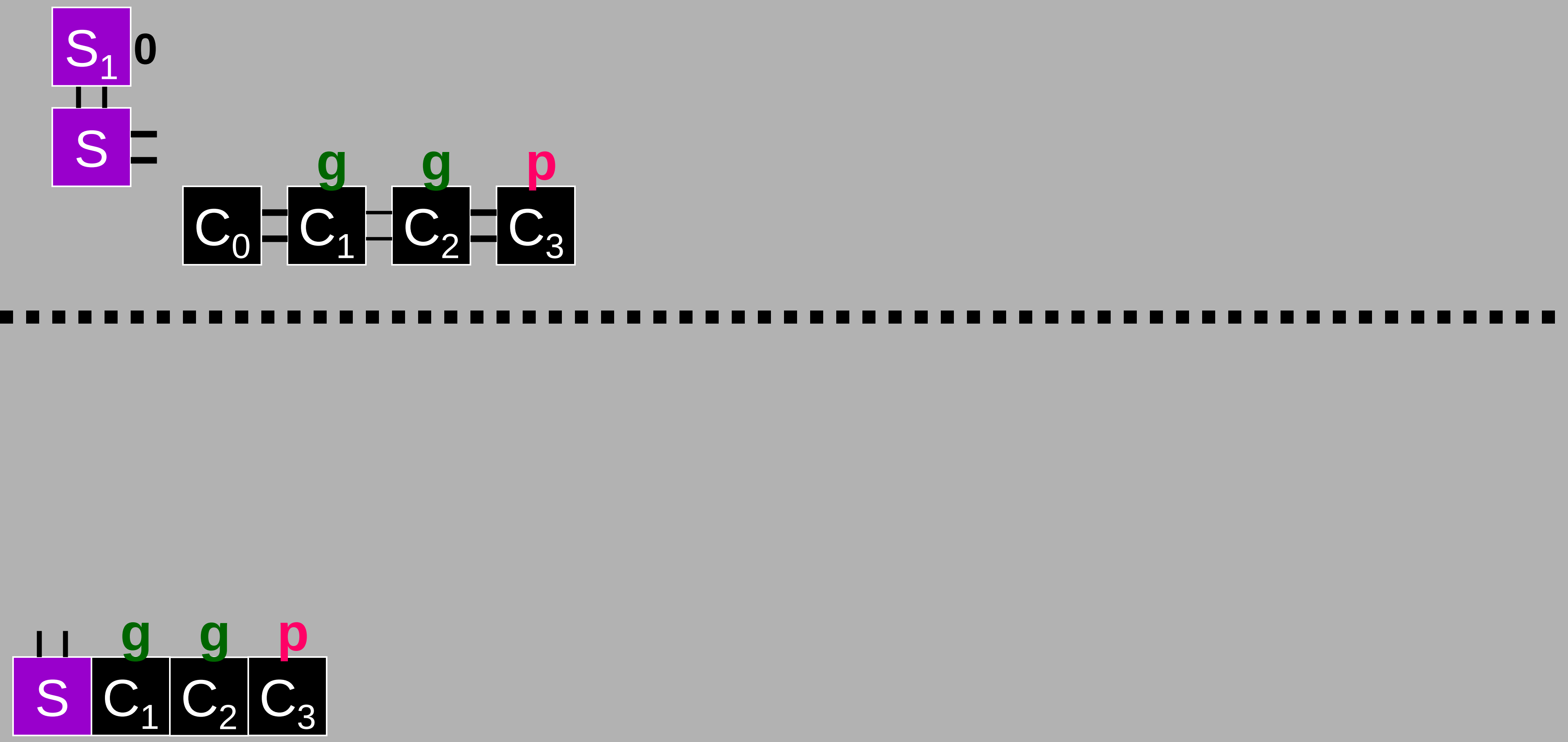
t = 2





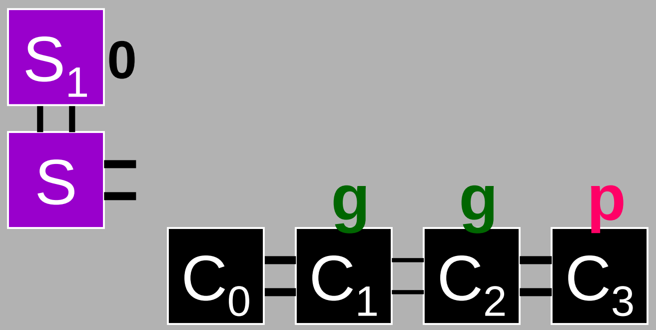
Build a 2 x 16 rectangle:

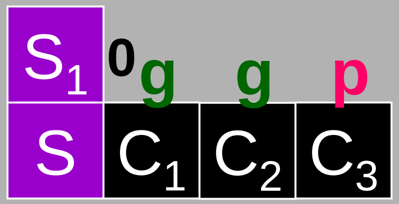
t = 2



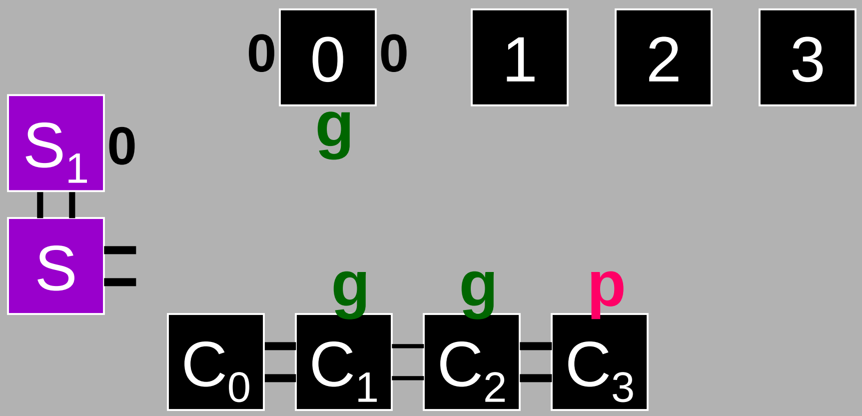
Build a 2 x 16 rectangle:

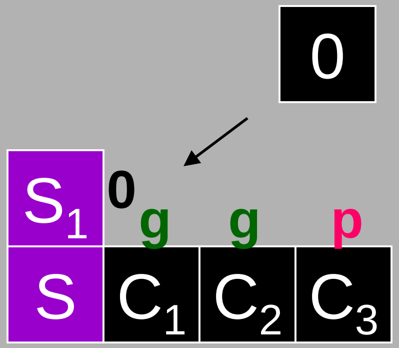
t = 2





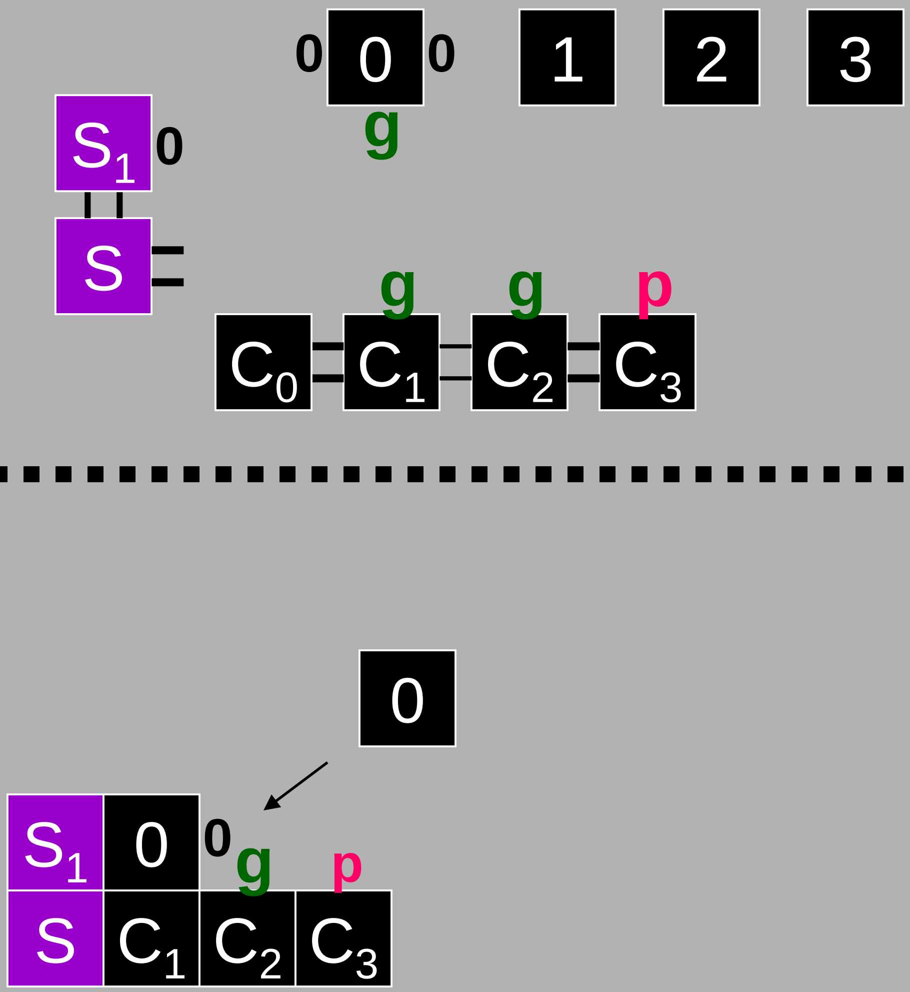
Build a 2 x 16 rectangle: t = 2





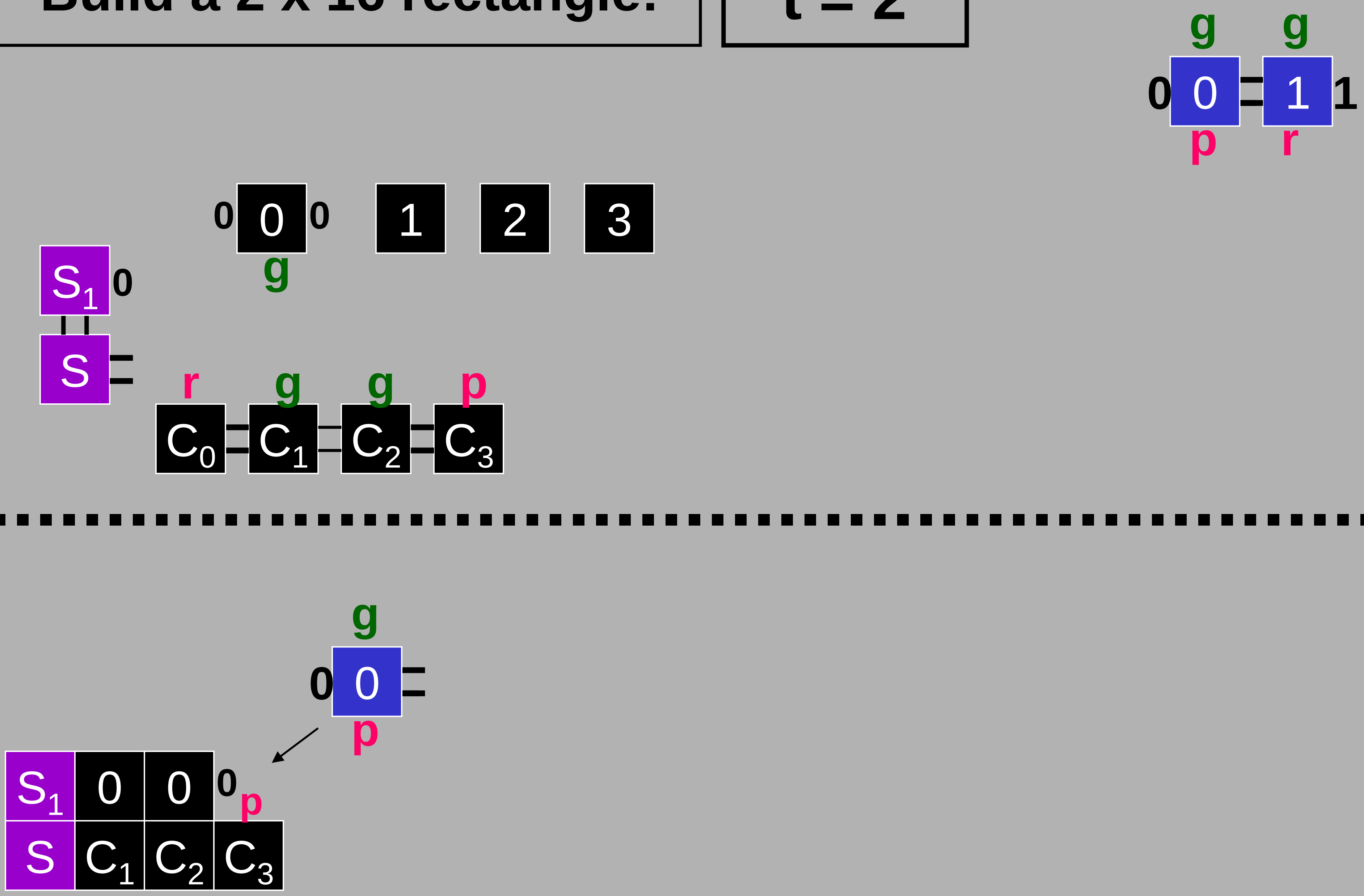
Build a 2 x 16 rectangle:

t = 2



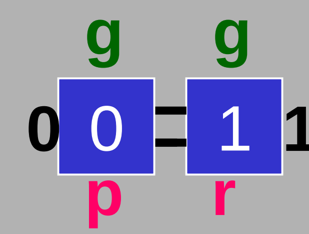
Build a 2 x 16 rectangle:

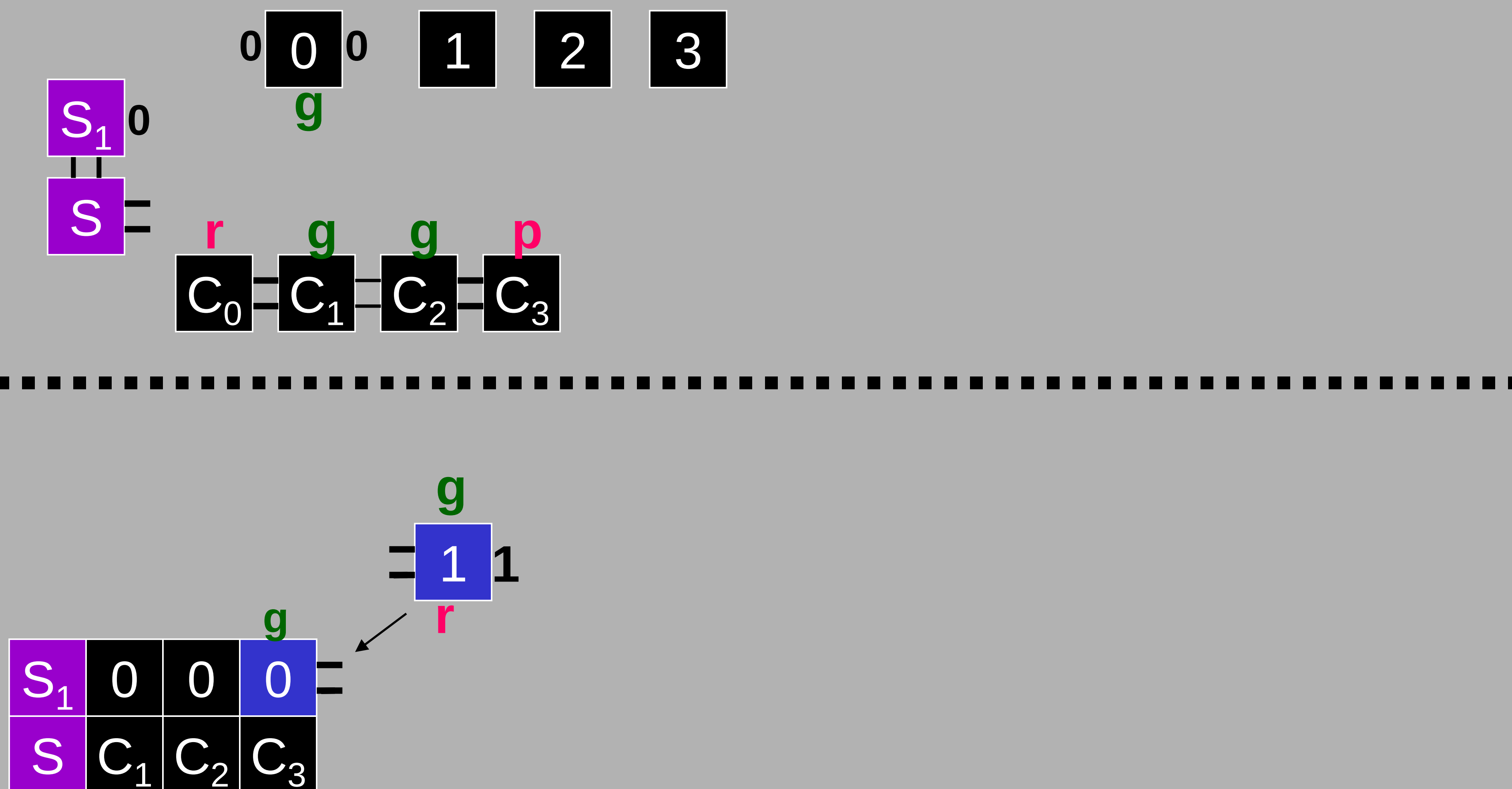
t = 2



Build a 2 x 16 rectangle:

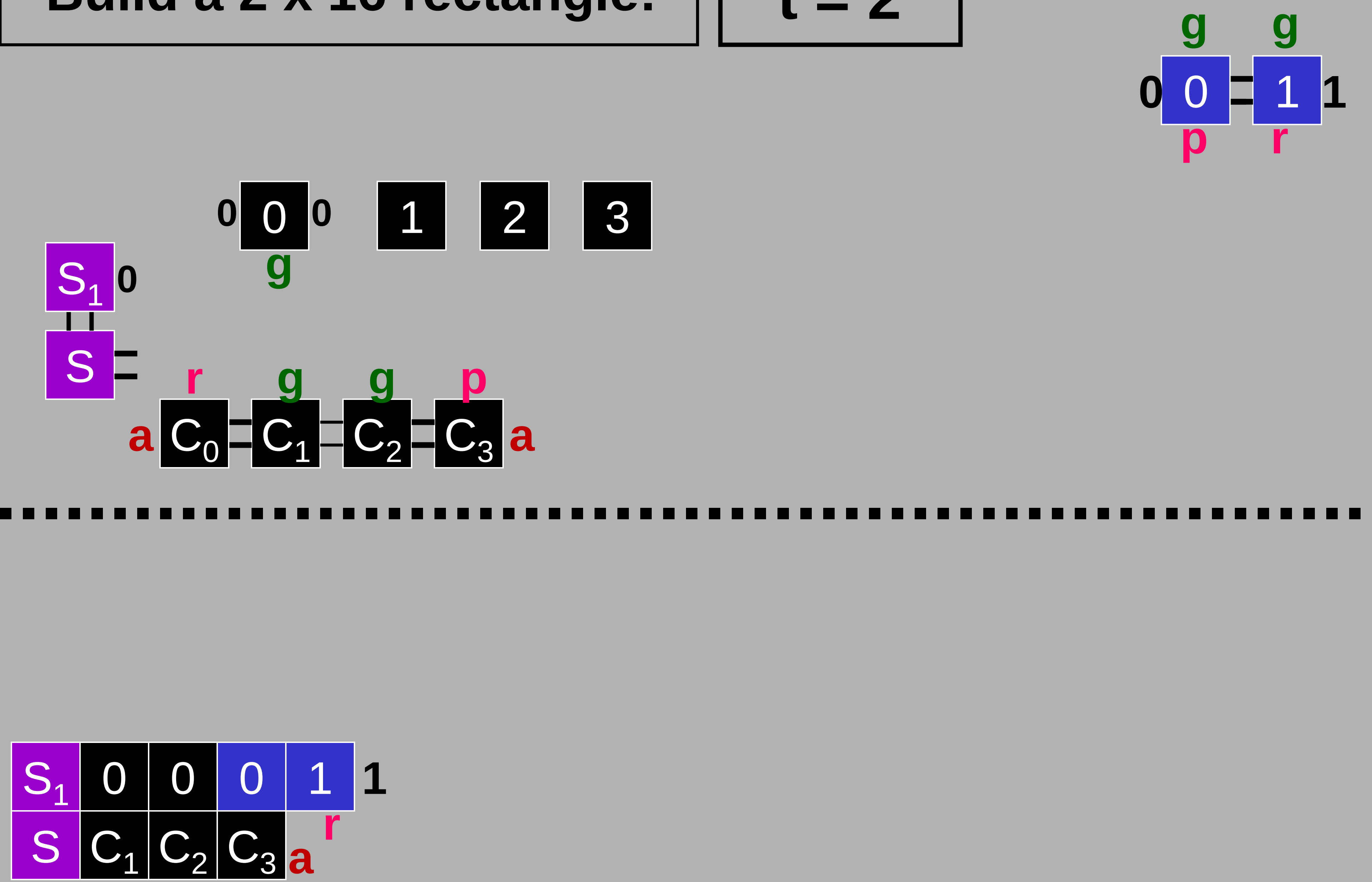
t = 2





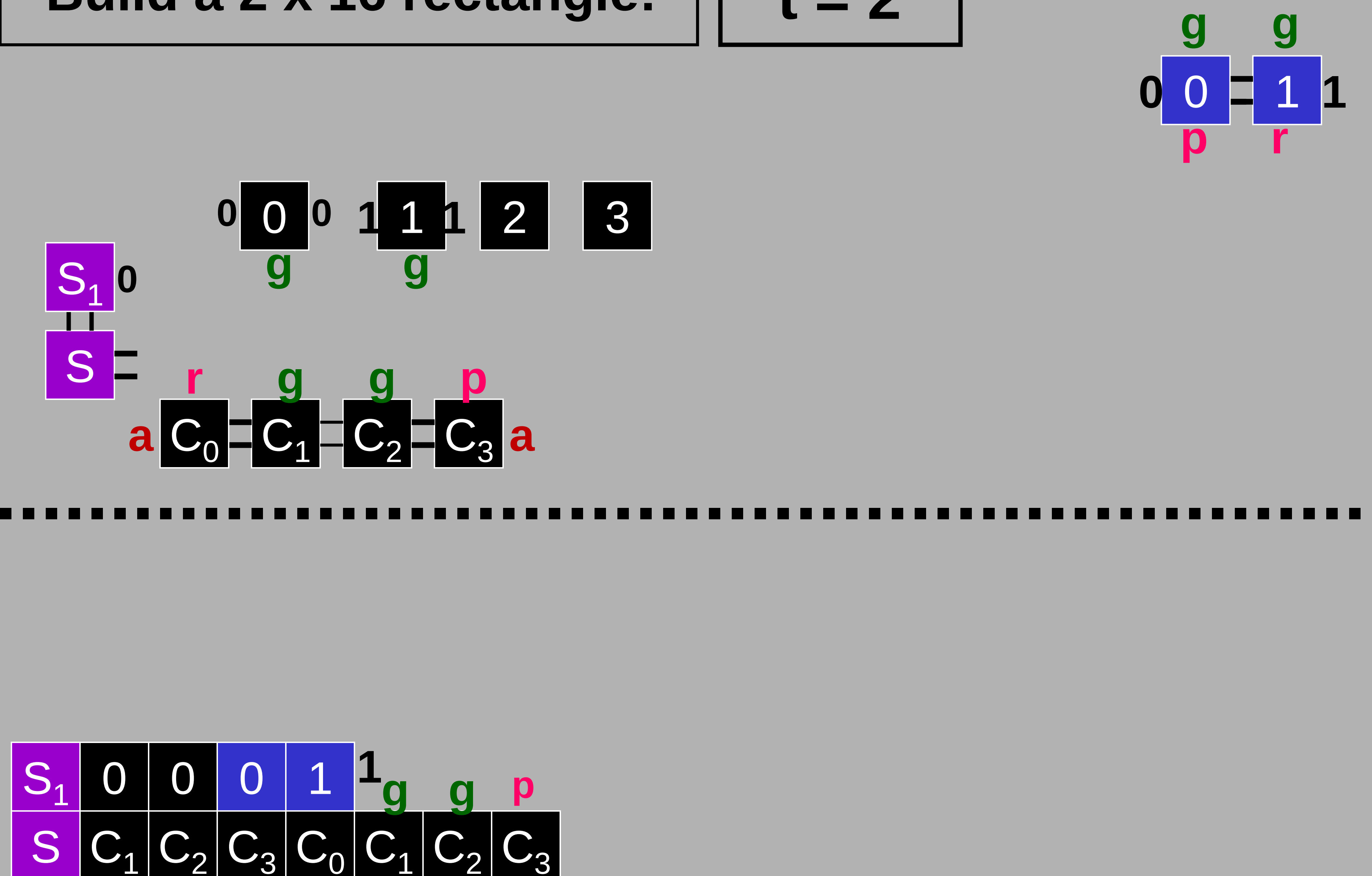
Build a 2 x 16 rectangle:

t = 2



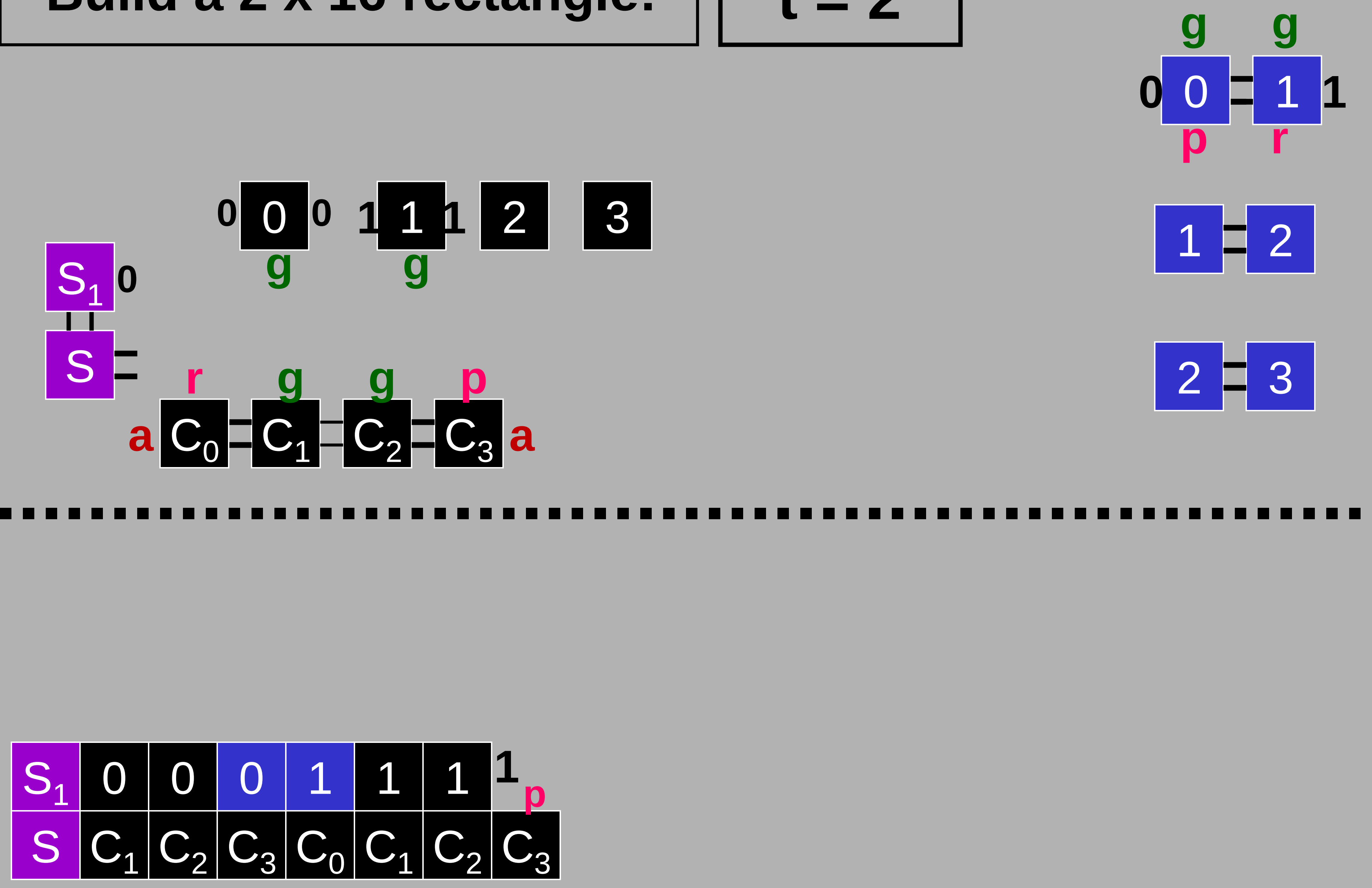
Build a 2 x 16 rectangle:

t = 2



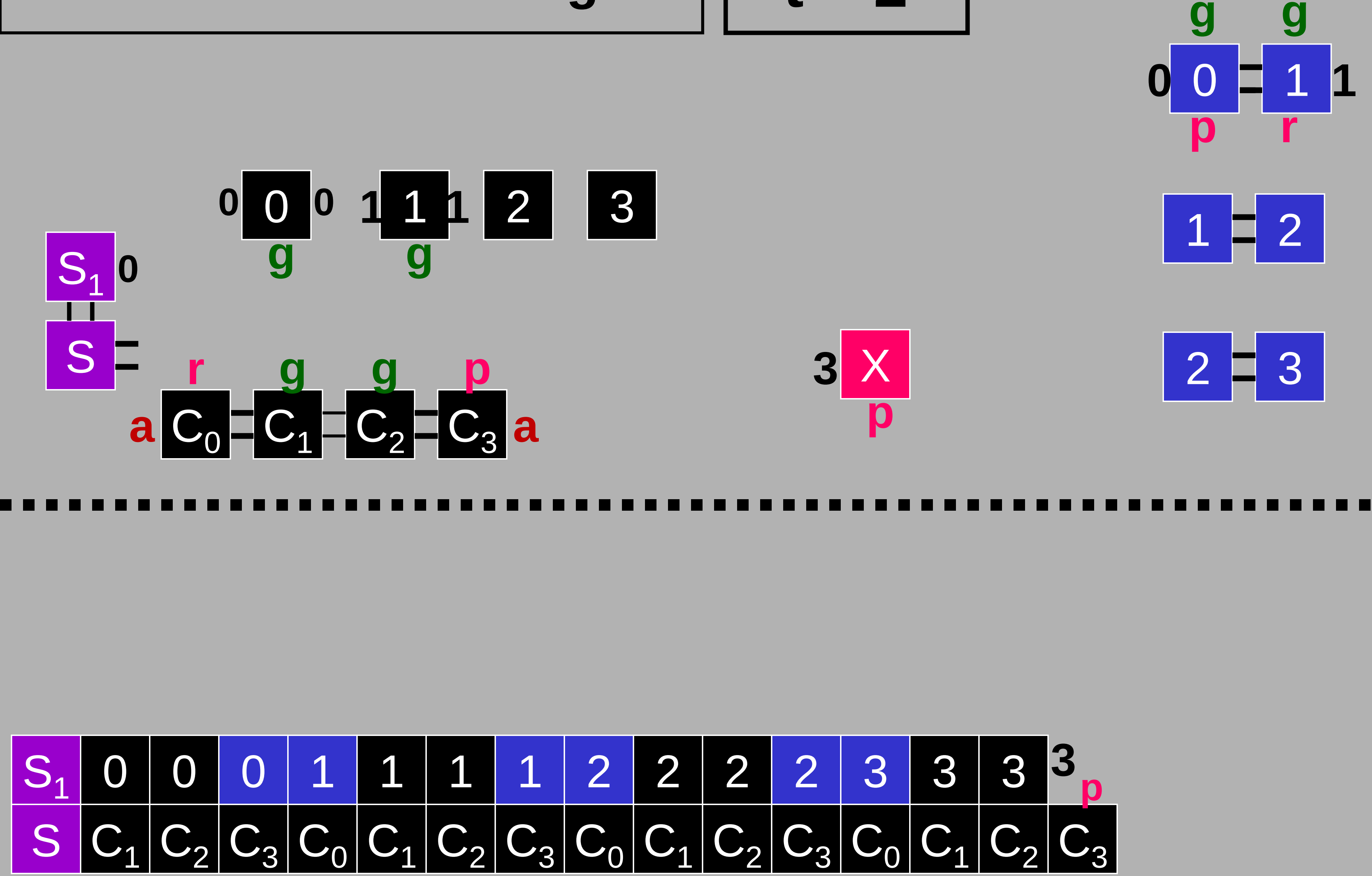
Build a 2 x 16 rectangle:

t = 2



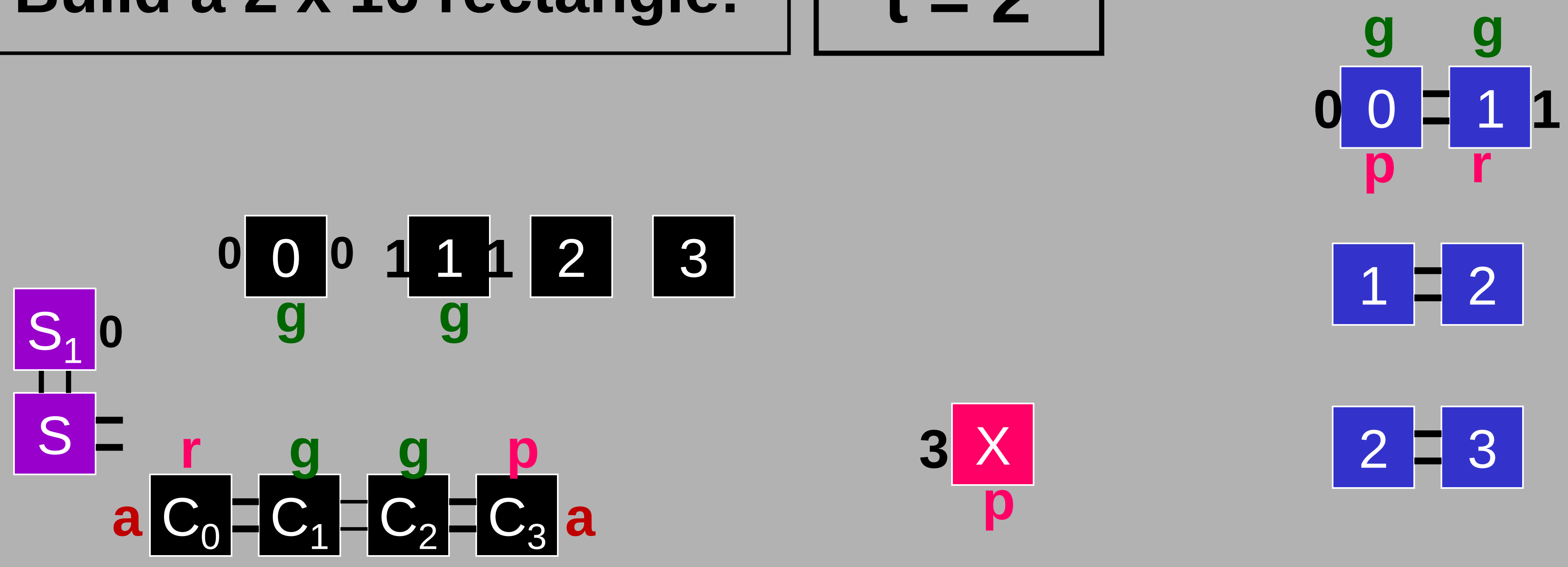
Build a 2 x 16 rectangle:

t = 2



Build a 2 x 16 rectangle:

t = 2



S_1	0	0	0	1	1	1	1	2	2	2	2	3	3	3	P	
S	C_1	C_2	C_3	C_0	C_1	C_2	C_3	C_0	C_1	C_2	C_3	C_0	C_1	C_2	C_3	

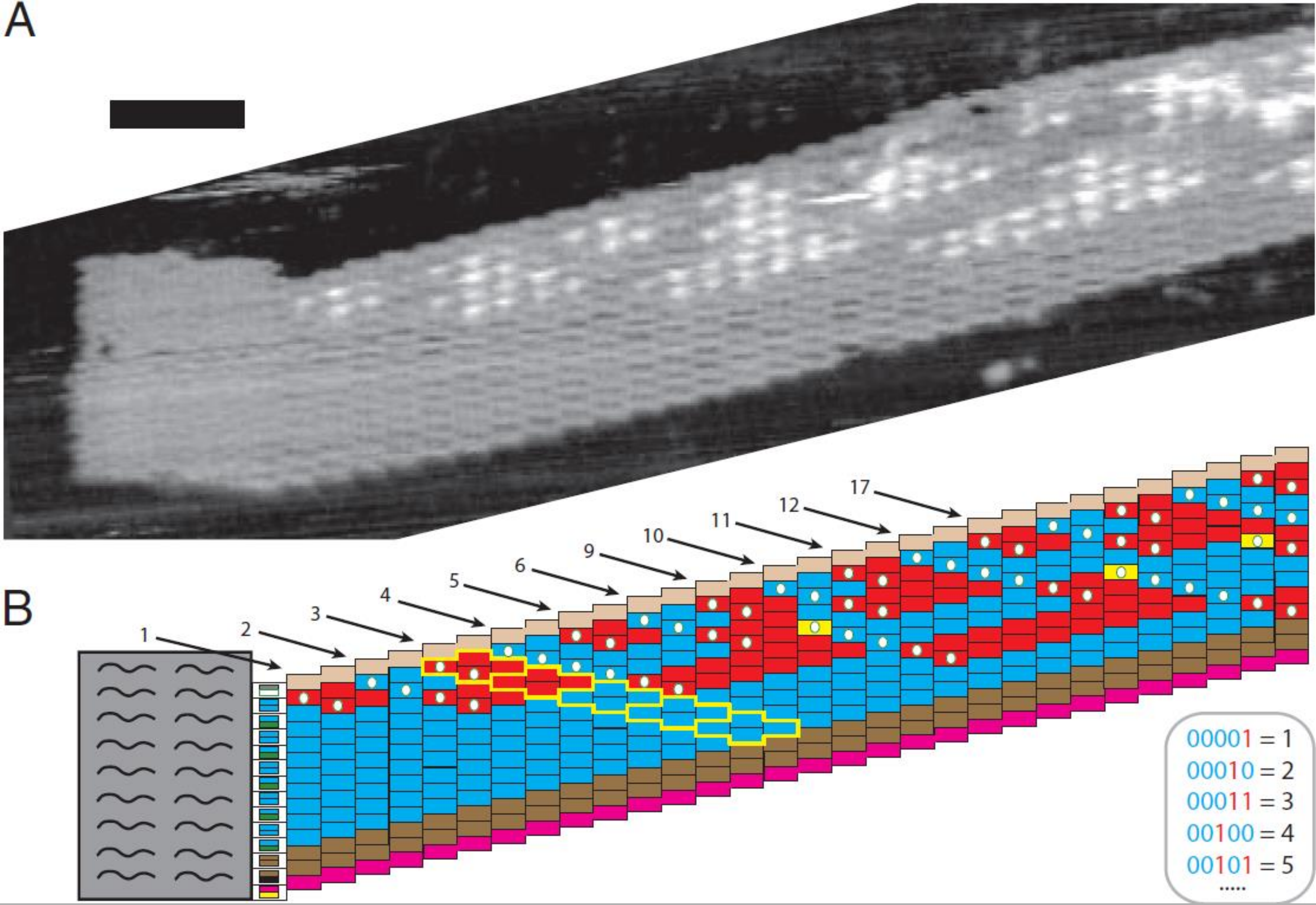
Build a 2 x 16 rectangle:

t = 2

S_1	0	0	0	1	1	1	1	2	2	2	2	3	3	3	P
S	C_1	C_2	C_3	C_0	C_1	C_2	C_3	C_0	C_1	C_2	C_3	C_0	C_1	C_2	C_3

2 x n lines

Lower Bound	Upper Bound
$\Omega(\sqrt{n})$	$O(\sqrt{n})$



Barish, Shulman, Rothemund, Winfree, 2009