Molecular Computation Homework

Topic: Probability and Randomization

1 Problems

1. Suppose you are given a biased coin that has $Pr[HEADS] = p$, with $p \geq a$ for some fixed $a$, without being given any other information about $p$. Devise a procedure for estimating $p$ by a value $p'$ such that you can guarantee that $Pr(|p - p'| > \epsilon) < \delta$, for any choice of the constants $0 < a, \epsilon, \delta < 1$. Let $N$ be the number of times you need to flip the biased coin to obtain the estimate. What is the smallest value of $N$ for which you can still give this guarantee?

2. Suppose you are given a coin for which the probability of HEADS, say $p$, is unknown. How can you use this coin to generate unbiased (i.e., $Pr[HEADS] = Pr[TAILS] = 1/2$) coin-flips? Give a scheme for which the expected number of flips of the biased coin for extracting one unbiased coin-flip is no more than $1/(p(1 - p))$. (Hint: Consider two consecutive flips of the biased coin.)