Optimal Squares with Flexible Glues

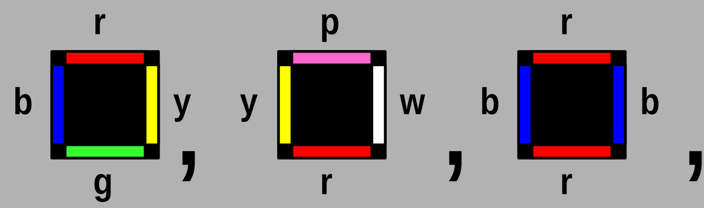
Tile Model of Self-Assembly

(Rothemund, Winfree STOC 2000)

Tile System: $\{t, G, T, s\}$

t : temperature, positive integer

G: glue function $G : \Sigma \times \Sigma \rightarrow \{0, 1, \ldots, t\}$

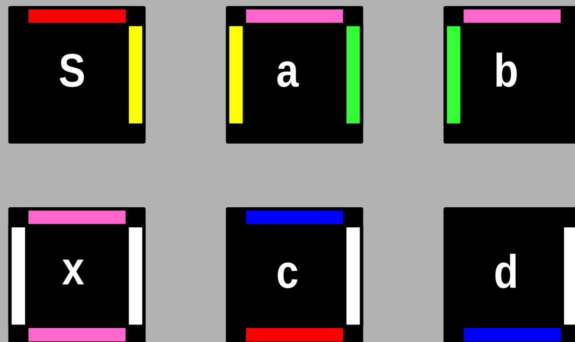
T: tileset $\{$ 

$, \ldots \}$

s: seed tile

How a tile system self assembles

T =



G(y,y) = 2

G(g,g) = 2

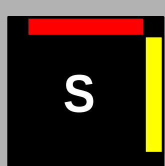
G(r, r) = 2

G(b,b) = 2

G(p,p) = 1

G(w,w) = 1

t = 2



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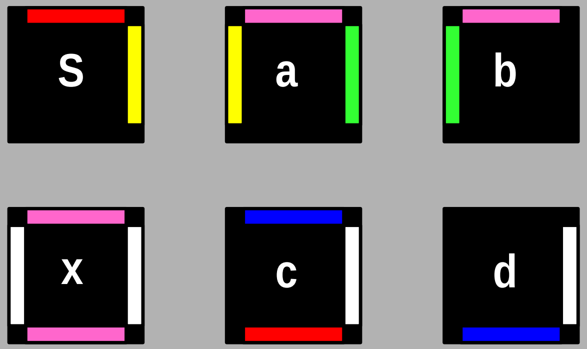

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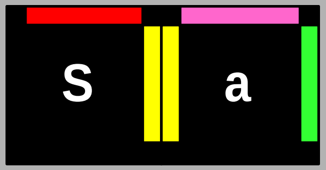

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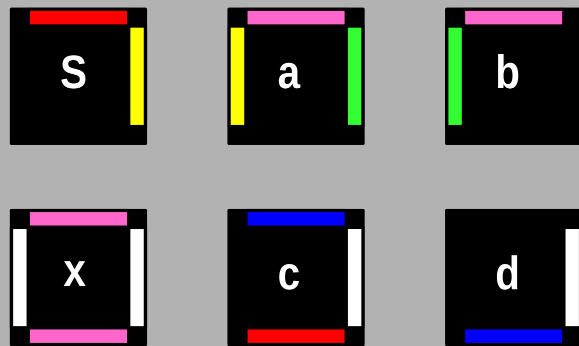
T = 

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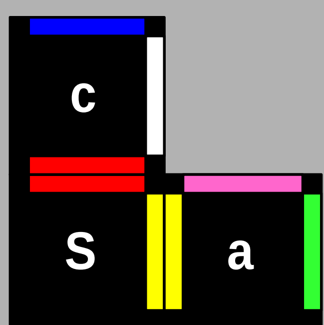

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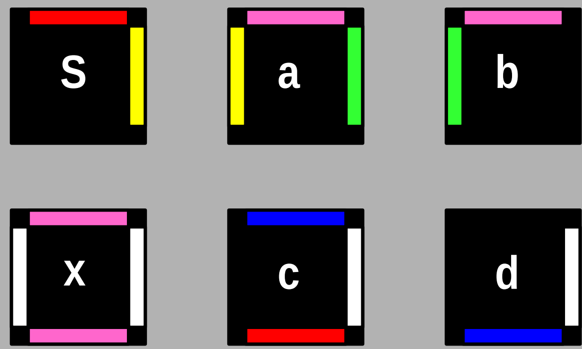

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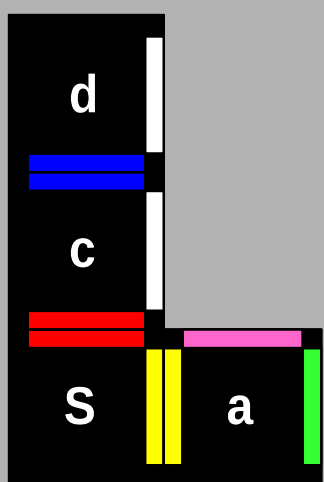

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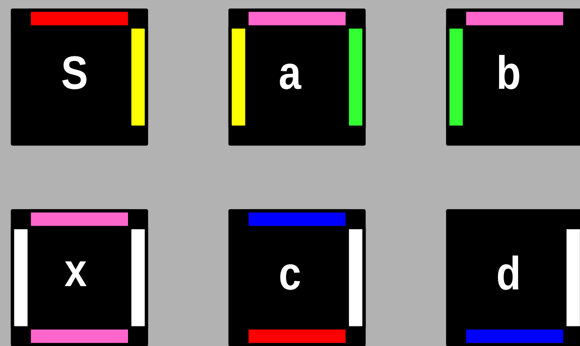
T = 

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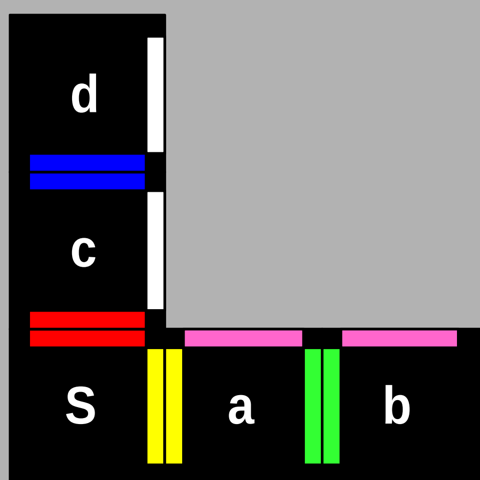

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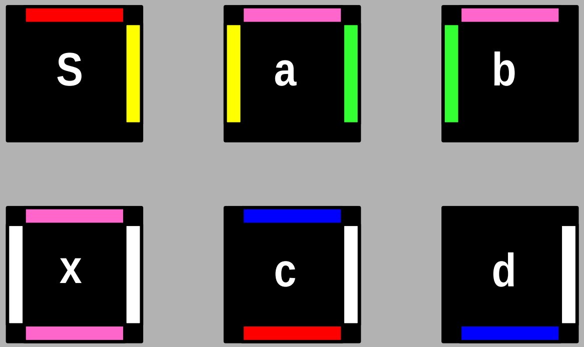

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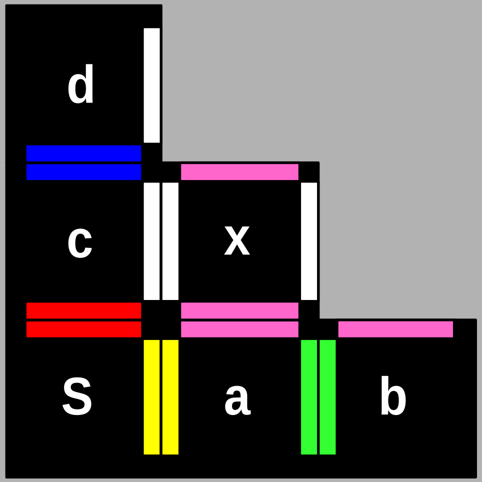

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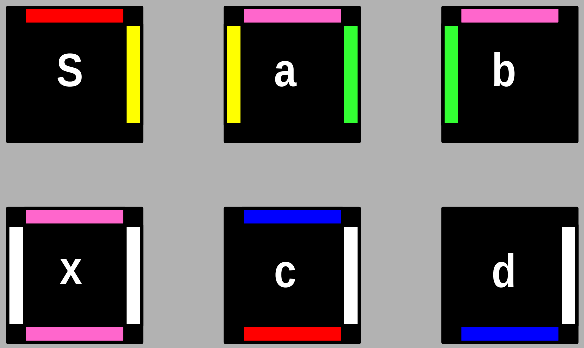

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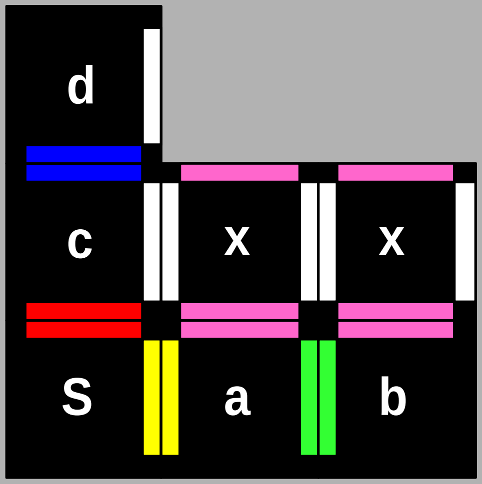

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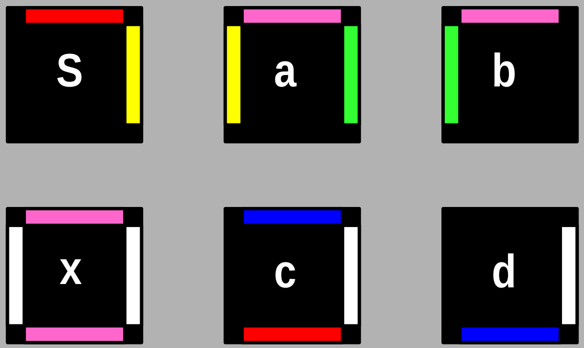
T = 

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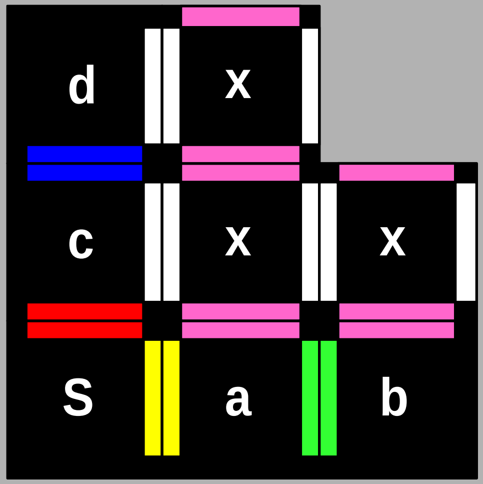

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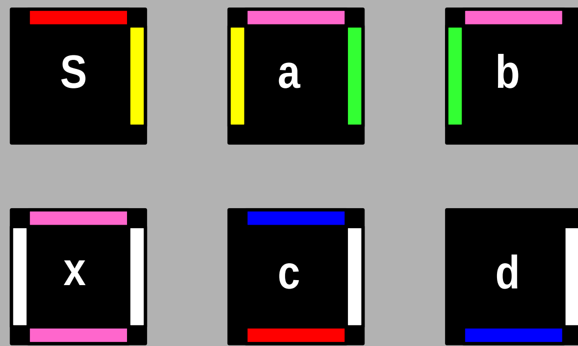
T = 

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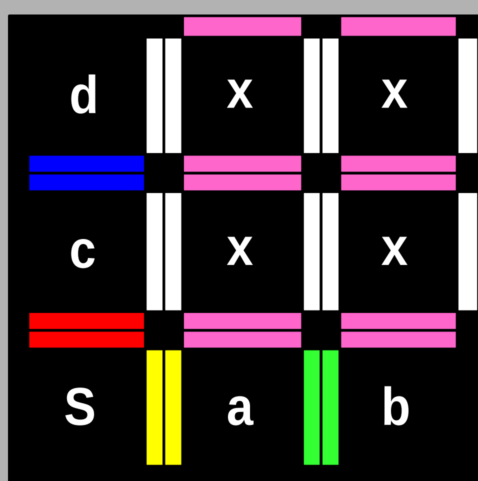
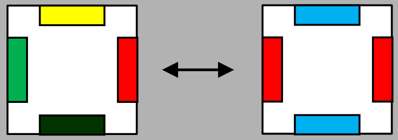


Focus

- Flexible Glue Model
  - Remove the restriction that G(x, y) = 0 for x!=y

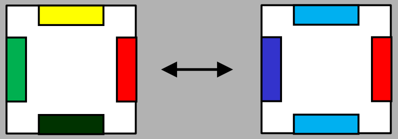
Basic model:
Only identical glues types stick

G(r, r) = 2



Flexible Glue model: Different glue types may be assigned positive interactions strength

G(r, b) = 2



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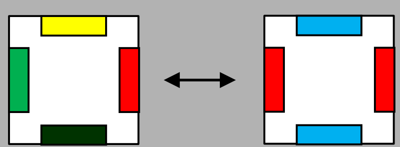

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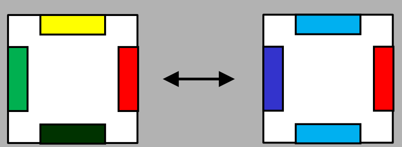


	nxn squares, Tile Complexity upper bound	lower bound
Basic	**O(log n/ loglog n)**	**Ω(log n/ loglog n) a.a**
Flexible	**?**	**? (Challenge: derive bound)**

N x N Squares --- Flexible Glue Model

Kolmogorov lower bounds:

Standard $\longrightarrow$ $\Omega(\frac{\log N}{\log\log N})$ (Rothemund, Winfree STOC 2000)

Flexible $\longrightarrow$ $\Omega(\sqrt{\log N})$

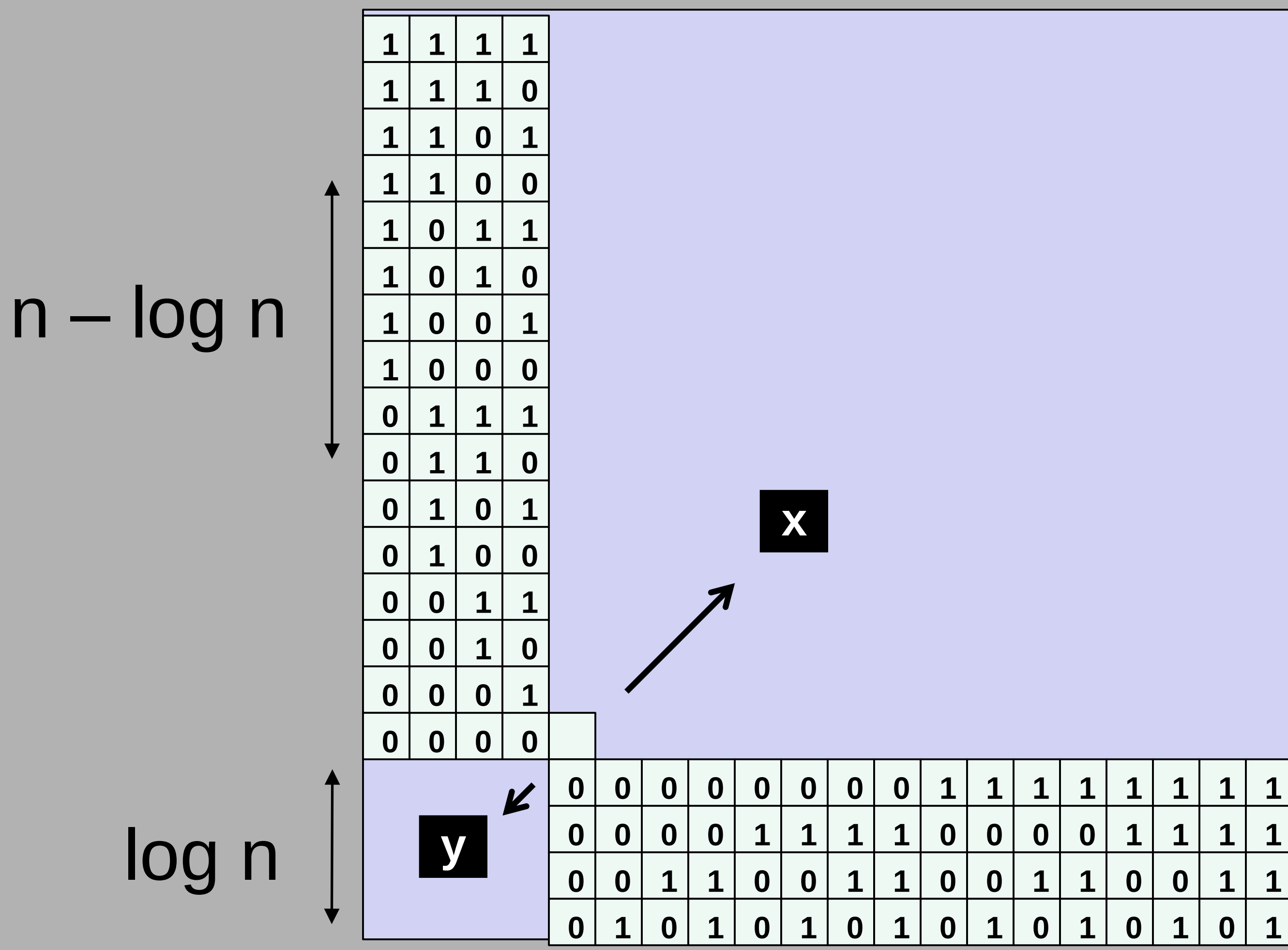
Standard Glue Function

	a	b	c	d	e	f
a	1	-	-	-	-	-
b	-	0	-	-	-	-
c	-	-	3	-	-	-
d	-	-	-	2	-	-
e	-	-	-	-	2	-
f	-	-	-	-	-	1

Flexible Glue Function

	a	b	c	d	e	f
a	1	0	2	0	0	1
b	0	0	1	0	1	0
c	0	0	3	0	1	1
d	2	2	2	2	0	1
e	0	0	0	1	2	1
f	1	1	2	2	1	1

How efficiently can you build an n x n square?



Tile Complexity

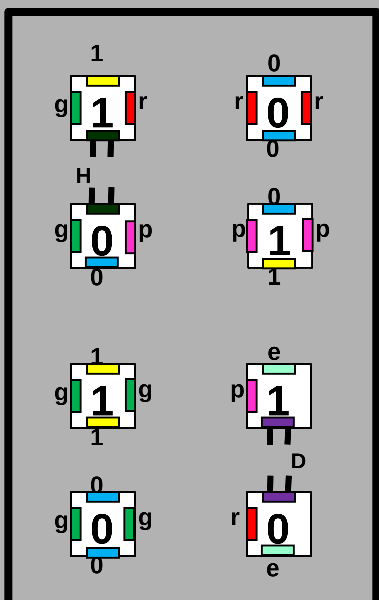
O(log n)

(Rothemund, Winfree 2000)

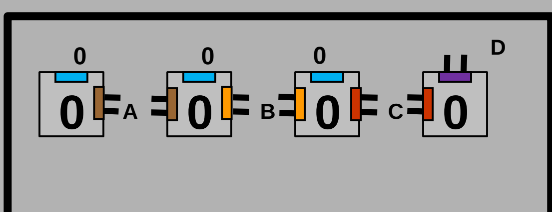
Binary Counter

(Rothemund, Winfree 2000)

Tile set:

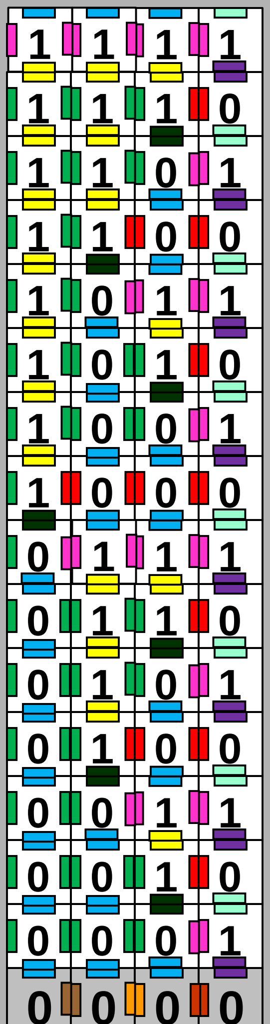


8 tile types



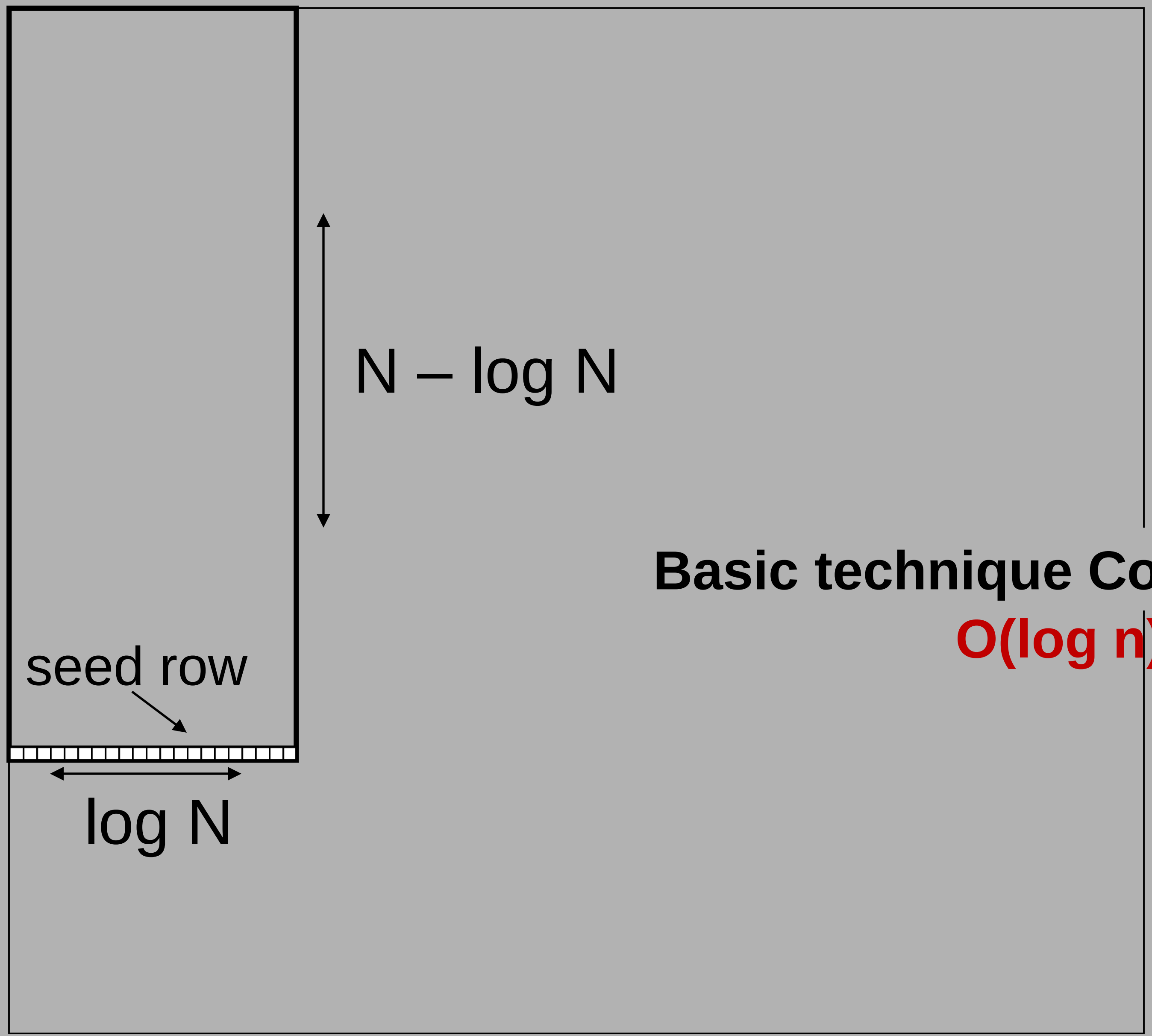
log n
tile types

Assembly:



n x log n rectangle:
O(log n)
tile complexity

N x N Square --- Flexible Glue Model



Basic technique Complexity:
O(log n)

Build a 2 x 16 rectangle:

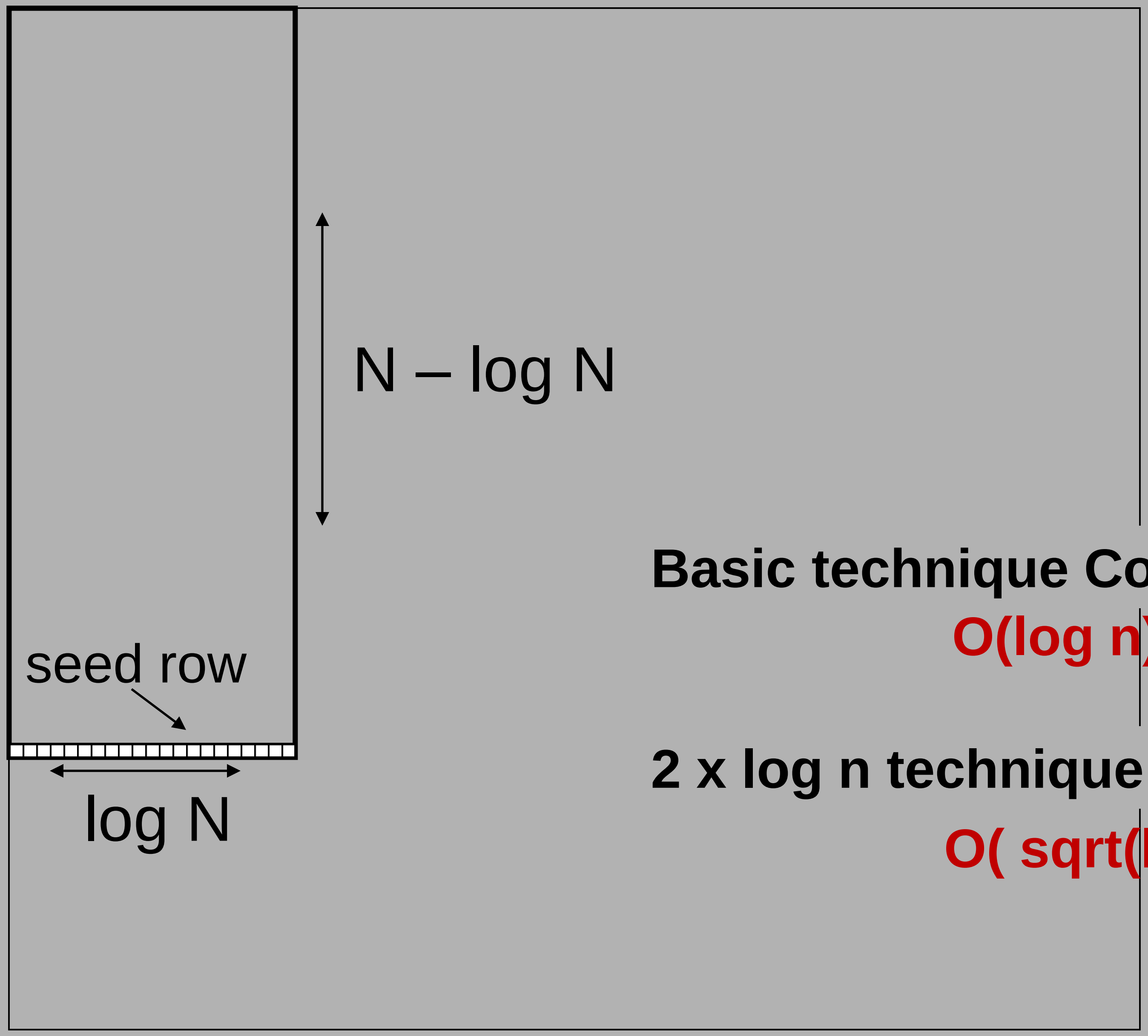
t = 2

S_1	0	0	0	1	1	1	1	2	2	2	2	3	3	3	P
S	C_1	C_2	C_3	C_0	C_1	C_2	C_3	C_0	C_1	C_2	C_3	C_0	C_1	C_2	C_3

2 x n lines

Lower Bound	Upper Bound
$\Omega(\sqrt{n})$	$O(\sqrt{n})$

N x N Square --- Flexible Glue Model



Basic technique Complexity:

O(log n)

2 x log n technique:

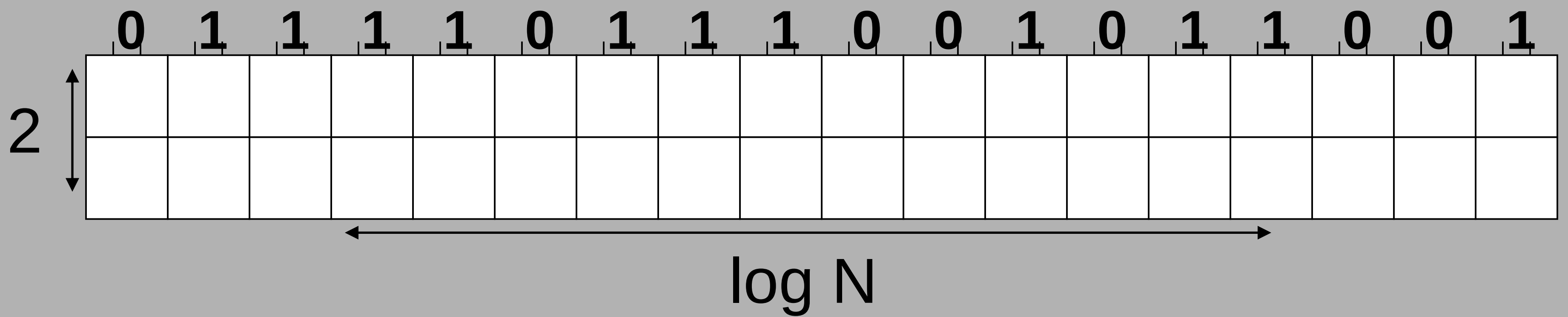
O(sqrt(log n))

N x N Square --- Flexible Glue Model

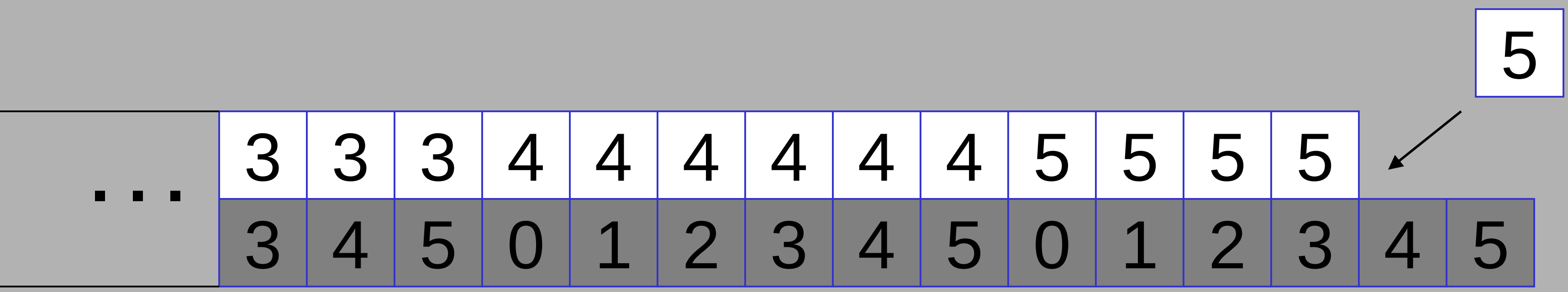
goal:

- seed binary counter to a given value

- $O(\sqrt{\log N})$

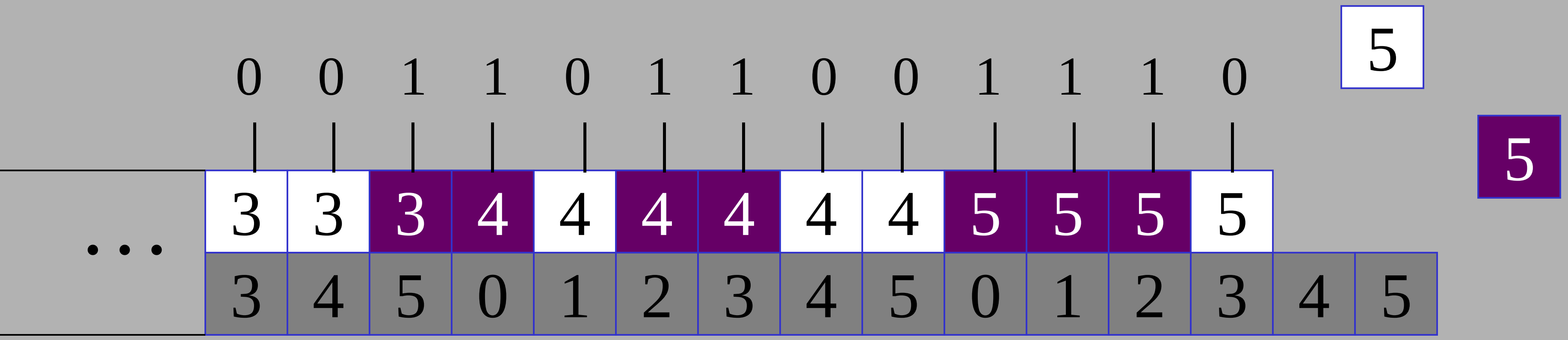


N x N Square --- Flexible Glue Model



N x N Square --- Flexible Glue Model

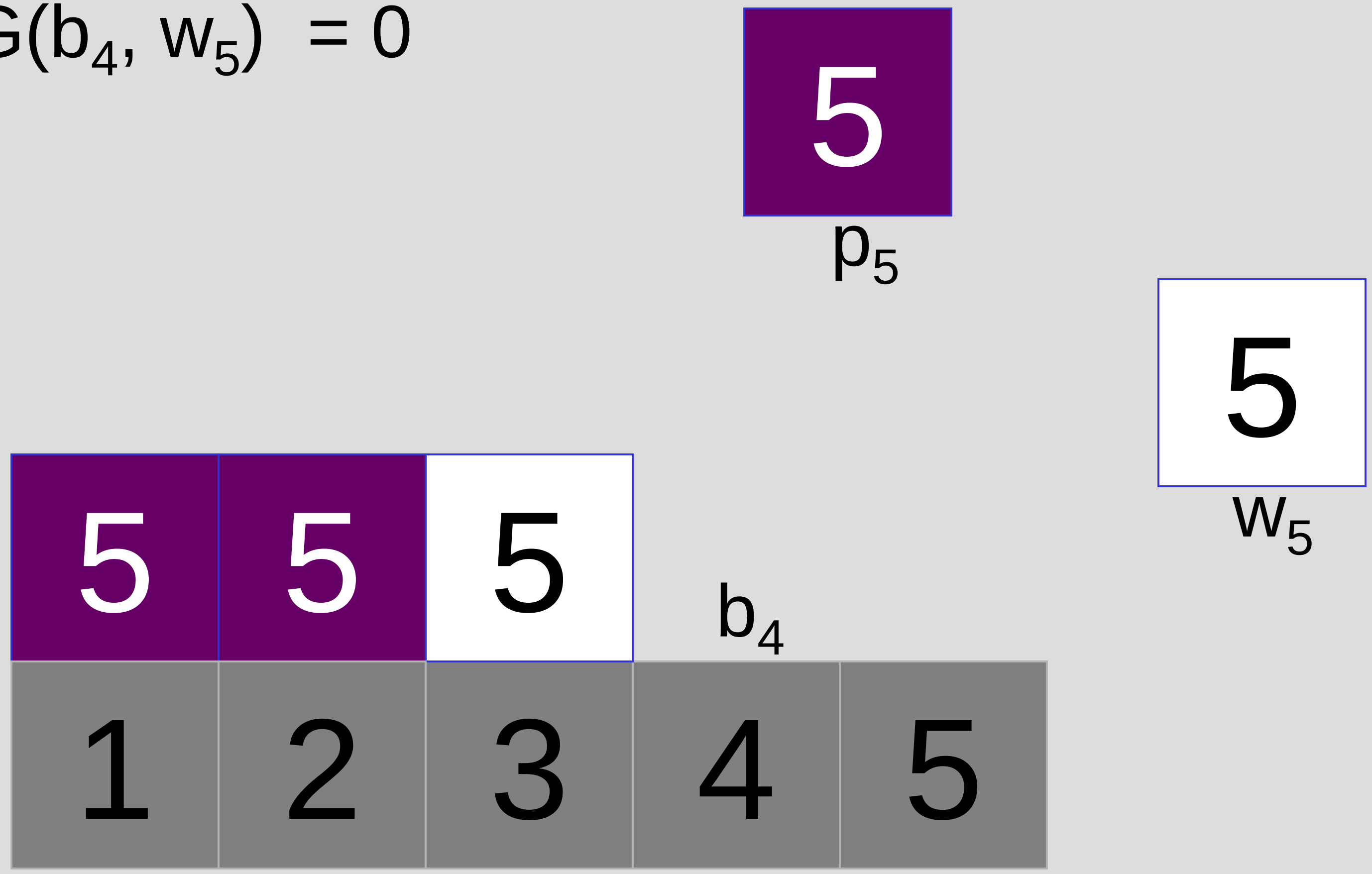
key idea:



N x N Square --- Flexible Glue Model

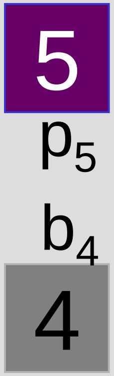
$G(b_4, p_5) = 1$

$G(b_4, w_5) = 0$



N x N Square --- Flexible Glue Model

- given B = 011011 110101 010111 …
- encode B into glue function

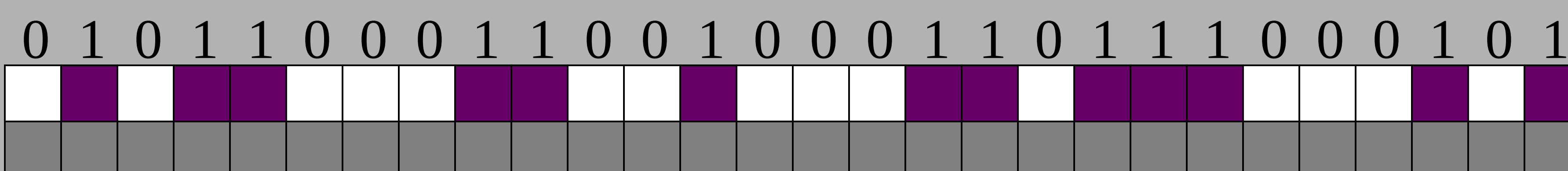


B = 011011 110101 010111 …

	p0	p1	p2	p3	p4	p5
b0	0	1	1	0	1	1
b1	1	1	0	1	0	1
b2	0	1	0	1	1	1
b3	0	0	1	0	1	0
b4	0	0	0	0	0	1
b5	1	1	1	1	1	0

N x N Square --- Flexible Glue Model

- build $2 \cdot \log N$ block
- Complexity: $O(\sqrt{\log N})$



0	1	0	1	1	0	0	0	1	1	0	0	1	0	0	0	1	1	0	1	1	1	0	0	1	1	1	0
0	1	0	1	1	0	0	0	1	1	0	0	1	0	0	0	1	1	0	1	1	1	0	0	1	1	0	1
0	1	0	1	1	0	0	0	1	1	0	0	1	0	0	0	1	1	0	1	1	1	0	0	1	1	0	0
0	1	0	1	1	0	0	0	1	1	0	0	1	0	0	0	1	1	0	1	1	1	0	0	1	0	1	1
0	1	0	1	1	0	0	0	1	1	0	0	1	0	0	0	1	1	0	1	1	1	0	0	1	0	1	0
0	1	0	1	1	0	0	0	1	1	0	0	1	0	0	0	1	1	0	1	1	1	0	0	1	0	0	1
0	1	0	1	1	0	0	0	1	1	0	0	1	0	0	0	1	1	0	1	1	1	0	0	1	0	0	0
0	1	0	1	1	0	0	0	1	1	0	0	1	0	0	0	1	1	0	1	1	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0	1	1	0	0	1	0	0	0	1	1	0	1	1	1	0	0	0	1	1	0
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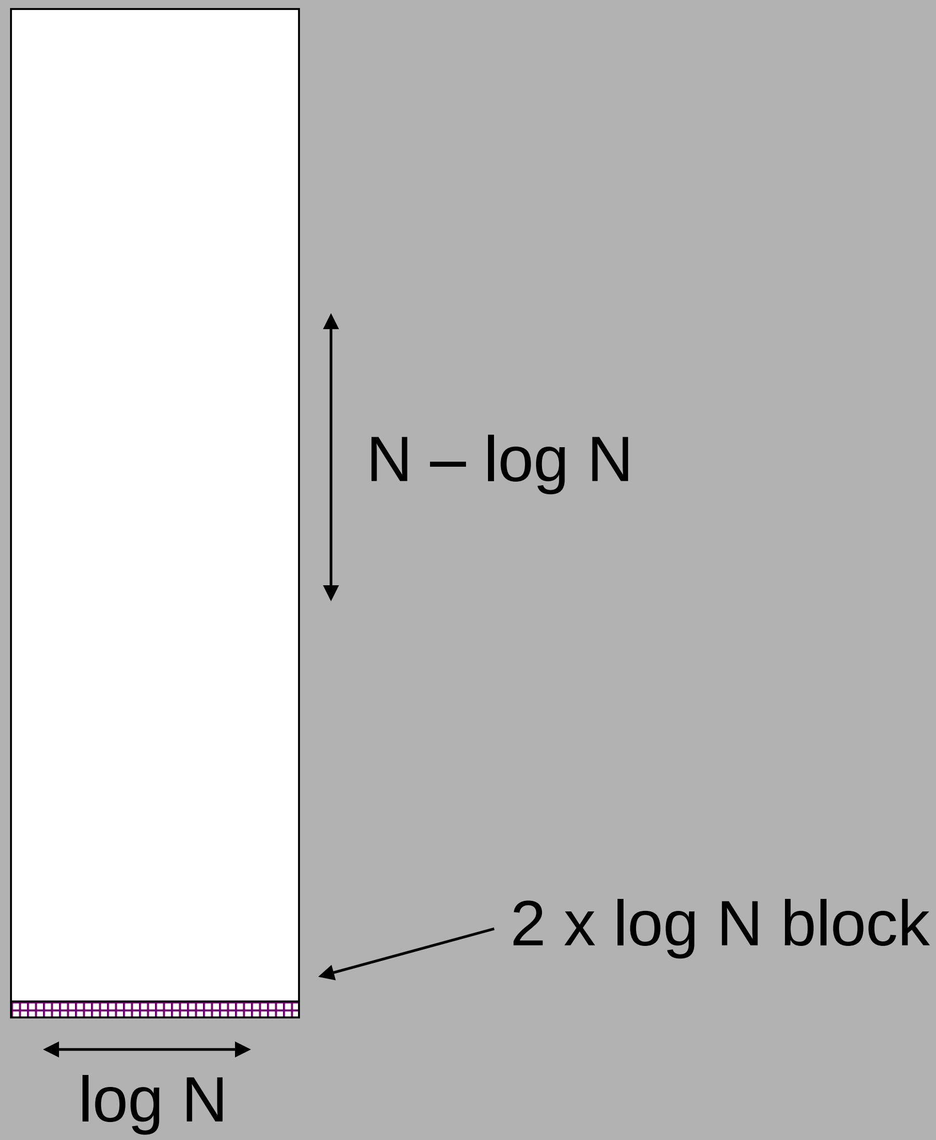
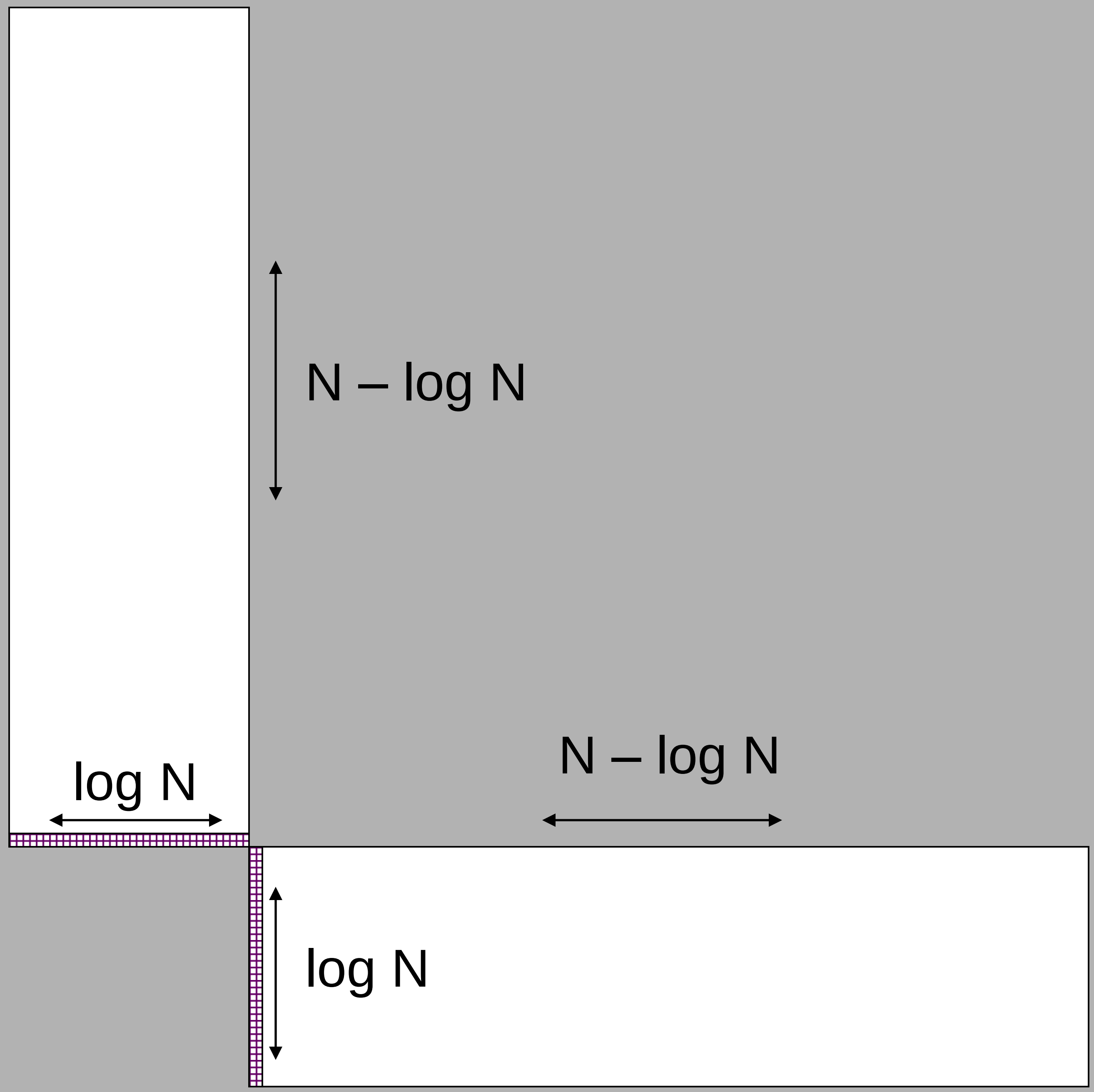
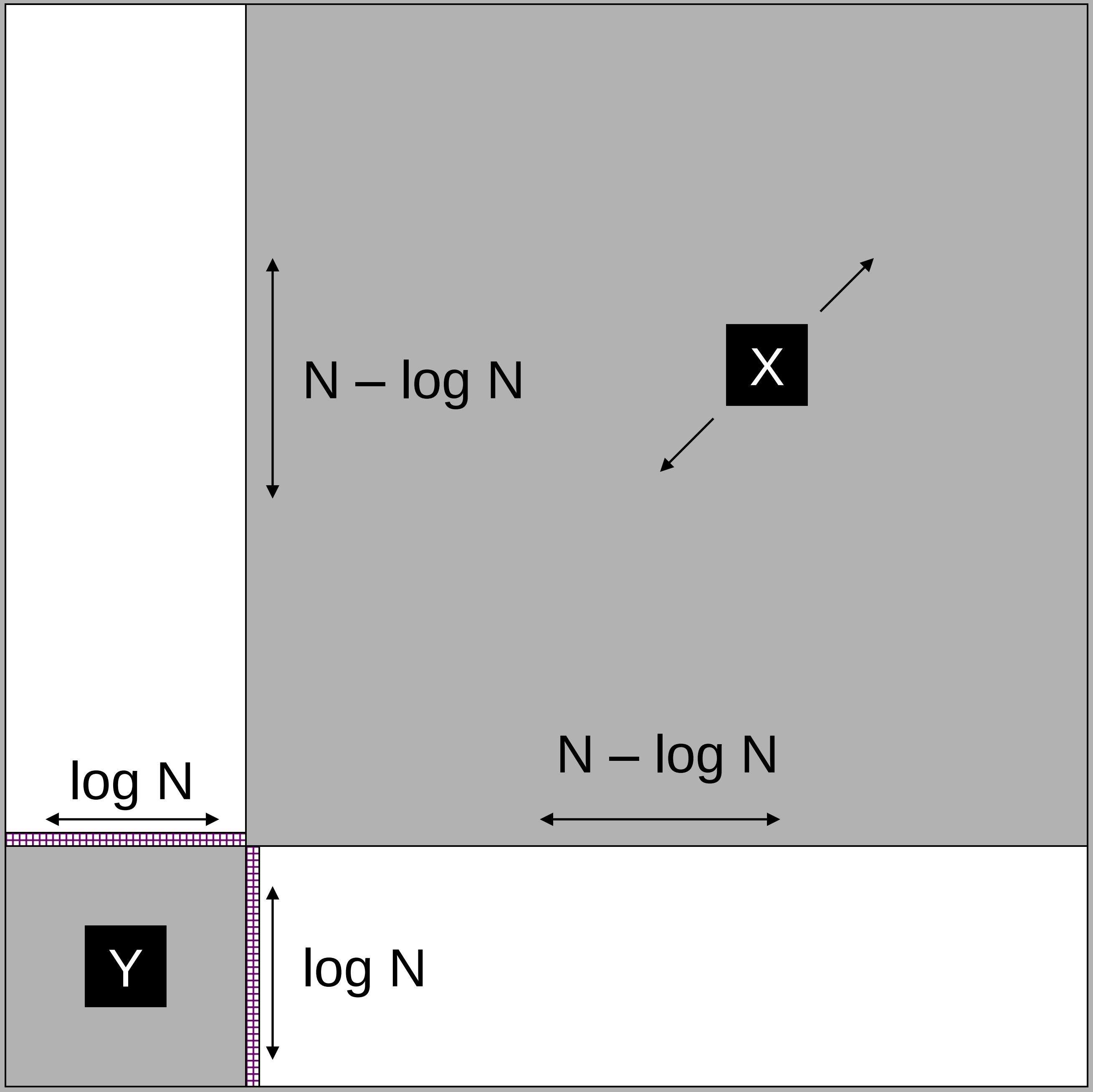
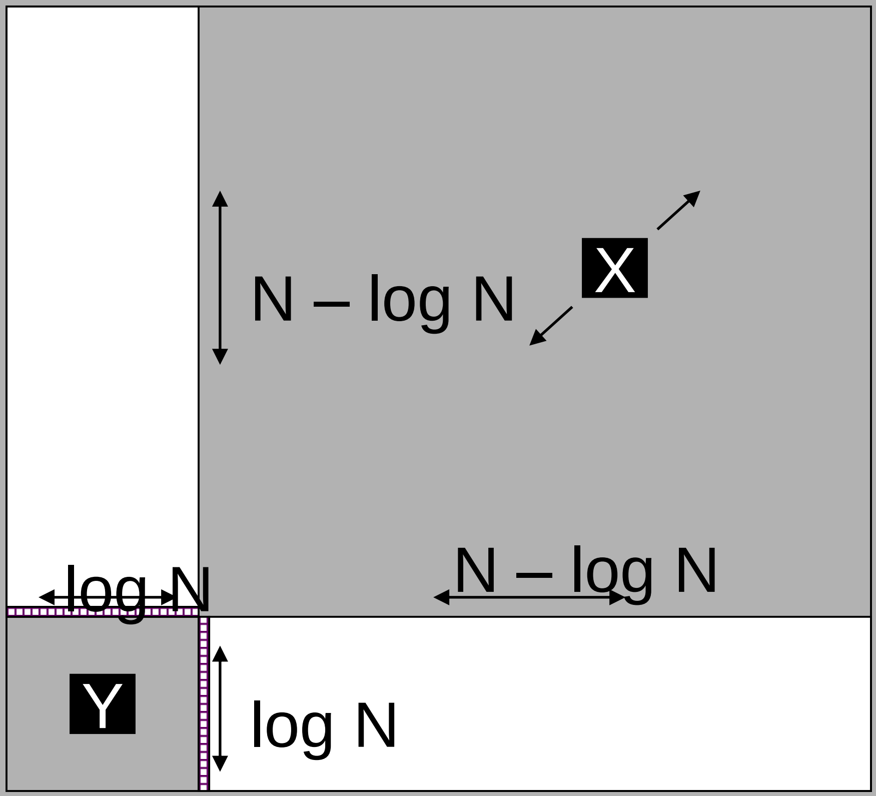

N – log N
2 x log N block
log N


N – log N
N – log N
log N
log N



Complexity:

$$\Theta(\sqrt{\log N})$$



	nxn squares, Tile Complexity upper bound	lower bound
Basic	$\Theta(\frac{\log N}{\log\log N})$	
Flexible	$\Theta(\sqrt{\log N})$	