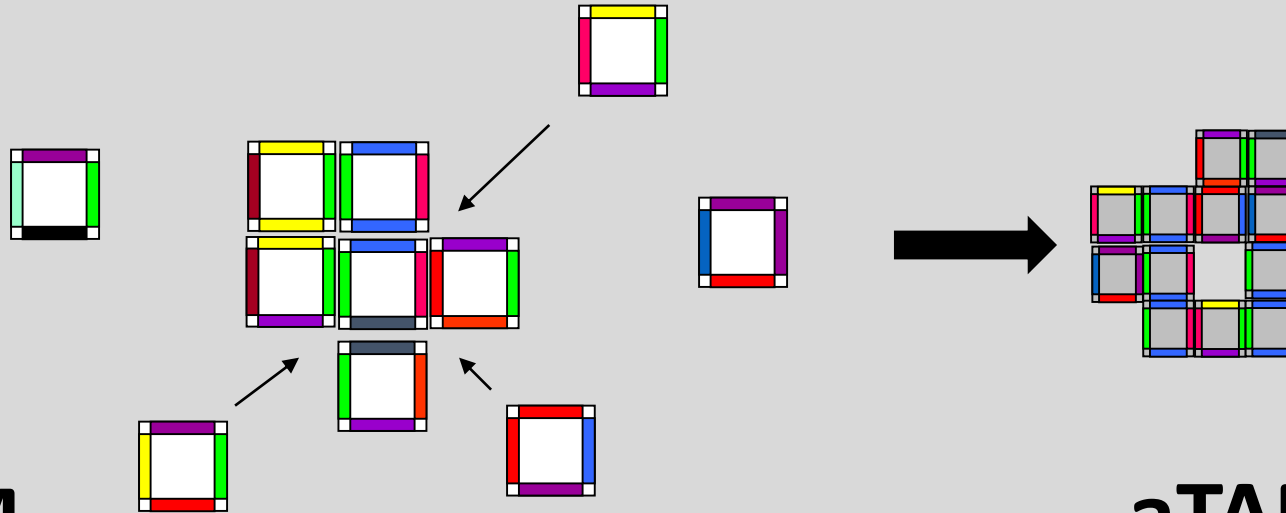


Unique Assembly Verification for High-Temperature Two-Handed Tile Self-Assembly



2HAM

2-Handed Assembly Model

Assembly Verification

	Dim.	Temp.
coNP-C [STACS 2013]	3D	2
coNP-C	2D	τ
?	2D	$O(1)$

aTAM

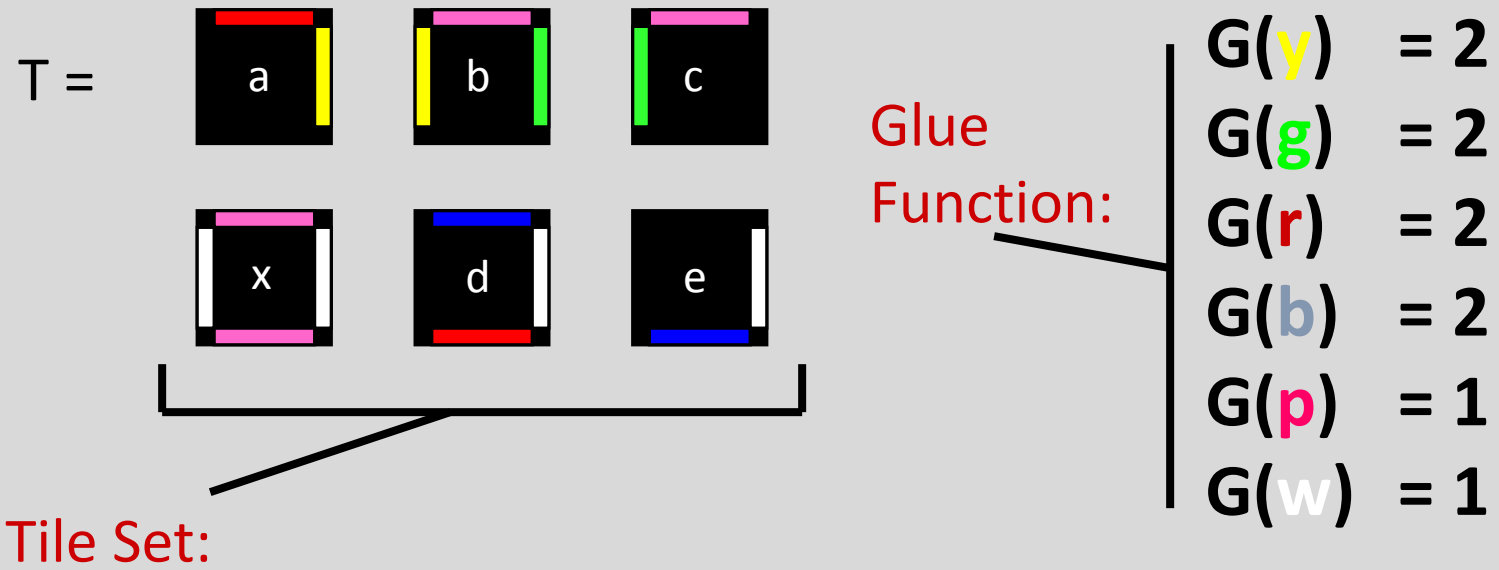
Abstract Tile Assembly Model

Assembly Verification

	Dim.	Temp.
P [Adleman et. Al. 2002]	2D/3D	any

2-Handed Tile Assembly Model (2HAM)

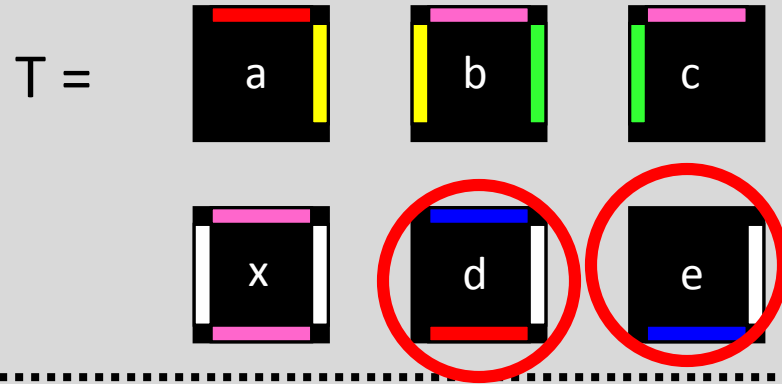
(Rothemund, Winfree, Adleman)



Temperature: $\tau = 2$

Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

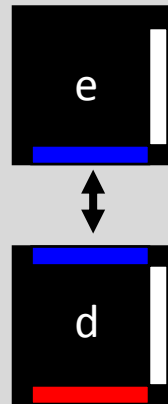
$$G(g) = 2$$

$$G(r) = 2$$

$$G(b) = 2$$

$$G(p) = 1$$

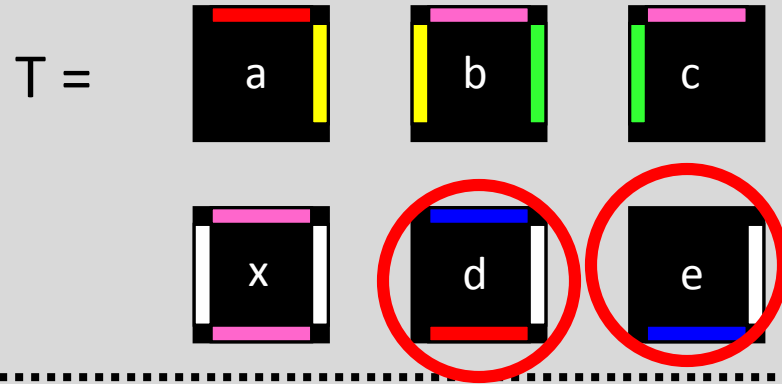
$$G(w) = 1$$



$$\tau = 2$$

Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

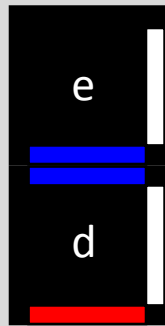
$$G(g) = 2$$

$$G(r) = 2$$

$$G(b) = 2$$

$$G(p) = 1$$

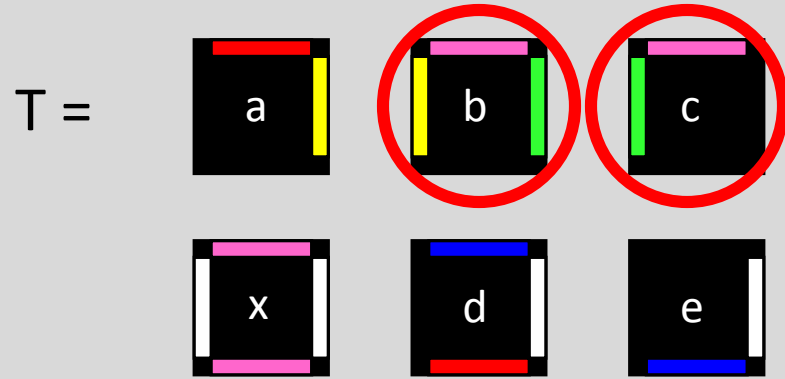
$$G(w) = 1$$



$$\tau = 2$$

Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

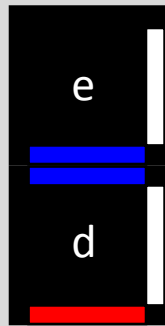
$$G(g) = 2$$

$$G(r) = 2$$

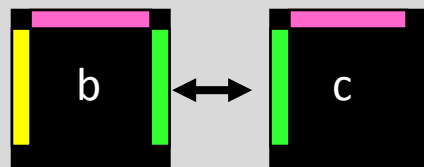
$$G(b) = 2$$

$$G(p) = 1$$

$$G(w) = 1$$

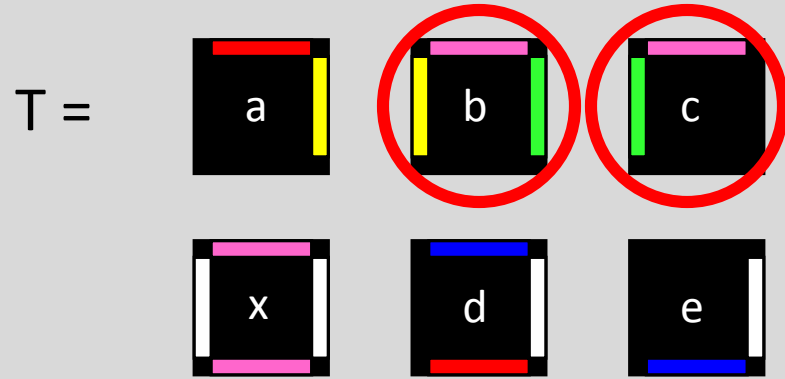


$$\tau = 2$$



Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

$$G(g) = 2$$

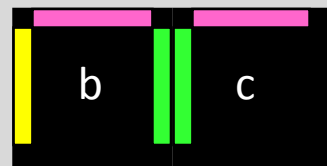
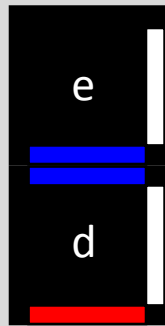
$$G(r) = 2$$

$$G(b) = 2$$

$$G(p) = 1$$

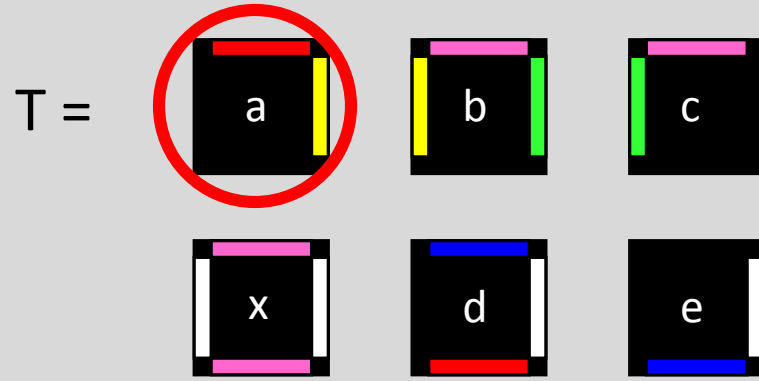
$$G(w) = 1$$

$$\tau = 2$$



Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

$$G(g) = 2$$

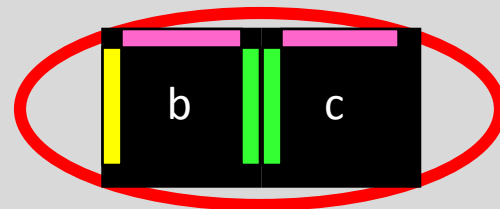
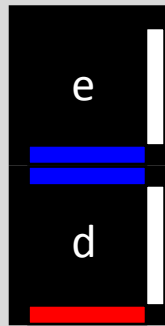
$$G(r) = 2$$

$$G(b) = 2$$

$$G(p) = 1$$

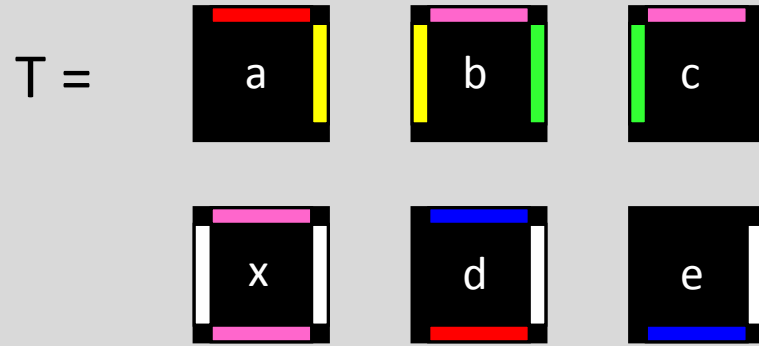
$$G(w) = 1$$

$$\tau = 2$$



Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

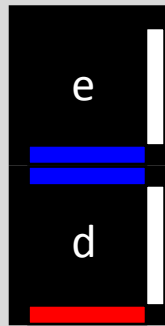
$$G(g) = 2$$

$$G(r) = 2$$

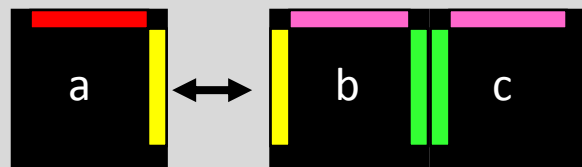
$$G(b) = 2$$

$$G(p) = 1$$

$$G(w) = 1$$

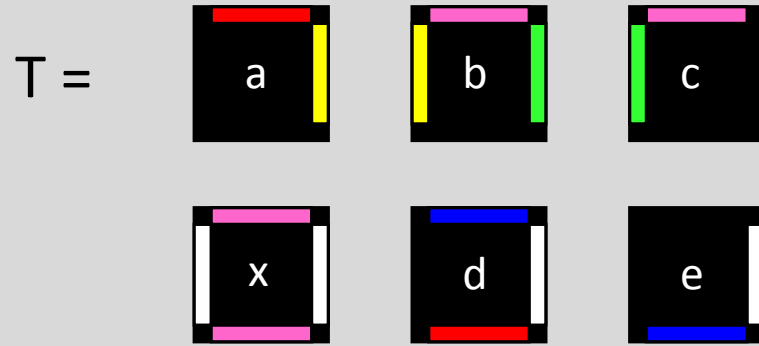


$$\tau = 2$$



Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

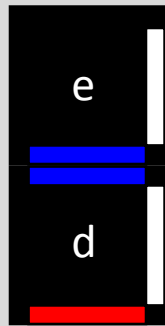
$$G(g) = 2$$

$$G(r) = 2$$

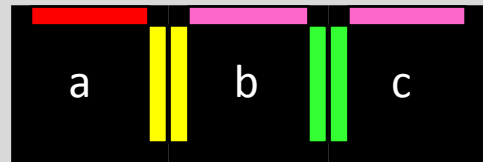
$$G(b) = 2$$

$$G(p) = 1$$

$$G(w) = 1$$

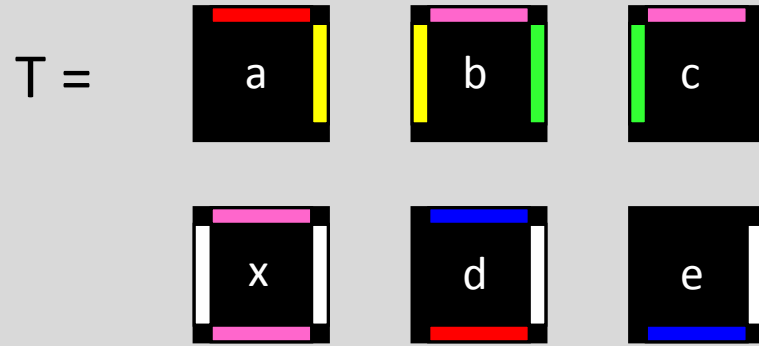


$$\tau = 2$$



Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

$$G(g) = 2$$

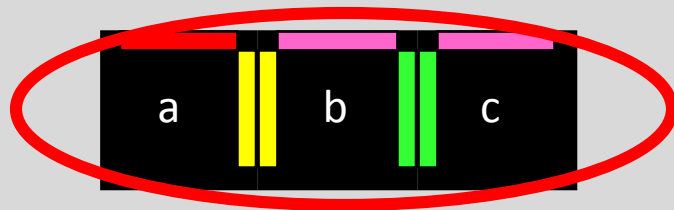
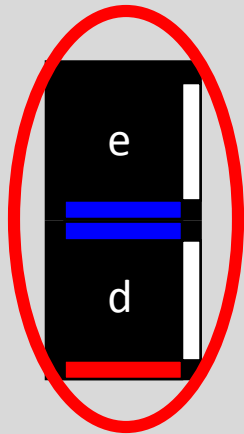
$$G(r) = 2$$

$$G(b) = 2$$

$$G(p) = 1$$

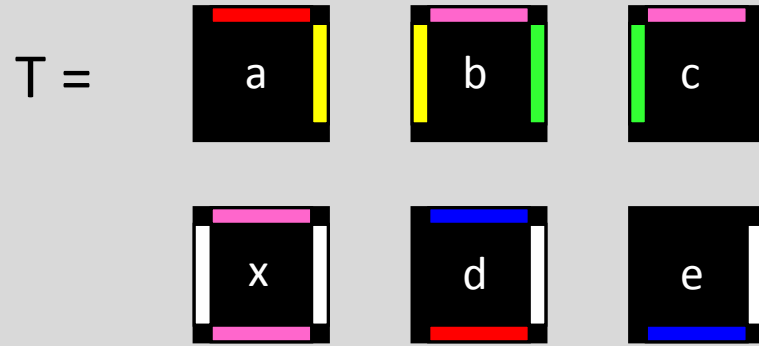
$$G(w) = 1$$

$$\tau = 2$$



Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

$$G(g) = 2$$

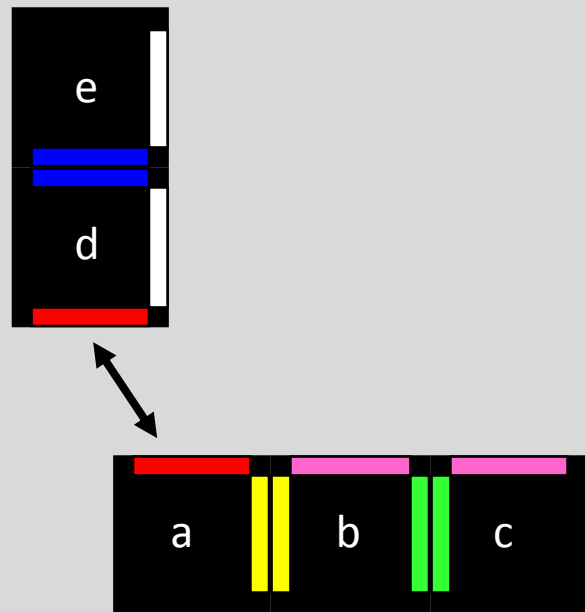
$$G(r) = 2$$

$$G(b) = 2$$

$$G(p) = 1$$

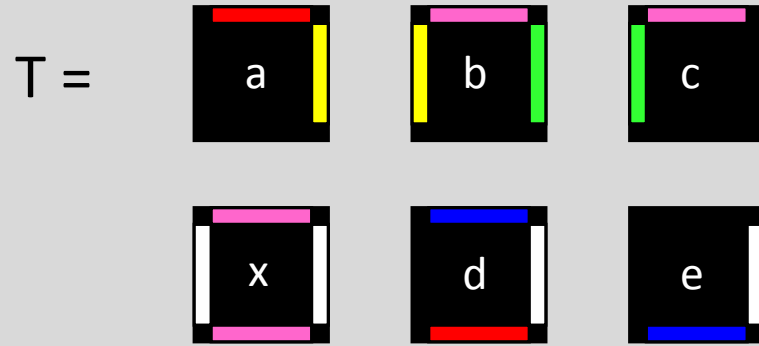
$$G(w) = 1$$

$$\tau = 2$$



Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

$$G(g) = 2$$

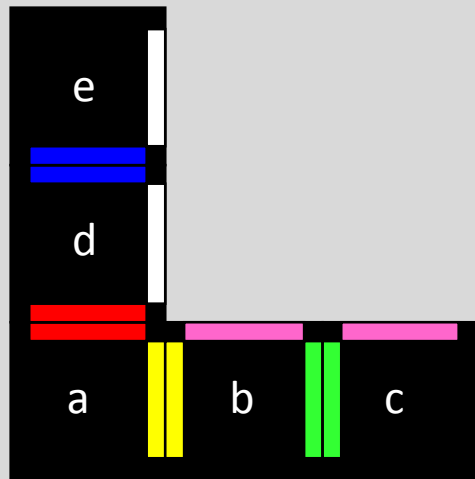
$$G(r) = 2$$

$$G(b) = 2$$

$$G(p) = 1$$

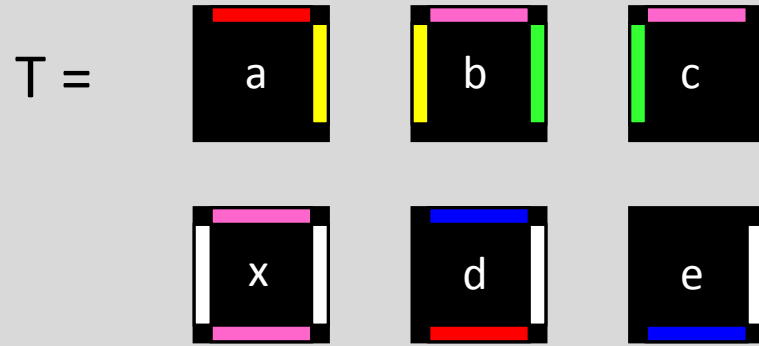
$$G(w) = 1$$

$$\tau = 2$$



Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

$$G(g) = 2$$

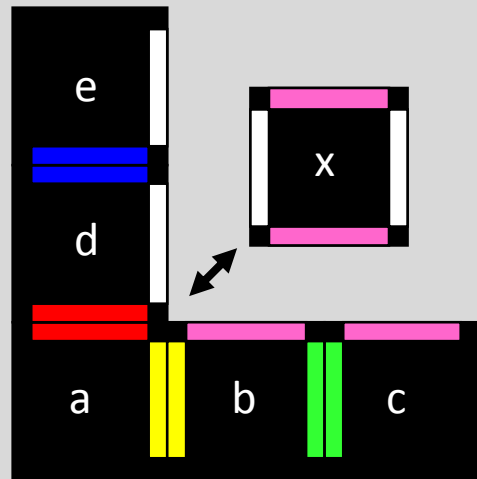
$$G(r) = 2$$

$$G(b) = 2$$

$$G(p) = 1$$

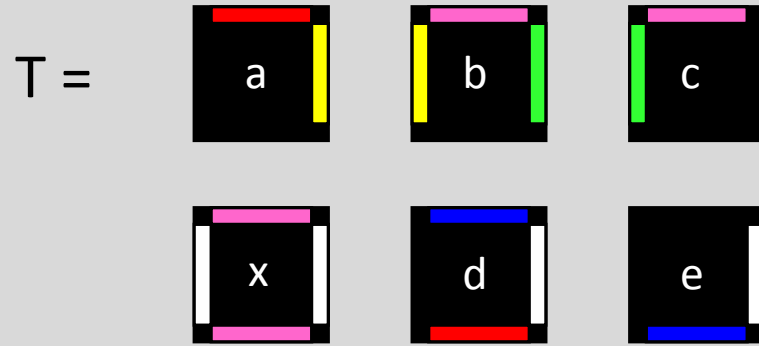
$$G(w) = 1$$

$$\tau = 2$$



Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

$$G(g) = 2$$

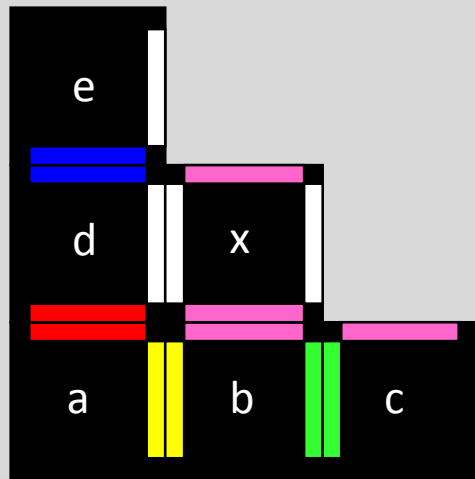
$$G(r) = 2$$

$$G(b) = 2$$

$$G(p) = 1$$

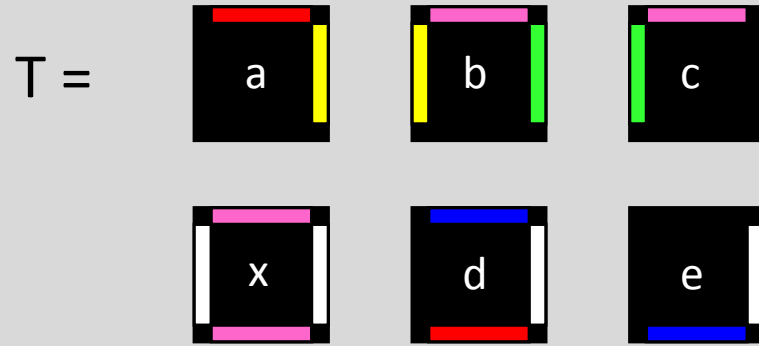
$$G(w) = 1$$

$$\tau = 2$$



Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

$$G(g) = 2$$

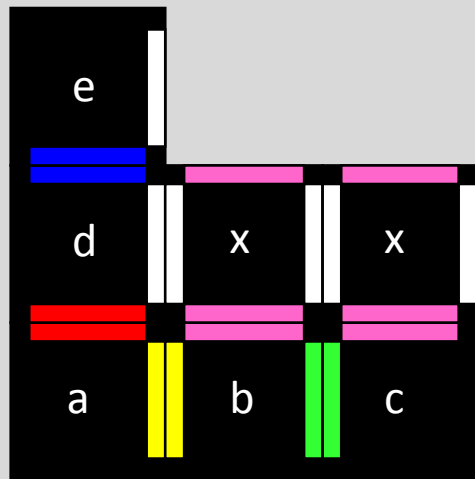
$$G(r) = 2$$

$$G(b) = 2$$

$$G(p) = 1$$

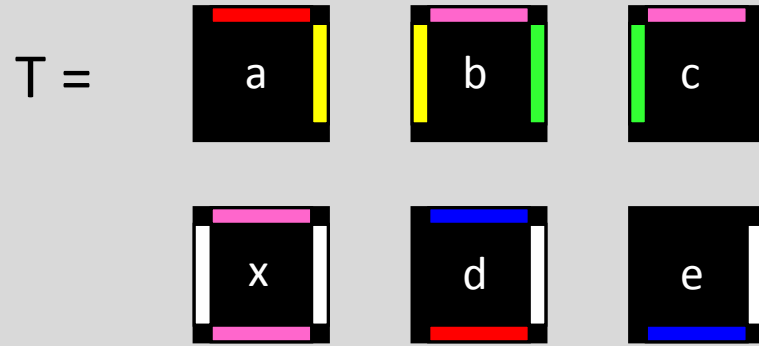
$$G(w) = 1$$

$$\tau = 2$$



Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

$$G(g) = 2$$

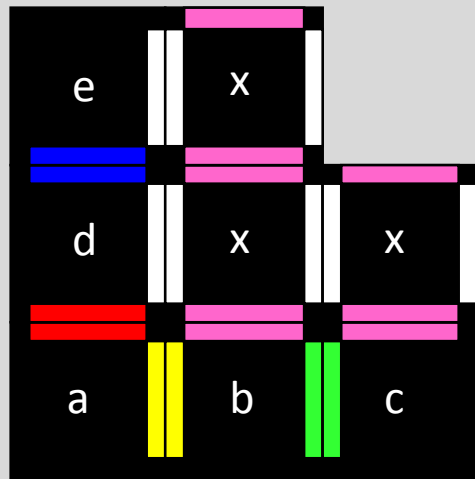
$$G(r) = 2$$

$$G(b) = 2$$

$$G(p) = 1$$

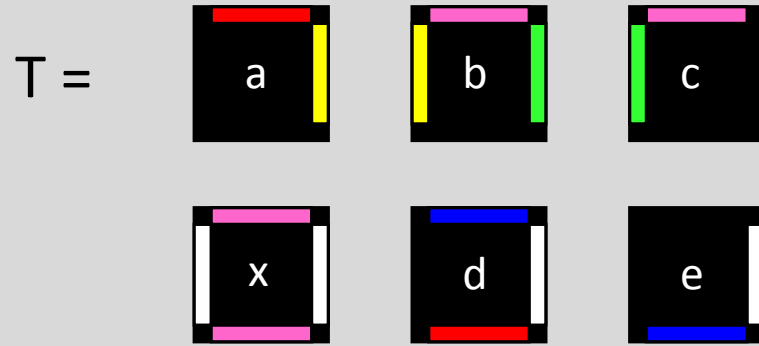
$$G(w) = 1$$

$$\tau = 2$$



Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

$$G(g) = 2$$

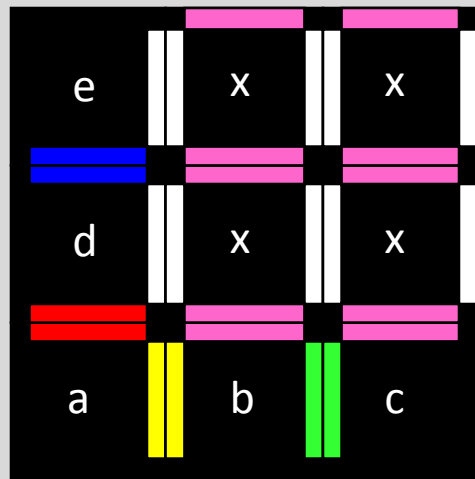
$$G(r) = 2$$

$$G(b) = 2$$

$$G(p) = 1$$

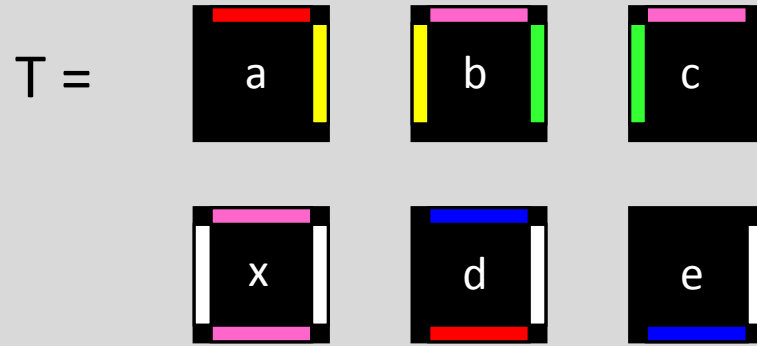
$$G(w) = 1$$

$$\tau = 2$$



Tile Assembly Model

(Rothemund, Winfree, Adleman)



$$G(y) = 2$$

$$G(g) = 2$$

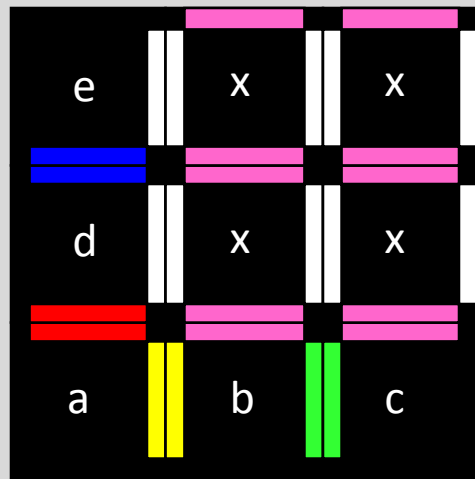
$$G(r) = 2$$

$$G(b) = 2$$

$$G(p) = 1$$

$$G(w) = 1$$

$$\tau = 2$$



Complexities within this model:

- **Verification:** Does a system build a given assembly?
- **Tile Complexity:** What is the minimum number of tile types needed to build a given shape?
 - For a given temperature τ ?

Complexities at High-Temperature

- **Verification**

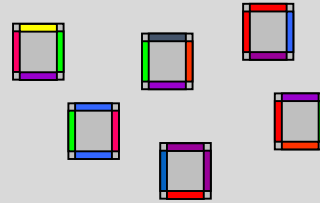
- Unique Assembly Verification is coNP-complete
 - In coNP
 - coNP-hard
- Related Problems and Open Problems

Unique Assembly Verification

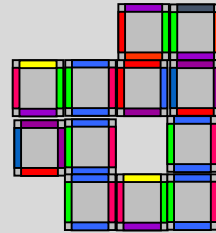
Unique Assembly Verification Problem (UAV Problem)

Input:

- A tile system Γ :



- An assembly A :



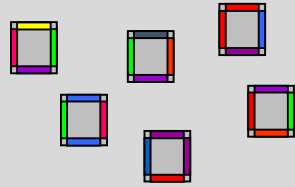
Output: Does Γ uniquely assemble A ?

Unique Assembly Verification

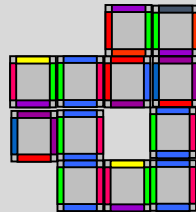
Unique Assembly Verification Problem (UAV Problem)

Input:

- A tile system Γ :



- An assembly A :



Output: Does Γ uniquely assemble A ?

UAV Complexity (2HAM model)

Complexity	Dim.	Temp.
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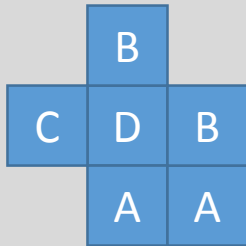
coNP-C	3D	2
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[Cannon et. Al. 2013]

coNP-C	2D	τ
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Membership in coNP

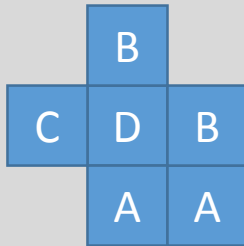
Is this uniquely assembled?



- Why is this not in NP?
 - Difficulty is verifying UNIQUE Assembly, i.e., that you build nothing else.

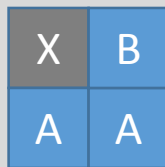
Membership in coNP

Is this uniquely assembled?

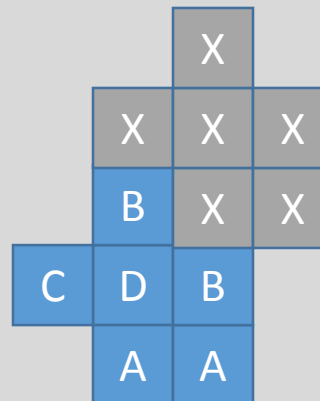


“No” certificates:

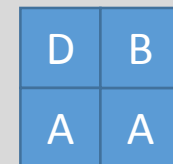
Incompatible producible:



Super-Assembly:

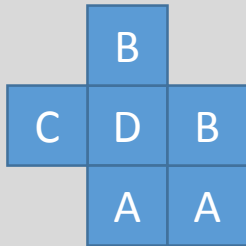


Terminal, producible sub-assembly?:



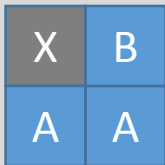
Membership in coNP

Is this uniquely assembled?

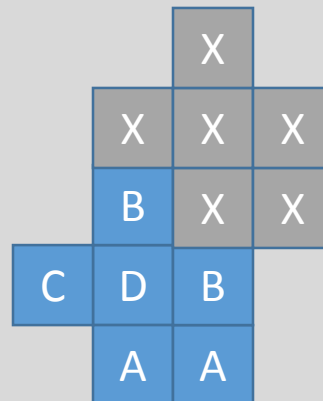


“No” certificates:

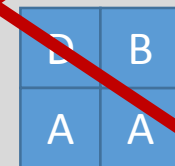
Incompatible producible:



Super-Assembly:

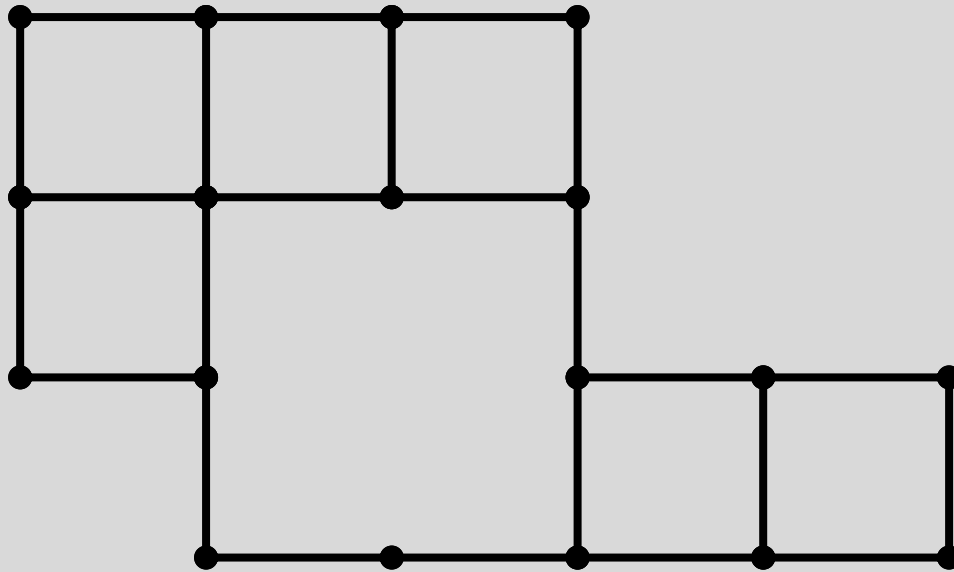


~~Terminal, producible sub-assembly?:~~



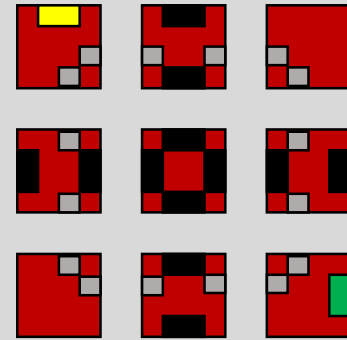
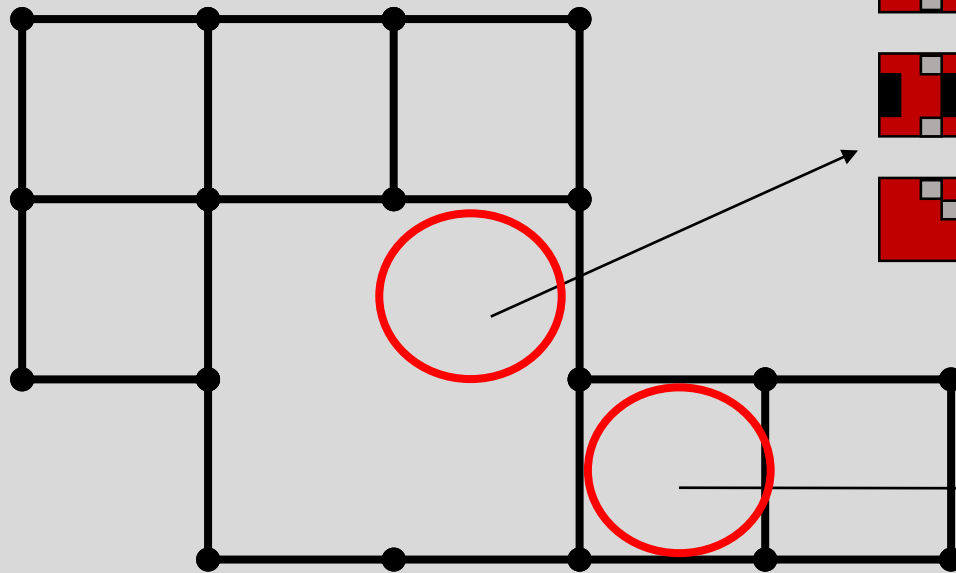
Unique Assembly Verification: **Hardness Reduction**

Reduction from the Hamiltonian Cycle problem for grid graphs

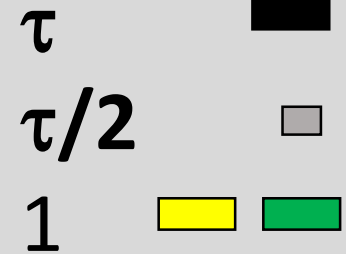


Unique Assembly Verification: **Hardness Reduction**

Reduction from the Hamiltonian Cycle problem for grid graphs

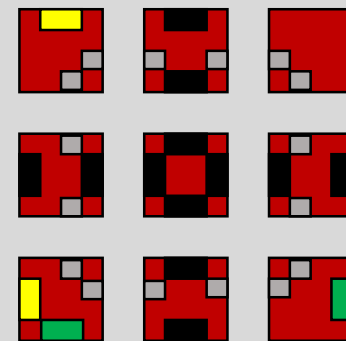


Glue Strength:



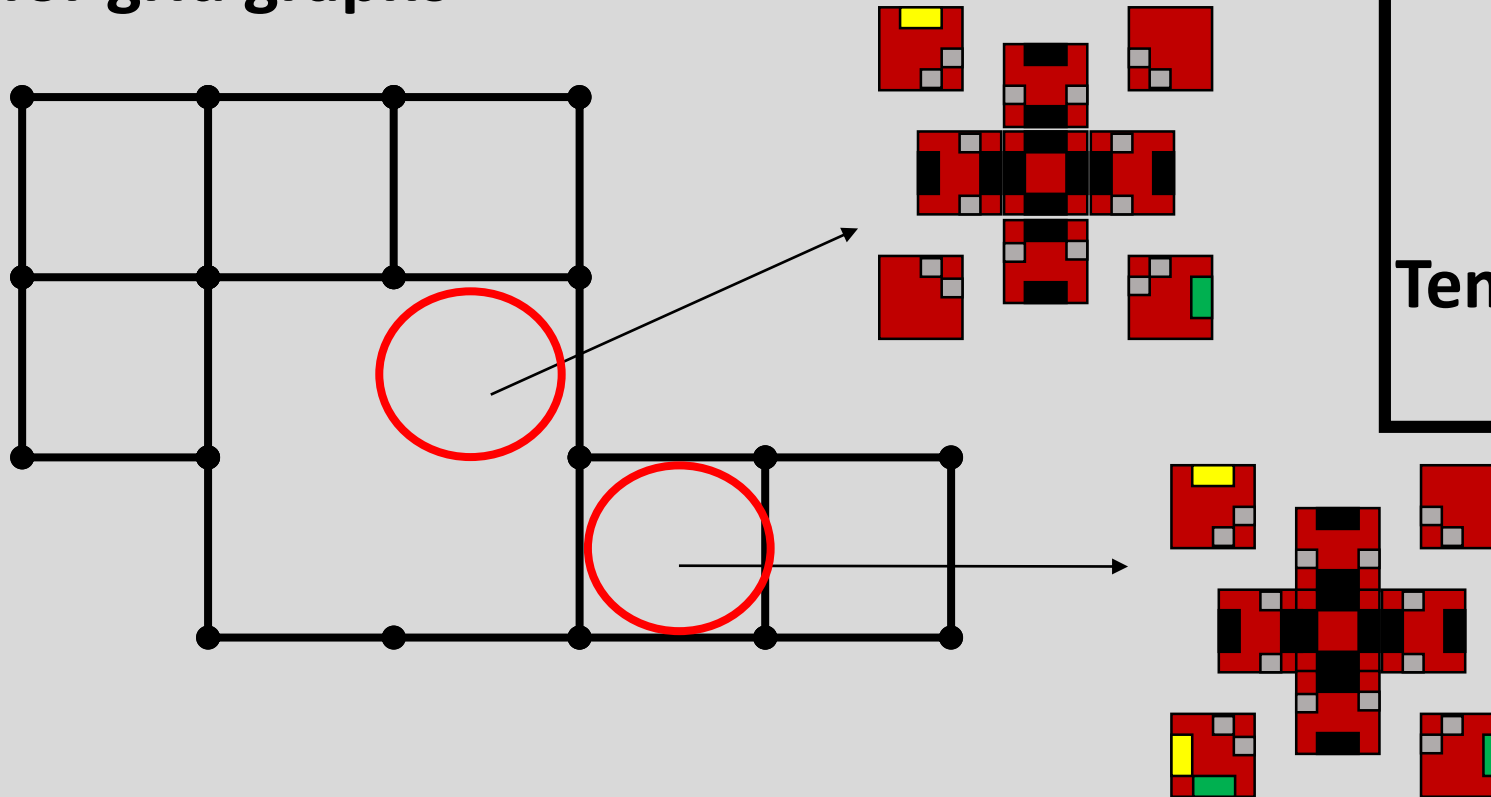
Temperature:

$$\tau = |V|$$




Unique Assembly Verification: **Hardness Reduction**

Reduction from the Hamiltonian Cycle problem for grid graphs



Glue Strength:

τ 

$\tau/2$ 

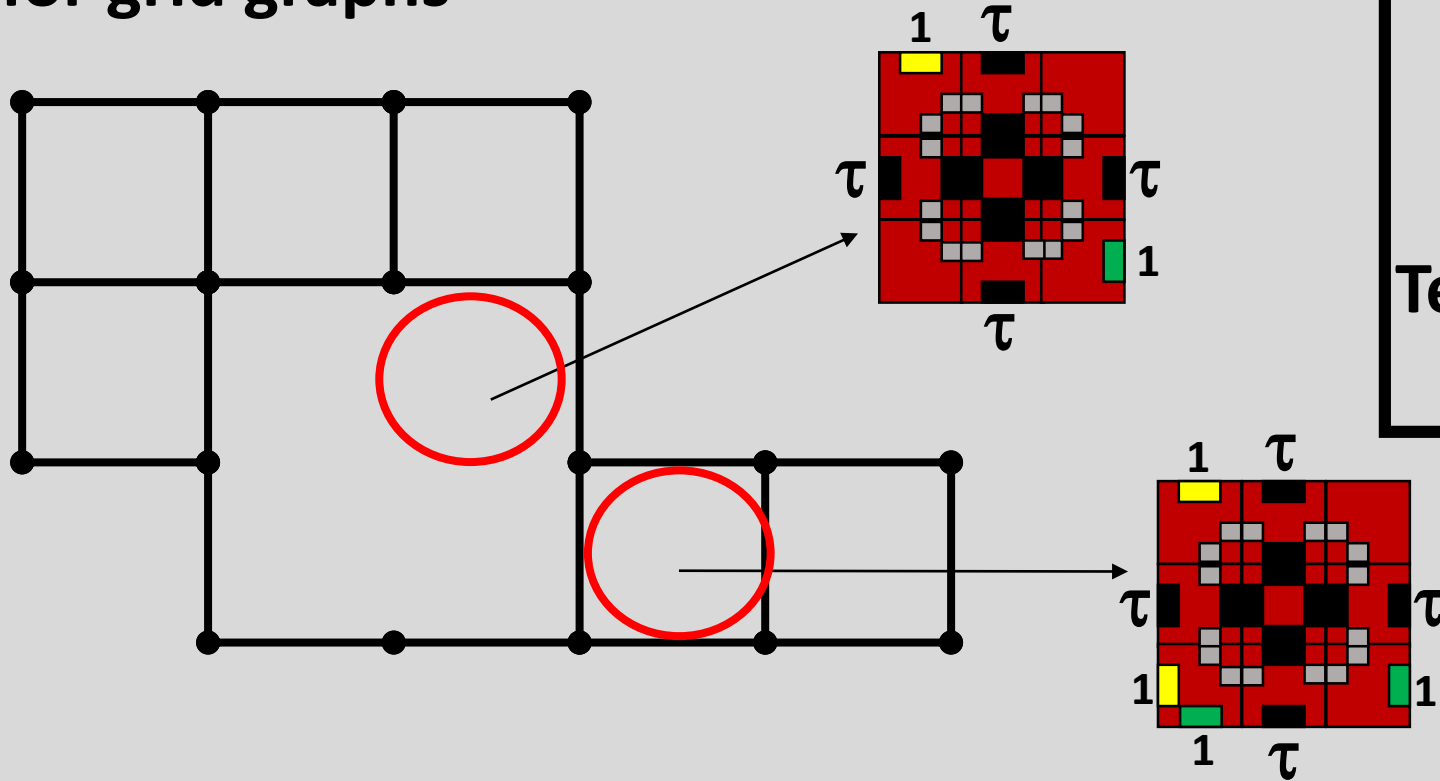
1  

Temperature:

$$\tau = |V|$$

Unique Assembly Verification: **Hardness Reduction**

Reduction from the Hamiltonian Cycle problem for grid graphs



Glue Strength:

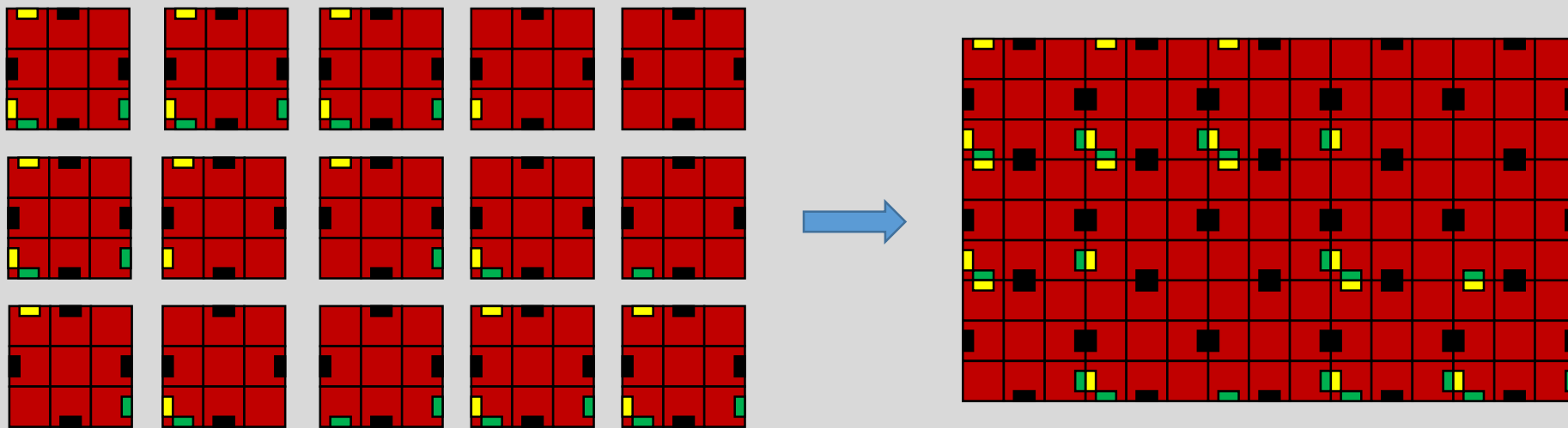
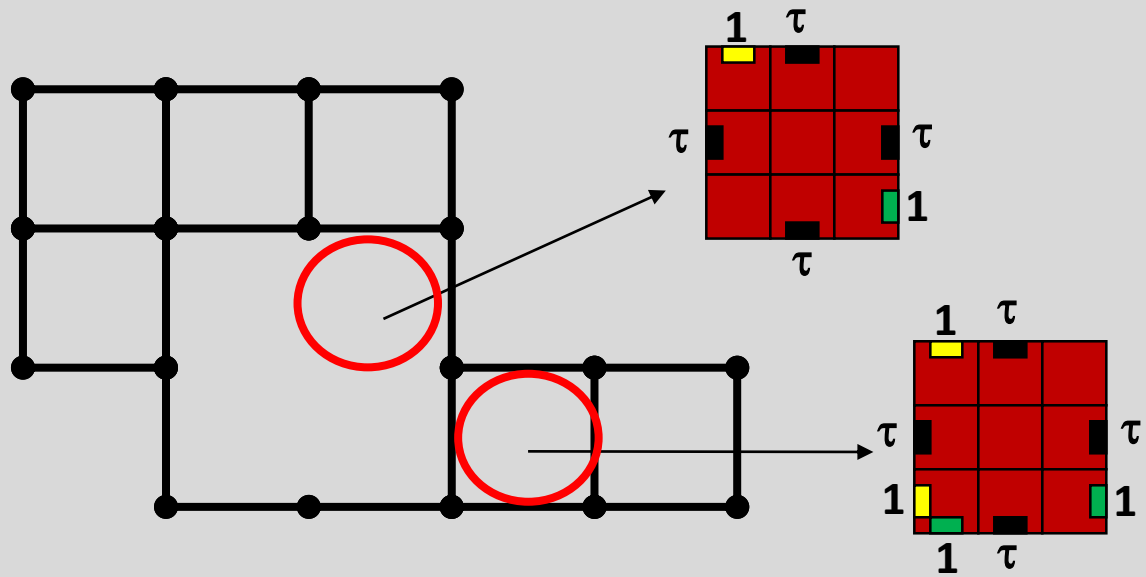
τ 

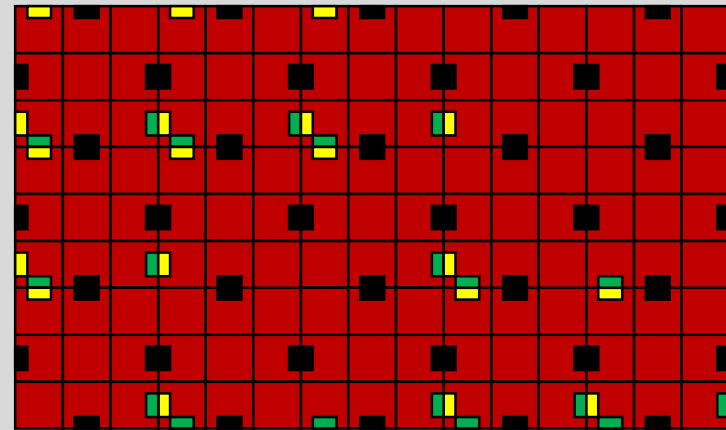
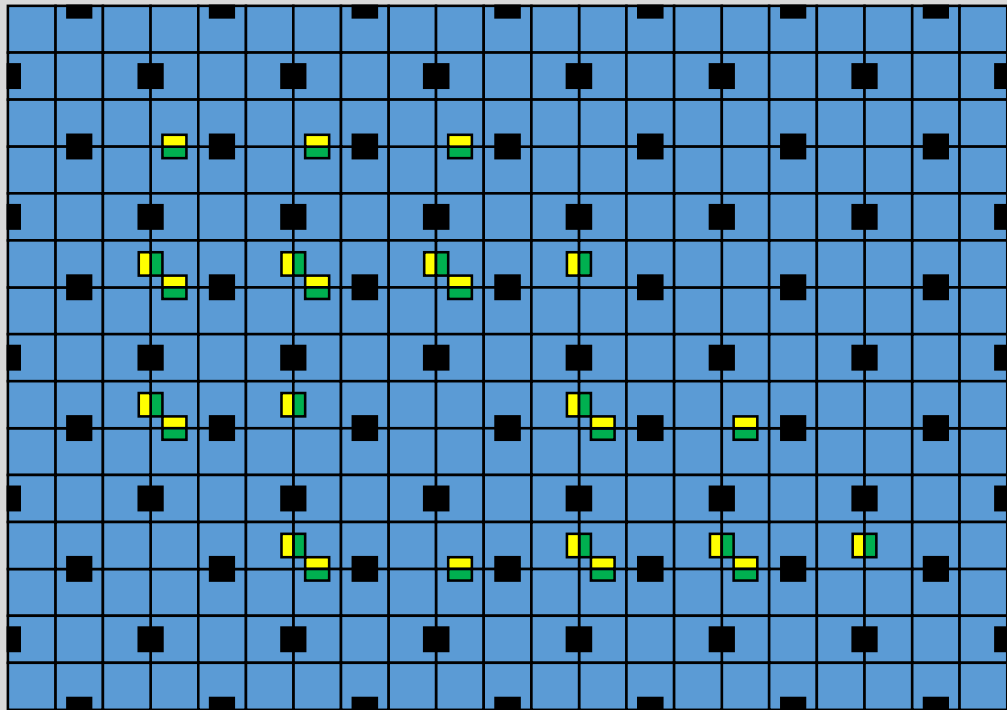
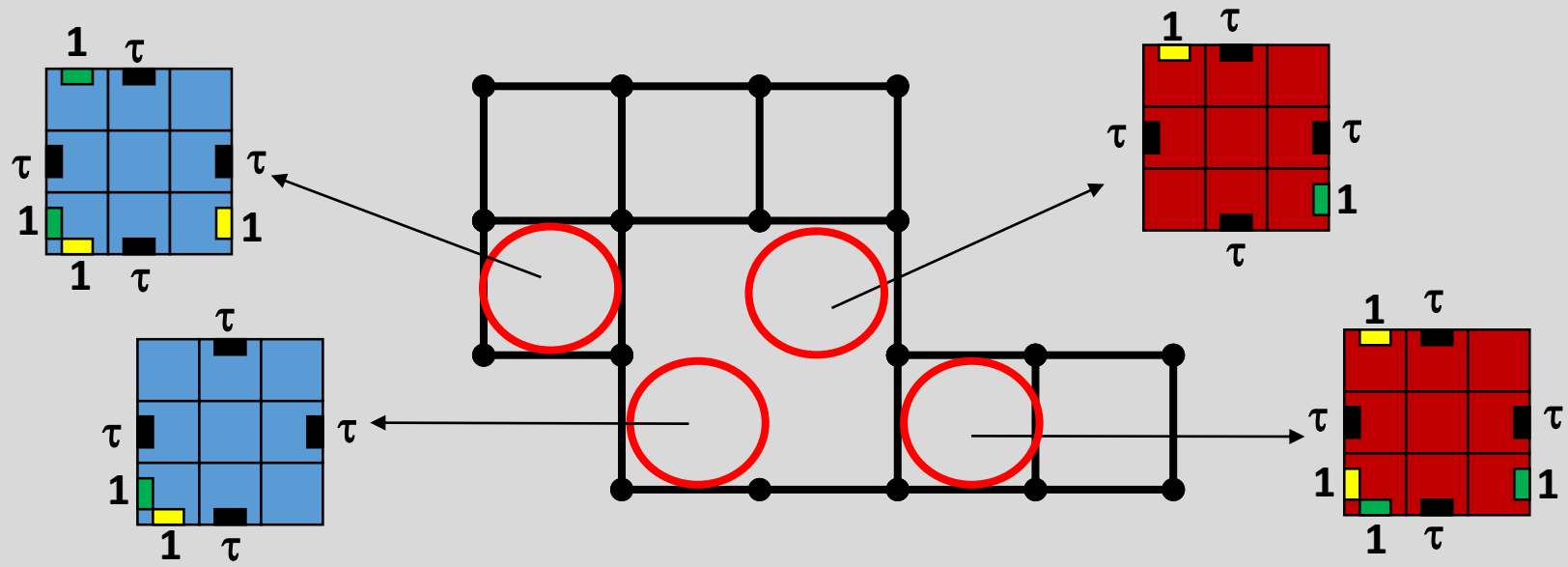
$\tau/2$ 

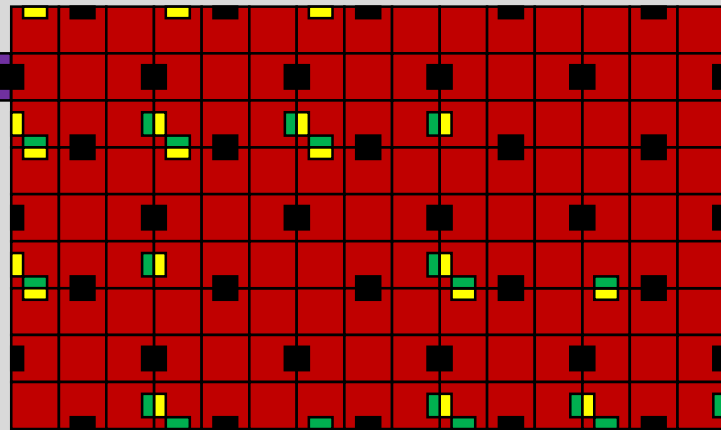
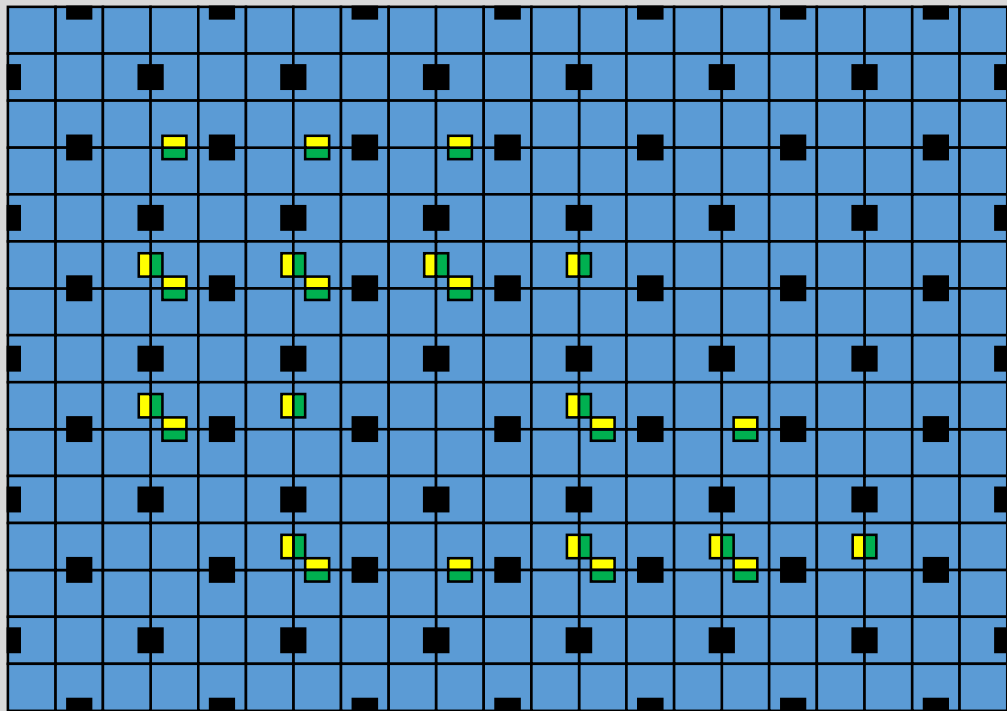
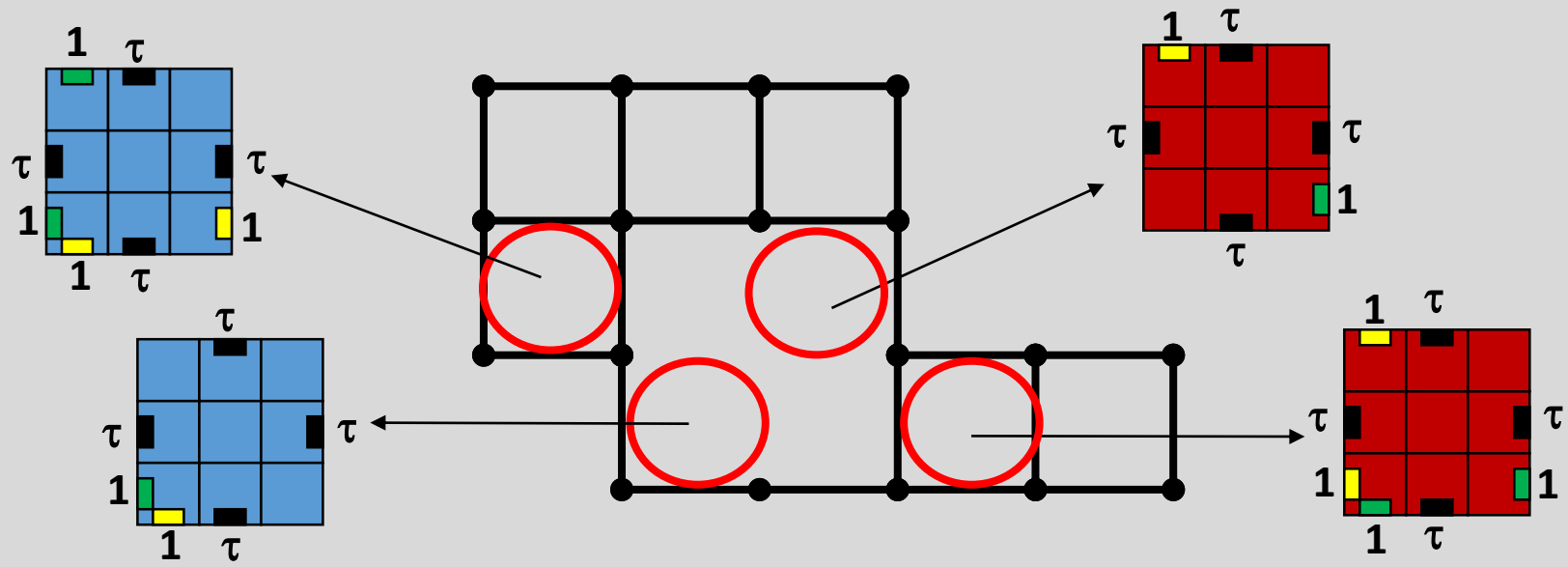
1  

Temperature:

$$\tau = |V|$$

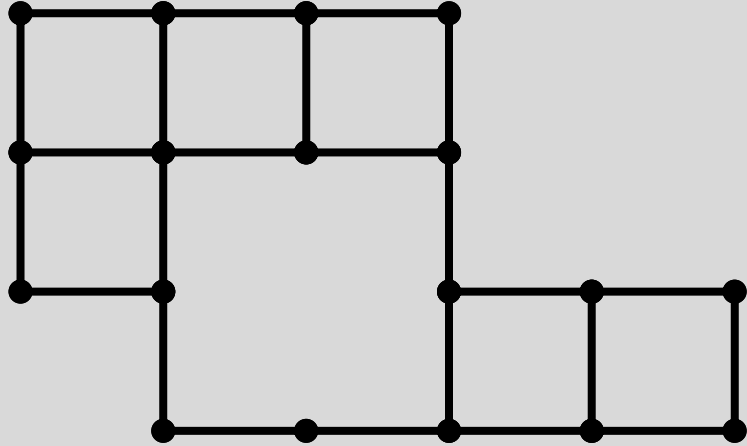




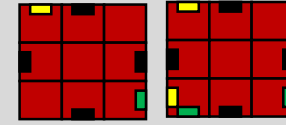


Unique Assembly Verification: **Hardness Reduction**

Grid Graph G :

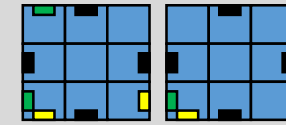


Tile System Γ :

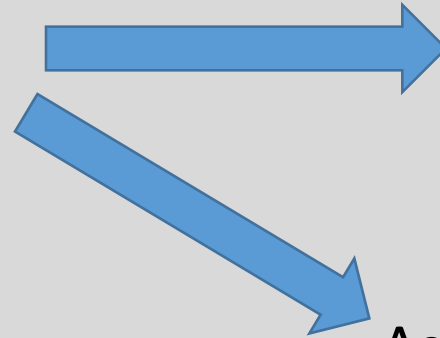


... Temperature

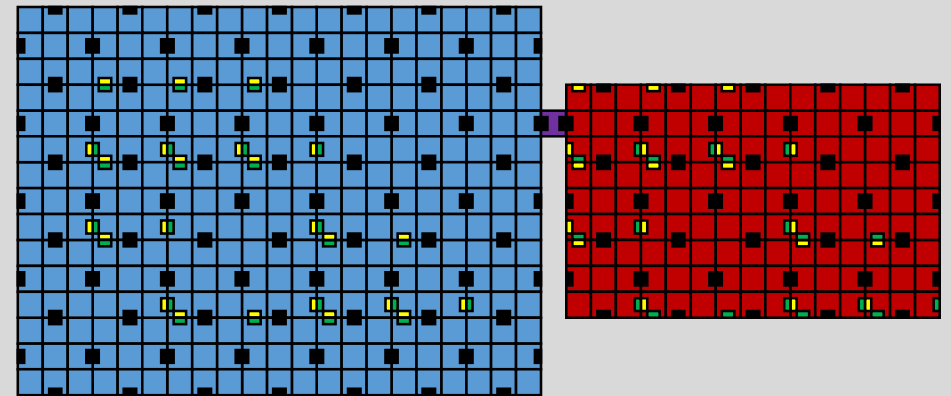
$$\tau = |V|$$



...

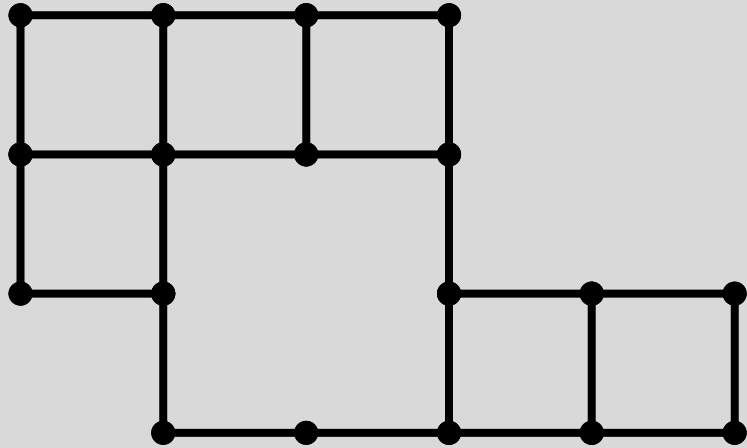


Assembly A :

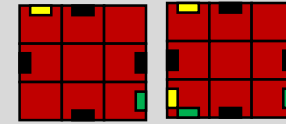


Unique Assembly Verification: **Hardness Reduction**

Grid Graph G :

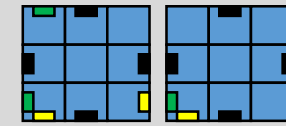


Tile System Γ :

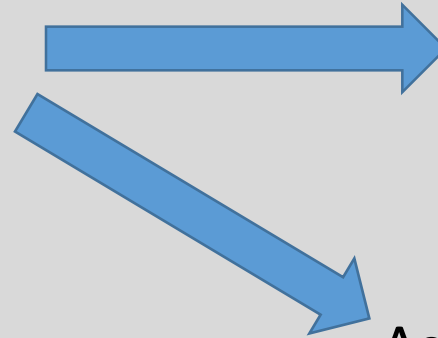


... Temperature

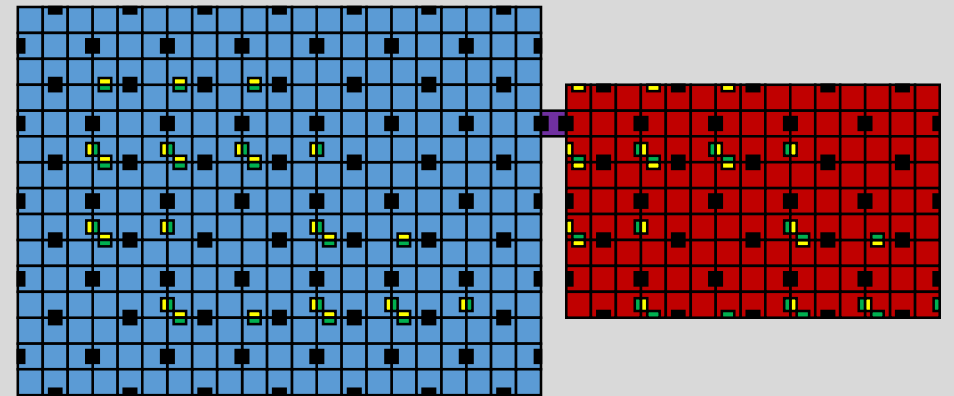
$$\tau = |V|$$



...



Assembly A :

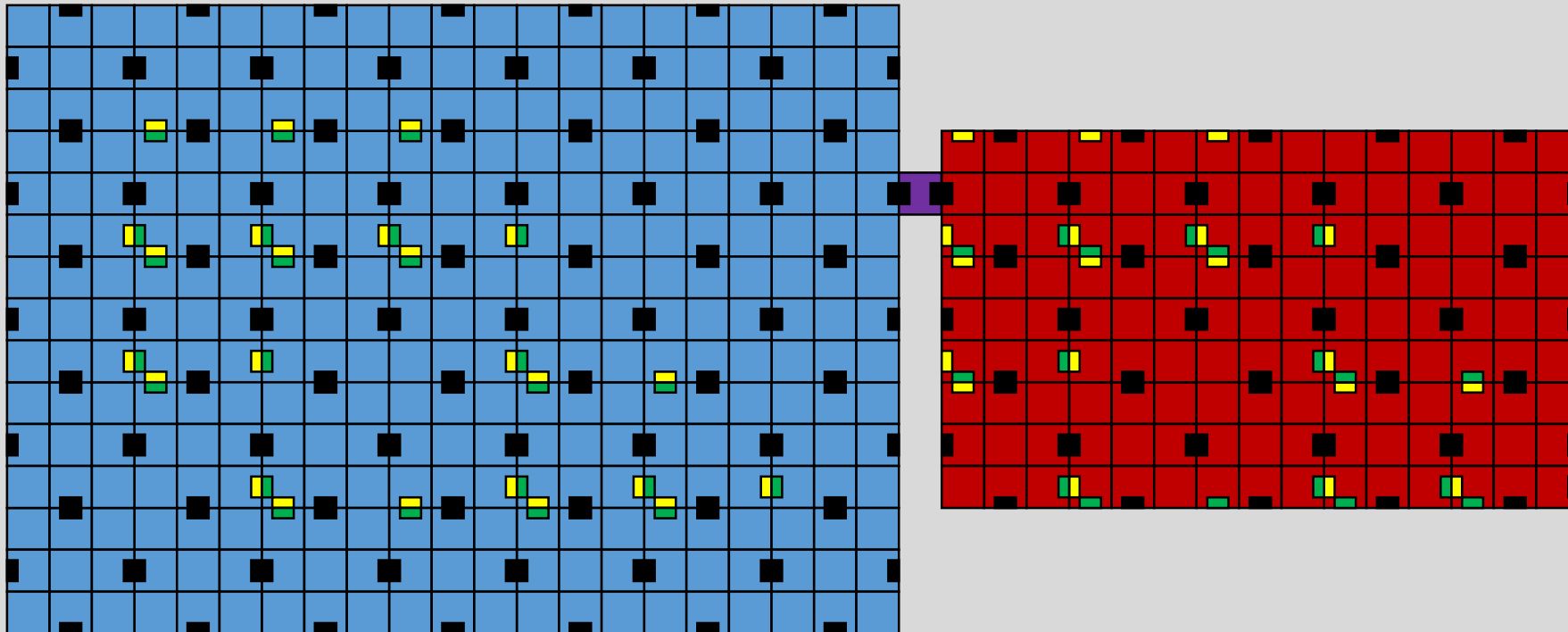
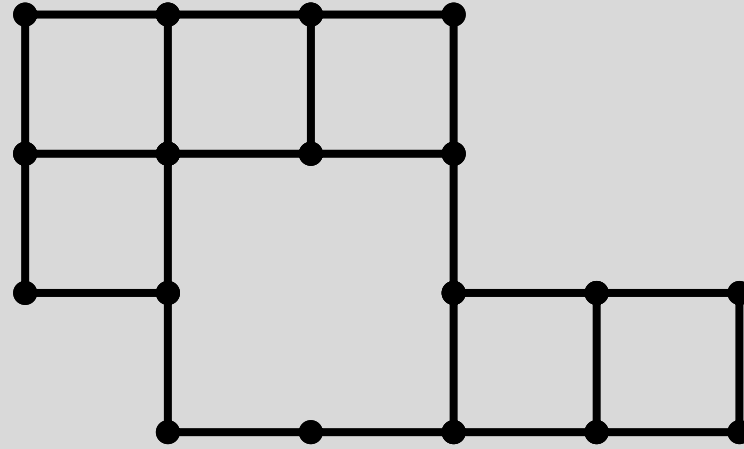


Γ uniquely produces assembly A

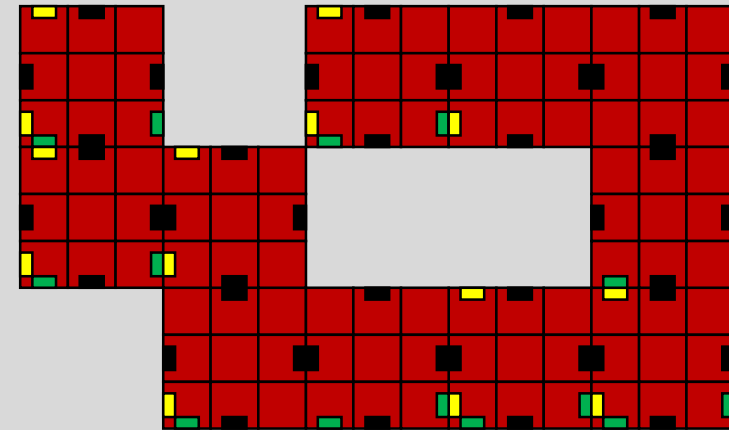
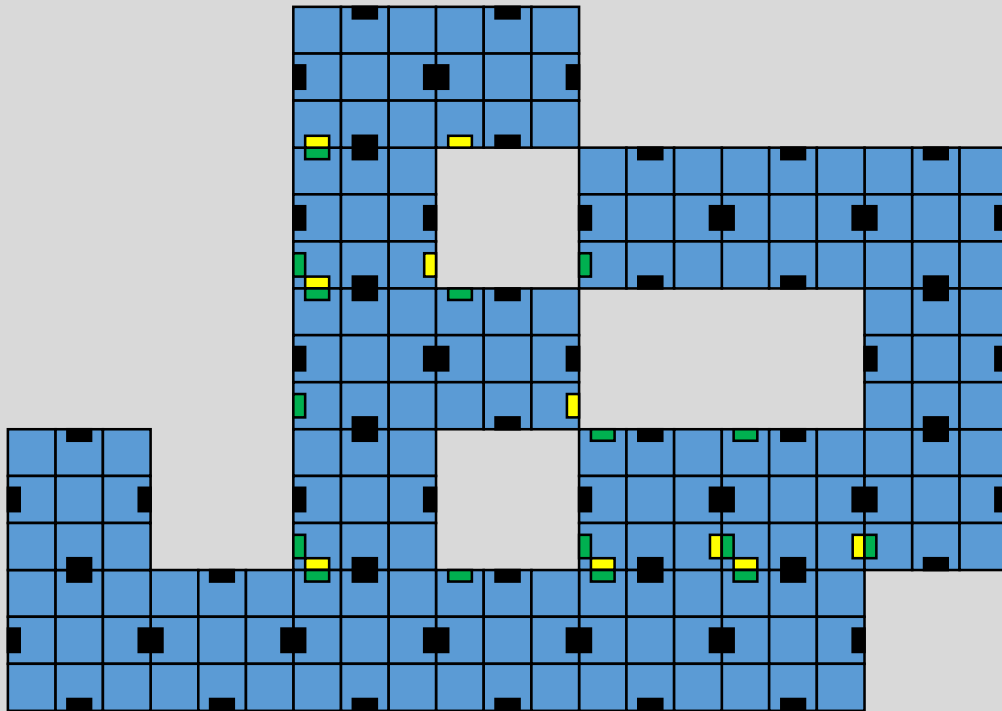
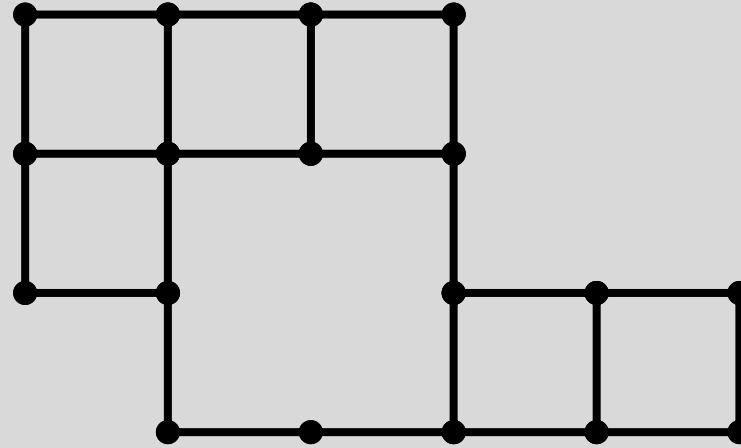
If and only if

G does NOT have a Hamiltonian cycle.

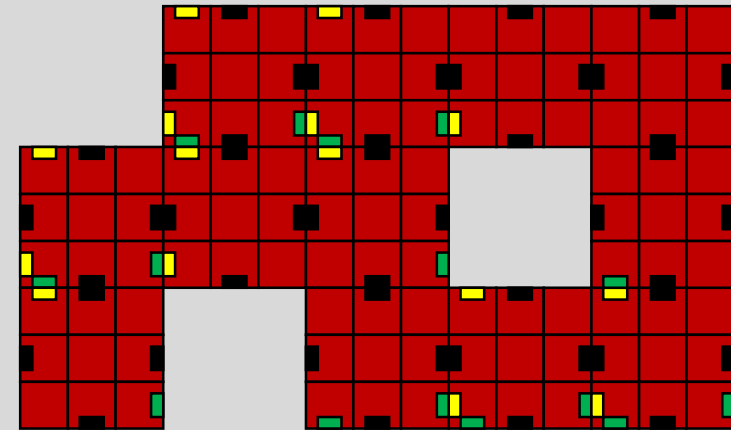
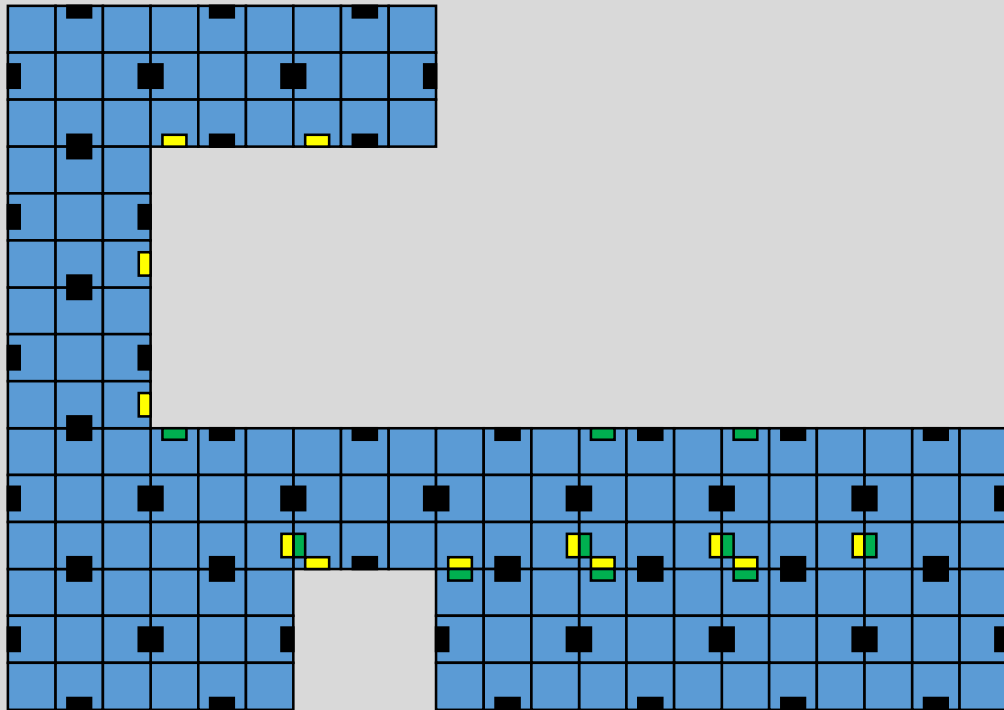
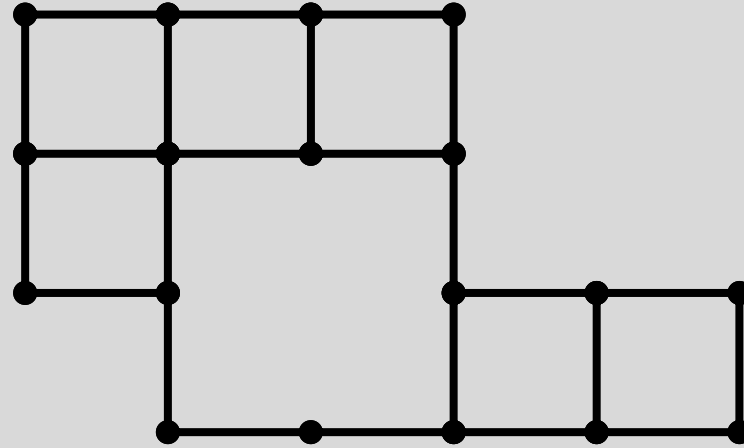
Hamiltonian Cycle => Rogue Assembly



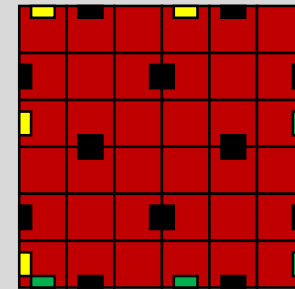
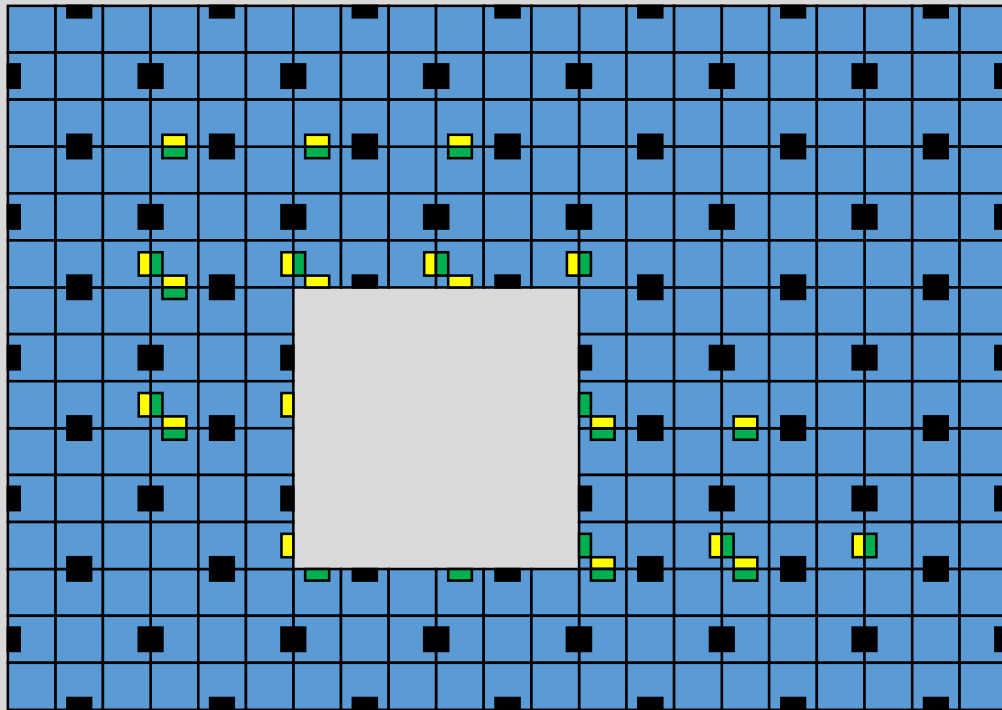
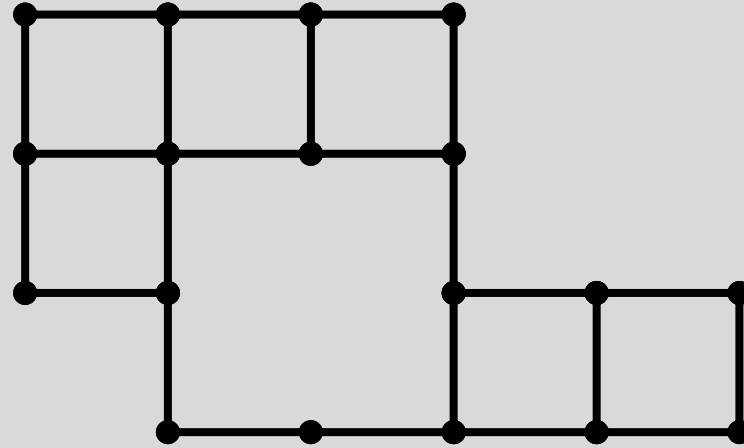
Hamiltonian Cycle => Rogue Assembly



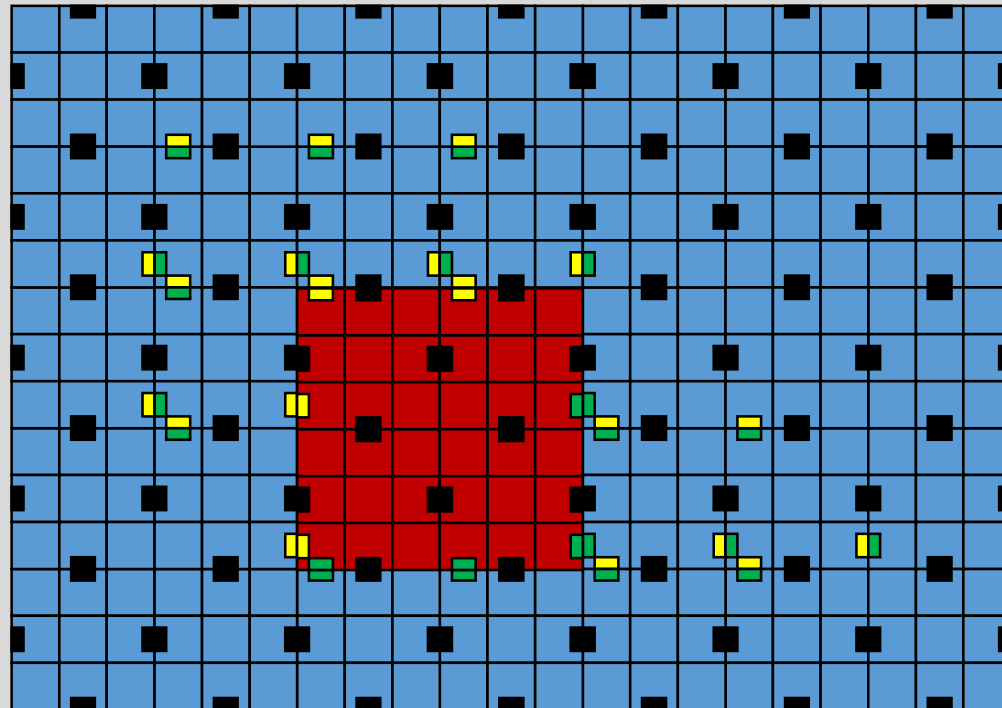
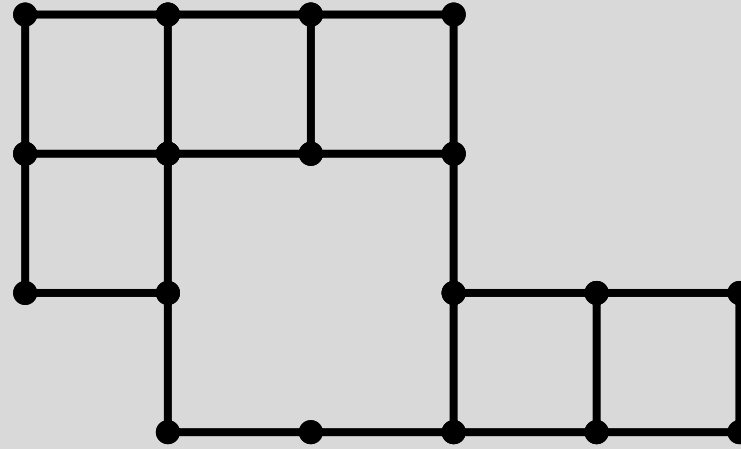
Hamiltonian Cycle => Rogue Assembly



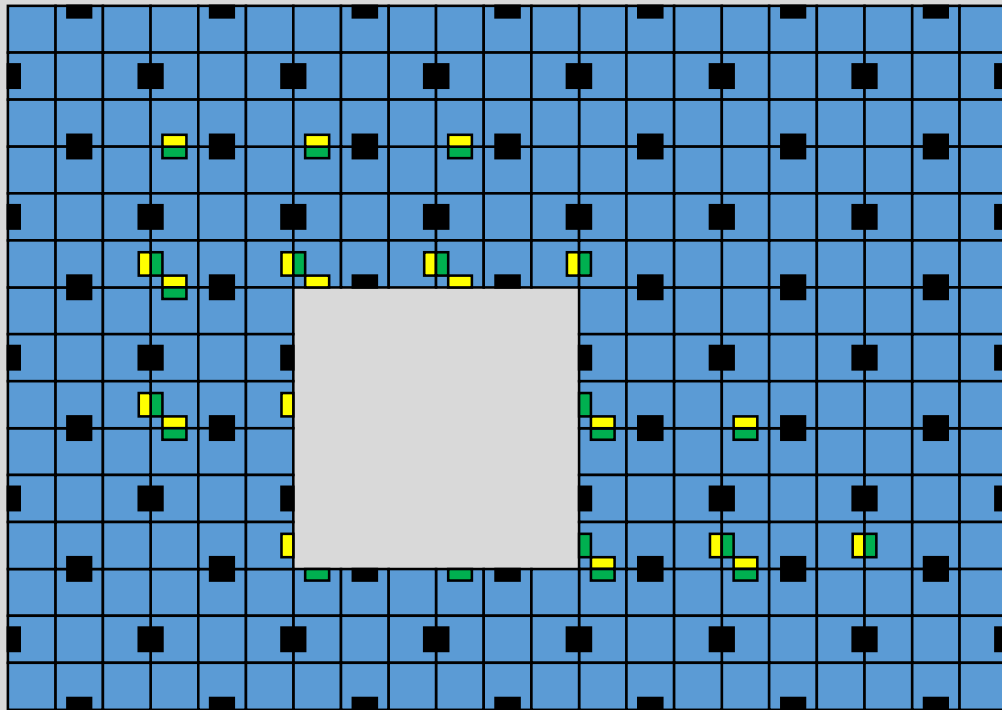
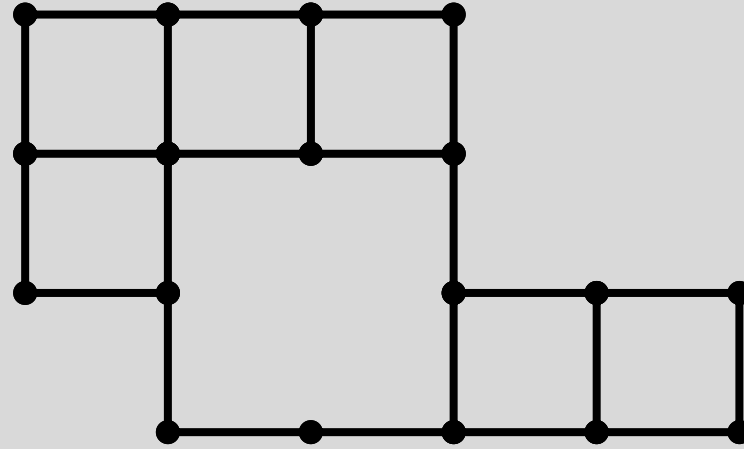
Hamiltonian Cycle => Rogue Assembly



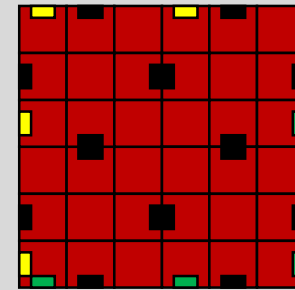
Hamiltonian Cycle => Rogue Assembly



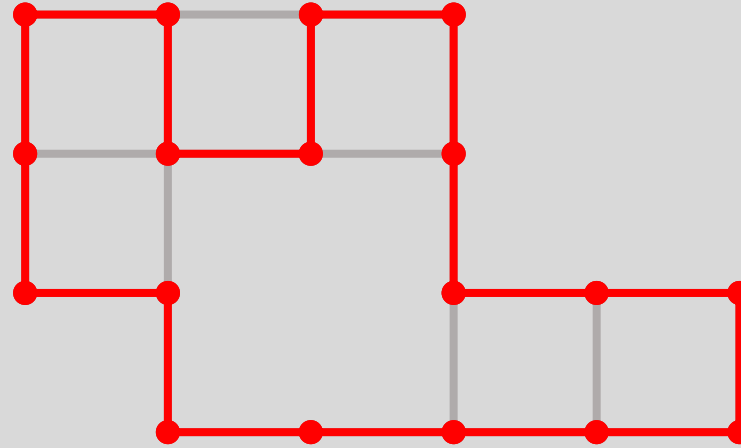
Hamiltonian Cycle => Rogue Assembly



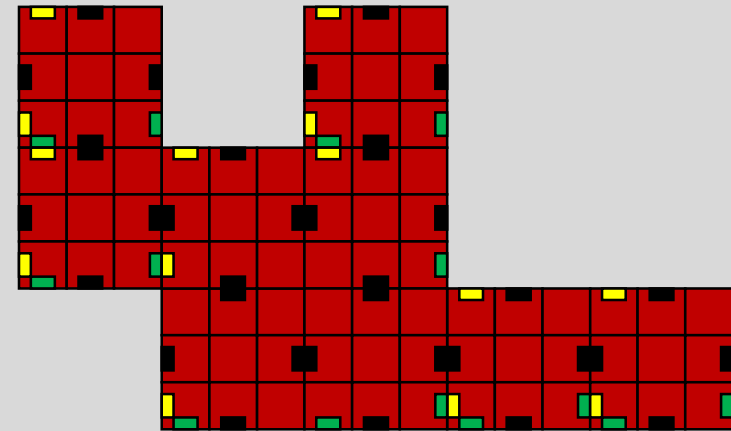
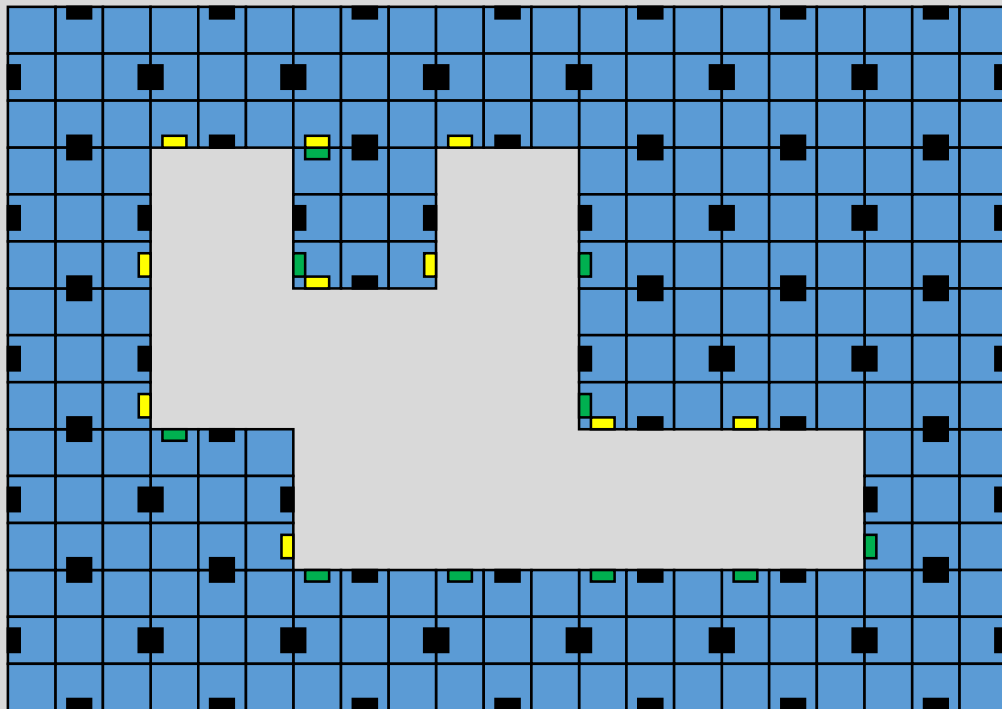
Temperature
 $\tau = |V|$



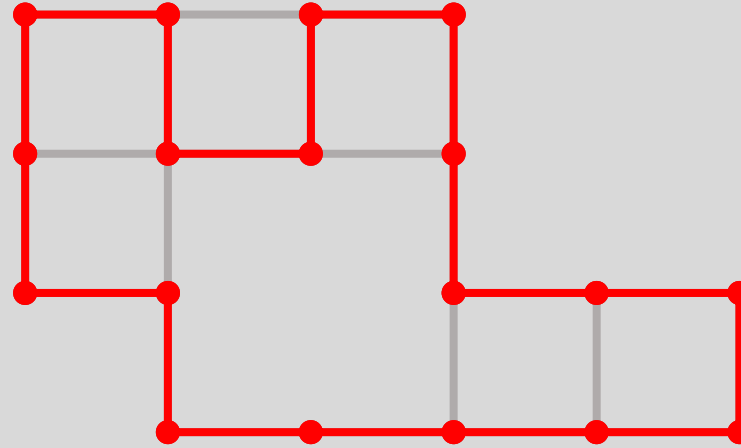
Hamiltonian Cycle => Rogue Assembly



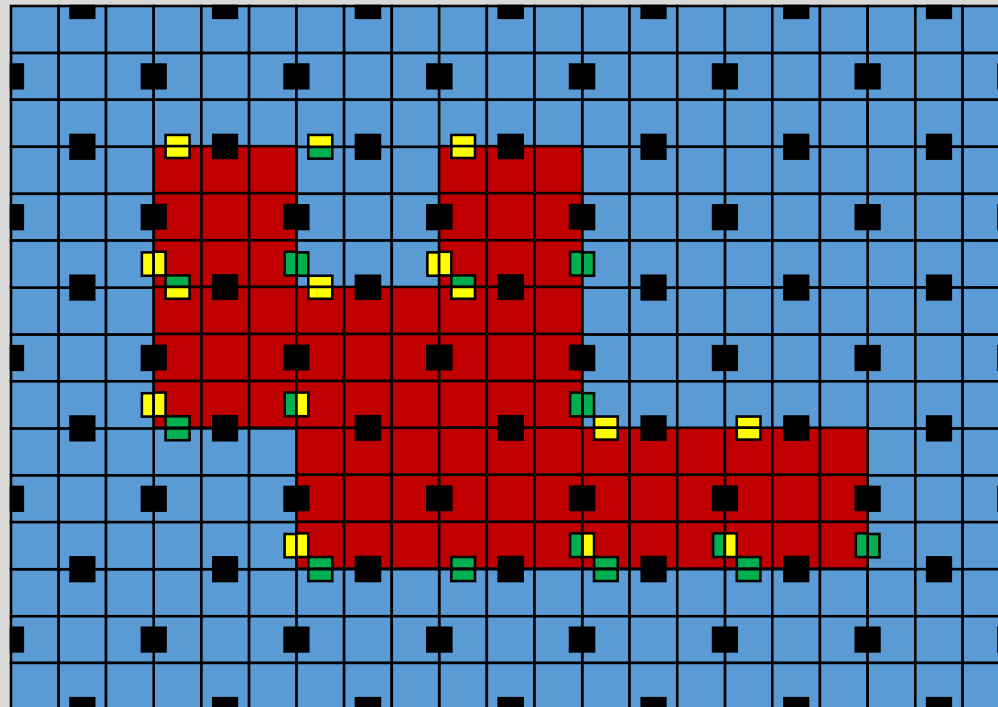
Exactly $\tau = |V|$ strength-1 yellow and green glues



Hamiltonian Cycle => Rogue Assembly

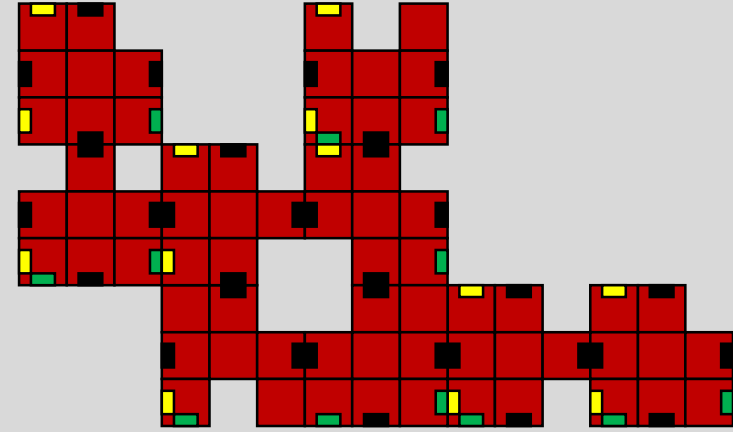


Exactly $\tau = |V|$ strength-1 yellow and green glues



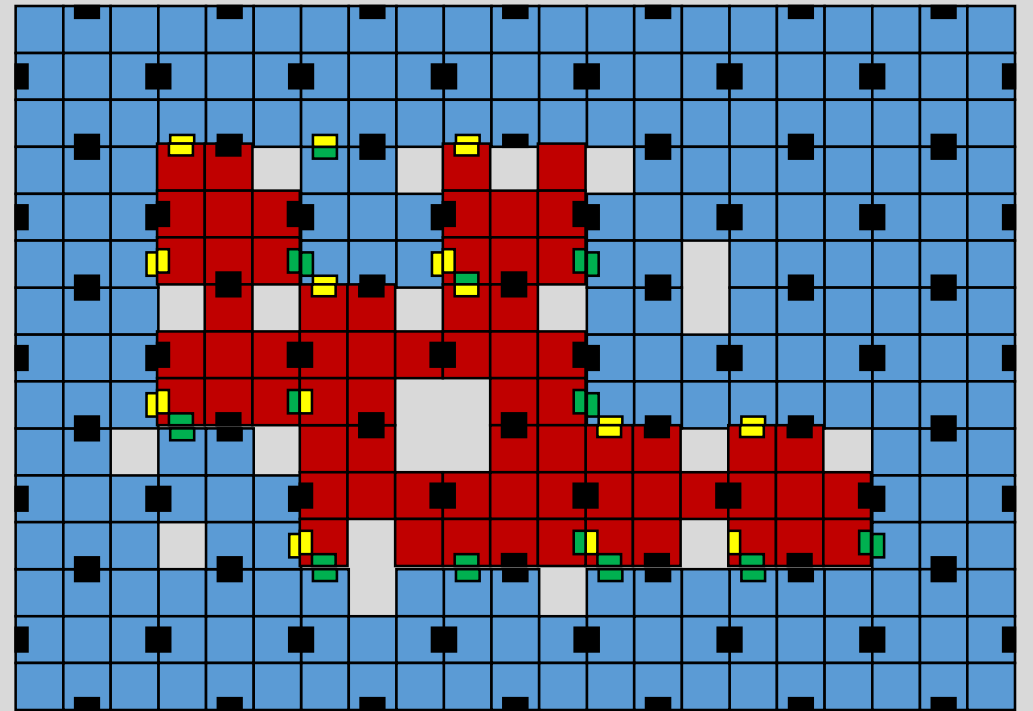
Rogue Assembly => Hamiltonian Cycle

- Rogue Assembly must be due to $|V|$ yellow/green glues
- Each present yellow/green glue implies presence of corresponding center tile



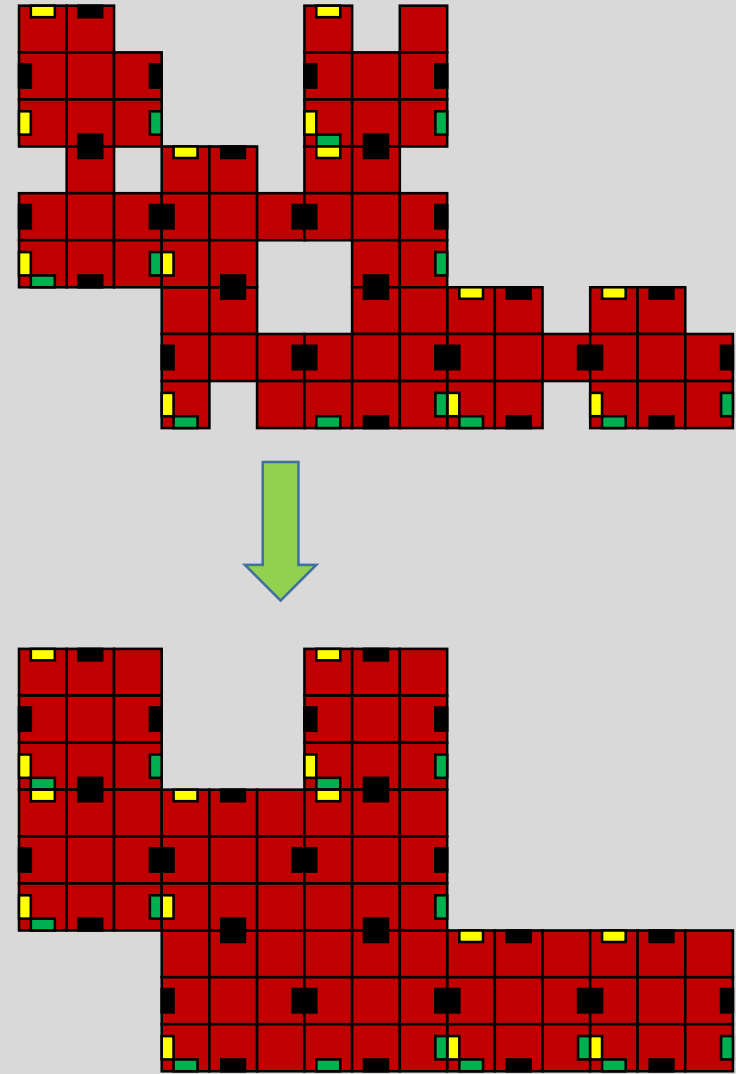
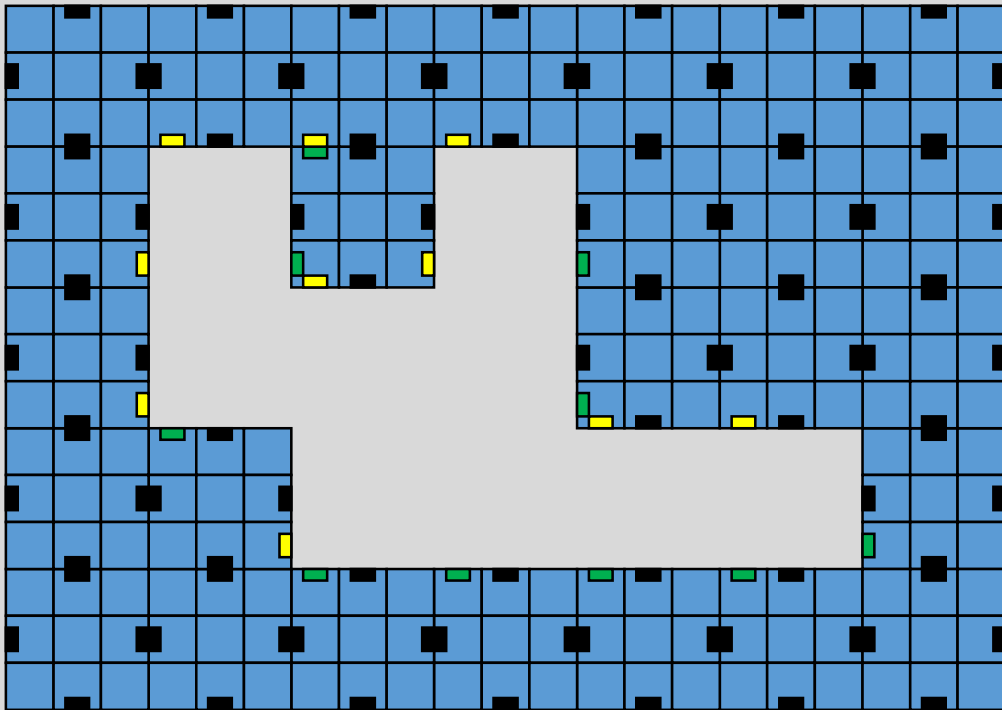
Rogue Assembly => Hamiltonian Cycle

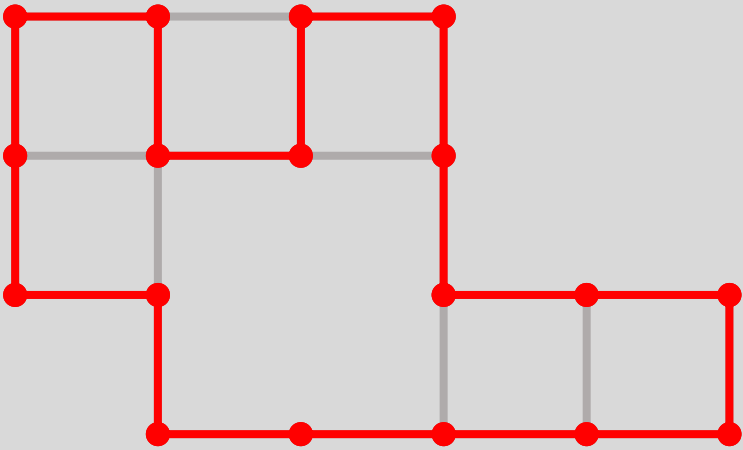
- Rogue Assembly must be due to $|V|$ yellow/green glues
- Each present yellow/green glue implies presence of corresponding center tile



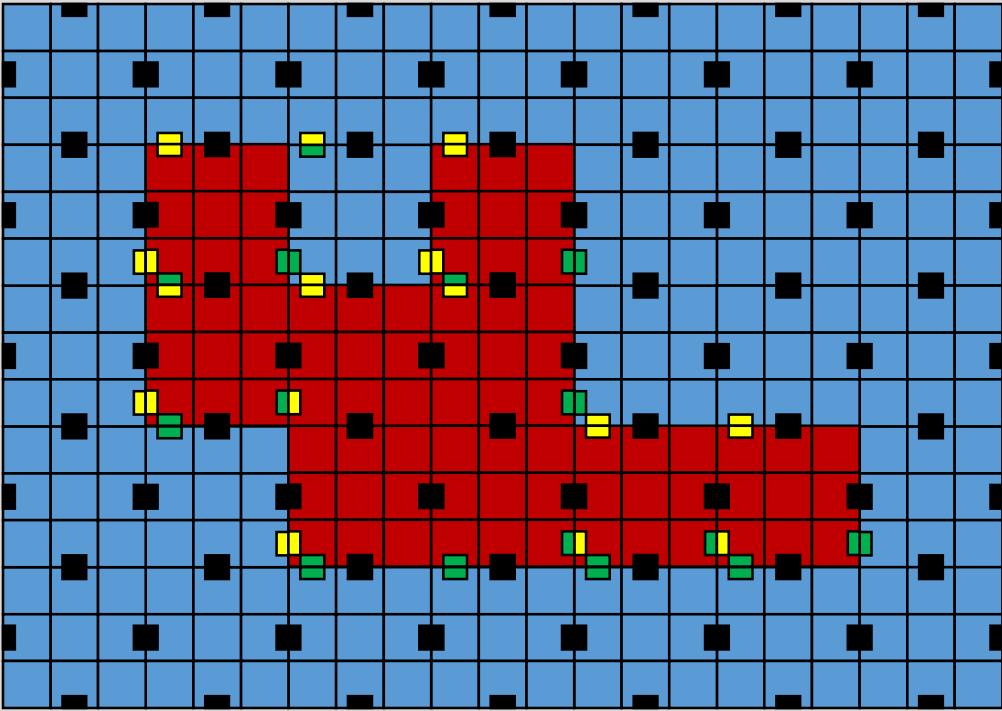
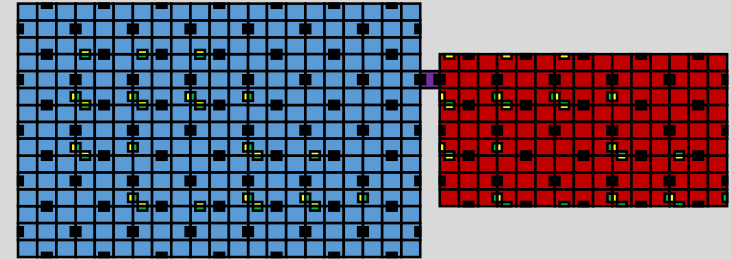
Rogue Assembly => Hamiltonian Cycle

- Rogue Assembly must be due to $|V|$ yellow/green glues
- Each present yellow/green glue implies presence of corresponding center tile
- Rogue assembly implies “clean” version is also rogue
- Clean versions exterior maps directly to a Hamiltonian cycle.

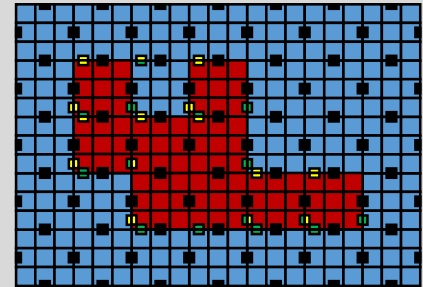




If G does NOT have a Hamiltonian cycle, then Γ uniquely assembles A

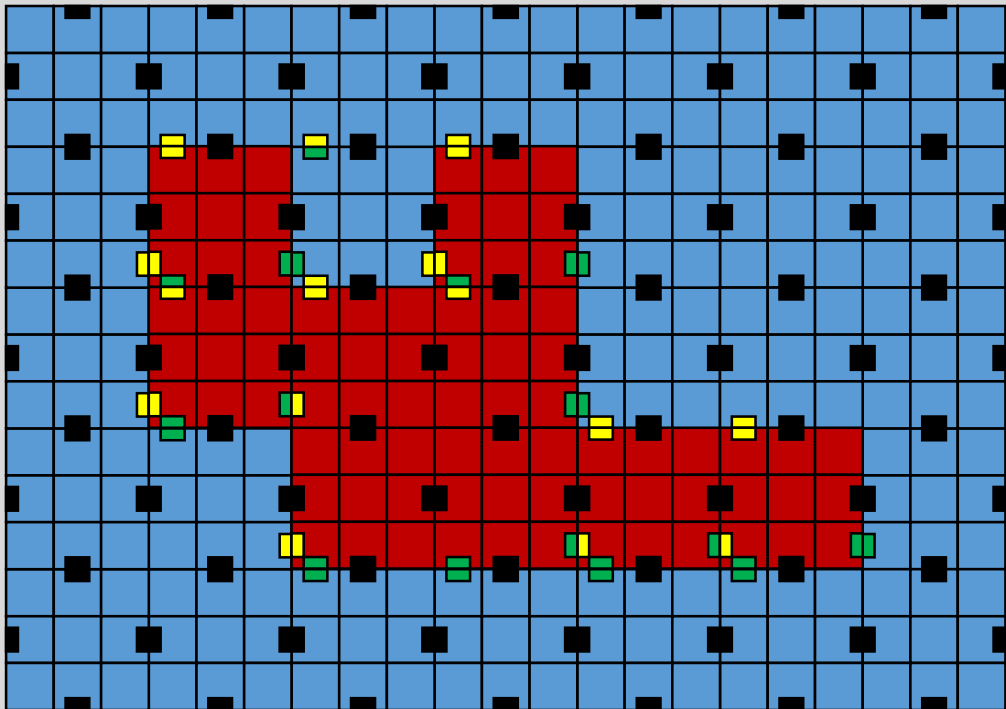
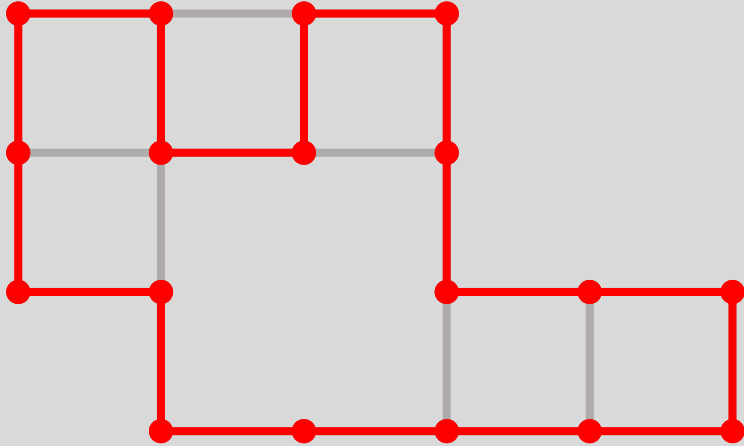


If G DOES have a Hamiltonian cycle, then Γ ALSO assembles things like this:



Theorem: Unique Assembly Verification is co-NP-complete

Unique Assembly Verification



UAV Complexity (2HAM model)

Complexity	Dim.	Temp.
coNP-C	3D	2
coNP-C	2D	τ
?	2D	$O(1)$

[Cannon et. Al. 2013]

Complexities at High-Temperature

- **Verification**

- Unique Assembly Verification is coNP-complete
- **Related Problems and Open Problems**

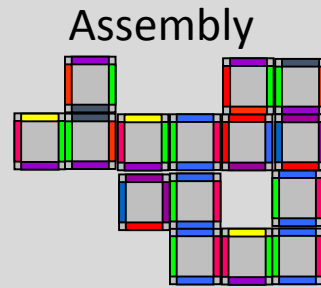
Related Problems

2HAM

2-Handed Assembly Model

Assembly Verification

	Dim.	Temp.
coNP-C [STACS 2013]	3D	2
coNP-C	2D	τ
?	2D	$O(1)$



Related Problems

2HAM

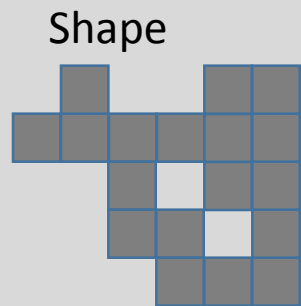
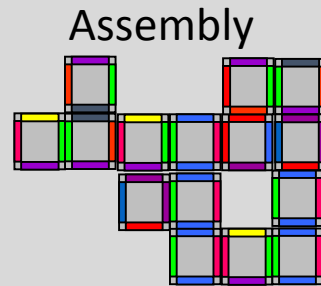
2-Handed Assembly Model

Assembly Verification

	Dim.	Temp.
coNP-C [STACS 2013]	3D	2
coNP-C	2D	τ
?	2D	$O(1)$

Shape Verification

	Dim.	Temp.
coNP ^{NP} -C [UCNC 2017]	2D	2



Related Problems

2HAM

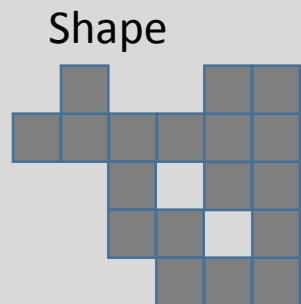
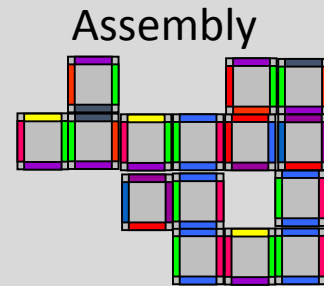
2-Handed Assembly Model

Assembly Verification

	Dim.	Temp.
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?	2D	$O(1)$

Shape Verification

	Dim.	Temp.
coNP ^{NP} -C [UCNC 2017]	2D	2



aTAM

Abstract Tile Assembly Model

Assembly Verification

	Dim.	Temp.
P [Adleman et. Al. 2002]	2D/3D	any

Related Problems

2HAM

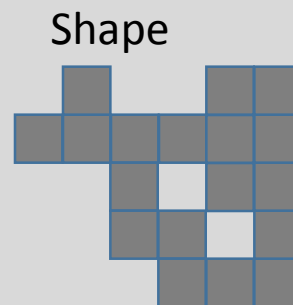
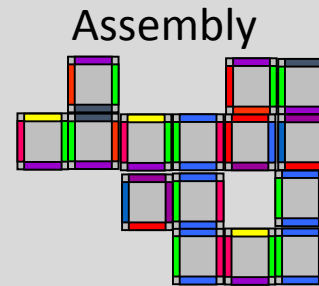
2-Handed Assembly Model

Assembly Verification

	Dim.	Temp.
coNP-C [STACS 2013]	3D	2
coNP-C	2D	τ
?	2D	$O(1)$

Shape Verification

	Dim.	Temp.
coNP ^{NP} -C [UCNC 2017]	2D	2



aTAM

Abstract Tile Assembly Model

Assembly Verification

	Dim.	Temp.
P [Adleman et. Al. 2002]	2D/3D	any

Shape Verification

	Dim.	Temp.
coNP-C [SODA 2004]	2D	2

Related Problems

2HAM

2-Handed Assembly Model

Assembly Verification

	Dim.	Temp.
coNP-C [STACS 2013]	3D	2
coNP-C	2D	τ
?	2D	$O(1)$

Shape Verification

	Dim.	Temp.
coNP ^{NP} -C [UCNC 2017]	2D	2

aTAM

Abstract Tile Assembly Model

Assembly Verification

	Dim.	Temp.
P [Adleman et. Al. 2002]	2D/3D	any

Shape Verification

	Dim.	Temp.
coNP-C [SODA 2004]	2D	2

Staged

Staged Assembly Model

Assembly Verification

	Dim.	Temp.
coNP ^{NP} -hard [UCNC 2017]	2D	2
Pspace-C ?	2D	2