Section 13.1

Find the gradient vector field of f:

22) $f(x,y) = \tan(3x-4y)$

Soln We have

$$\nabla f(x,y) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

$$= \left\langle \frac{\partial}{\partial x}(\tan(3x-4y)), \frac{\partial}{\partial y}(\tan(3x-4y)) \right\rangle$$

$$= \langle 3\sec^2(3x-4y), -4\sec^2(3x-4y) \rangle$$

$$= 3\sec^2(3x-4y)\, i - 4\sec^2(3x-4y) \cdot j$$

(Ans)

24) $f(x,y,z) = x\ln(y-2z)$.

Soln

The gradient vector field is

$$\nabla f(x,y,z) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

$$= \left\langle \ln(y-2z), \frac{x}{y-2z}, \frac{x}{y-2z}(-2) \right\rangle$$

$$= \ln(y-2z)\, i + \frac{x}{y-2z}\, j - \frac{2x}{y-2z} K.$$

(Ans)

~~29~~ 30) At time $t=1$, ~~the particle~~ a particle is located at position $(1,3)$. If it moves in a velocity field

$$\vec{F}(x,y) = \langle xy-2, y^2-10 \rangle$$

find its approximate location at time $t=1.05$.

Sol^n At $t=1$ the particle is at $(1,3)$ so its velocity ~~at~~ is $F(1,3) = \langle 1\cdot 3-2,\ 3^2-10\rangle = \langle 1,-1\rangle$.

After 0.05 units of time, the particle's change in location should be approximately $0.05\,F(1,3) = 0.05\langle 1,-1\rangle = \langle 0.05,-0.05\rangle$, so the particle should be approximately at the point $(1.05,\ 2.95)$.

Ans