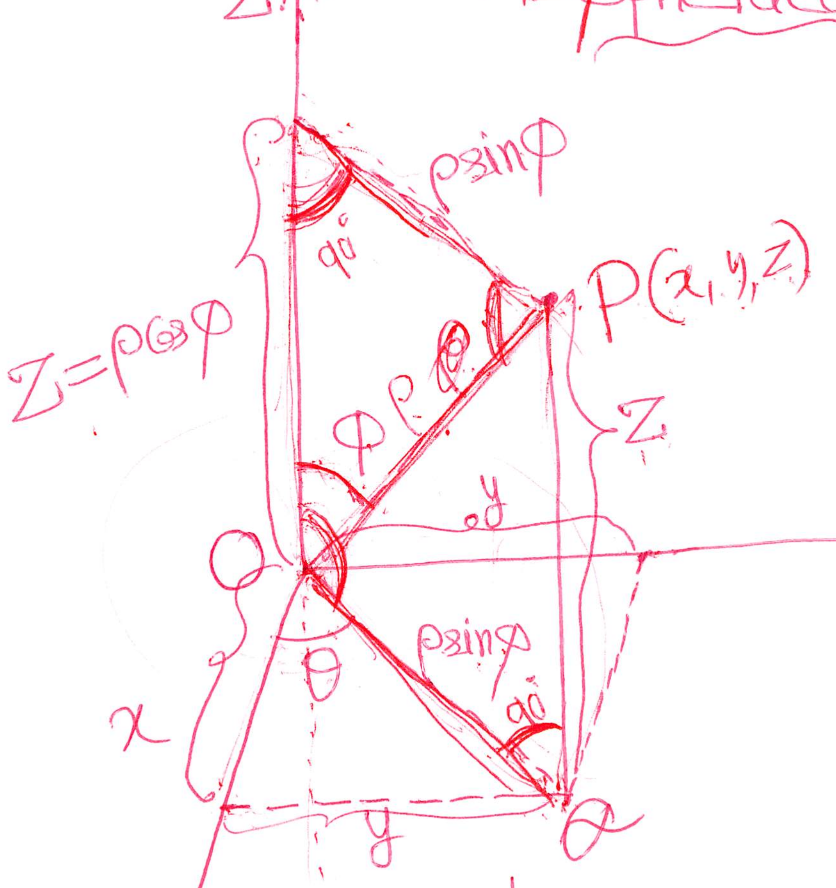
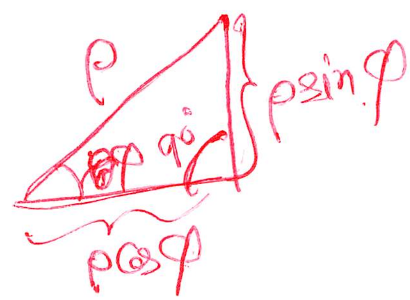
Section 12.7 Spherical Coordinates



$0 \le \varphi \le \pi$

$0 \le \theta \le 2\pi$



We have

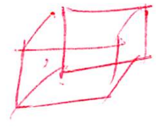
$$x = \overline{OQ}\cos\theta = \rho\sin\varphi\cos\theta$$

$$y = \overline{OQ}\sin\theta = \rho\sin\varphi\sin\theta$$

$$z = \rho\cos\varphi$$

$dV = dx\,dy\,dz$

$=$ length x width x height



Notice that

$$x^2+y^2+z^2=\rho^2$$

In this case

$$dV = \rho^2\sin\varphi\, d\rho\, d\theta\, d\varphi$$

where dV is the volume of the spherical wedge as shown in page 738 of ~~you~~ your book. Notice that here spherical wedge is the counterpart of the rectangular box considered in the previous section.

Thus, we have

$$\iiint_E f(x,y,z)\,dV = \int_{\varphi=c}^{d}\int_{\theta=\alpha}^{\beta}\int_{\rho=a}^{b} f(\rho\sin\varphi\cos\theta,\ \rho\sin\varphi\sin\theta,\ \rho\cos\varphi)\cdot\rho^2\sin\varphi\,d\rho\,d\theta\,d\varphi$$

where E is a spherical wedge given by

$$E = \{(\rho,\theta,\varphi):\ a\le\rho\le b,\ \alpha\le\theta\le\beta,\ c\le\varphi\le d\}.$$