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Find the shortest distance from the point $(2,0,-3)$ to the plane $x+y+z=1$.

<u>Sol^n</u> Let d be the distance of the point $(2,0,-3)$ from ~~the plane~~ a point (x,y,z) on the plane $x+y+z=1$.

Then,
$$d=\sqrt{(x-2)^2+(y-0)^2+(z+3)^2}$$
$$=\sqrt{(x-2)^2+y^2+(4-x-y)^2}$$

since $x+y+z=1$
$\Rightarrow z=1-x-y$

Notice that d will be minimum if ~~d^2 is mini~~ $d^2=(x-2)^2+y^2+(4-x-y)^2$ is minimum. Let
$$f(x,y)=(x-2)^2+y^2+(4-x-y)^2$$

We have
$$f_x=\frac{\partial f}{\partial x}=2(x-2)+2(4-x-y)(-1)$$
$$=2x-4-8+2x+2y$$
$$=4x+2y-12$$

and
$$f_y=\frac{\partial f}{\partial y}=2y+2(4-x-y)(-1)$$
$$=2y-8+2x+2y$$
$$=2x+4y-8$$

Now
$$f_x=0,\ f_y=0$$
$$\Rightarrow 4x+2y-12=0 \ \&\ 2x+4y-8=0$$
$$\Rightarrow 2x+y=6 \ \&\ x+2y=4$$
$$\Rightarrow x=\frac{8}{3},\ y=\frac{2}{3}.$$

$$2x + y = 6$$
$$2x + 4y = 8$$
$$(-)\quad -3y = -2$$
$$\Rightarrow y = \frac{2}{3}$$
$$2x = 6 - \frac{2}{3} = \frac{16}{3}$$
$$\Rightarrow x = \frac{8}{3}$$

Hence $\left(\frac{8}{3}, \frac{2}{3}\right)$ is a critical point. Since the shortest distance exists, we can say $f(x, y)$ is minimum at the point $\left(\frac{8}{3}, \frac{2}{3}\right)$.
Hence, the shortest distance is given by

$$d = \sqrt{(x-2)^2 + y^2 + (4-x-y)^2}$$
$$= \sqrt{\left(\frac{8}{3}-2\right)^2 + \left(\frac{2}{3}\right)^2 + \left(4-\frac{8}{3}-\frac{2}{3}\right)^2}$$
$$= \sqrt{\frac{4}{9} + \frac{4}{9} + \frac{4}{9}}$$
$$= \sqrt{\frac{12}{9}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \text{ unit}$$

Ans

#32 Page 676 Find the point on the plane $x - 2y + 3z = 6$ that is closest to the point $(0, 1, 1)$.

Sol^n Let (x, y, z) be the point on the plane $x - 2y + 3z = 6$ that is closest to the point $(0, 1, 1)$.

Let d be the distance of the point (3)
(x, y, z) from $(0, 1, 1)$. Then,

$$d = \sqrt{(x-0)^2 + (y-1)^2 + (z-1)^2}$$

$$= \sqrt{x^2 + (y-1)^2 + \left(2 - \frac{x}{3} + \frac{2y}{3} - 1\right)^2}$$

$\left[x - 2y + 3z = 6 \Rightarrow z = \frac{1}{3}(6 - x + 2y) = 2 - \frac{x}{3} + \frac{2y}{3} \right]$

$$= \sqrt{x^2 + (y-1)^2 + \left(1 - \frac{x}{3} + \frac{2y}{3}\right)^2}$$

Let

$$f(x, y) = d^2$$

$$\Rightarrow f(x, y) = x^2 + (y-1)^2 + \left(1 - \frac{x}{3} + \frac{2y}{3}\right)^2$$

Now

$$f_x = 2x + 2\left(1 - \frac{x}{3} + \frac{2y}{3}\right)\left(-\frac{1}{3}\right)$$

$\left[\frac{d(x^n)}{dx} = n x^{n-1}\right]$

$$= 2x - \frac{2}{3} + \frac{2x}{9} - \frac{4y}{9}$$

$$= \frac{20x}{9} - \frac{4y}{9} - \frac{2}{3}$$

& $$f_y = 2(y-1) + 2\left(1 - \frac{x}{3} + \frac{2y}{3}\right) \cdot \frac{2}{3}$$

$$= 2y - 2 + \frac{4}{3} - \frac{4}{9}x + \frac{8y}{9}$$

$$= -\frac{4x}{9} + \frac{26y}{9} - \frac{2}{3}$$

Now $f_x = 0$ & $f_y = 0$

$$\Rightarrow \frac{20x}{9} - \frac{4y}{9} = \frac{2}{3} \quad \text{—(1)} \quad \& \quad -\frac{4x}{9} + \frac{26y}{9} = \frac{2}{3} \quad \text{—(2)}$$

Soling (1) & (2) we have

$x = \frac{5}{14}$, $y = \frac{2}{7}$, and then

$$z = 2 - \frac{5}{42} + \frac{2}{3} \cdot \frac{2}{7}$$

$$= 2 - \frac{5}{42} + \frac{4}{21} = \frac{29}{14}$$

$\frac{\cancel{20}x}{\cancel{9}} - \frac{4y}{9} = \frac{2}{3}$ (3)

$-\frac{\cancel{20}x}{\cancel{9}} + \frac{130y}{9} = \frac{10}{3}$ (4)

$(+)$ $\frac{126}{9}y = \frac{12}{3}$

$\Rightarrow 14y = 4$

$\Rightarrow y = \frac{2}{7}$

$\frac{20x}{9} - \frac{4}{9}\cdot\frac{2}{7} = \frac{2}{3}$

$\Rightarrow \frac{20x}{9} - \frac{8}{63} = \frac{2}{3}$

$\Rightarrow \frac{20x}{9} = \frac{2}{3} + \frac{8}{63}$

$= \frac{42+8}{63}$

$\Rightarrow \frac{\cancel{20}x}{\cancel{9}} = \frac{\cancel{50}}{\cancel{63}_{7}}$

$\Rightarrow x = \frac{5}{14}$

$\frac{2}{1} - \frac{5}{42} + \frac{4}{21}$

$= \frac{84 - 5 + 8}{42}$

$= \frac{87}{42} = \frac{29}{14}$

Hence the closest point is $\left(\frac{5}{14}, \frac{2}{7}, \frac{29}{14}\right)$

(A8)