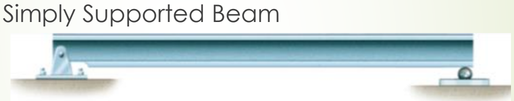
MECE 3321: Mechanics of Solids Chapter 6

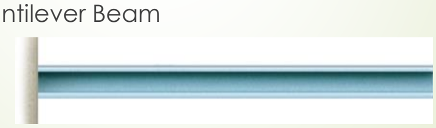
Samantha Ramirez

Beams

- **Beams** are long straight members that carry loads perpendicular to their longitudinal axis
- Beams are classified by the way they are supported
  - Simply Supported Beam



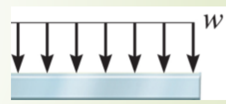
  - Cantilever Beam



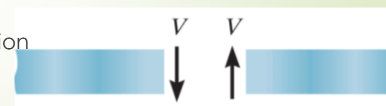
  - Overhanging Beam

Shear & Bending Moment Diagrams

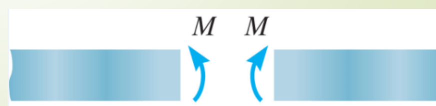
- **Shear and bending moment diagrams** are graphical representations of the internal shears and moments within a beam.
- They can be constructed by establishing a sign convention.
  - Positive Distributed Load



  - Positive Internal Shear
    - Causes a clockwise rotation



  - Positive Internal Moment
    - Compression on top
      - Can hold water

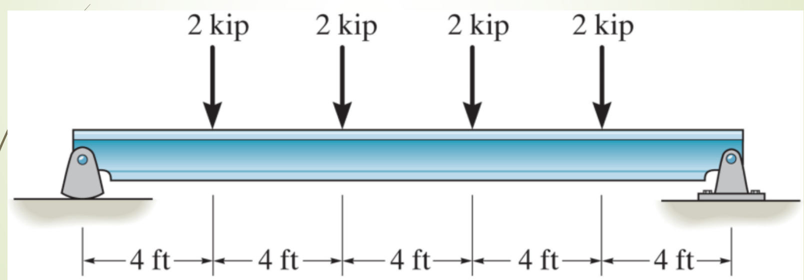


How to Analyze Shear & Moment Diagrams

1. Statics
   - FBD
   - Reactions
2. Solids
   - Cut between concentrated forces or moments
     - Note distance, x, from the beam's left end
   - FBD of each section
   - Solve for V and M
3. Shear & Moment Diagrams
   - Plot the shear diagram (V vs x)
   - Plot the moment diagram (M vs x)

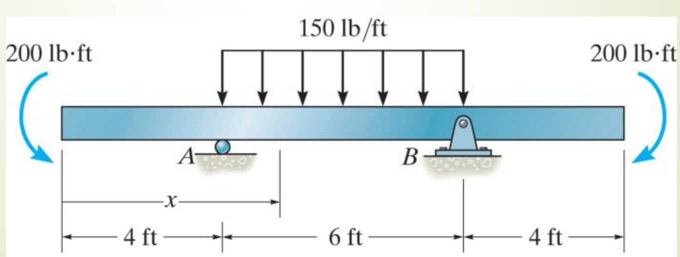
Problem 6-4

- Draw the shear and moment diagrams for the beam.



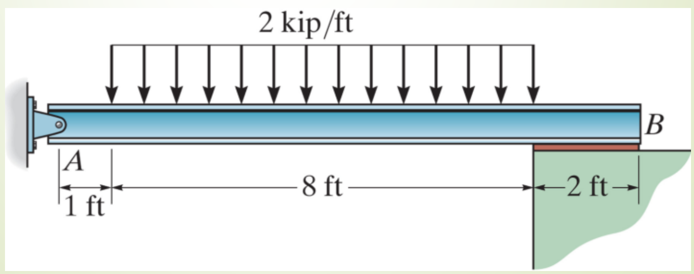
Problem 6-25

- Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x, where 4 ft < x < 10 ft.



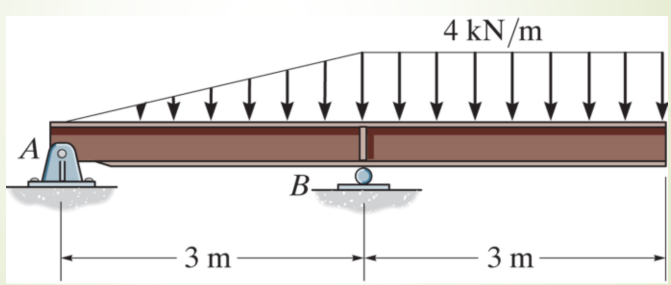
Problem 6-30

- The beam is bolted or pinned at A and rests on a bearing pad at B that exerts a uniform distributed loading on the beam over its 2-ft length. Draw the shear and moment diagrams for the beam if it supports a uniform loading of 2 kip/ft.



Problem 6-22

- Draw the shear and moment diagrams for the overhang beam.



Graphical Method for Constructing V & M Diagrams

$$\frac{dV}{dx} = -w(x)$$

- The slope of the shear curve is equal to the negative of the intensity of the distributed load.

$$\frac{dM}{dx} = V(x)$$

- The slope of the moment curve is equal to the intensity of the shear force.

$$\Delta V = \int -w(x)dx$$

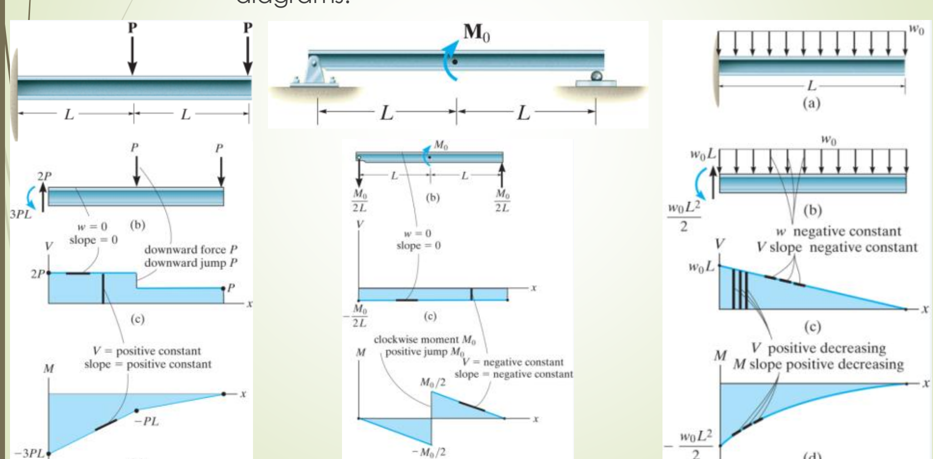
- The change of shear is equal to the negative of the area under the distributed load.

$$\Delta M = \int V(x)dx$$

- The change of moment is equal to the area under the shear diagram.

Discontinuities

- Discontinuities (jumps) at points where concentrated loads or moments are applied are present in V & M diagrams.

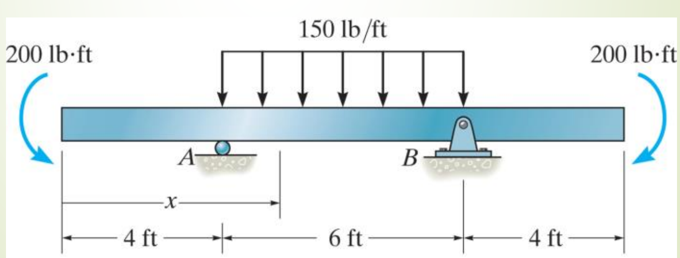


How to Analyze V & M Diagrams Using the Graphical Method

1. Statics
   - FBD
   - Reaction Forces
2. Establish V & M at the ends of the member
3. Use 4 relations to draw the diagrams
   - V vs x
   - M vs x

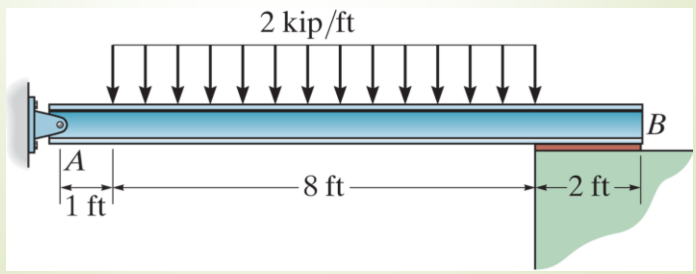
Problem 6-25

- Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x, where 4 ft < x < 10 ft.



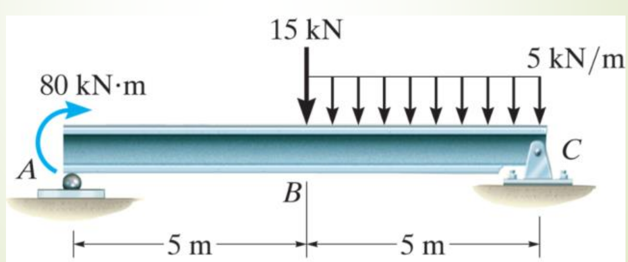
Problem 6-30

- The beam is bolted or pinned at A and rests on a bearing pad at B that exerts a uniform distributed loading on the beam over its 2-ft length. Draw the shear and moment diagrams for the beam if it supports a uniform loading of 2 kip/ft.



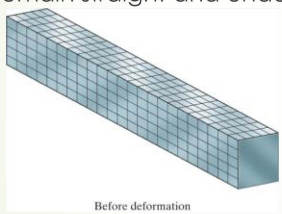
Example 6.4

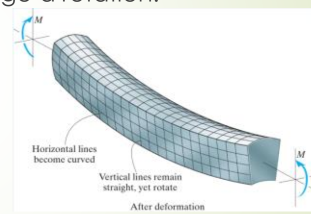
- Draw the shear and moment diagrams for the beam shown.



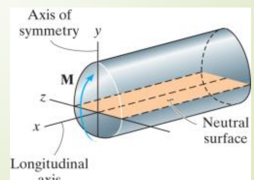
Bending Deformation of a Straight Member

- When a bending moment is applied, the longitudinal lines become curved and the vertical transverse lines remain straight and undergo a rotation.



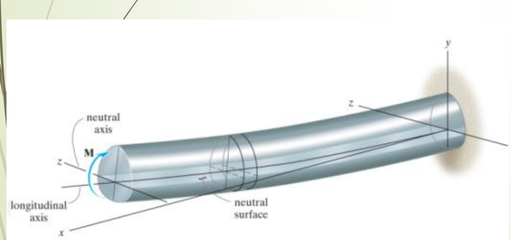


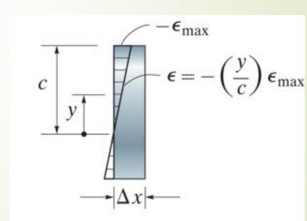
- The neutral surface is found between the stretched and compressed surfaces and does not experience any change in length.



Normal Strain Distribution in Bending

- Longitudinal strain varies linearly from zero at the neutral axis to a maximum at the outer fibers of the beam.

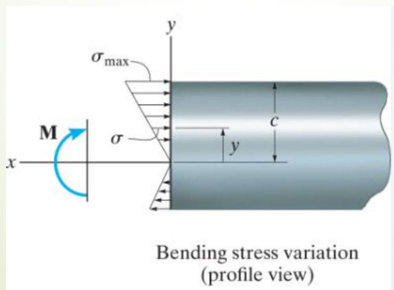




Normal strain distribution

Normal Stress Distribution in Bending

- Assuming a homogeneous material and linear elastic deformation, the stress also varies in a linear fashion over the cross-sectional area.



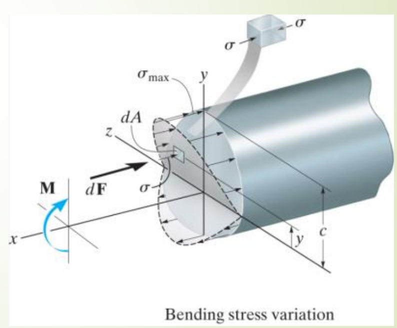
Bending stress variation (profile view)

$$\sigma = -\left(\frac{y}{c}\right)\sigma_{max}$$

Flexure Formula

$$\sigma_{max} = \frac{Mc}{I}$$

- σ_{max}: the maximum normal stress in a member which occurs at a point on the cross-sectional area farthest away from the neutral axis
- M: the resultant internal moment
- c: the perpendicular distance from the neutral axis to where σ_{max} acts (point farthest from the neutral axis)
- I: the moment of inertia of the cross-sectional area about the neutral axis



Bending stress variation

Flexure Formula

- Similarly,

$$\sigma = -\frac{My}{I}$$

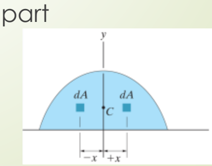
- where y is the perpendicular distance from the neutral axis to the point of interest
- This equation is valid for beams with cross-sectional areas symmetric about the y-axis.
- Note: For linear elastic materials, the neutral axis passes through the centroid

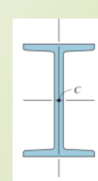
Centroids

- The centroid of an area refers to the point that defines the geometric center for the area.
- In cases where the area has an axis of symmetry, the centroid will lie along this axis.
- Composite Area: The area can be sectioned or divided into several parts having simpler shapes

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} \qquad \bar{y} = \frac{\sum \tilde{y}A}{\sum A}$$

- $\tilde{x}, \tilde{y}$: the algebraic distances (or x & y coordinates) for the centroid of each composite part
- A: the area of each composite part



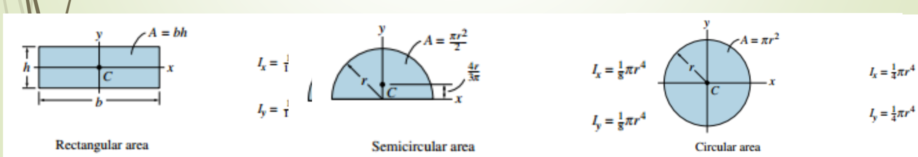


Moments of Inertia

- A geometric property that is calculated about an axis.

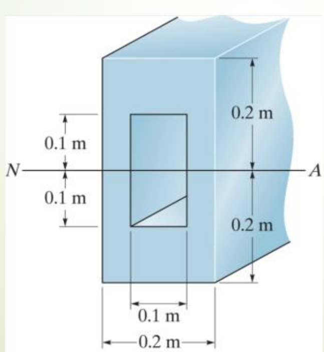
$$I_x = \bar{I}_{x'} + Ad_y^2 \qquad\qquad I_y = \bar{I}_{y'} + Ad_x^2$$

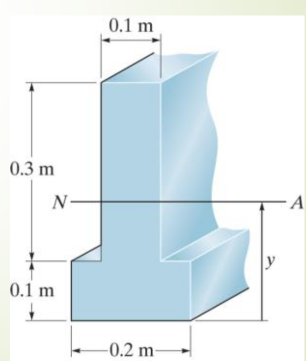
- The term Ad² is zero if the axis passes through the area's centroid
- Composite areas can be used to calculate the moment of inertia of complex shapes
- You can subtract the moment of inertia of an empty area from the moment of inertia of the entire area.



Determining Centroids & Moments of Inertia

- Determine the centroid and moment of inertia for each of the following cross-sections about the neutral axis.



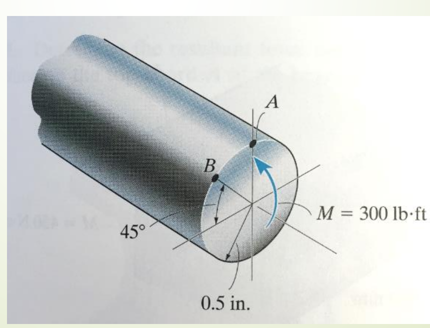


Procedure for Analysis

1. Internal Moment
   - FBD/Statics
   - FBD/Solids
2. Section Property
   - Moment of Inertia
3. Normal Stress
   - Specify the location y
   - Apply Flexure Formula

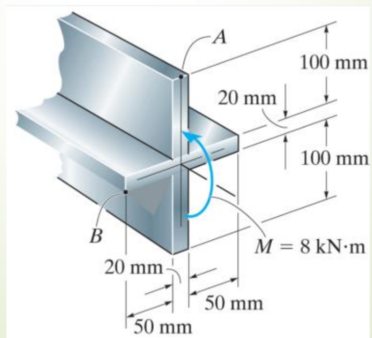
Example 1

- The steel rod having a diameter of 1 in is subjected to an internal moment of M=300 lb-ft. Determine the stress created at points A and B. Also, sketch a 3-D view of the stress distribution acting over the cross section.



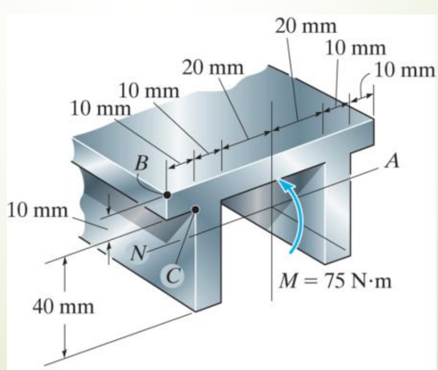
Problem 6-56

- The aluminum strut has a cross-sectional area in the form of a cross. If it is subjected to the moment M = 8 kNm, determine the bending stress acting at points A and B, and show the results acting on volume elements located at these points.



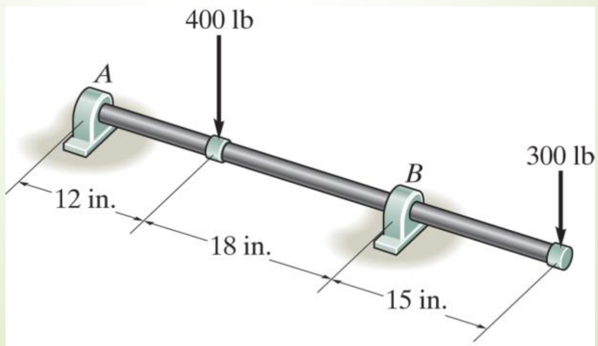
Problem 6-59

- The aluminum machine part is subjected to a moment of M = 75 Nm. Determine the maximum tensile and compressive bending stresses in the part.



Problem 6-73

- Determine the smallest allowable diameter of the shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces, and the allowable bending stress is σ_{allow}=22 ksi.



Problem 6-94

- The beam has a rectangular cross section as shown. Determine the largest load P that can be supported on its overhanging ends so that the bending stress does not exceed σ_{max}=10 MPa.