MECE 3321: Mechanics of Solids Chapter 4

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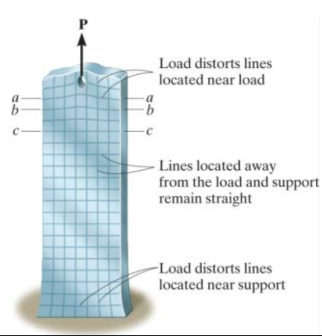
Saint-Venant's Principle

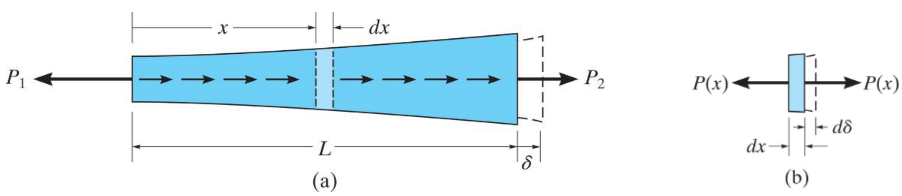
- The stress and strain at points in a body sufficiently away from the region of load application will be the same as the stress and strain produced by any applied loadings that have the same statically equivalent resultant.



Deformation of an Axially Loaded Member

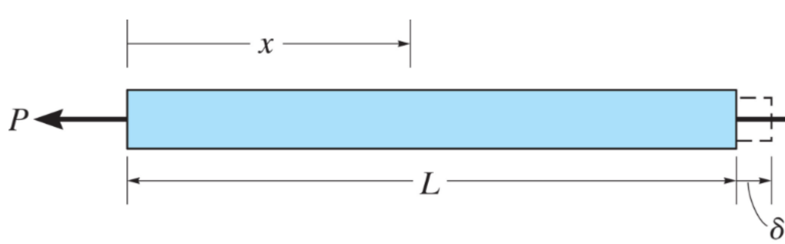
$$\delta = \int_0^L \frac{P(x)dx}{A(x)E(x)}$$

- δ = displacement of one point on the bar relative to the other point
- L = original length of the bar
- P(x) = internal axial force at the section, located a distance x from one end
- A(x) = cross-sectional area of the bar expressed as a function of x
- E(x) = modulus of elasticity for the material expressed as a function of x



Deformation of an Axially Loaded Member

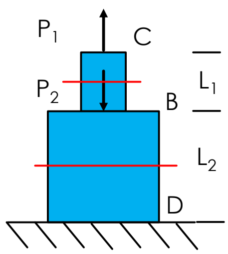
- For a member subjected to a constant load P and with constant cross-sectional area:



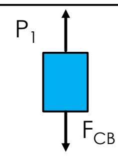
$$\delta = \frac{PL}{AE}$$

- P is positive in tension
- δ is positive if it is an elongation
- Linear elastic material behavior
  - Hooke's Law

Axially Loaded Member with Different Areas and Loads

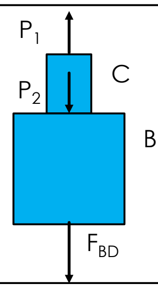


$$\boxed{\delta_{C/D} = \delta_{C/B} + \delta_{B/D}}$$



$$F_{CB} = P_1$$

$$\boxed{\delta_{C/B} = \frac{F_{CB} L_1}{A_{CB} E_{CB}}}$$



$$F_{BD} = P_1 - P_2$$

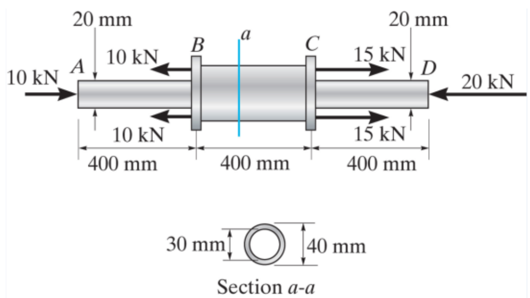
$$\boxed{\delta_{B/D} = \frac{F_{BD} L_2}{A_{BD} E_{BD}}}$$

Procedure for Analysis

- Statics (if necessary)
  - FBD
  - Equilibrium Equations
- Solids
  - FBD
  - Internal Loads
    - Tensile loadings and elongation are positive
  - Calculate Displacement

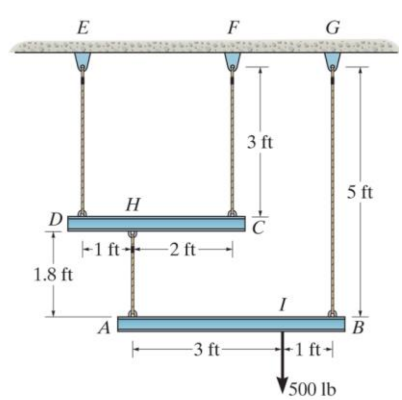
Problem F4-2

- Segments AB and CD of the assembly are solid circular rods, and segment BC is a tube. If the assembly is made of 6061-T6 aluminum, determine the displacement of end D with respect to end A.



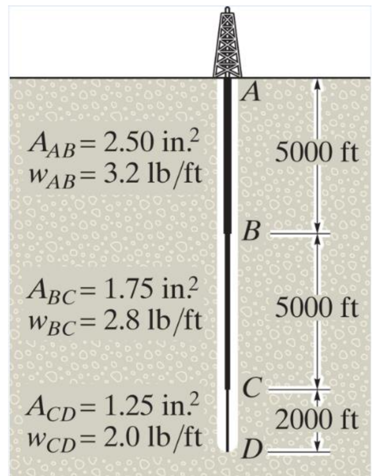
Problem 4-12

- The load is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the angle of tilt of each member after the 500-lb load is applied. The members were originally horizontal, and each wire has a cross-sectional area of 0.025 in^2.



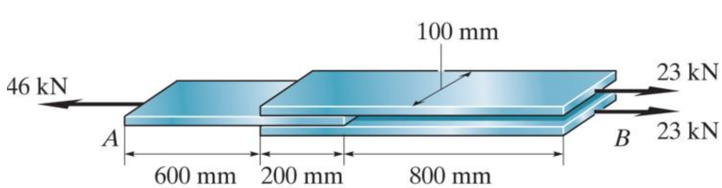
Problem 4-20

- The A992 steel drill shaft of an oil well extends 12000 ft into the ground. Assuming that the pipe used to drill the well is suspended freely from the derrick at A, determine the maximum average normal stress in each pipe segment and the elongation of its end D with respect to the fixed end at A. The shaft consists of three different sizes of pipe, AB, BC, and CD, each having the length, weight per unit length, and cross-sectional area indicated.



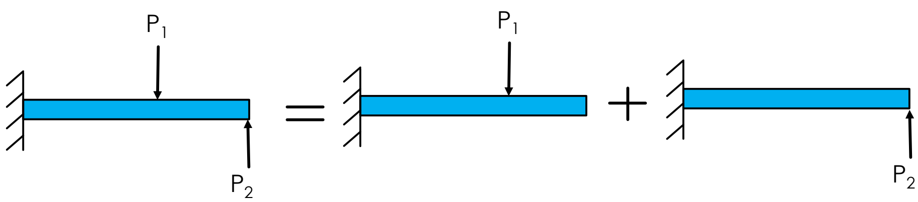
Problem 4-122

- The joint is made from three A992 steel plates that are bonded together at their seams. Determine the displacement of end A with respect to end B when the joint is subjected to the axial loads shown. Each plate has a thickness of 5 mm.



Principle of Superposition

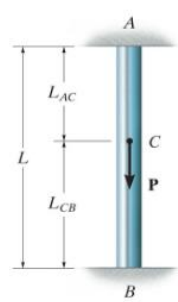
- The resultant stress or displacement at the point can be determined by algebraically summing the stress or displacement caused by each load component applied separately to the member.

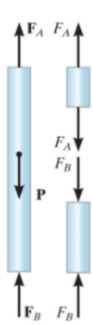


- The principle of superposition is valid only when the relationship between stress, strain, and loads is linear and the original geometry or configuration of the member is not significantly changed.

Statically Indeterminate Situations

- A statically indeterminate member cannot be solved using only the equilibrium equations.

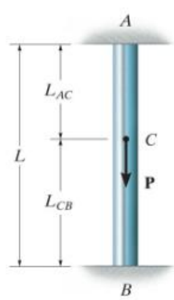




- The geometry of the deformation can be used in order to find additional equations to solve the problem.
  - Compatibility equation
  - Load-Displacement Relationship

Option 1: By Deformation Condition

- We know that in magnitude $\delta_{C/A} = \delta_{C/B}$.



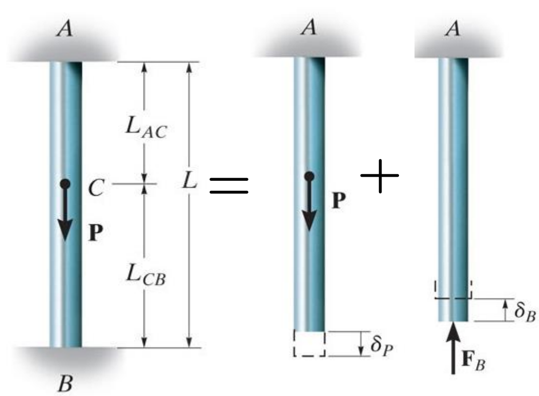
$$\delta = \frac{PL}{AE}$$

$$\frac{F_A L_{AC}}{A_{AC}E_{AC}} - \frac{F_B L_{BC}}{A_{BC}E_{BC}} = 0$$

$$+\uparrow \sum F_y = 0:$$

$$F_A + F_B - P = 0$$

Option 2: By Superposition



$$\delta_P - \delta_B = 0$$

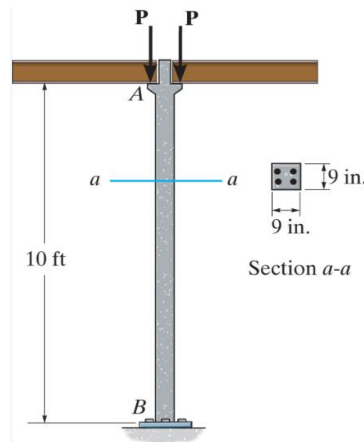
$$\frac{PL_{AC}}{AE} - \frac{F_B L}{AE} = 0$$

$$+\uparrow \sum F_y = 0$$

$$F_A + F_B - P = 0$$

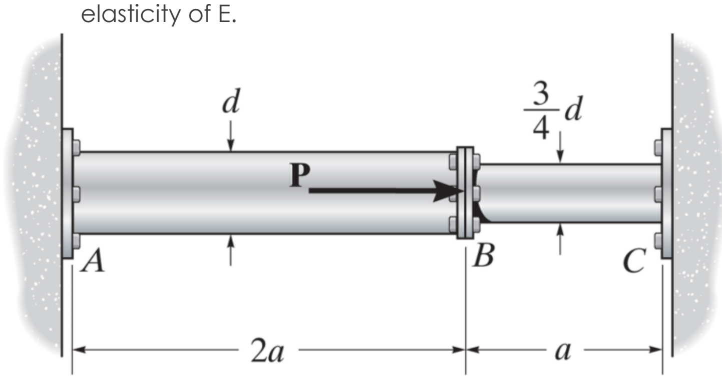
Problem 4-34

- If column AB is made from high strength pre-cast concrete and reinforced with four ¾ in diameter A-36 steel rods, determine the average normal stress developed in the concrete and in each rod. Set P= 75 kip.



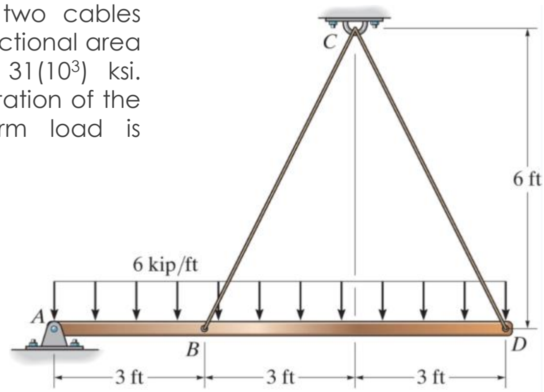
Problem 4-36

- Determine the support reactions at the rigid supports A and C. The material has a modulus of elasticity of E.



Problem 4-52

- The rigid bar is originally horizontal and is supported by two cables each having a cross-sectional area of 0.5 in^2, and E = 31(10^3) ksi. Determine the slight rotation of the bar when the uniform load is supplied.



Thermal Stress

- Usually expansion or contraction is linearly related to temperature change

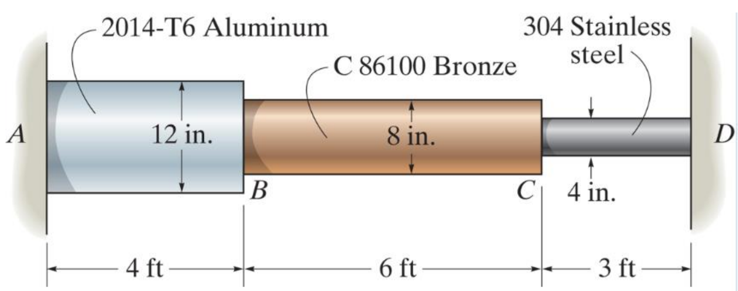
$$\delta_T = \int_0^L \alpha \Delta T dx \qquad if\ \Delta T\ is\ constant, \qquad \delta_T = \alpha \Delta T L$$

  - δ_T : the algebraic change in temperature of the member
  - α : the linear coefficient of thermal expansion
  - ΔT: the algebraic change in temperature of the member
  - L: the original length of the member

- If the member is statically indeterminate, the thermal displacements will be constrained by the supports.
  - Thermal stresses

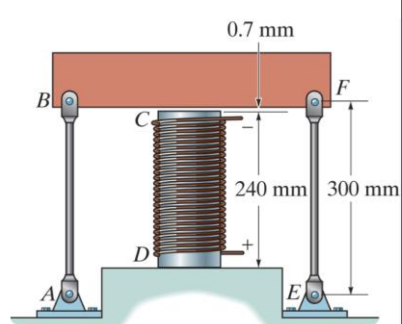
Problem 4-69

- The assembly has the diameters and material makeup indicated. If it fits securely between its fixed supports when the temperature is T_1=70°F, determine the average normal stress in each material when the temperature reaches T_2=110°F.



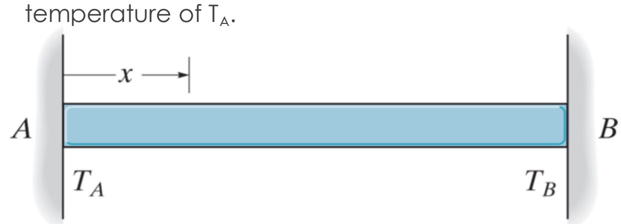
Problem 4-85

- The center rod CD of the assembly is heated from T_1=30°C to T_2=180°C using electrical resistance heating. Also, the two end rods AB and EF are heated from T_1=30°C to T_2=50°C. At the lower temperature T_1 the gape between C and the rigid bar is 0.7 mm. Determine the force in rods AB and EF caused by the increase in temperature. Rods AB and EF are made of steel, and each has a cross-sectional area of 125 mm². CD is made of aluminum and has a cross sectional area of 375 mm². E_{St}=200 GPa, E_{Al}=70 GPa, α_{St}=12(10-6)/°C, and α_{Al}=23(10-6)/°C.



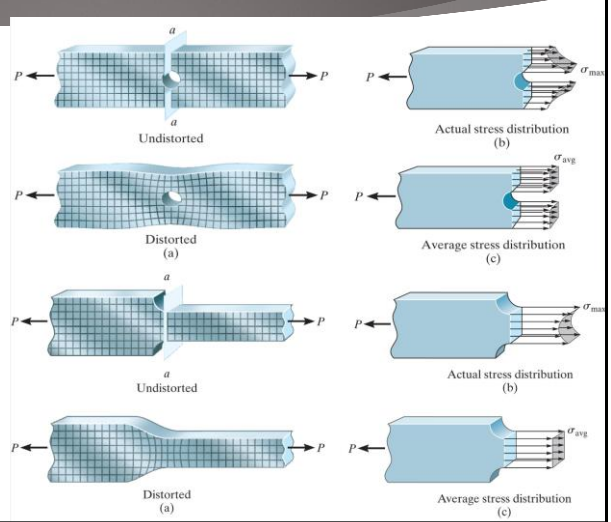
Problem 4-77

- The bar has a cross-sectional area A, length L, modulus of elasticity E, and coefficient of thermal expansion a. The temperature of the bar changes uniformly along its length from T_A at A to T_B at B so that at any point x along the bar $T=T_A+x(T_B-T_A)/L$. Determine the force the bar exerts on the rigid walls. Initially no axial force is in the bar and the bar has a temperature of T_A.



Stress Concentrations

- Holes and sharp transitions at a cross section will create stress concentrations.
- In engineering practice, only the maximum stress at these sections must be known.
- The maximum normal stress occurs at the *smallest* cross-sectional area.



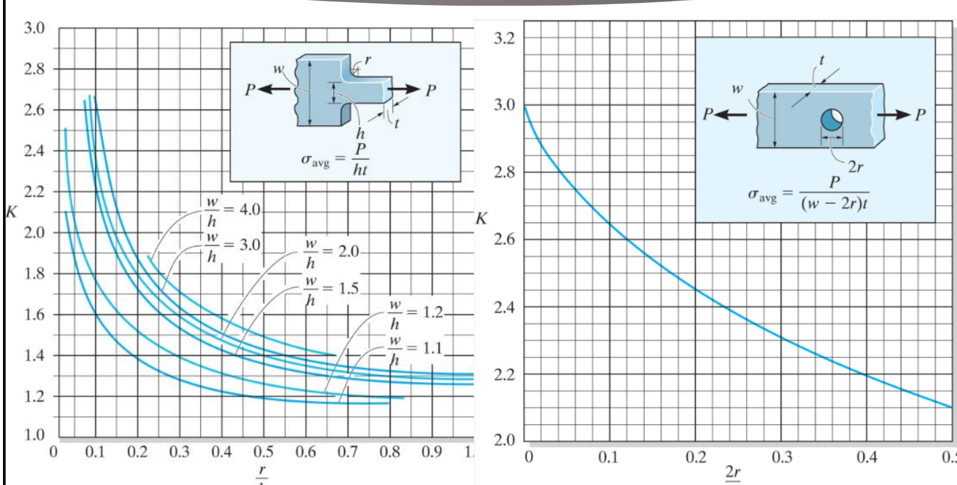
Stress Concentration Factor

- A stress concentration factor "K" is used to determine the maximum stress at sections where the cross-sectional area changes.

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

- K is found from graphs in handbooks of stress analysis
- σ_{avg} is the average normal stress at the smallest cross section

Stress Concentration Factor



Problem 4-92

- Determine the maximum normal stress developed in the bar when it is subjected to a tension of P=2 kip.