MECE 3321
MECHANICS OF SOLIDS
CHAPTER 1

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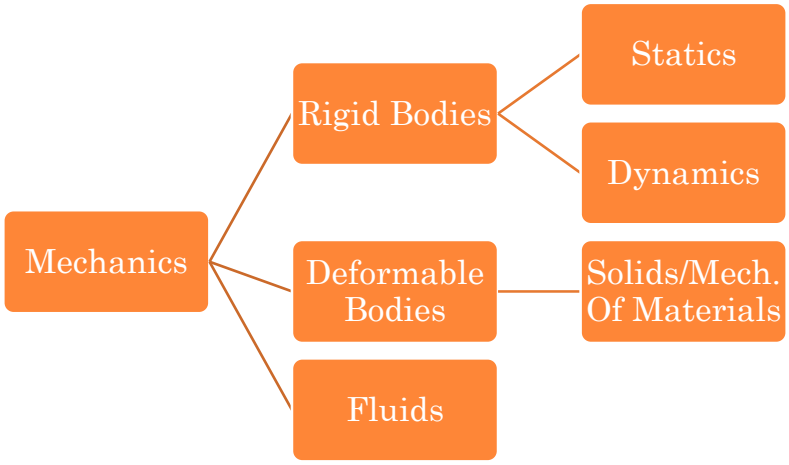
WHAT IS MECHANICS OF MATERIALS?



What is Mechanics of Materials?

- Mechanics
  - The branch of physics concerned with the state of rest or motion of material objects that are subjected to the action of forces or by thermal disturbances.
- Mechanics of Materials
  - A branch of mechanics that studies the relationship between **external loads** applied to a deformable body and the intensity of **internal forces**.
  - Involves computing the deformation of the body based on the determination and understanding of the mechanical behavior of the materials being used.

Statics Review: External Loads

- Can be applied of a body as distributed or concentrated surface loading or as body forces which act throughout the volume.

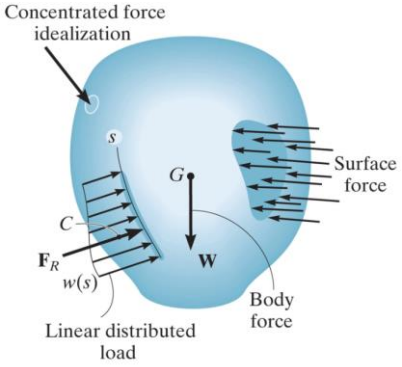


Figure: 01_01

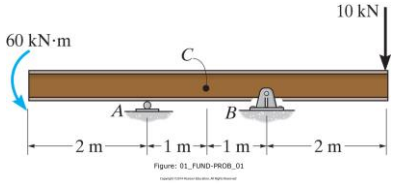


Figure: 01_FUND-PROB_01

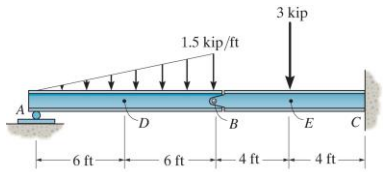


Figure: 01_PROB_005

Statics Review: Connections

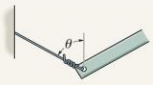
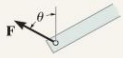
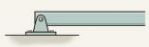
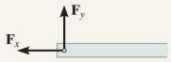
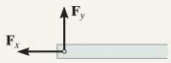
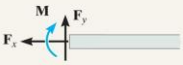
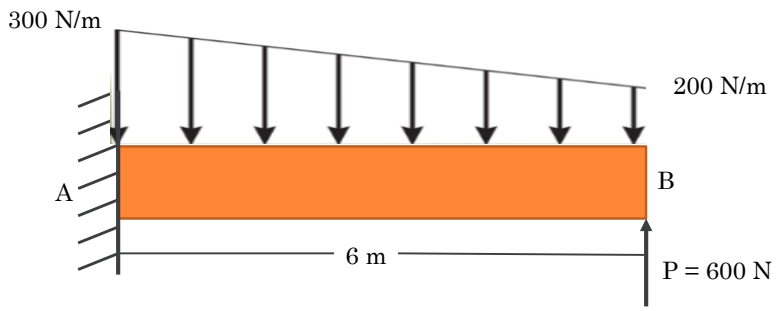
Type of connection	Reaction	Type of connection	Reaction
 Cable	 One unknown: F	 External pin	 Two unknowns: F_x, F_y
 Roller	 One unknown: F	 Internal pin	 Two unknowns: F_x, F_y
 Smooth support	 One unknown: F	 Fixed support	 Three unknowns: F_x, F_y, M

Figure: 01_TABLE_01

- If the support prevents translation in a given direction, then a force must be developed on the member in that direction.
- Likewise, if rotation is prevented, a couple moment must be exerted on the member.

Example 1

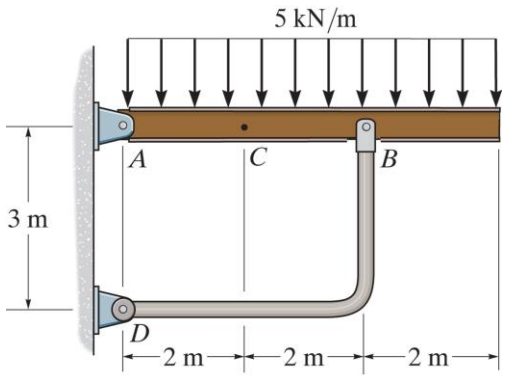
- Determine the reaction at point A.



- Linear distributed loadings produce a resultant force having a magnitude equal to the area under the load diagram, and having a location that passes through the centroid of this area.

EXAMPLE 2: PROBLEM F1-6

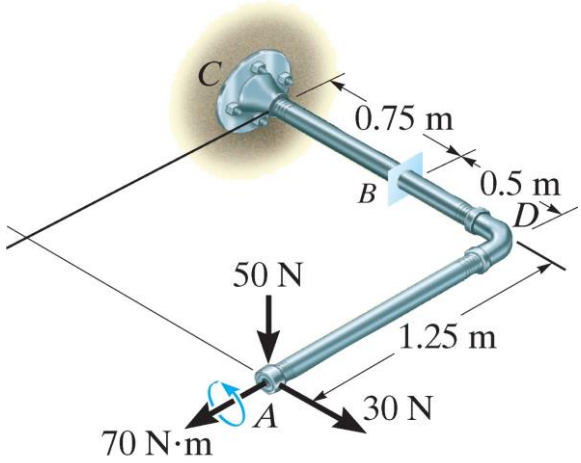
- Determine the reaction at point A.



- Two-force member
  - A member that has pin supports at both ends and is subjected to no load in between, it is called a two-force member.
- Two-force principal
  - If only two forces act on a body that is in equilibrium, these two forces must be equal in magnitude and opposite in sense.

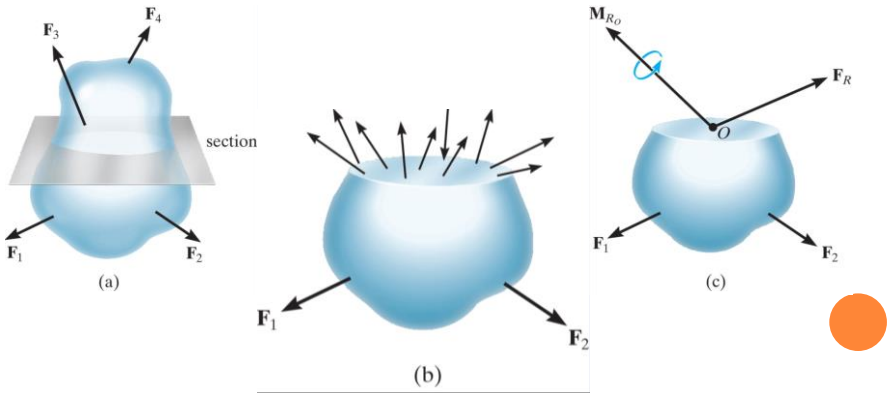
EXAMPLE 3

- Determine the reactions at point C.



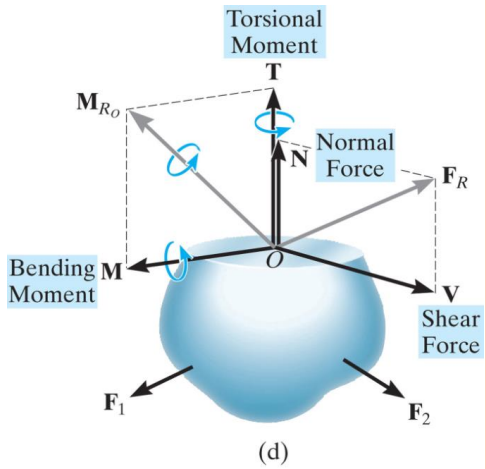
Internal Resultant Loadings

- The resultant force and moment which are necessary to hold the body together when the body is subjected to external loads.



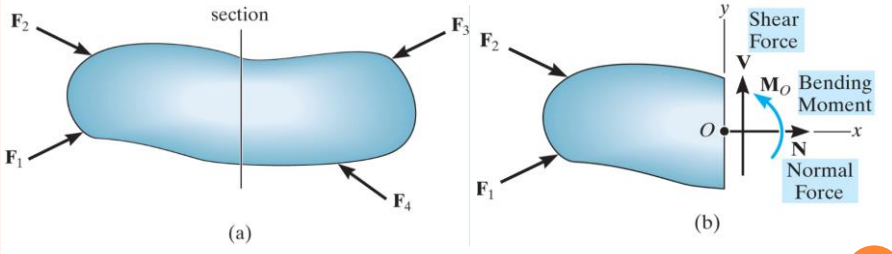
Internal Resultant Loadings

- **Normal Force (N)**
  - Acts perpendicular to the area due to external loads pushing or pulling on the two segments of the body
- **Shear Force (V)**
  - Lies in the plane of the area due to external loads causing the two segments of the body to slide over one another (2 forces)
- **Bending Moment (M)**
  - Caused by the external loads bending the body about an axis lying within the plane of the area (2 moments)
- **Torsional Moment or Torque (T)**
  - Caused by the external loads twisting one segment of the body with respect to the other about an axis perpendicular to the area



Coplanar Loadings

- When only coplanar forces act on the body, the internal resultant loadings are the normal force (N), shear force (V), and bending moment (M).

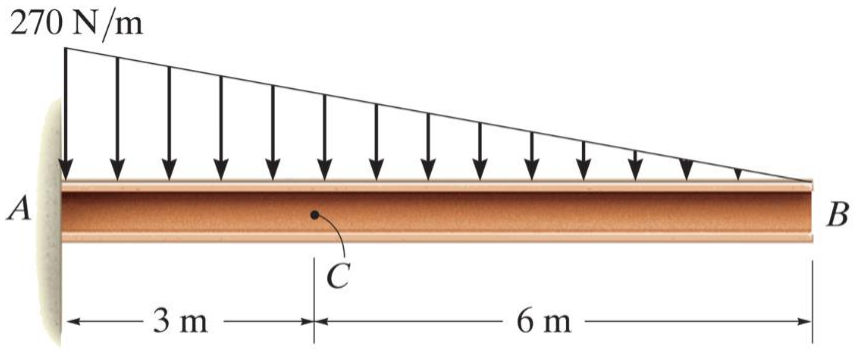


How To Find Internal Loadings

- When analyzing a body that is a member or part of a machine or structure, the machine or structure might have to be disassembled to determine the forces and moments acting on the body before computing the internal loadings.
- Procedure
  1. Draw a FBD and determine the reactions at the body's connections (Statics)
  2. Pass an imaginary section through the body (to cut the body) at the location where the internal loadings are to be determined.
  3. Draw a FBD of one of the segments of the cut body and clearly indicate the internal resultant loadings (N, V, M, T) acting on the cross-section.
  4. Use the equilibrium equations to find the internal resultant loadings.

EXAMPLE 1.1

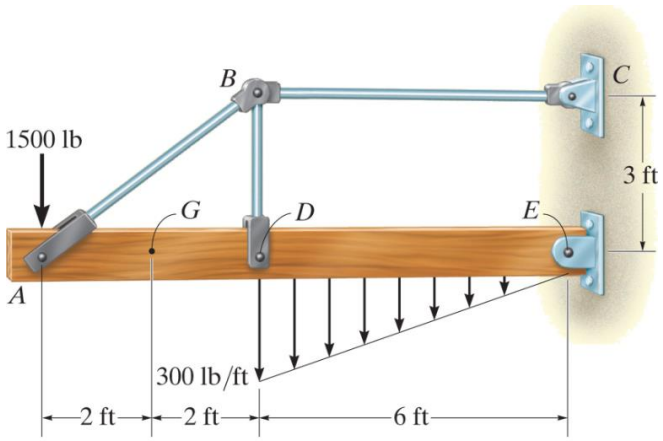
- Determine the resultant internal loadings acting on the cross section at C of the cantilevered beam.



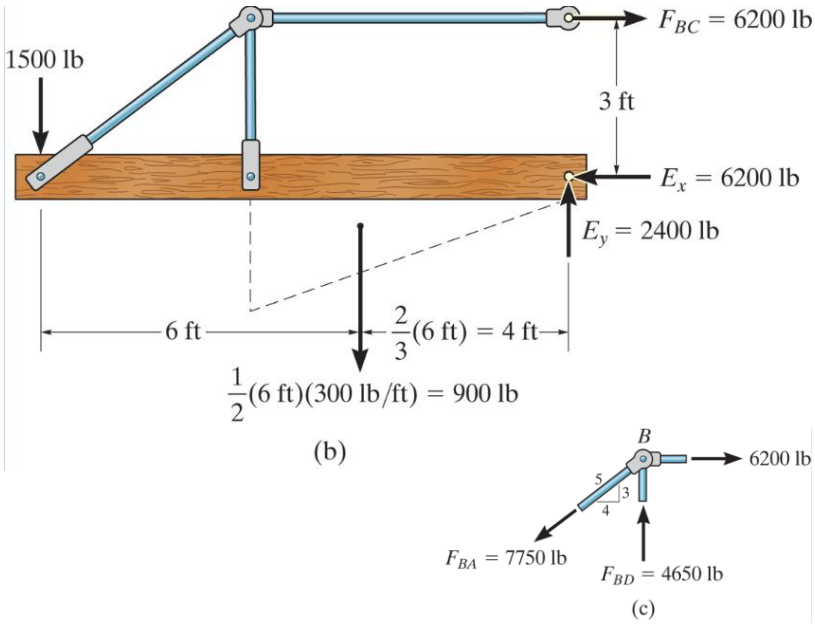
(a)

EXAMPLE 1.3

- Determine the resultant internal loadings acting on the cross section at G of the beam shown below. Each joint is a pin connection.

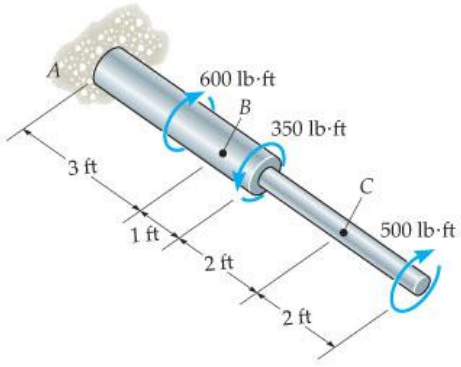


EXAMPLE 1.3



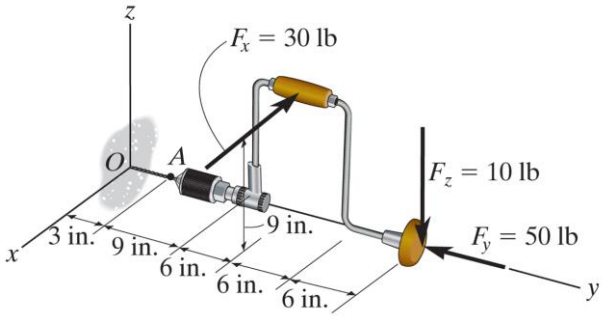
PROBLEM 1.3 (8TH ED)

- Determine the resultant internal torque acting on the cross section through point B and C.



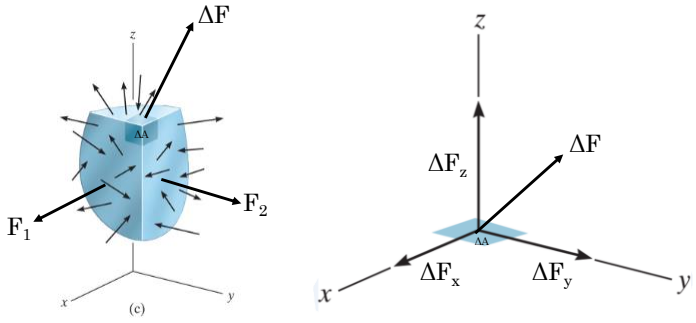
PROBLEM 1-28

- The brace and drill bit is used to drill a hole at O. If the drill bit jams when the brace is subjected to the forces shown, determine the resultant internal loadings acting on the cross section of the drill bit at A.



STRESS

- The intensity of the internal force acting on a specific plane (area) passing through a point.



- ΔA is an infinitesimal size area with a uniform force acting on it.

Stress

- **Normal Stress (σ)** is the intensity of force, or force per unit area, acting normal to ΔA.

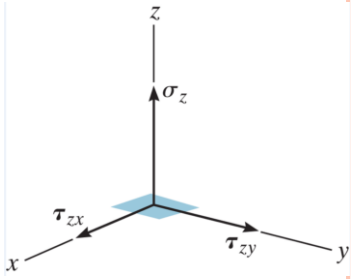
$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

- **Shear Stress (τ)** is the intensity of force, or force per unit area, acting parallel to ΔA.

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$

- Subscripts
  - First letter is the orientation of force normal to ΔA
  - Second letter is the orientation of force causing shear

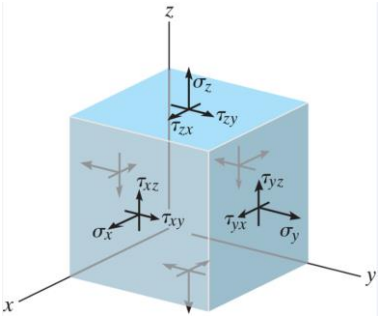


Stress States

$\sigma_x , \sigma_y , \sigma_z , \tau_{xy} , \tau_{xz} , \tau_{yx} , \tau_{yz} , \tau_{zx} , \tau_{zy}$

$\tau_{xy} = \tau_{yx} , \tau_{xz} = \tau_{zx} , \tau_{yz} = \tau_{zy}$

$\sigma_x , \sigma_y , \sigma_z , \tau_{xy} , \tau_{xz} , \tau_{yz}$



Units

N/m^2 (Pa)

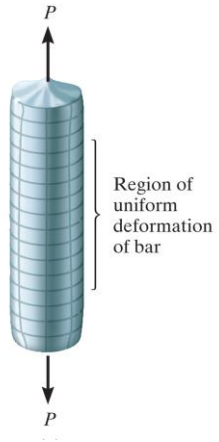
1000 N/m^2 (kPa)

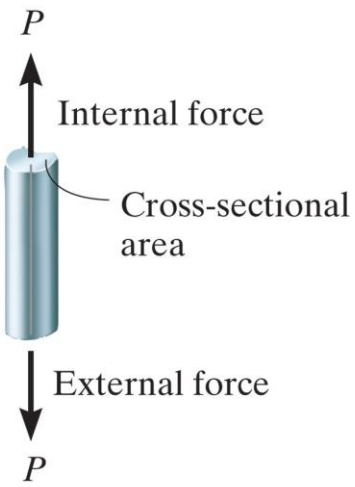
lb_f/in^2 (psi)

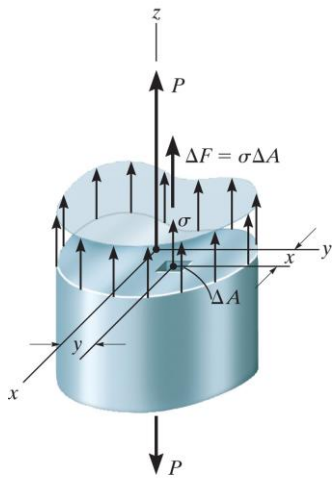
1000 lb_f/in^2 (ksi)

Average Normal Stress

- Axially Loaded Bar (Uniaxial Tensile Test)
  - Assumptions
    - Homogeneous material
    - Isotropic material
    - Bar remains straight and cross-section flat
    - P is applied along the centroid axis







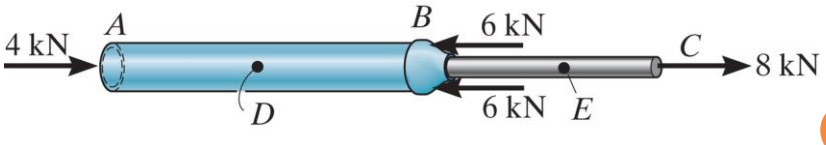
$$\int dF = \int \sigma dA$$

$$P = \sigma_{Avg} A$$

$$\sigma_{Avg} = \frac{P}{A}$$

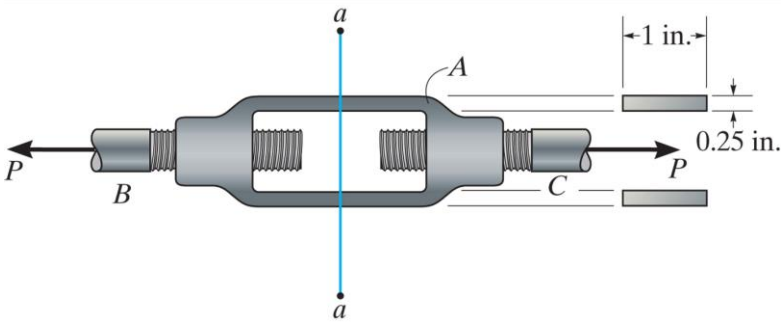
PROBLEM 1.34

- The built-up shaft consists of a pipe AB and solid rod BC. The pipe has an inner diameter of 20 mm and an outer diameter of 28 mm. The rod has a diameter of 12 mm. Determine the average normal stress at D and E and represent the stress on a volume element located at each of these points.



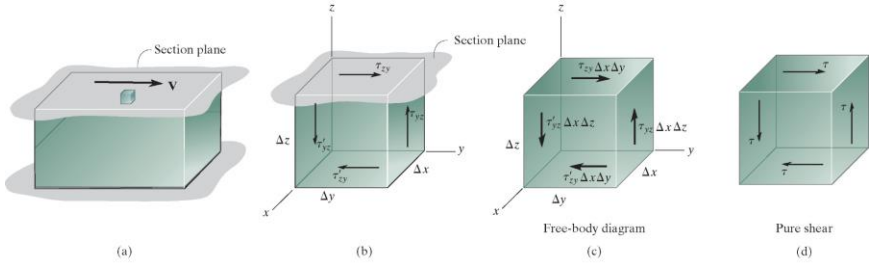
PROBLEM 1-35

- If the turnbuckle is subjected to an axial force of P=900 lb, determine the average normal stress developed in section a-a and in each bolt shank at B and C. Each bolt shank has a diameter of 0.5 in.



Average Shear Stress

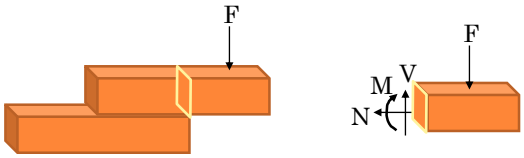
- Shear Stress is the force per unit area that acts in a plane of the cross-sectional area.



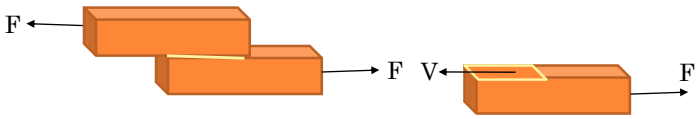
$$\tau_{Avg} = \frac{V}{A}$$

Single Shear

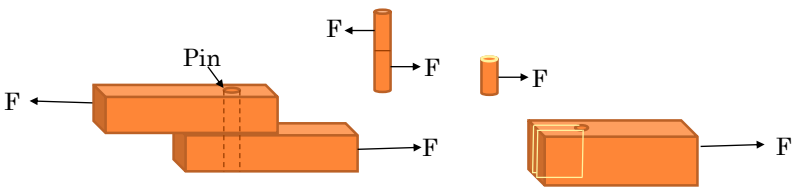
- Situation #1



- Situation #2

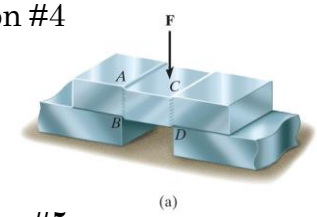


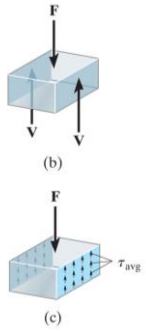
- Situation #3



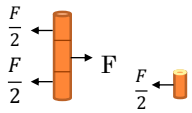
Double Shear

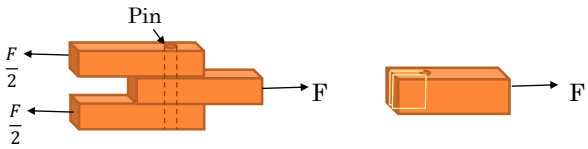
- Situation #4





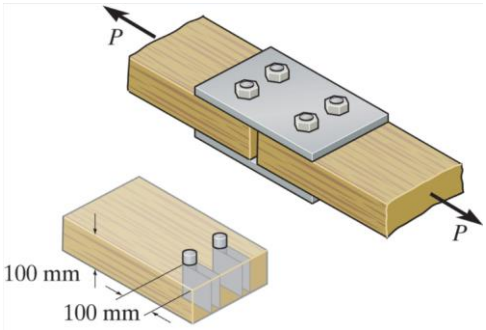
- Situation #5





Problem 1-52

- If the joint is subjected to an axial force of P=9 kN, determine the average shear stress developed in each of the 6-mm diameter bolts between the plates and the members and along each of the four shaded shear planes.



Factor of Safety

- Factor of Safety (FS) is a ratio of the failure load, F_{fail}, divided by the allowable load, F_{allow}. (FS>1)

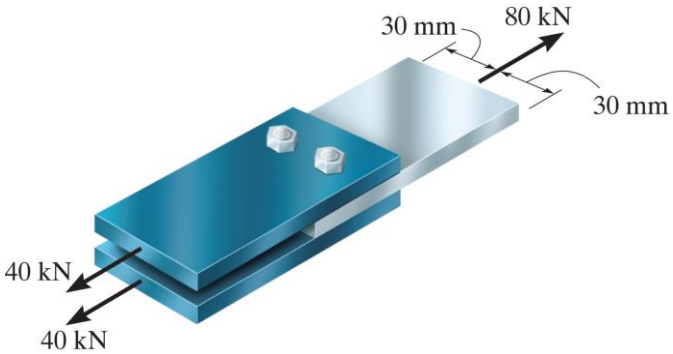
$$FS = \frac{F_{Fail}}{F_{Allow}}$$

- If the load applied to the member is linearly related to the stress developed within the member, then

$$FS = \frac{N_{Fail}}{N_{Allow}} = \frac{\sigma_{Fail}}{\sigma_{Allow}} \qquad FS = \frac{V_{Fail}}{V_{Allow}} = \frac{\tau_{Fail}}{\tau_{Allow}}$$

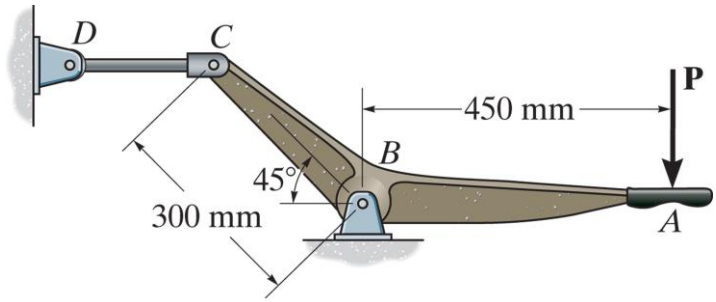
Problem 1.71

- The joint is fastened together using two bolts. Determine the required diameter of the bolts if the failure shear stress for the bolts is τ_{fail}=350 MPa. Use a factor of safety for shear of FS=2.5.



PROBLEM 1.65

- Determine the maximum vertical force P that can be applied to the bell crank so that the average normal stress developed in the 10 mm diameter rod, CD, and the average shear stress developed in the 6 mm diameter double sheared pin B not exceed 175 MPa and 75 MPa, respectively.



PROBLEM 1.99

- To the nearest 1/16", determine the required thickness of member BC and the diameter of the pins at A and B if the allowable normal stress for member BC is σ_{Allow}=29 ksi and the allowable shear stress for the pins is τ_{Allow}=10 ksi.