

# Graphs

## Terminology

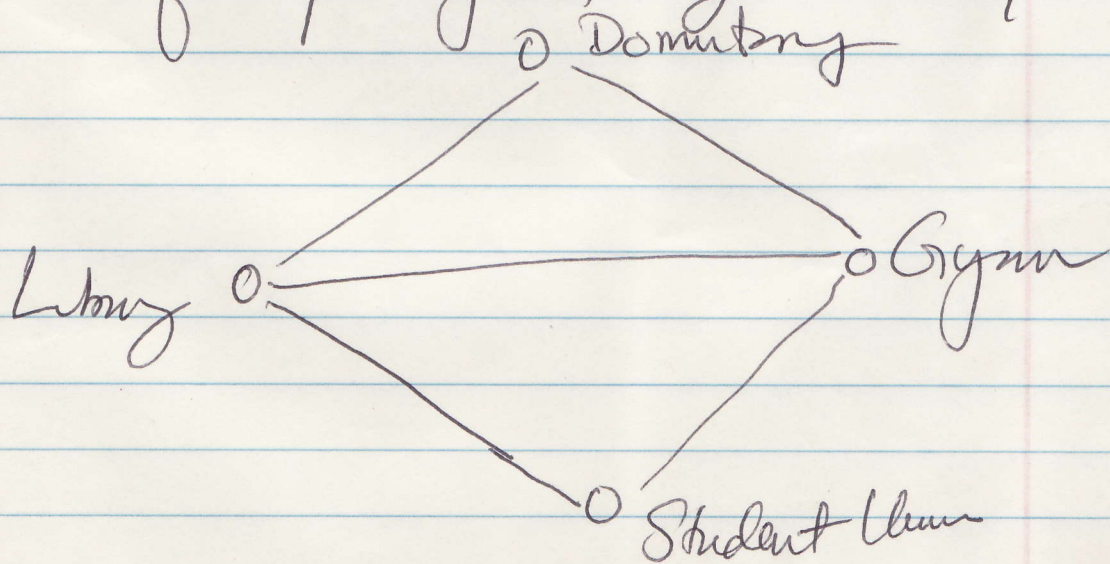
Graph  $G$  consists of two sets:

a set  $V$  of vertices or nodes

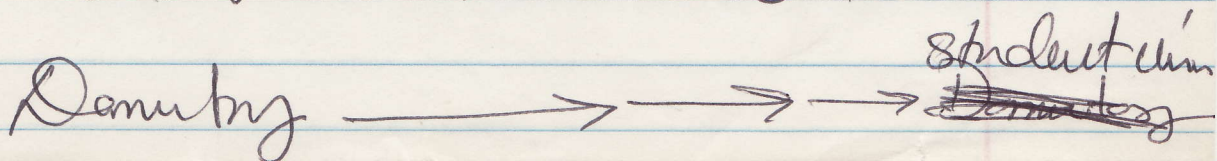
a set  $E$  of edges that connect the vertices.

A subgraph is a subset of graph's vertices and subset of its edges.

Two vertices of a graph are adjacent if they are joined by an edge.



A path between two vertices is a sequence of edges that begins at one vertex and ends at another.

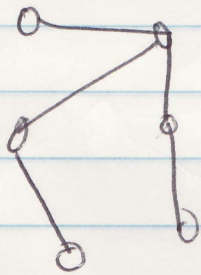


Hamiltonian  $\rightarrow$   $\rightarrow$  Student clues

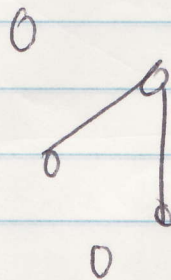
It is a simple path since the path <sup>does not</sup> pass through the same vertex twice.

Hamiltonian  $\rightarrow$   $\rightarrow$   $\rightarrow$  Hamiltonian  
a cycle.

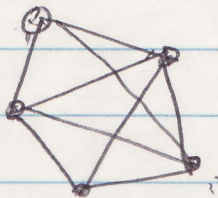
A graph is connected if there is a path between every pair of vertices.



Connected graph

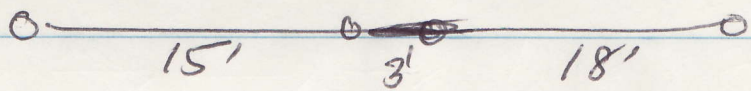


Disconnected graph

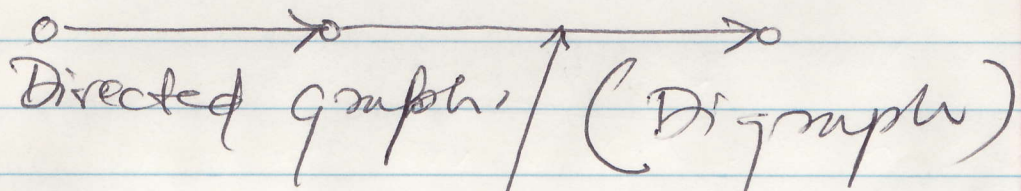


A complete graph

You can label the edges of a graph. When these represent a numeric values, the graph is called a weighted graph.

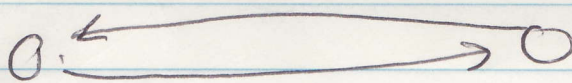


Undirected graph.



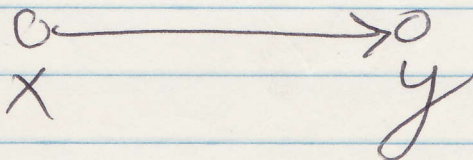
Directed graph (Digraph)

Directed edge.



Directed graph  
with 2 edges.

If there is a directed edge from vertex  $x$  to vertex  $y$  then  $y$  is adjacent to  $x$ .



## Graph ADT:

Create ( $G$ )

IsEmpty ( $G$ )

InsertVertex ( $G, v, success$ )

InsertEdge ( $G, v_1, v_2, success$ )

DeleteVertex ( $G, v, success$ )

DeleteEdge ( $G, v_1, v_2, success$ )

Retrieve ( $G, searchkey, v, success$ ) (return value)

Replace ( $G, searchkey, v, success$ ) replace vertex

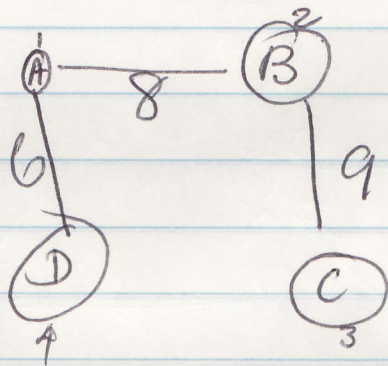
IsEdge ( $G, v_1, v_2$ )

Returns if there is an edge between  $v_1, v_2$



If it is weighted graph instead of T, or F put in true values.

## Adjacency matrix



	1	2	3	4
	A	B	C	D
1 A	0	8	0	6
2 B	8	0	9	0
3 C	0	9	0	0
4 D	6	0	0	0

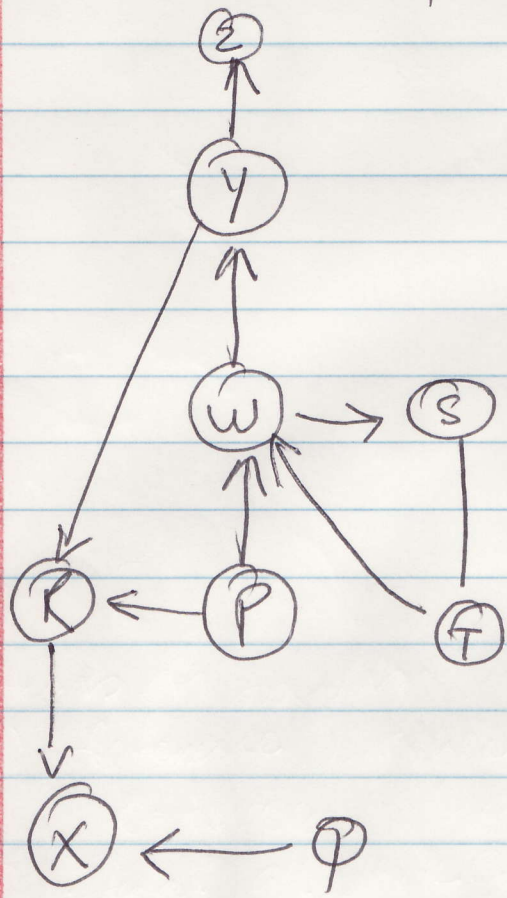
Operations on graphs.

- ① determine whether there is an edge between two vertices.
- ② Find all vertices adjacent to a given vertex.

just look at  $\text{Matrix}[i, j]$  given vertices  $i$  &  $j$ .

Adjacency list  
 a linked list.

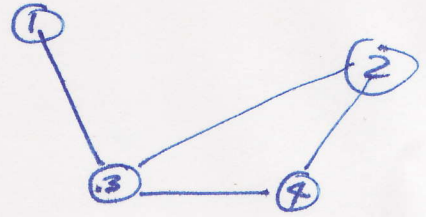
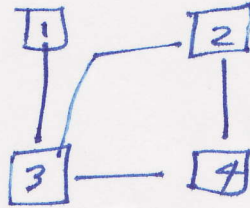
Requires less space than adjacency matrix



P	<del>Q</del>	→ R	→ W
Q		→ X	→ P
R	R	→ X	
S	S	→ T	
T		→ W	
W		→ S	→ Y
X			
Y	<del>Y</del>	→ R	→ Z
Z			

Find all vertices adjacent to a given vertex - can be found easily by this traversing this list.

A graph representation is made of the network.



To reach	Next hop
1	-
2	(1, 3)
3	(4, 3)
4	(1, 3)

Link between 1 & 3.

This is for node 1 All must go through 1, 3

To reach	Next hop
1	2, 3
2	-
3	2, 3
4	2, 4

For Node 2

To reach	Next hop
1	3, 1
2	3, 2
3	-
4	3, 4

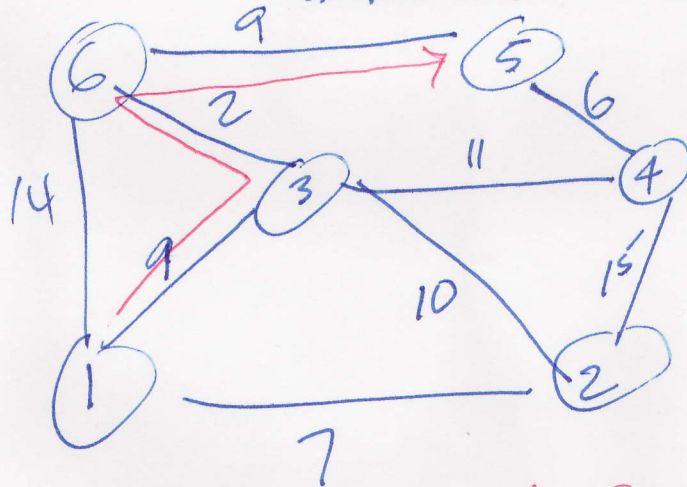
For Node 3

1	4, 3
2	4, 2
3	4, 3
4	-



Shortest Path. Uses Dijkstra's algorithm  
 is a graph search algorithm used for routing.  
 For a given node in a graph, the algorithm finds  
 path with ~~best~~ lowest cost, or shortest path,  
 Algorithm.

1. Mark every node a distance value  
 Initial node gets zero and all others  
 gets infinity, initially.
2. Mark all nodes unvisited
3. For the current node consider all its unvisited  
 neighbors and calculate their distance. If  
 the distance is less than previously recorded, ~~mark~~ <sup>update</sup>
4. After considering all neighbors, ~~mark~~ <sup>update</sup> Mark it  
 visited
5. Set the unvisited node with the  
 smallest distance from the initial node  
 as the next current node.



Want to go from 1 to 5

$1 \text{ to } 3 = 9$   
 $1 \text{ to } 2 = 7 \rightarrow 2 \text{ to } 3 = 7 + 10 = 13$  Compare to 1 to 3  
 $2 \text{ to } 4 = 7 + 15 = 22$  Keep  
 $1 \text{ to } 6 = 14 \rightarrow 6 \text{ to } 5 = 14 + 9 = 23$   
 $\rightarrow 3 \text{ to } 6 = 9 + 2 = 11$  which is less than 1 to 6  
 $\rightarrow 6 \text{ to } 5 = 11 + 9 = 20$  which is less than 1 to 5  
 So keep 1 to 3  
 So keep 6 to 5

Want to go from 1 to 4  
 $1-3-4 = 20$   
 ~~$1-2-4 = 22$~~