Incompressible Flow Over Finite wings

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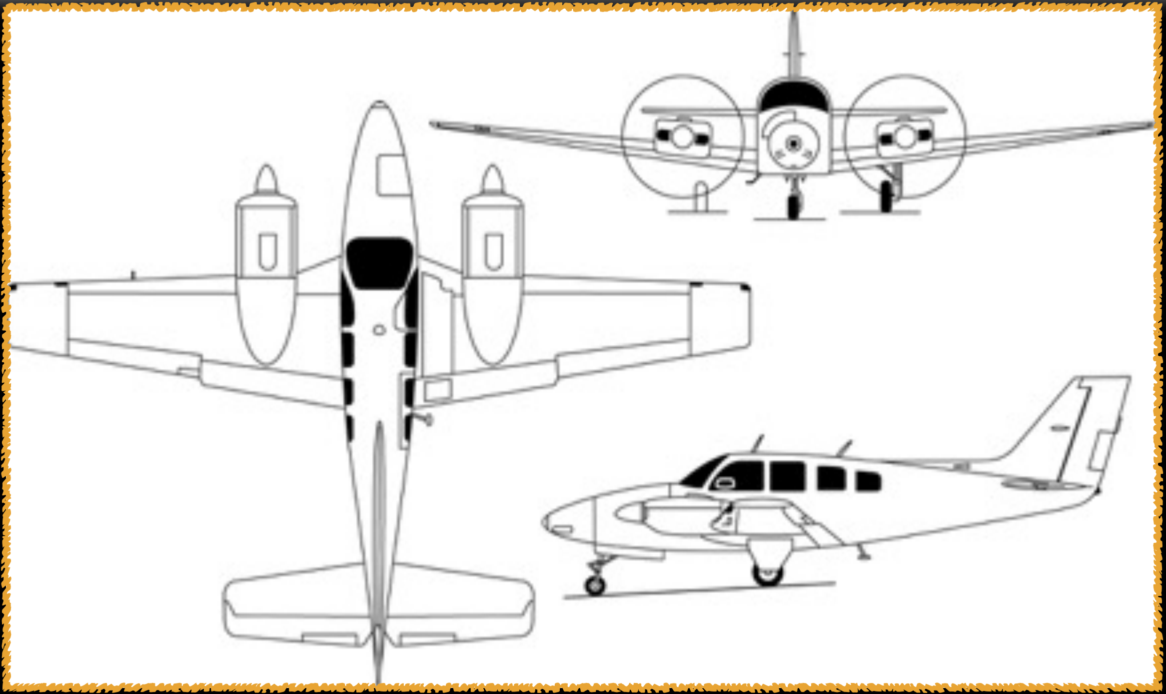
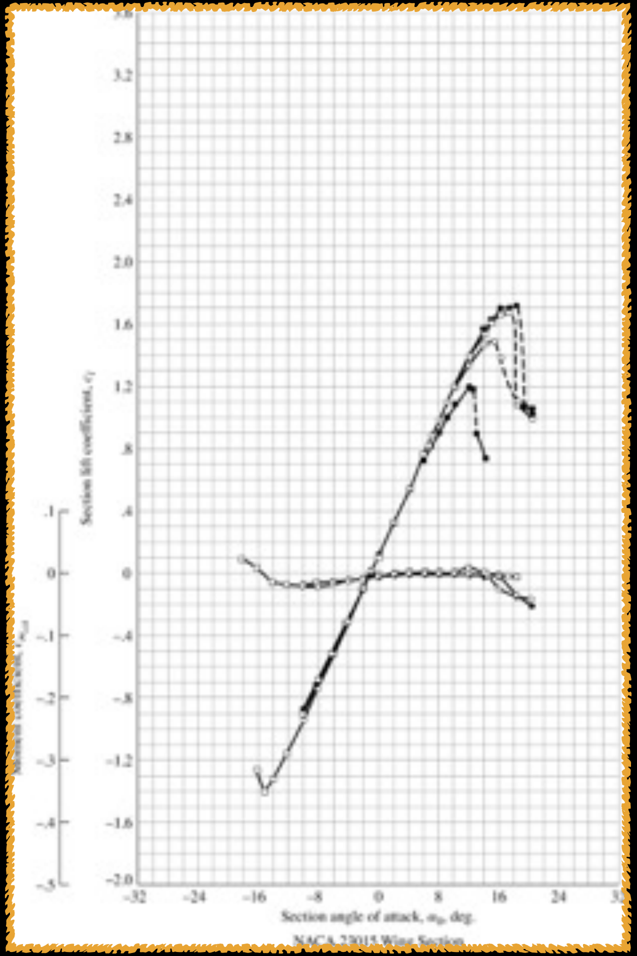
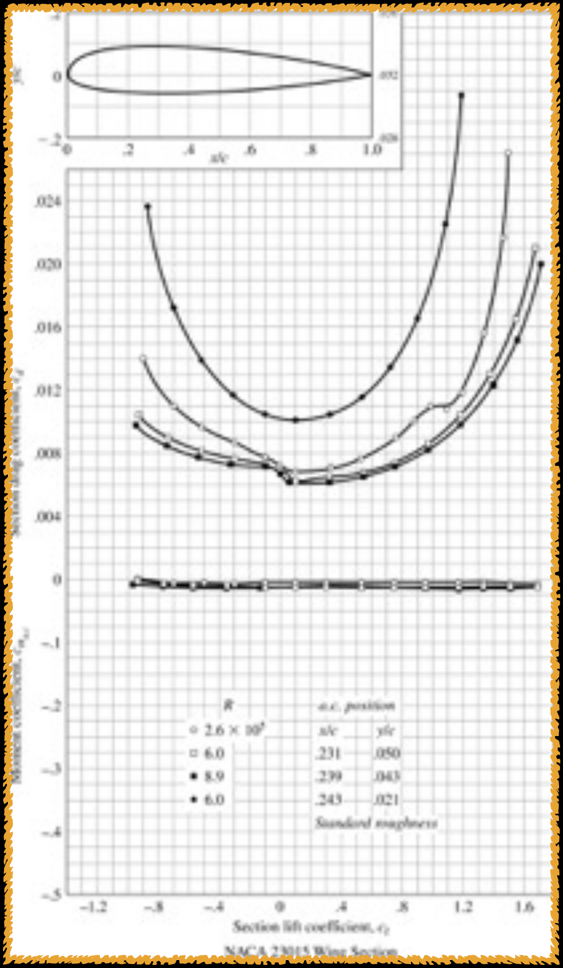
Introduction

- *Beechcraft Baron-58 aircraft*
  - *Airfoil - NACA 23015*
  - *Angle of attack = 4 deg.*
  - $c_l = 0.54$ *and* $c_d = 0.0068$



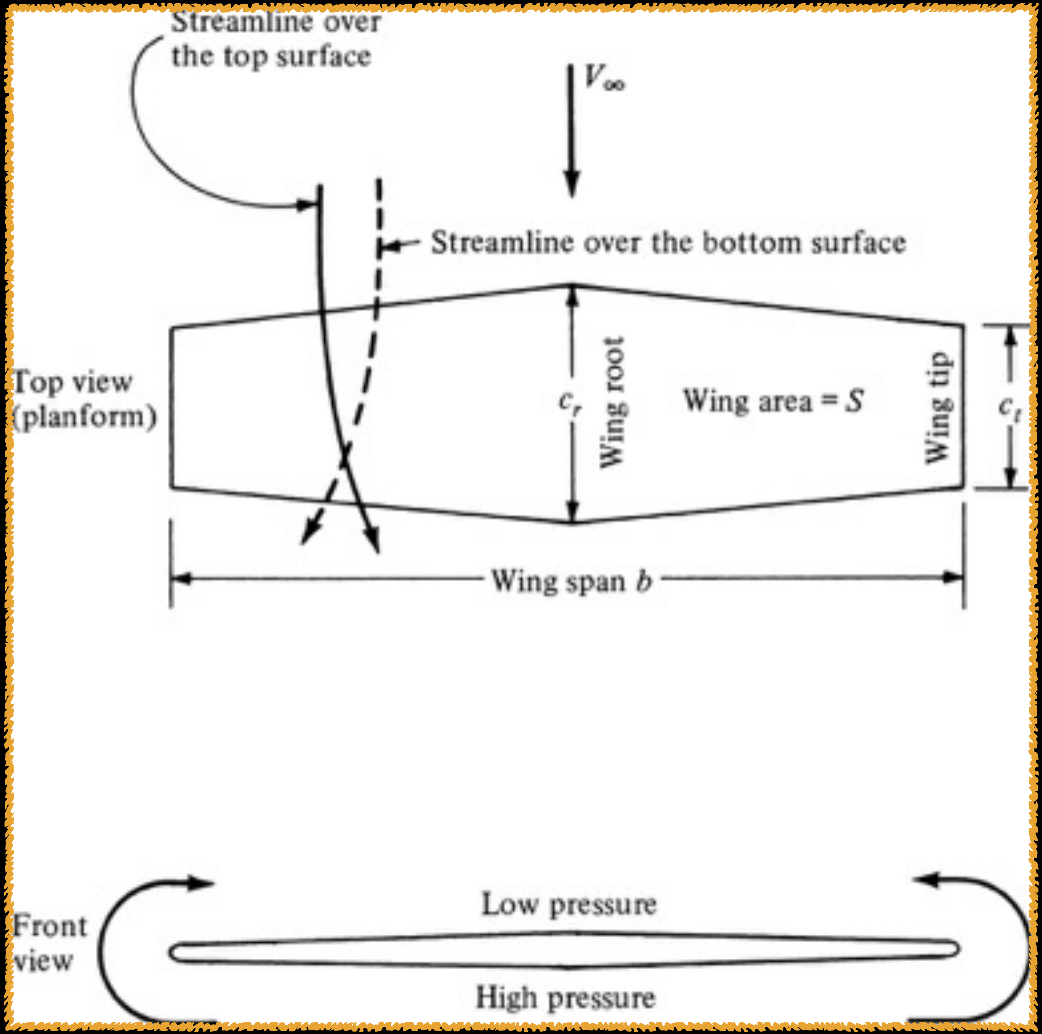




- *Are the lift and drag coefficients for the Beechcraft wing same as the airfoil?*
  - *NO!!*

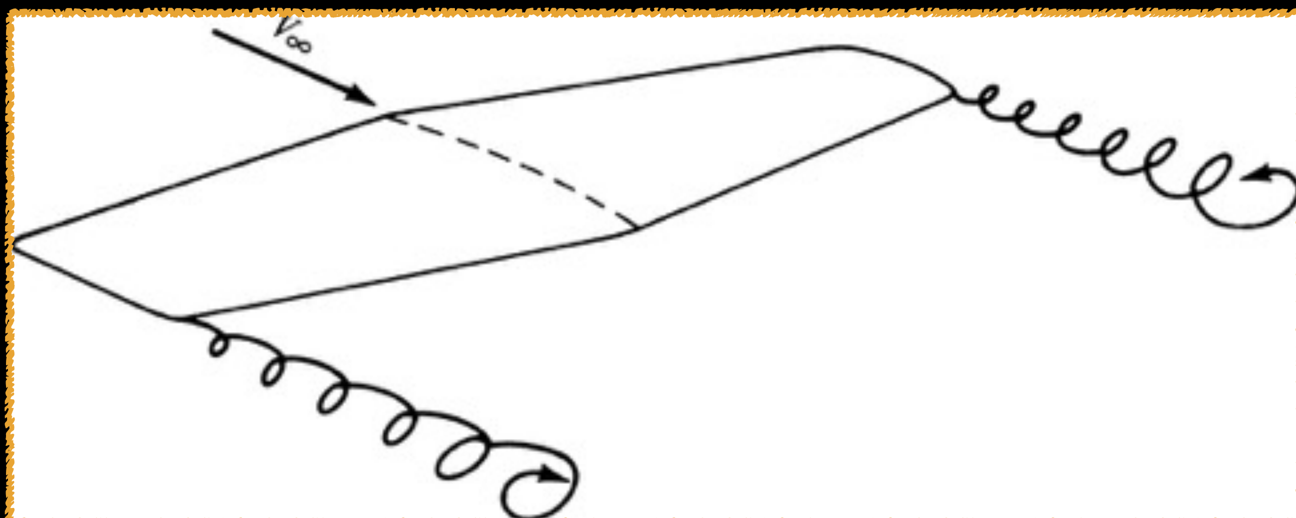
Downwash and Induced Drag

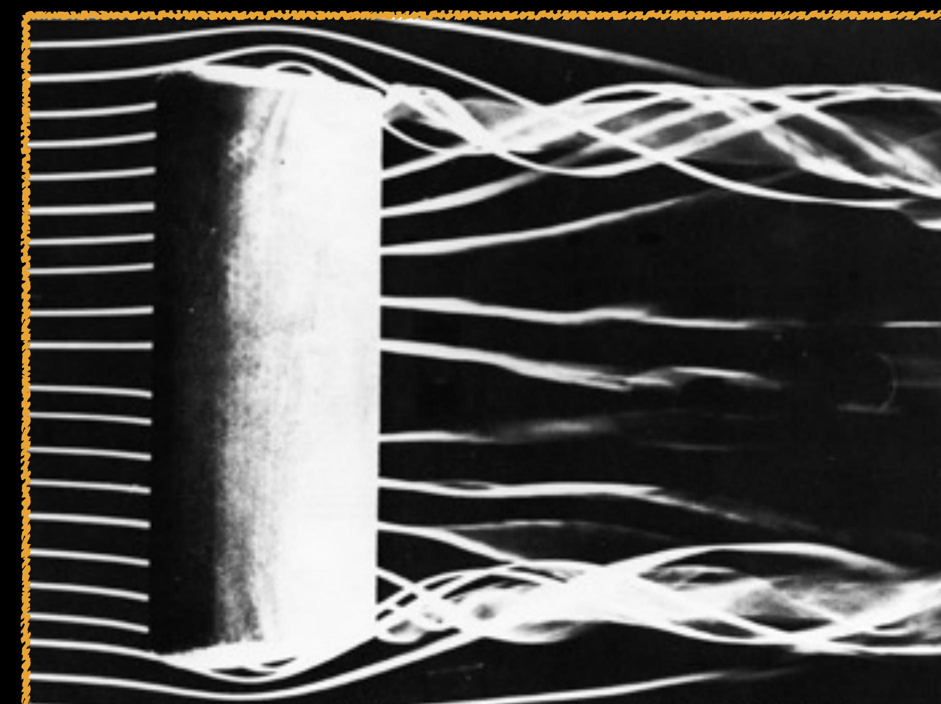
- *Why are the aerodynamic characteristics of finite wings different from its airfoil sections?*
  - *Flow over an airfoil is two-dimensional.*
  - *In contrast, flow over a finite wing is three-dimensional.*



Downwash and Induced Drag

- *The tendency for the flow to leak around the wing tips produces a trailing vortex at each wing tip.*

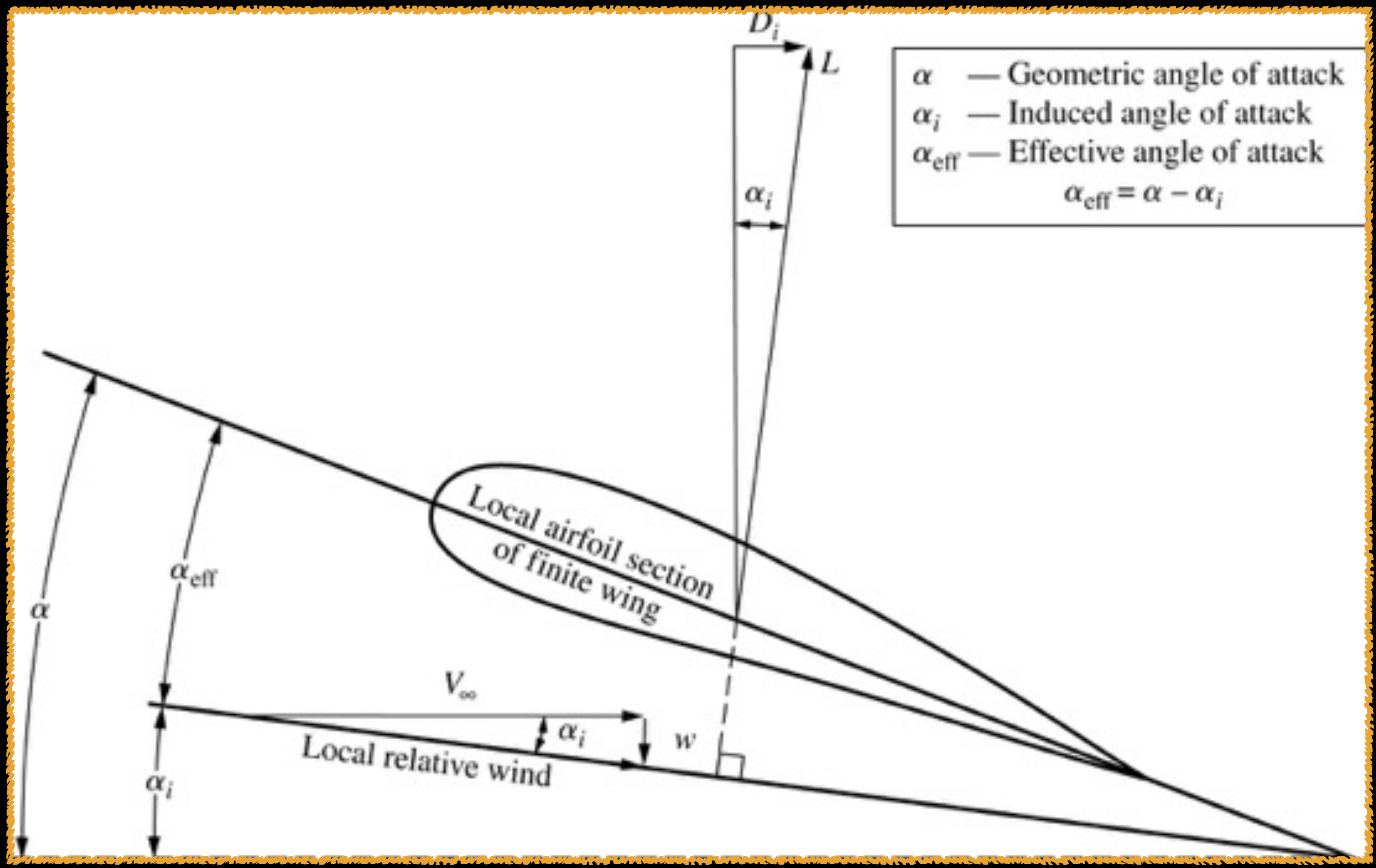




Source: Van Dyke, Milton. An Album of Fluid Motion. The Parabolic Press. Stanford, CA. 1982

- *Trailing vortices at each wing tip.*
  - *The two vortices drag the surrounding air inducing a velocity component in the downward direction - downwash.*
  - *The downwash combines with the local freestream to create a local relative wind.*

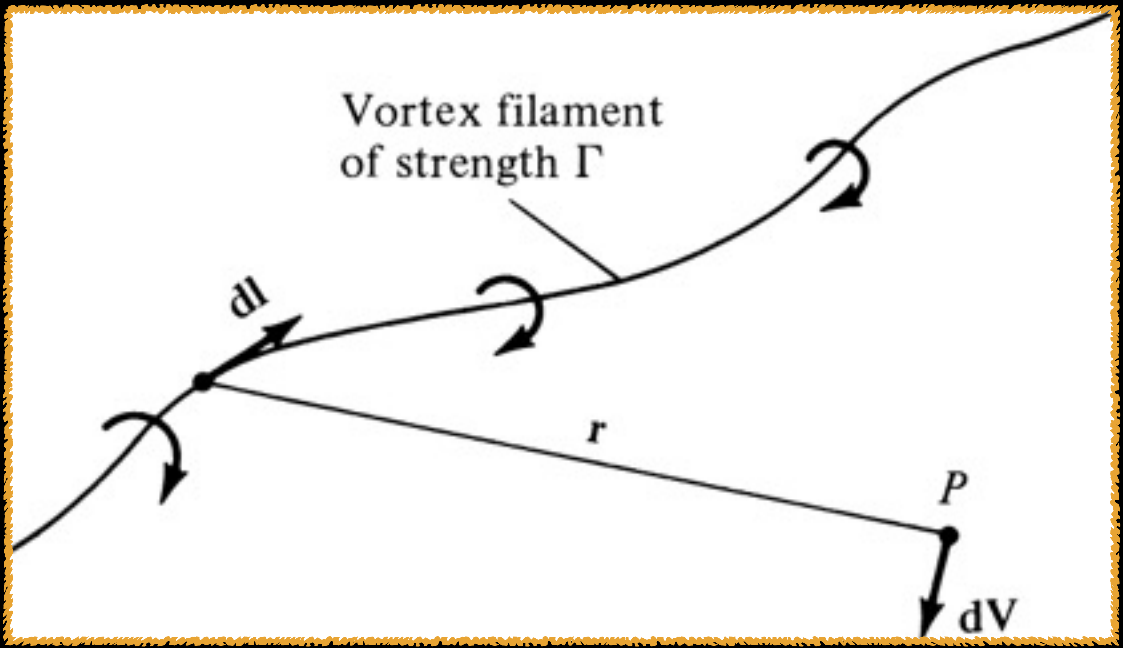
Downwash and Induced Drag



- *The downwash has two important effects:*
  - *The effective angle of attack is reduced.*
  - *Induced drag is created due to the tilting of the local lift vector.*
  - *The total drag = friction drag + pressure drag + induced drag.*

The Vortex Filament Theorem

- *Establish a rational aerodynamic theory for a finite wing.*



- *The curved filament induces a flow field in the surrounding space.*
- *Circulation taken about any closed path enclosing the filament is constant.*

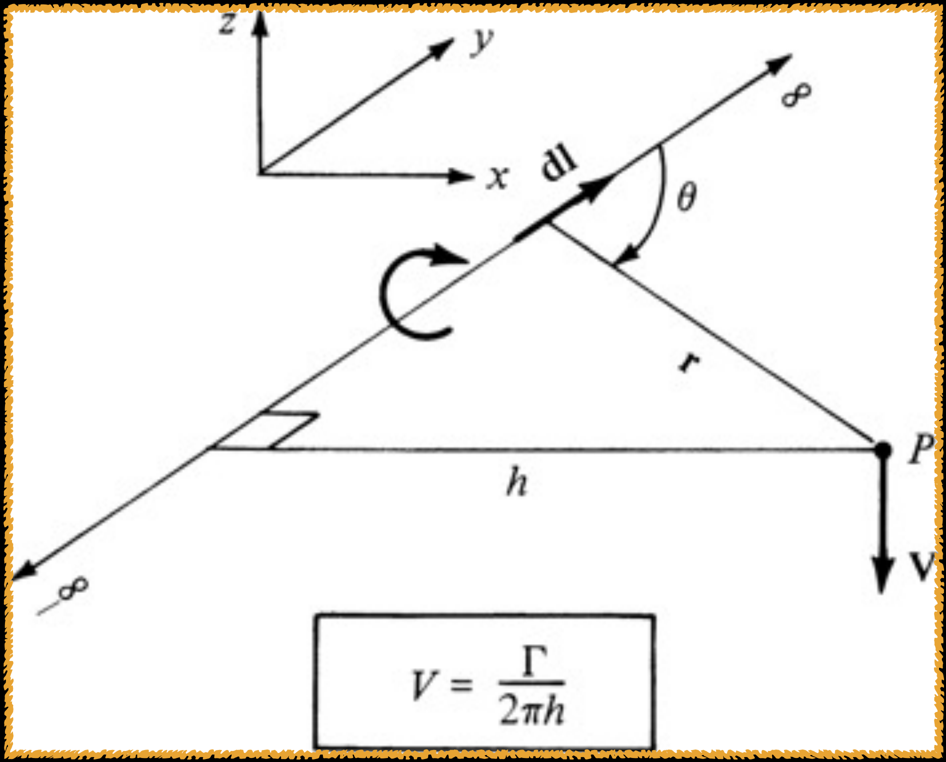
- *Consider a segment dl. It induces a velocity at P equal to:*

$$dV = \frac{\Gamma}{4\pi}\frac{dl \times r}{|r|^3}$$

Biot-Savart Law

The Vortex Filament Theorem

- *When a number of vortex filaments are used in conjunction with a uniform free stream, it is possible to synthesize the flow over a finite wing.*



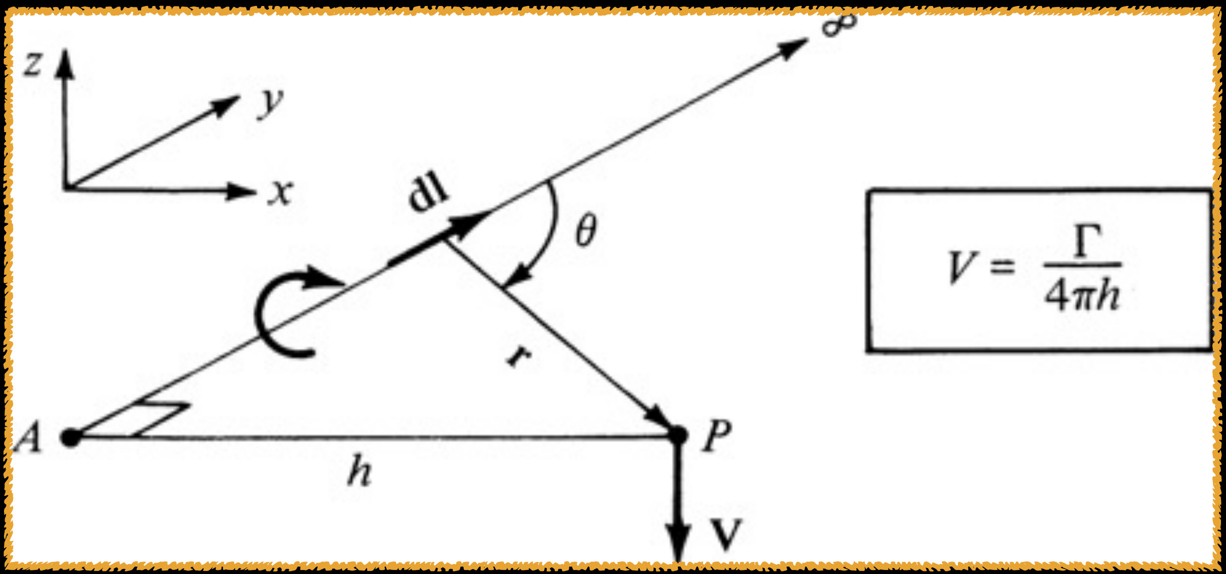
Biot-Savart Law to a straight vortex filament of infinite length

- *Velocity induced at P by the entire vortex filament is given by:*

$$V = \int_{-\infty}^{\infty} \frac{\Gamma}{4\pi} \frac{dl \times r}{|r|^3} = \frac{\Gamma}{2\pi h}$$

- *The above result is the same as that for a point vortex in a 2D flow.*

The Vortex Filament Theorem



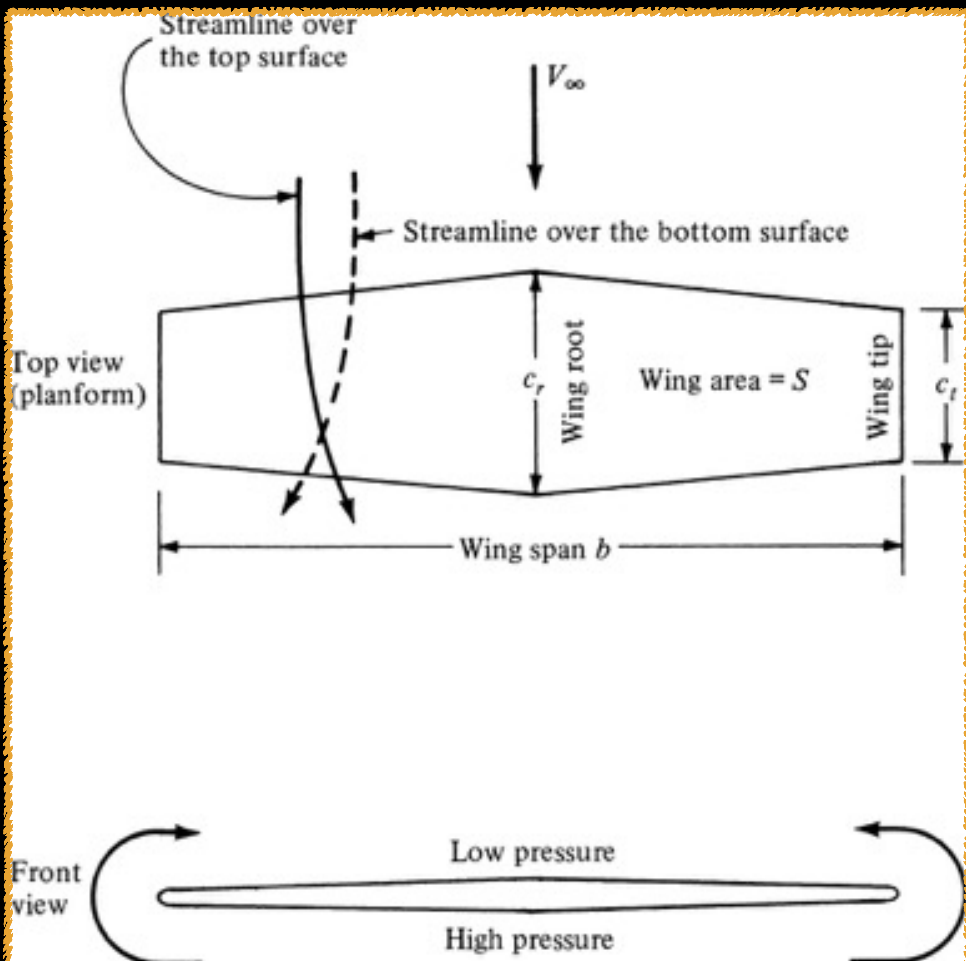
Semi-infinite vortex filament

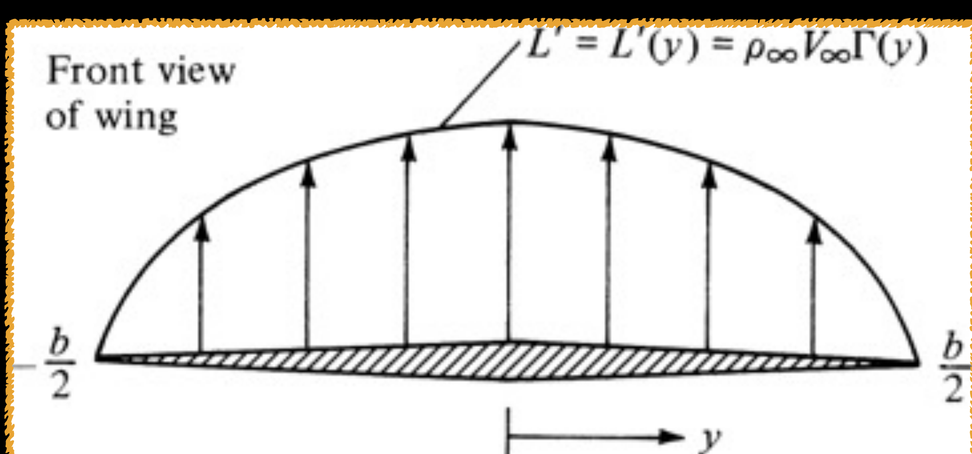
- *Velocity induced at P by the vortex filament is given by:*

$$V = \int_A^{\infty} \frac{\Gamma}{4\pi} \frac{dl \times r}{|r|^3} = \frac{\Gamma}{4\pi h}$$

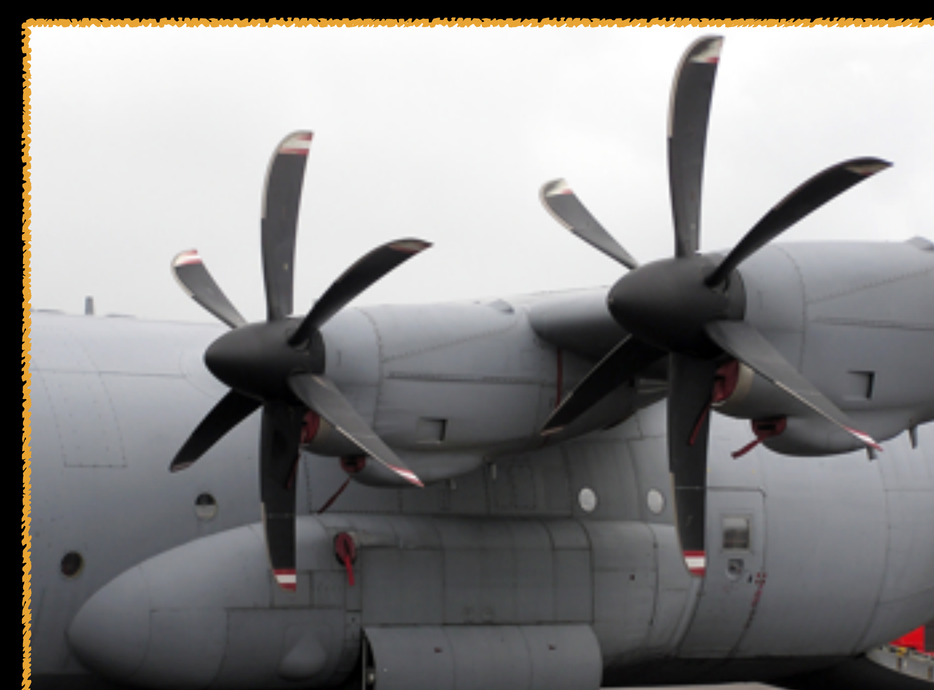
- *Helmholtz's Vortex Theorems (basic principles of vortex behavior)*
  - *The strength of a vortex filament is constant along its length.*
  - *A vortex filament cannot end in a fluid; it must extent to the boundaries of the fluid or form a closed path.*

Lift Distribution on a Finite Wing

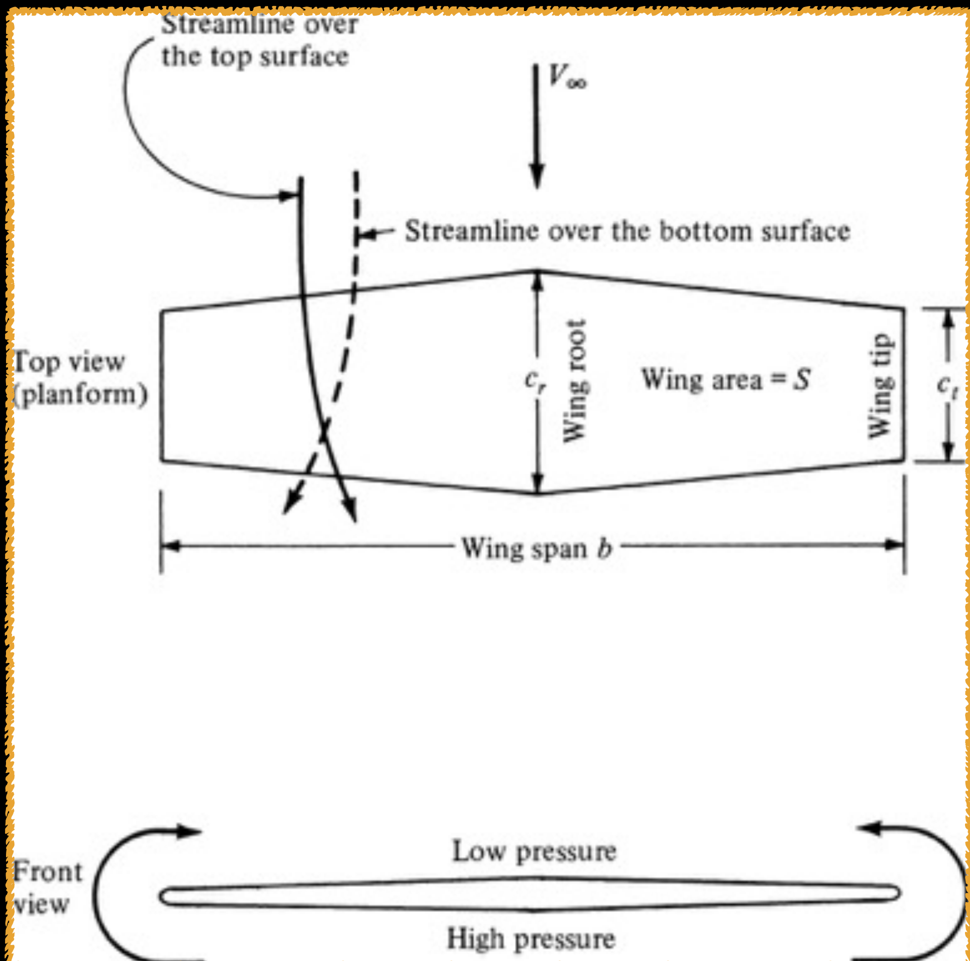


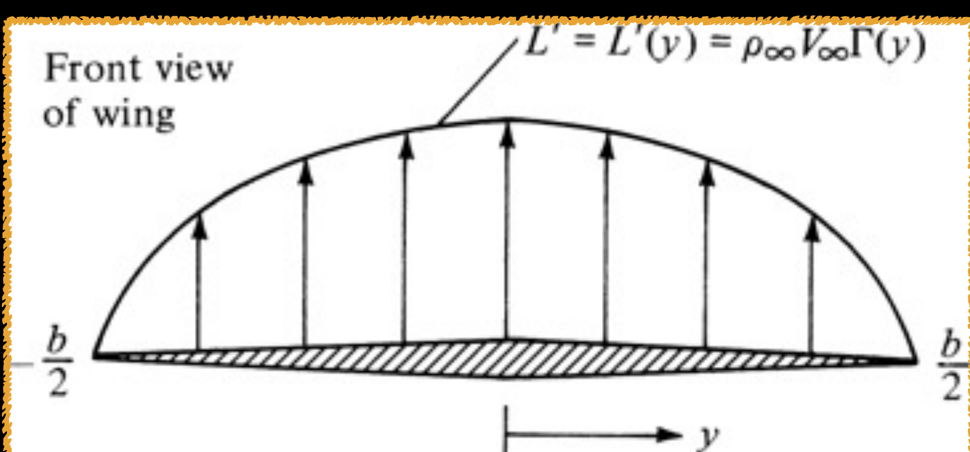


- *Consider a given spanwise location y_1, where the local chord is c.*
  - *The lift per unit span can vary along the span.*
  - *Different spanwise locations can have different angles of attack (geometric twist).*



Lift Distribution on a Finite Wing

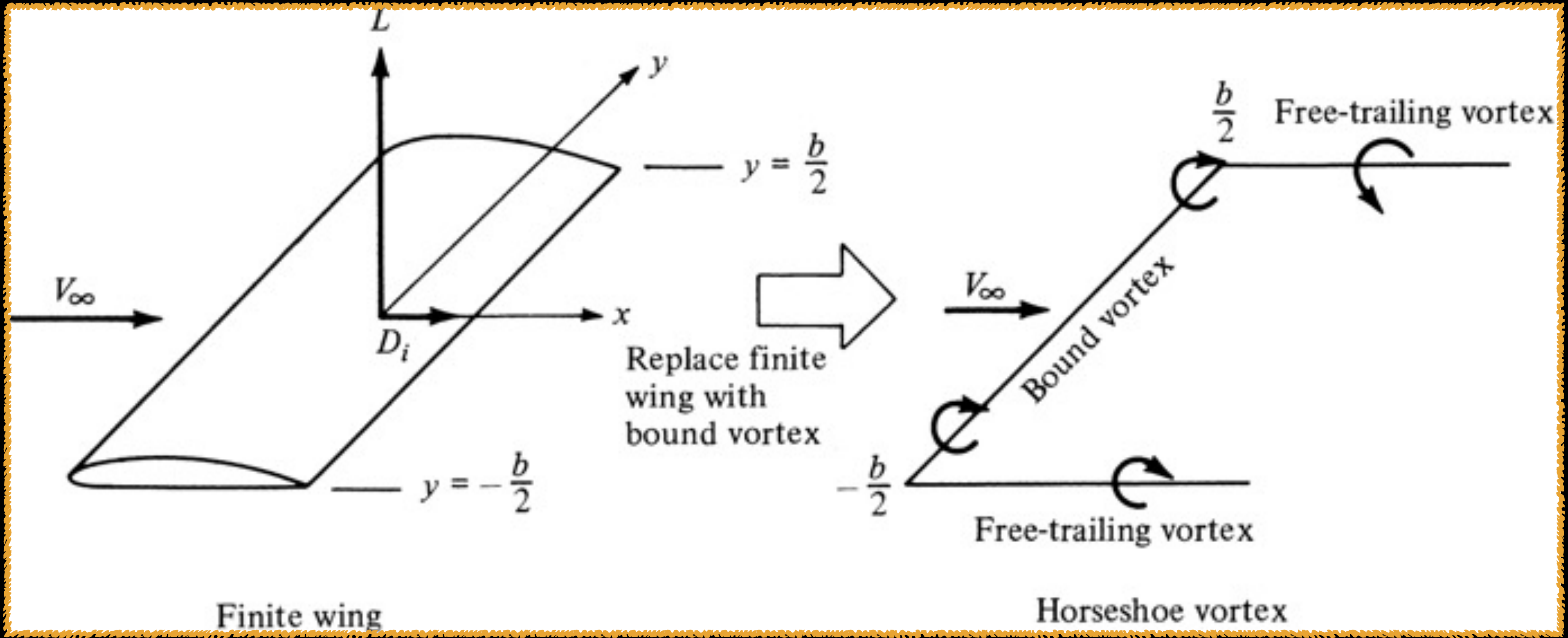




- *Consider a given spanwise location y_1, where the local chord is c.*
  - *Wings can also have different airfoil section spanwise (aerodynamic twist).*
  - *Pressure equalization occurs at y = -b/2 and b/2, and consequently there is no lift at these locations.*
  - *Our objective is to estimate this lift distribution, total lift and induced drag for the finite wing.*

Prandtl's Lifting Line Theory

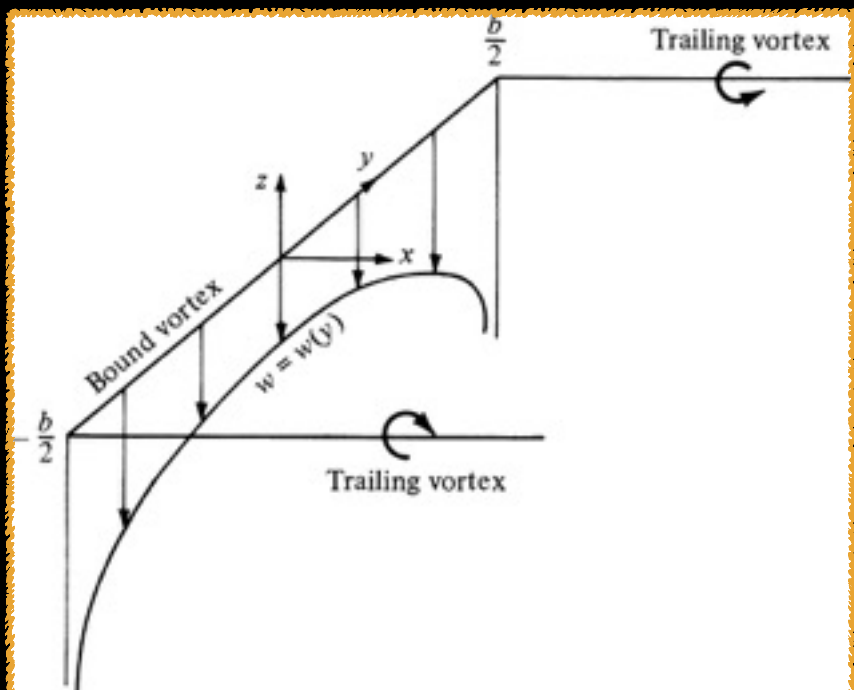
- *The theory is useful for predicting the aerodynamic characteristics of finite wings.*



- *The finite wing is replaced with a bound vortex.*
- *Due to Helmholtz's theorem, a vortex filament cannot end in the fluid.*
  - *Therefore, assume the vortex filament continues as two free vortices trailing downstream from the wing tips to infinity.*
- *Bound vortex + Trailing vortices —> Horseshoe Vortex.*

Prandtl's Lifting Line Theory

- *Consider the downwash induced along the bound vortex by the horseshoe vortex.*



- *The velocity at any point along the bound vortex induced by the trailing vortex is:*

$$w(y) = -\frac{\Gamma}{4\pi(b/2+y)} - \frac{\Gamma}{4\pi(b/2-y)}$$

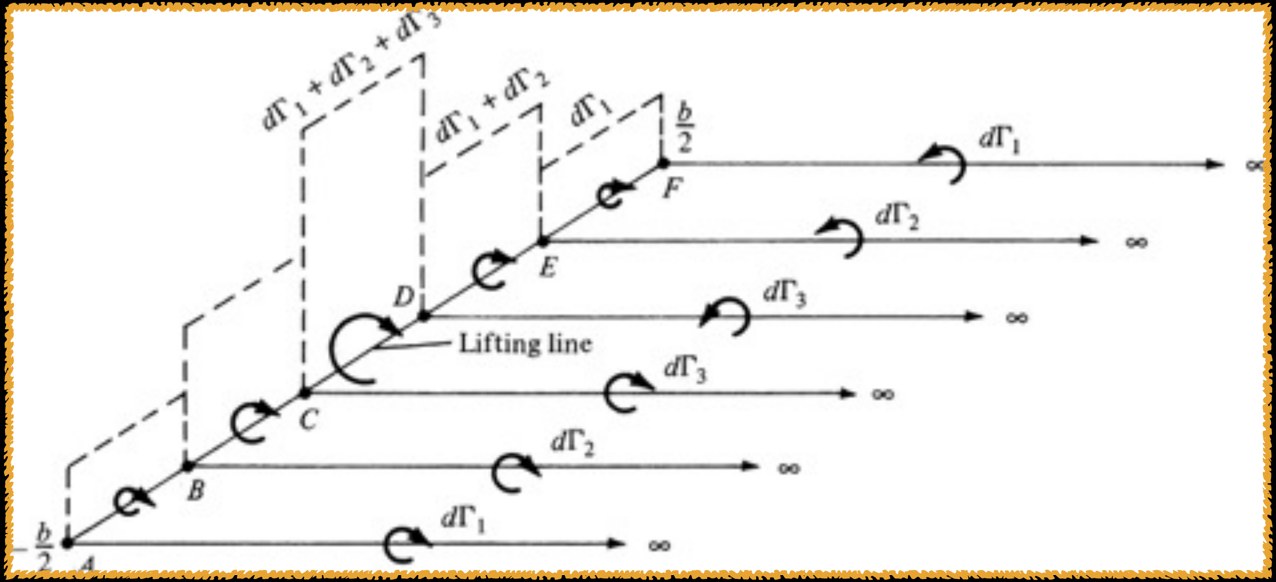
left vortex   right vortex

$$w(y) = -\frac{\Gamma b}{4\pi\left[(b/2)^2 - y^2\right]}$$

- *The downwash approaching infinity value at the tips is disconcerting.*

Prandtl's Lifting Line Theory

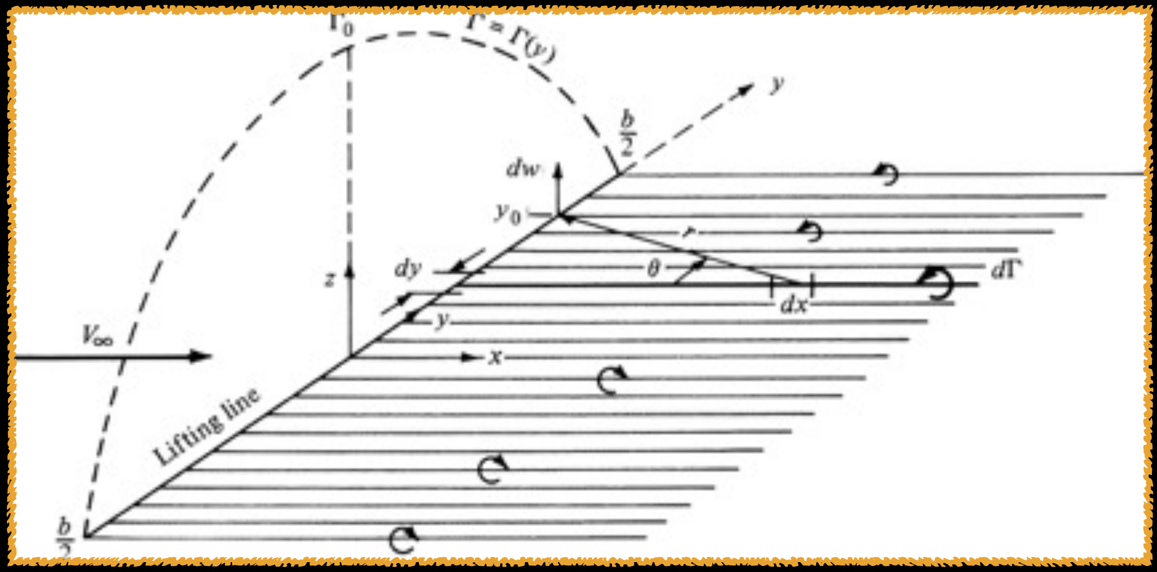
- *Instead of representing the wing by a single horseshoe vortex, superimpose using a large number of horseshoe vortices.*
  - *Each horseshoe with a different length of the bound vortex.*
  - *All bound vortices coincident along a single line - Lifting Line.*



- *The series of trailing vortices represents pairs of vortices.*
  - *Each pair is associated with a given horseshoe vortex.*
- *The strength of each trailing vortex is equal to the change in circulation along the lifting line.*

Prandtl's Lifting Line Theory

- *Let us extrapolate to the case where an infinite number of horseshoe vortices are superimposed along the lifting line.*
  - *Each horseshoe has vanishingly small strength.*

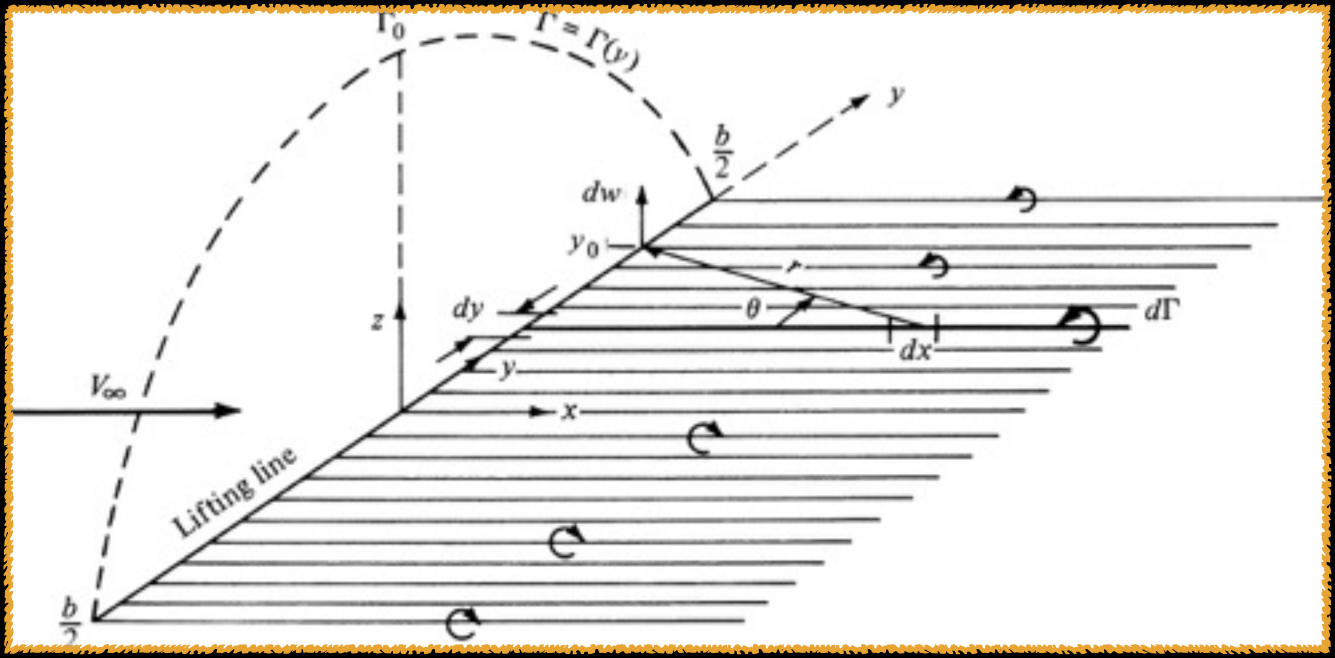


- *The finite number of trailing vortices in the earlier case have become a continuous vortex sheet.*
- *The total strength of the sheet integrated across the span of the wing is zero (because of pairs of trailing vortices of equal but opposite strengths).*

Prandtl's Lifting Line Theory

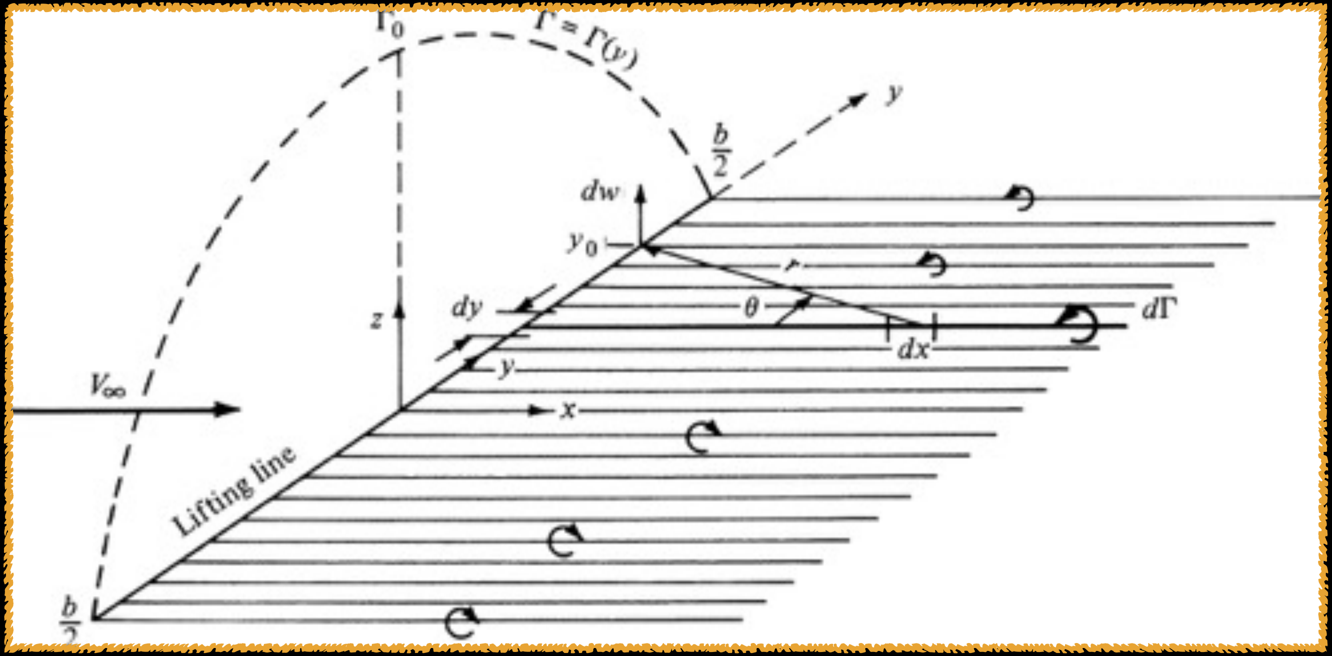
- *Let us single out an infinitesimally small segment of the lifting line dy.*
  - *The circulation, and the change in circulation at y are respectively:*

$$\Gamma(y) \text{ and } d\Gamma = (d\Gamma/dy)dy$$



- *In turn, the strength of the trailing vortex at y must equal the change in circulation along the lifting line.*

Prandtl's Lifting Line Theory



- *The velocity dw induced at* y_0 *by the entire semi-infinite trailing vortex located at y is:*

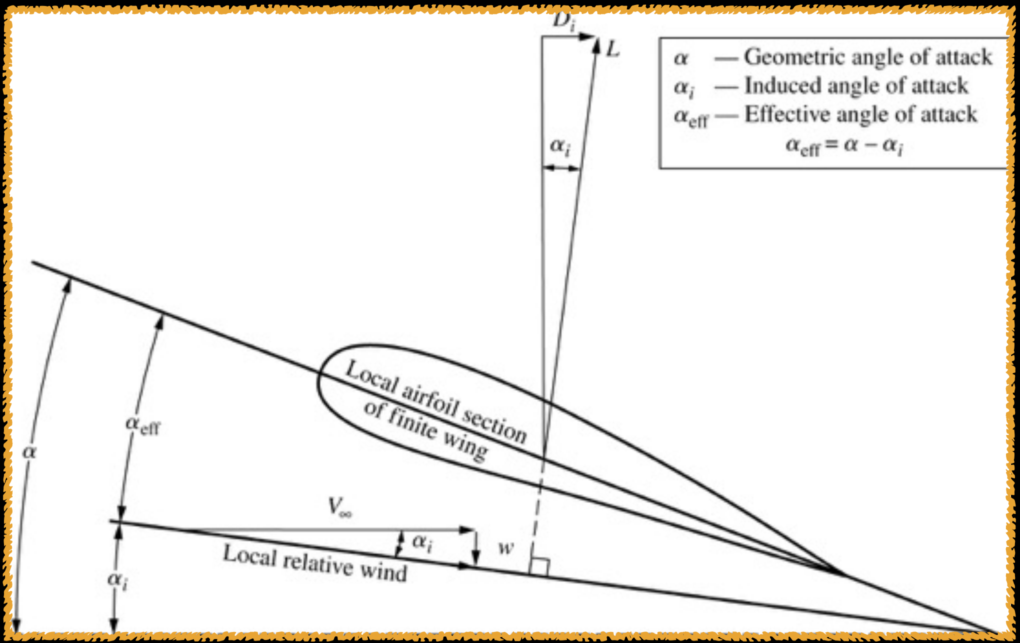
$$dw = -\frac{(d\Gamma/dy)dy}{4\pi(y_0 - y)}$$

- *The total velocity induced at* y_0 *by the entire trailing vortex sheet is:*

$$w(y_0) = -\frac{1}{4\pi}\int_{-b/2}^{b/2}\frac{(d\Gamma/dy)dy}{(y_0 - y)}$$

Prandtl's Lifting Line Theory

- *However, our central problem remains to be solved: we want to calculate* $\Gamma(y)$ *for a given finite wing.*



$$\alpha_i(y_0) = tan^{-1}\left(\frac{-w(y_0)}{V_\infty}\right)$$

$$\boxed{\text{For small angles, } \alpha_i(y_0) = \left(\frac{-w(y_0)}{V_\infty}\right)}$$

- *The geometric angle of attack is given by:*

$$\alpha(y_0) = \frac{\Gamma(y_0)}{\pi V_\infty c(y_0)} + \alpha_{L=0}(y_0) + \frac{1}{4\pi V_\infty}\int_{-b/2}^{b/2} \frac{(d\Gamma/dy)dy}{y_0 - y}$$

Prandtl's Lifting Line Theory Equation

Prandtl's Lifting Line Theory - Elliptic Lift Dist.

- *Consider a circulation distribution given by:*

$$\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

- *The circulation, and hence lift, goes to zero at the wing tips.*
  - *What are the aerodynamic properties of a finite wing with such an elliptic lift distribution?*

Constant downwash over the span

$$w(\theta_0) = -\frac{\Gamma_0}{2b}$$

Constant induced angle of attack over the span

$$\alpha_i = -\frac{\Gamma_0}{2bV_\infty} = \frac{C_L}{\pi(AR)}$$

Induced drag proportional to square of lift coefficient

$$C_{D,i} = \frac{C_L^2}{\pi(AR)}$$

Prandtl's Lifting Line Theory - Elliptic Lift Dist.

Induced drag

$$C_{D,i} = \frac{C_L^2}{\pi(AR)}$$

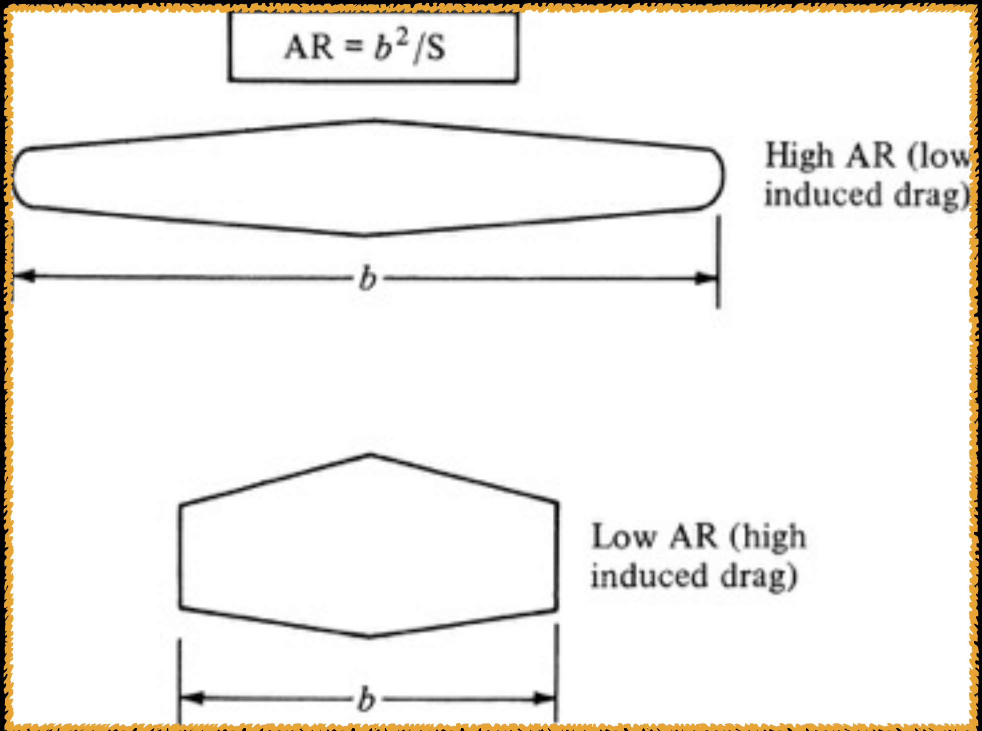
- *The dependence of induced drag on lift is not surprising.*
  - *Induced drag is a consequence of the presence of wing-tip vortices produced by the pressure difference between lower and upper wing surfaces.*
  - *Lift is also produced by this same pressure difference and hence the dependence.*
- *Induced drag is the price for generation of lift. An aircraft cannot generate lift for free.*
  - *Power required for an aircraft to overcome induced drag is the power required to generate the lift of the aircraft.*
  - *Induced drag is high at take-off, landing , and about 25% of total drag at cruising speeds.*

Prandtl's Lifting Line Theory - Elliptic Lift Dist.

Induced drag

$$C_{D,i} = \frac{C_L^2}{\pi(AR)}$$

- *Induced drag is inversely proportional to the aspect ratio.*
  - *To reduce induced drag, we want a finite wing with the highest possible aspect ratio.*
  - *Design of high aspect ratio wings with sufficient structural strength is difficult.*
  - *Therefore aspect ratio is a compromise between aerodynamic and structural requirements.*



Prandtl's Lifting Line Theory - Elliptic Lift Dist.

- *Another property of an elliptic lift distribution:*
  - *Consider a wing with no geometric twist (angle of attack is constant along the span).*
  - *Also, assume no aerodynamic twist (zero-lift angle of attack is constant along the span).*

The local section lift coefficient is $c_l = a_0(\alpha_{eff} - \alpha_{L=0})$

The lift per unit span is then given by $L'(y) = q_\infty c c_l$

Solving for the chord, we have $c(y) = \dfrac{L'(y)}{q_\infty c_l}$

- The above equations show that for an elliptic lift distribution:
  - *The chord must vary elliptically along the span.*
  - *The wing planform is elliptical.*

Prandtl's Lifting Line Theory - General Lift Dist.

- *Consider the transformation:* $y = -\frac{b}{2}cos\theta$
- *The elliptic lift distribution is then given by:*

$$\Gamma(\theta) = \Gamma_0 sin\theta = 2bV_\infty \sum_1^N A_n sin(n\theta)$$

- *So the angle of attack evaluated at a given spanwise location is:*

$$\alpha(\theta_0) = \frac{2b}{\pi c(\theta_0)} \sum_1^N A_n sin(n\theta_0) + \alpha_{L=0}(\theta_0) + \sum_1^N nA_n \frac{sin(n\theta_0)}{sin\theta_0}$$

- *The only unknowns in the above equation are the Fourier coefficients.*
  - *They can be evaluated by considering 'n' spanwise locations.*

Prandtl's Lifting Line Theory - General Lift Dist.

- *Now that* $\Gamma(\theta)$ *is known, the lift coefficient can be calculated:*

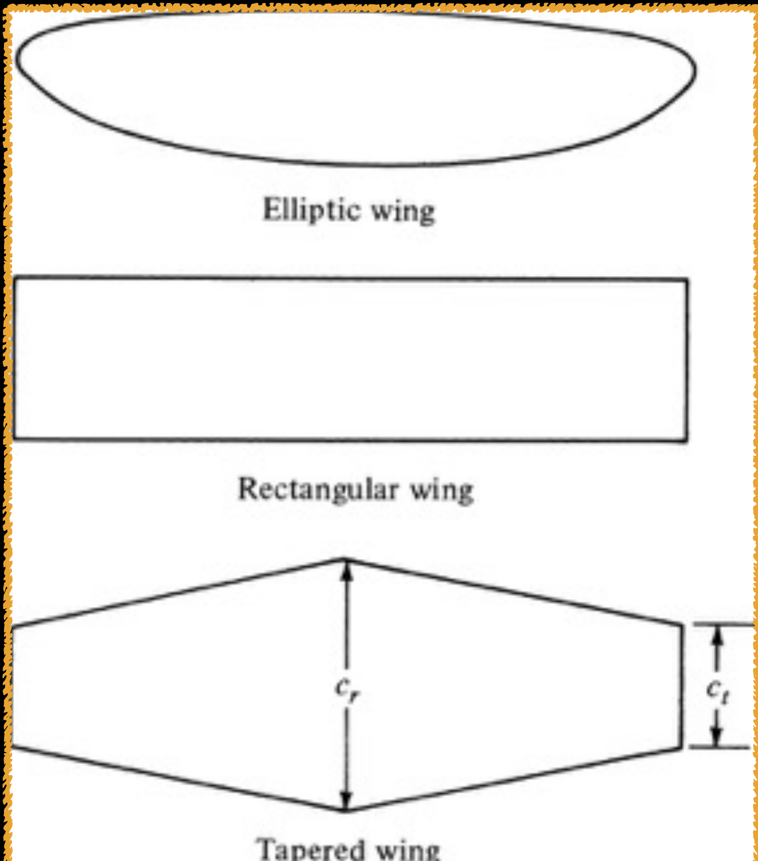
$$C_L = A_1 \pi (AR)$$

- *The induced drag coefficient is then calculated to be:*

$$C_{D,i} = \frac{C_L^2}{\pi(AR)}(1+\delta), \text{ where}$$

$$\delta = \sum_2^N n(A_n/A_1)^2$$

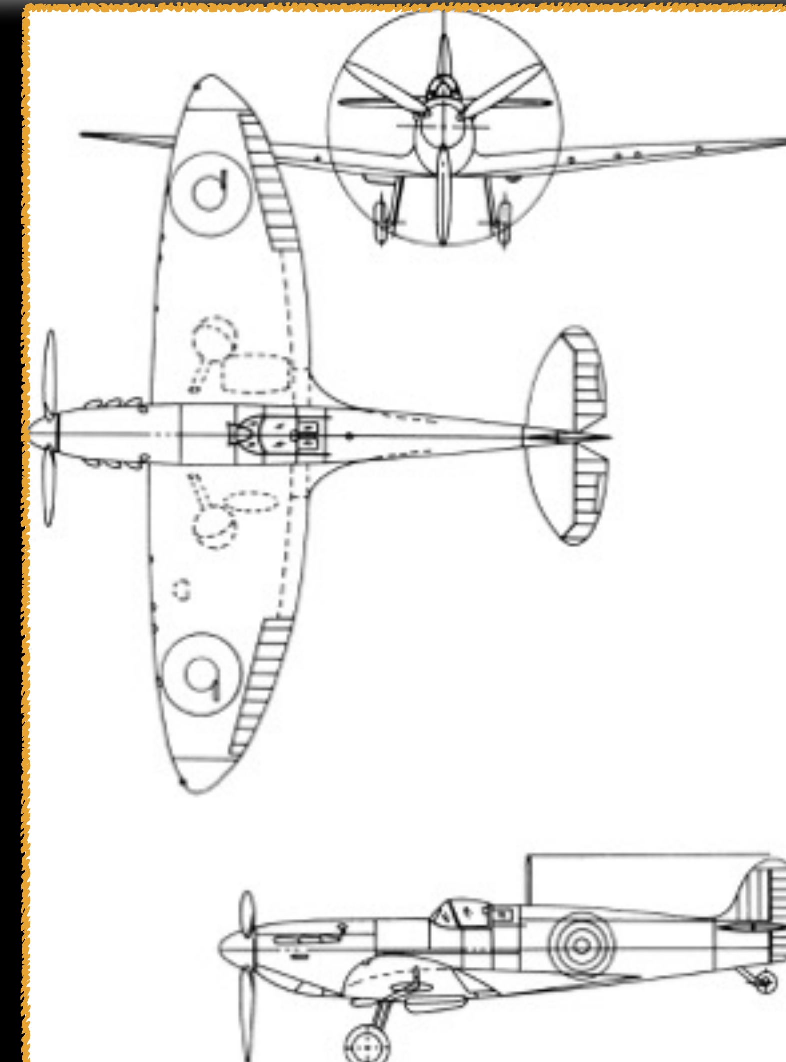
Prandtl’s Lifting Line Theory - General Lift Dist.



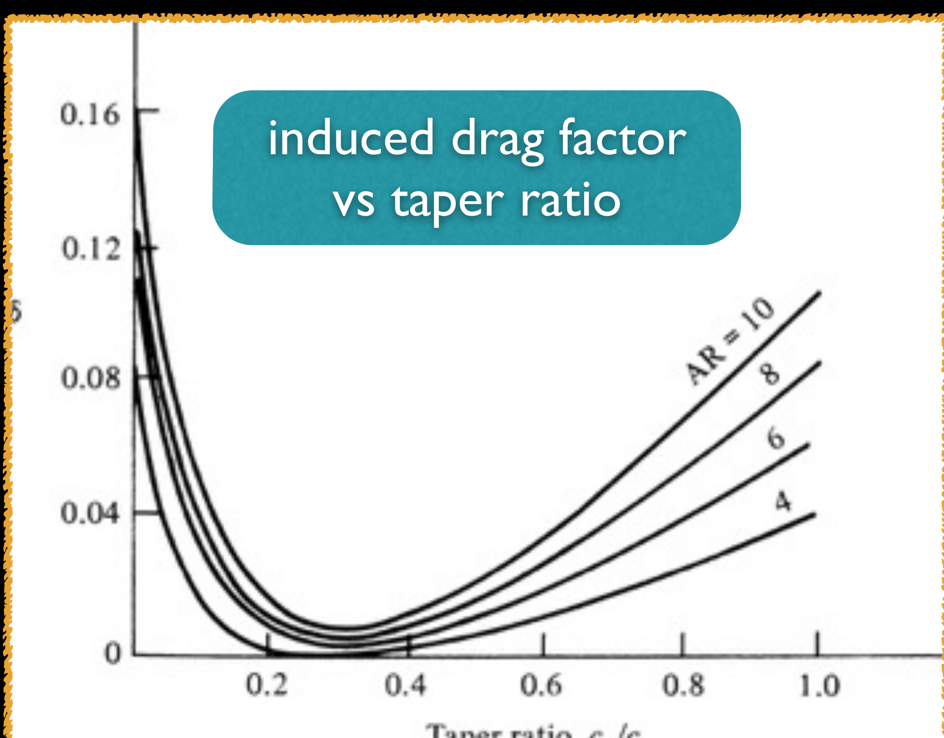
expensive to manufacture

less than optimum lift

compromise between elliptic & rectangular



Supermarine Spitfire



induced drag factor vs taper ratio

- *Obtain taper ratio such that the lift distribution closely approximates the elliptic case.*

Effect of Aspect ratio

- *AR typically varies from 6 to 22 for standard subsonic aircraft.*
  - *The primary design factor for minimizing induced drag is to make the aspect ratio as large as possible.*
- *The total drag is: profile drag + induced drag*

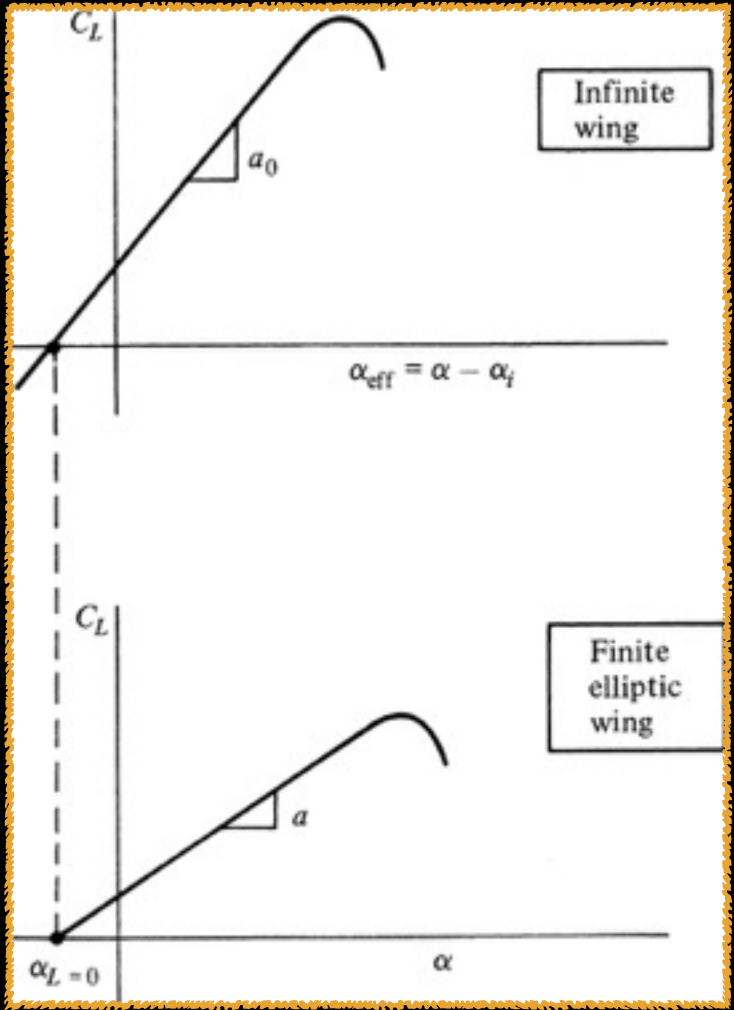
$$C_D = c_d + \frac{C_L^2}{\pi e AR}$$

- *Consider two wings with different aspect ratios but same airfoil cross-section.*
  - *It is possible to scale the data of a wing with one aspect ratio to correspond to the case of another aspect ratio.*

$$C_{D,1} = C_{D,2} + \frac{C_L^2}{\pi e}\left[\frac{1}{5} - \frac{1}{AR_2}\right]$$

Effect of Aspect ratio

- *There are two primary differences between airfoil and finite wing properties.*
  - *One: finite wing generates induced drag.*
  - *Two: The lift slope of a finite wing is less than that of an infinite wing.*



$$a = \frac{a_0}{1 + (a_0/\pi AR)(1+\tau)}$$

- *τ is a function of Fourier coefficients and typically range between 0.05 and 0.25.*