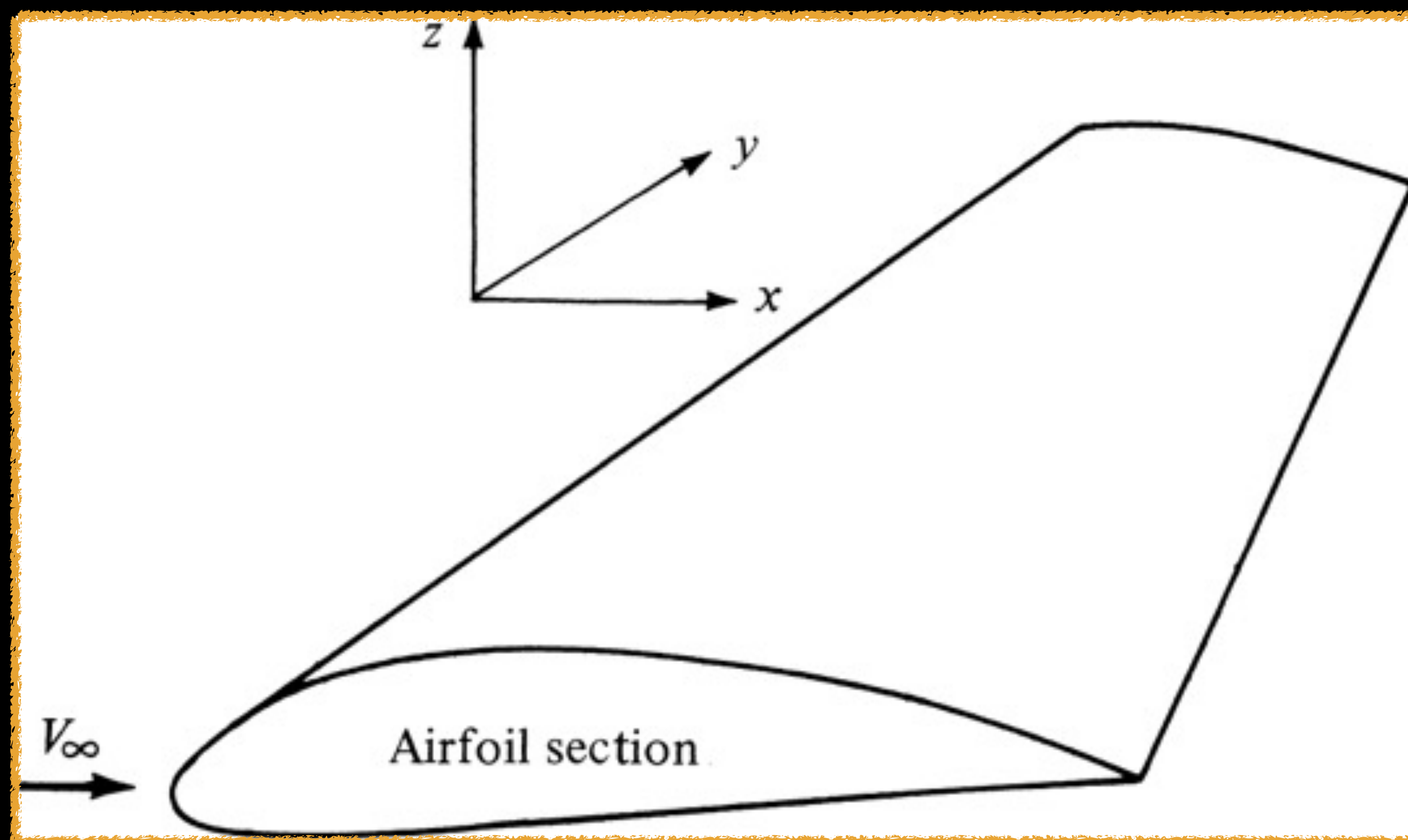
Incompressible Flow Over Airfoils

Isaac Choutapalli
Department of Mechanical Engineering
The University of Texas - Pan American
Edinburg, TX

Introduction

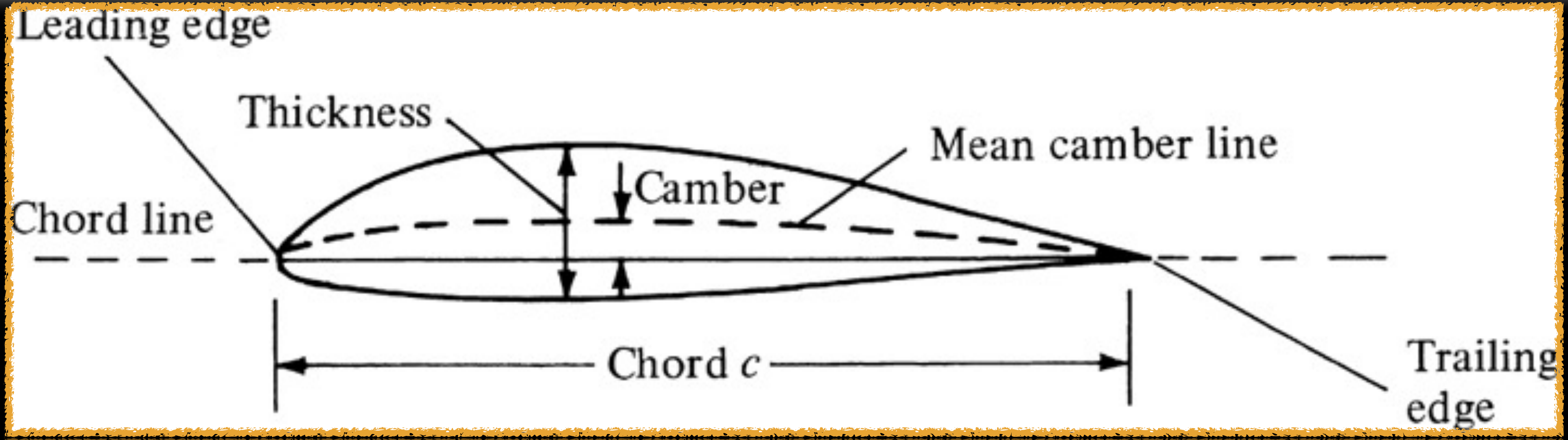
- *We will focus on how to obtain airfoil properties.*
    - Circulation theory
    - Source Panel Method
    - Design and Performance
- *Aerodynamic consideration of wings:*
    - Section of a wing - *airfoil*
    - The complete *finite wing*
- *In this chapter, we will deal with airfoils.*
- *In the next chapter, we will deal with finite wings*

What is an Airfoil?



- *The wing extends in the y-direction.*
- *Any section of the wing cut by a plane parallel to xz-plane is called an airfoil.*

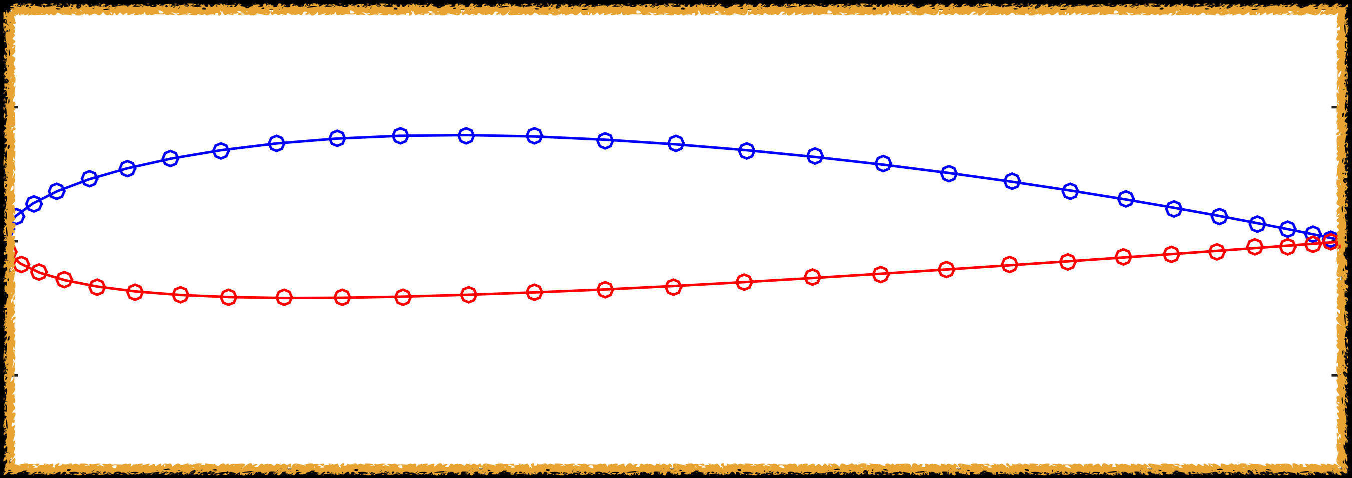
Airfoil Nomenclature



- *Mean camber line - locus of points halfway between upper and lower surfaces*
- *Chord - Straight line connecting the leading and trailing edges.*
- *Camber - Maximum distance between the mean camber line and the chord line.*
- *Thickness - Distance between the upper and lower surfaces measured perpendicular to the chord line.*
- *Leading-edge is generally circular with a radius of 0.02c.*

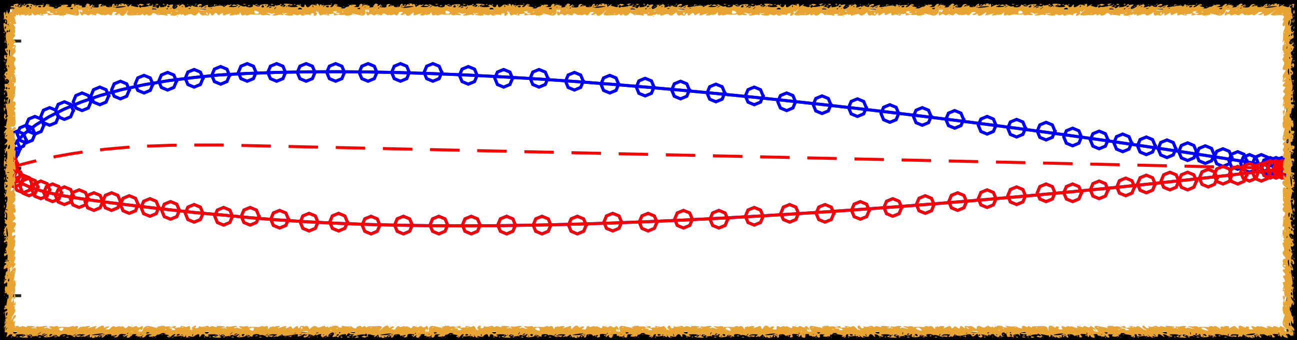
NACA Airfoils

- *NACA 4-digit series (e.g. NACA 2412)*
  - *1st digit: maximum camber in hundredths of chord (0.02c or 2%).*
  - *2nd digit: location of maximum camber from the leading edge along the chord in tenths of chord (0.4c or 40%).*
  - *Last two digits: maximum thickness in hundredths of chord (0.12c or 12%).*



NACA Airfoils

- *NACA 5-digit series (e.g. NACA 23012)*
  - *1st digit* x *(3/2) gives design lift coefficient in tenths of chord (0.3).*
  - *(Next two digits)/2 gives: location of maximum camber from the leading edge along the chord in hundredths of chord (0.15c or 15%).*
  - *Last two digits: maximum thickness in hundredths of chord (0.12c or 12%).*

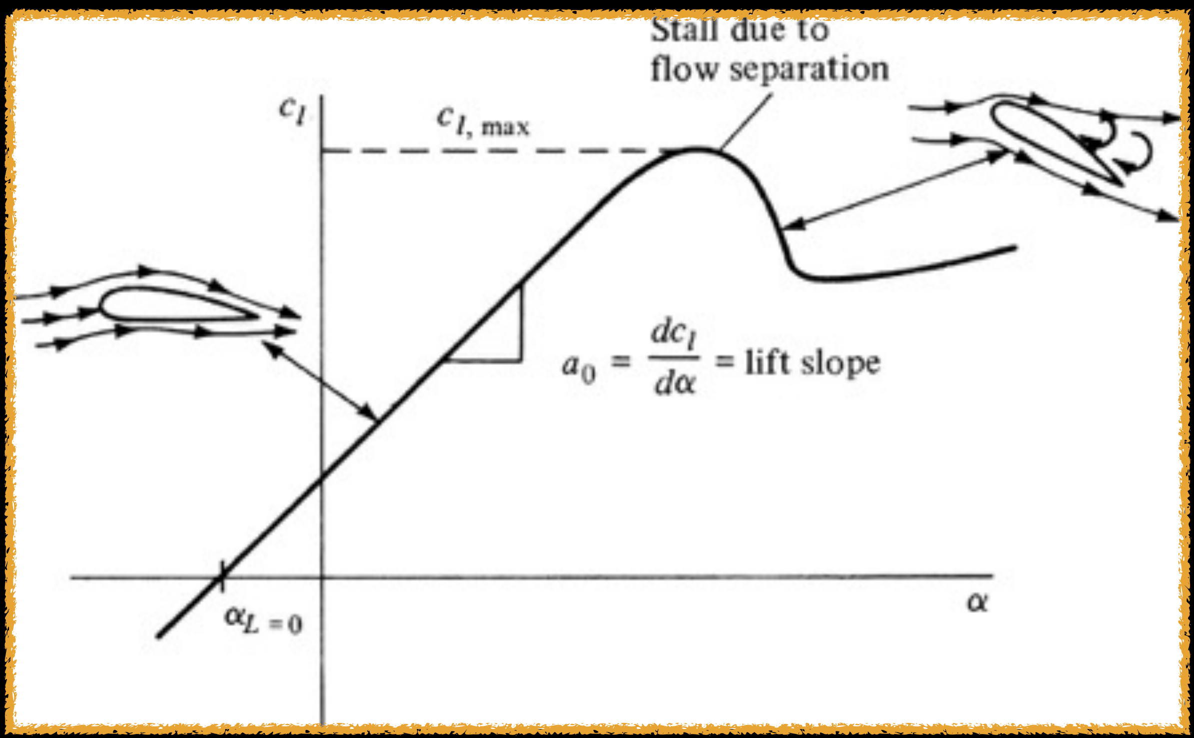


NACA Airfoils

- *NACA 6-digit series (e.g. NACA 65-218)*
  - *Very widely used laminar flow airfoils*
  - *1st digit: represents the series.*
  - *2nd digit: Gives location of minimum pressure in tenths of the chord from the leading edge (0.5c).*
  - *3rd digit: Design lift coefficient in tenths of the chord (0.2).*
  - *Last two digits: maximum thickness in hundredths of chord (0.18c or 18%).*

Airfoil Characteristics

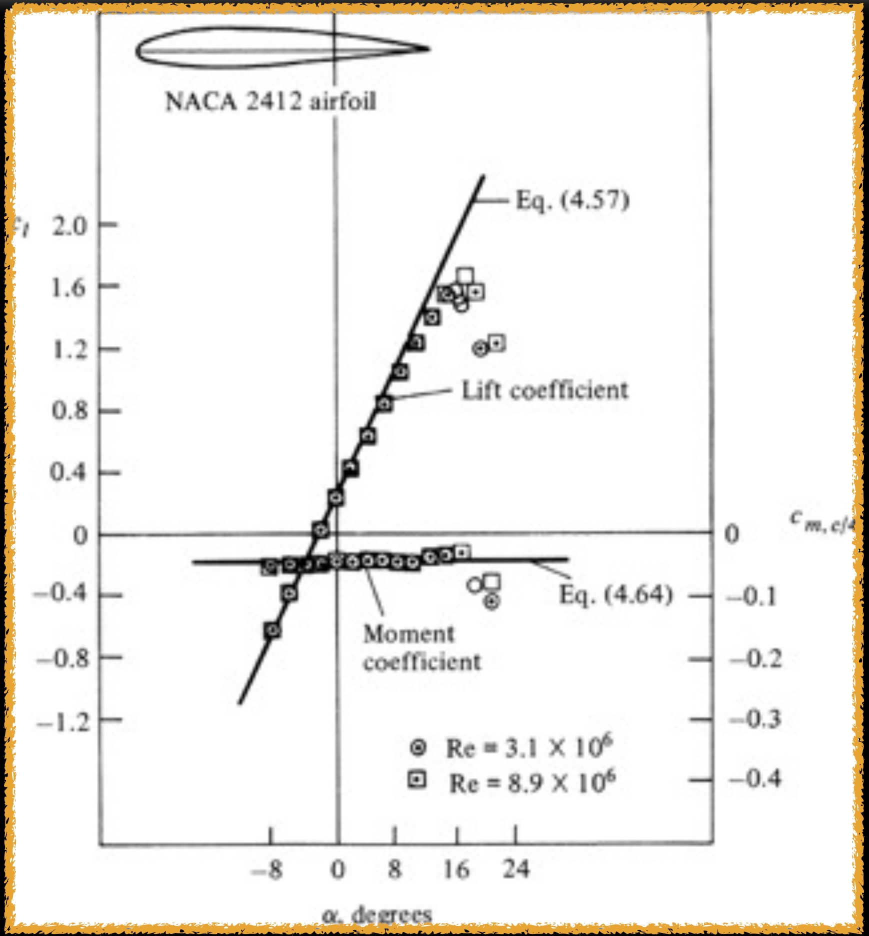
- *Airfoil characteristics are typically lift, drag and moment coefficients.*



- *Lift slope: lift coefficient varies linearly angle of angle of attack.*
- *Zero-lift angle of attack: the value of angle of attack when lift is zero.*

- *Using inviscid flow theory, we can predict lift slope and zero angle of attack. But the maximum lift coefficient can only be calculated using viscous flow theory.*

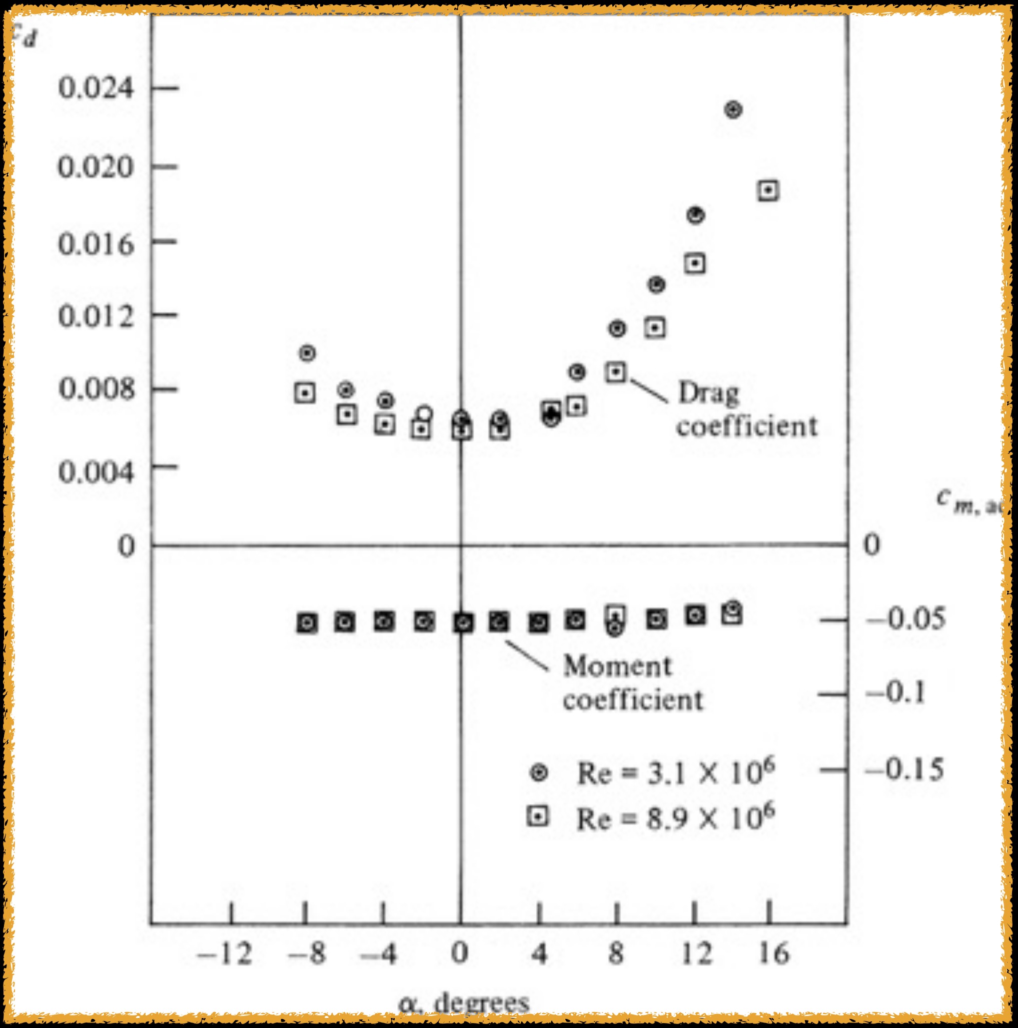
Lift and Moment Coefficients



- *These coefficients are taken about the quarter-chord location.*
- *Note the difference for the two Re cases wrt lift-slope and max. lift coefficient.*
- *Max. lift coefficient depends on Re (viscous effects).*

- *Moment coefficient does not vary for this airfoil with angle of attack.*
  - *Center of pressure: location where the resultant of a distributed load acts.*
  - *Aerodynamic Center: location where the pitching moment is relatively constant with angle of attack.*

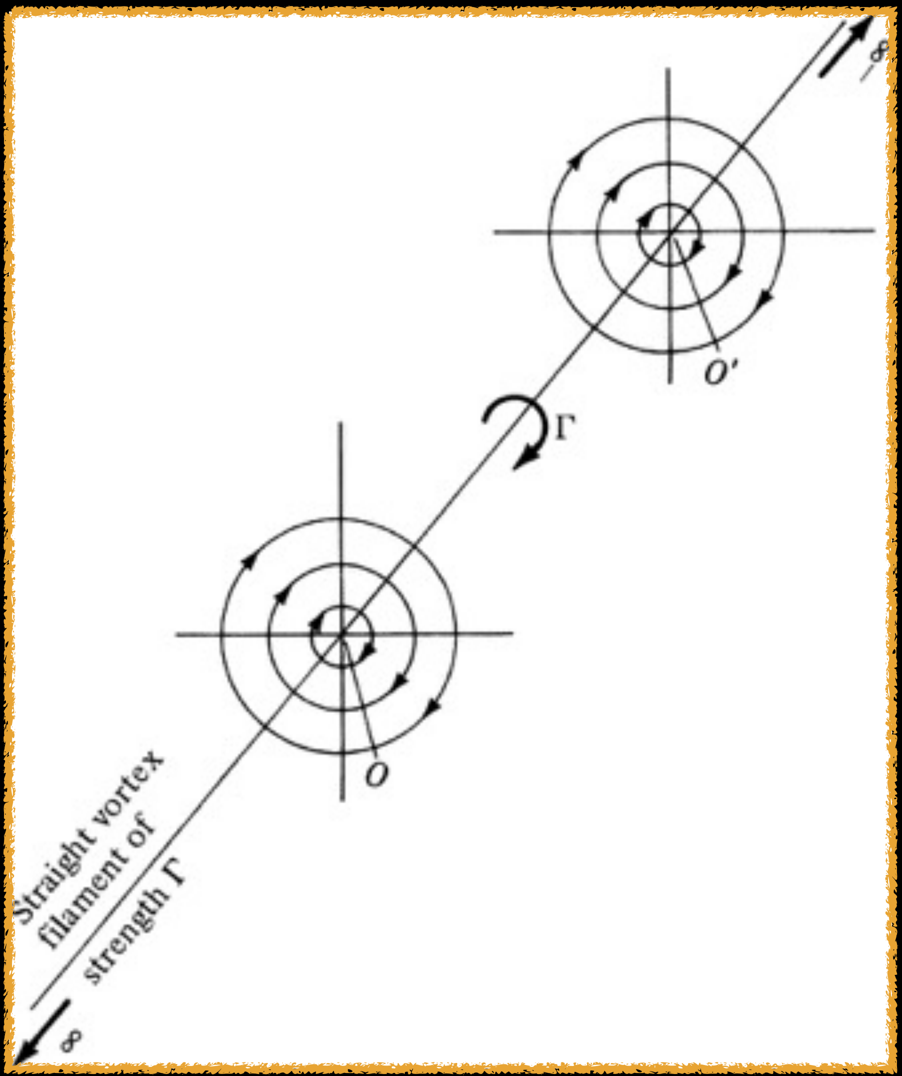
Profile Drag



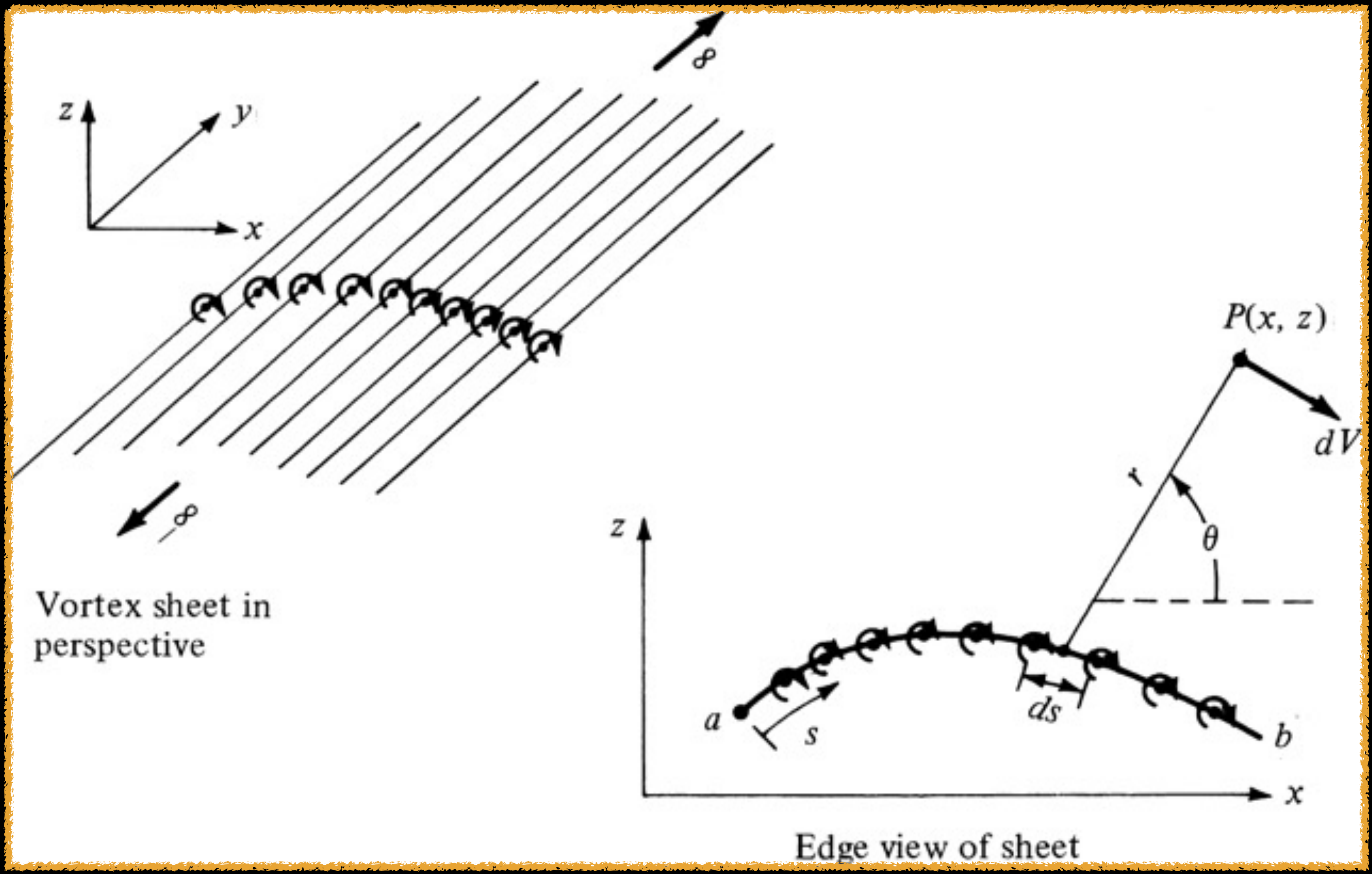
- *Profile drag: friction drag + pressure drag*
- *Profile drag is dependent on Re.*
- *Moments coefficients for moments taken about the aerodynamic center very weakly dependent (nearly independent) on the angle of attack for NACA 2412.*

Examples!!

Theoretical Solutions for Low-Speed Airfoils

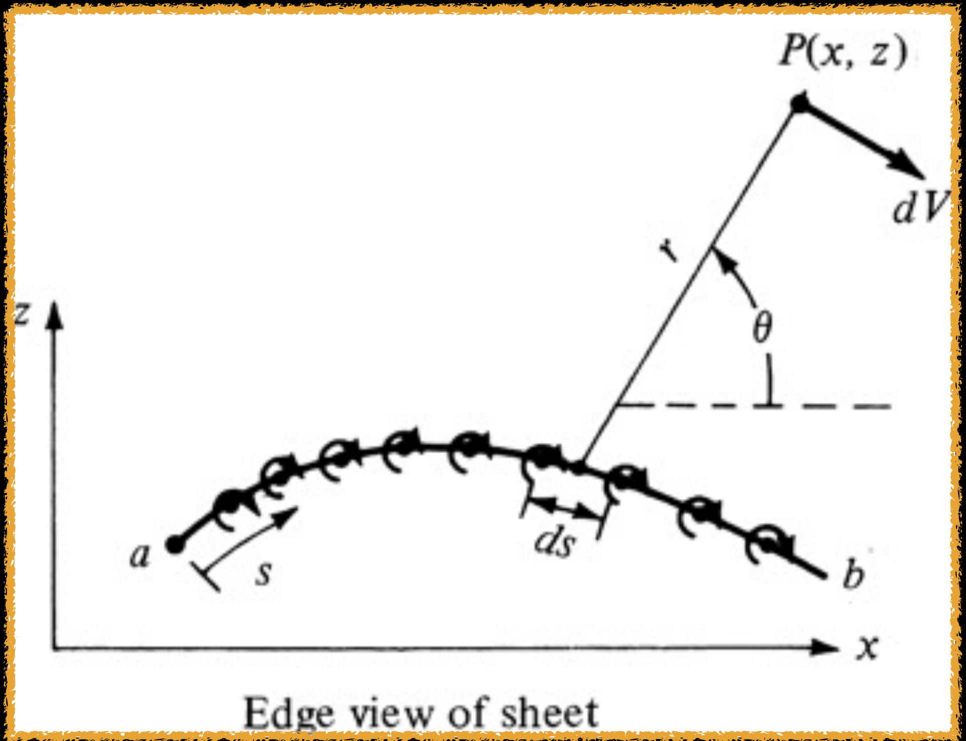


Vortex Filament



Vortex Sheet

Theoretical Solutions for Low-Speed Airfoils



induced velocity $dV = -\frac{\gamma ds}{2\pi r}$

velocity potential $d\Phi = -\frac{\gamma ds}{2\pi}\theta$

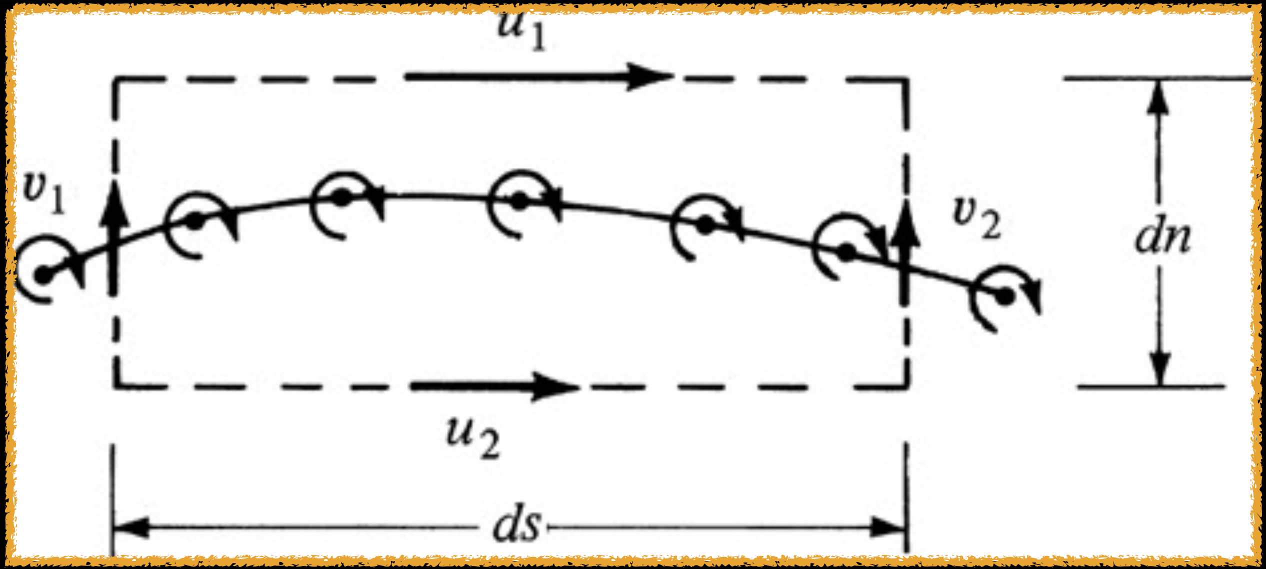
- *Velocity potential at P due to entire vortex sheet is:*

$$\Phi(x, z) = -\frac{1}{2\pi}\int_a^b \theta\gamma ds$$

- *Circulation around entire vortex sheet (sum of strengths of elemental vortices) is:*

$$\Gamma = \int_a^b \gamma ds$$

Circulation around an Airfoil



$$\Gamma = -(v_2 dn - u_1 ds - v_1 dn + u_2 ds) = \gamma ds$$

$$\Gamma = (u_1 - u_2)ds + (v_1 - v_2)dn = \gamma ds$$

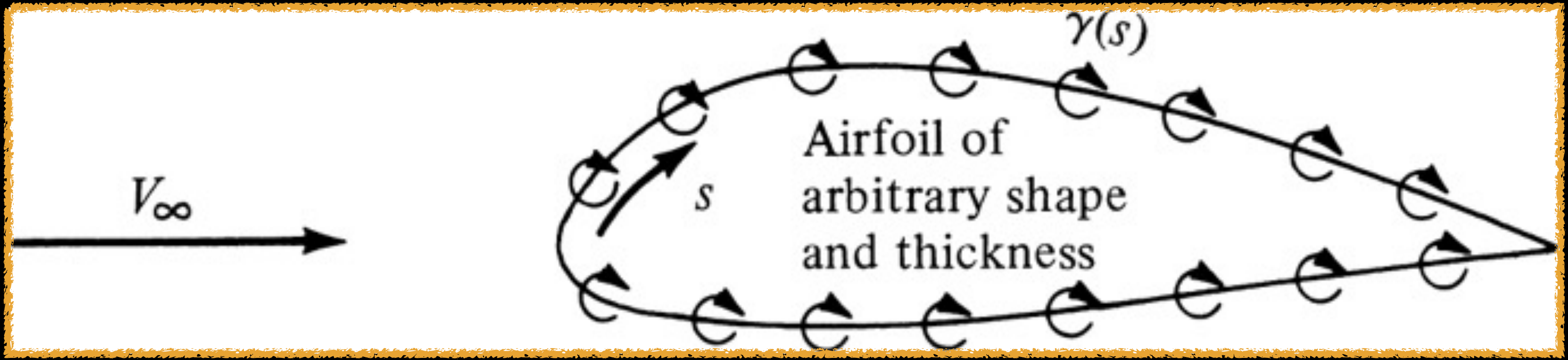
$$\text{assuming } dn \to 0, \gamma ds = (u_1 - u_2)ds$$

$$\Rightarrow \;\; \Gamma = u_1 - u_2$$

- *The local jump in tangential velocity across a vortex sheet is equal to the local sheet strength.*

Circulation around an Airfoil

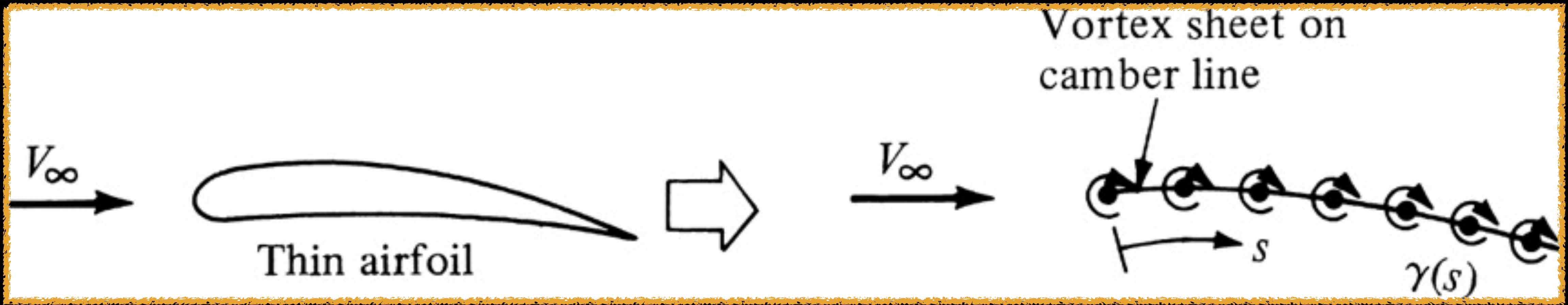
- *The concept of vortex sheet is instrumental in the analysis of low-speed characteristics of an airfoil.*



- *Replace airfoil surface with a vortex sheet of variable strength.*
- *Calculate strength of elemental vortices as a function of 's' such that when vortex induced velocity is aded to freestream, the streamline would represent the airfoil surface.*
- *The circulation around the airfoil is then given by:*

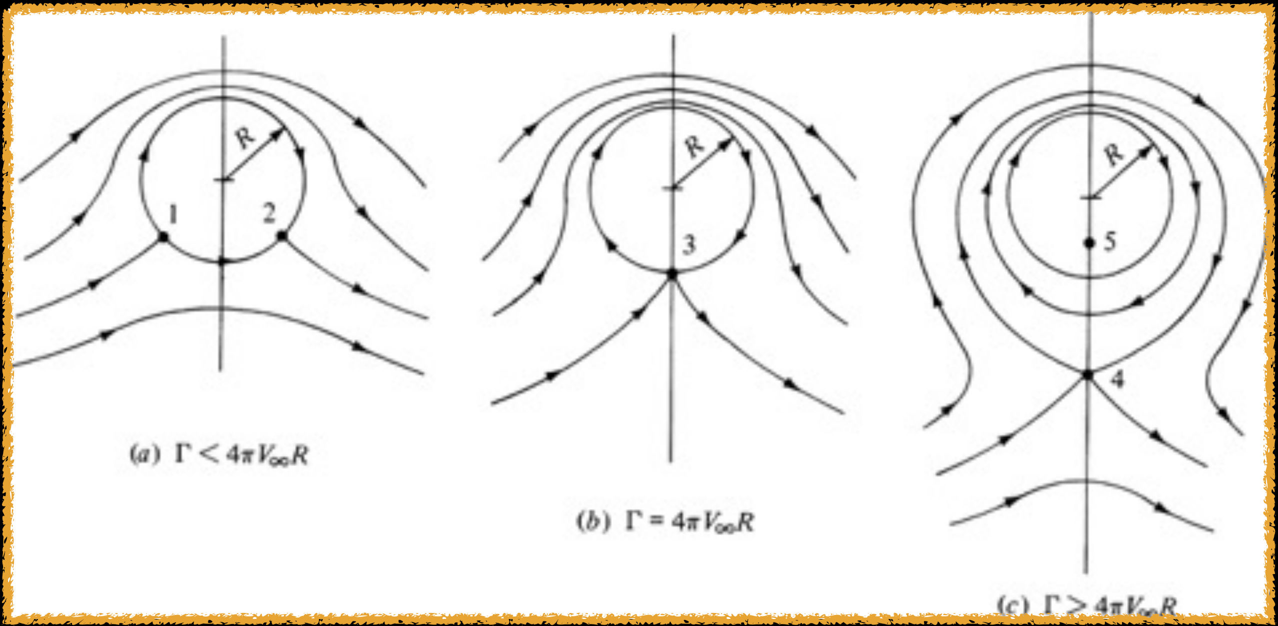
$$\Gamma = \int \gamma ds \;\Rightarrow\; L' = \rho_\infty V_\infty \Gamma \quad \textit{(Kutta-Joukowski theorem)}$$

Thin Airfoil Approximation

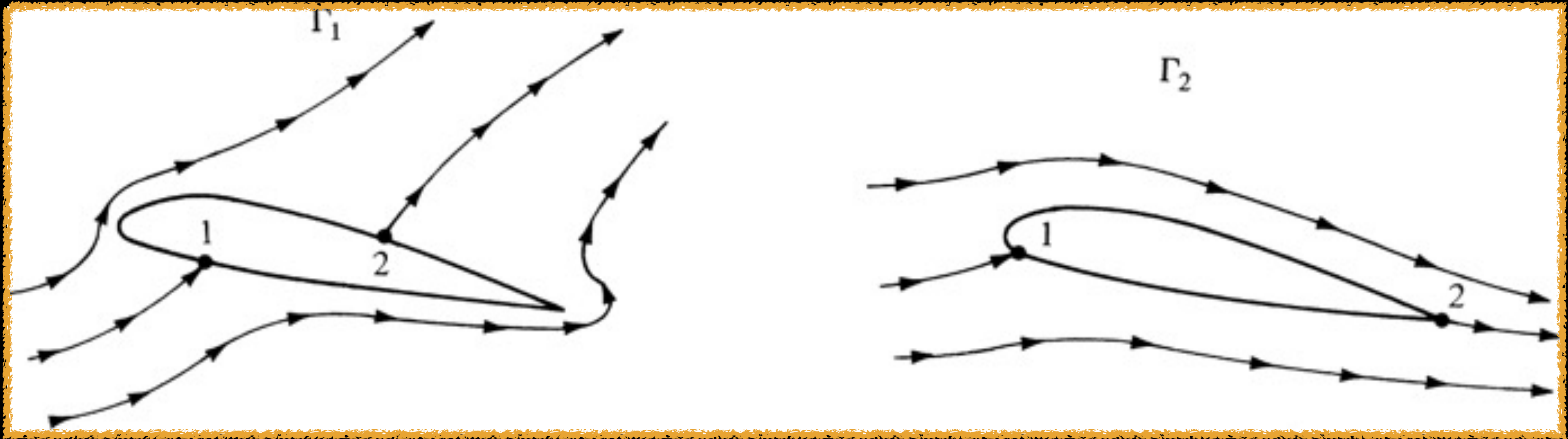


- *Imagine the airfoil is made very thin (top and bottom surfaces coincide).*
- *Airfoil can be represented with a single vortex sheet distributed over the camber line.*
- *The strength of the vortex sheet can be calculated such that when the induced velocity is added to the free stream velocity, the camber line become a streamline of the flow.*

The Kutta Condition

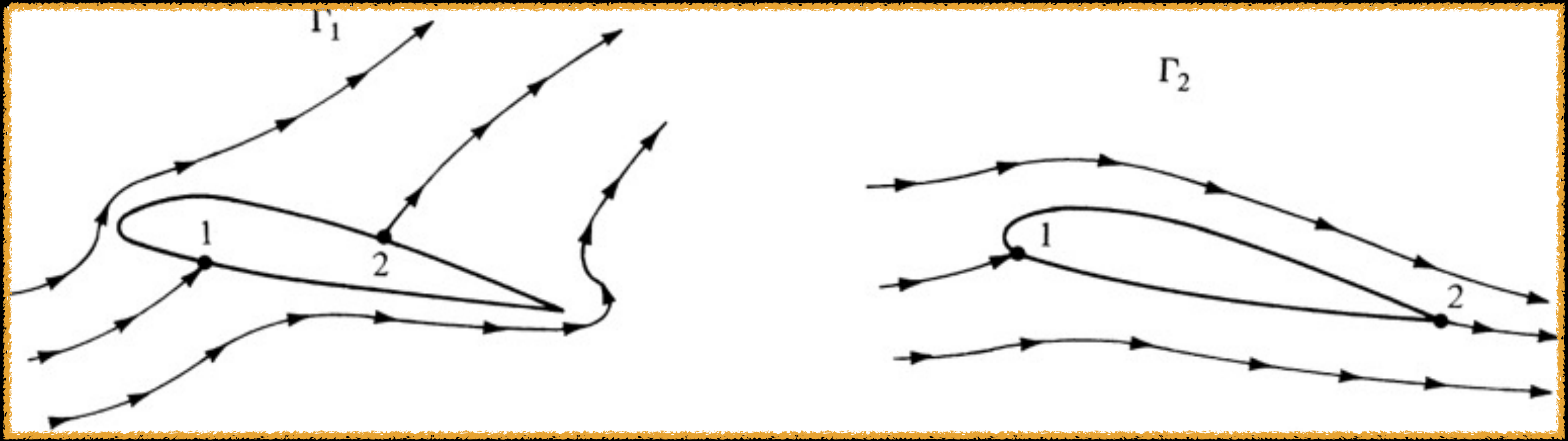


* *Infinite number of potential flow solution are possible depending on the choice of circulation magnitude.*



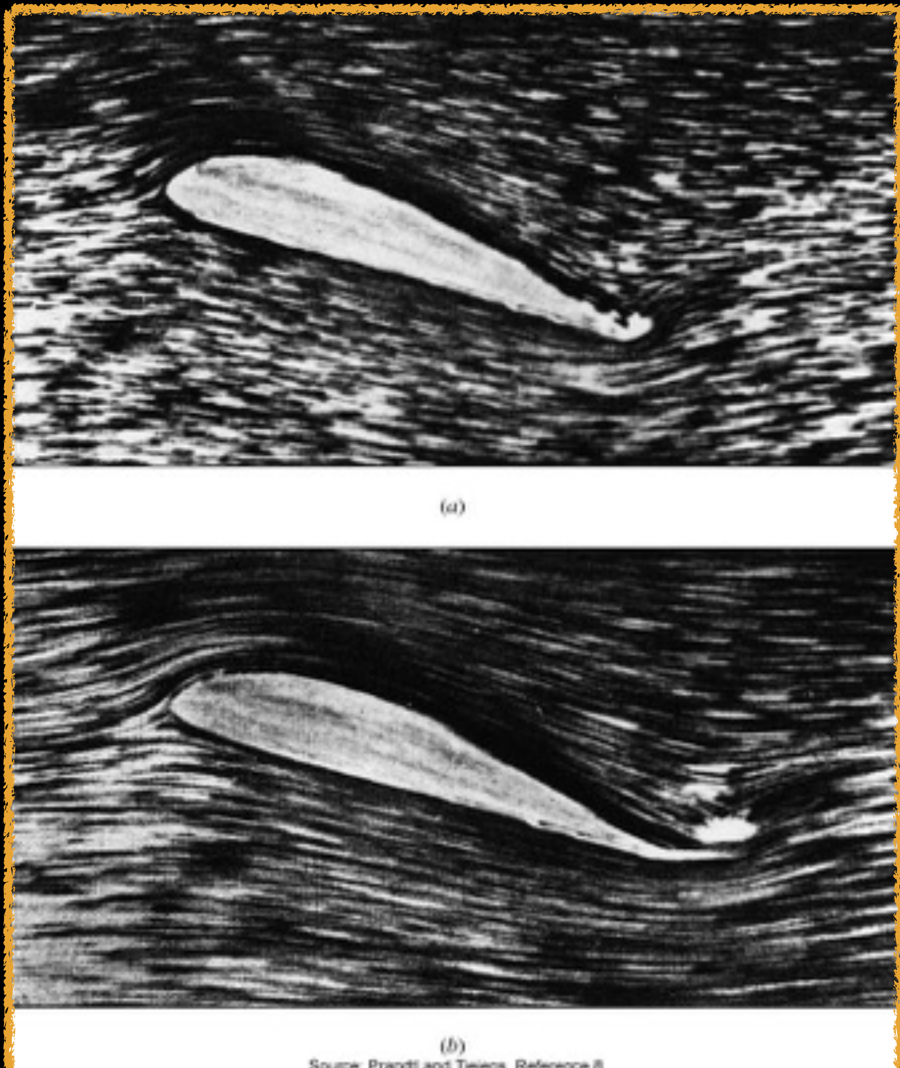
* *Similarly, for an airfoil, infinite number of potential flow solutions are possible.*
* *So, which one down pick?*

The Kutta Condition



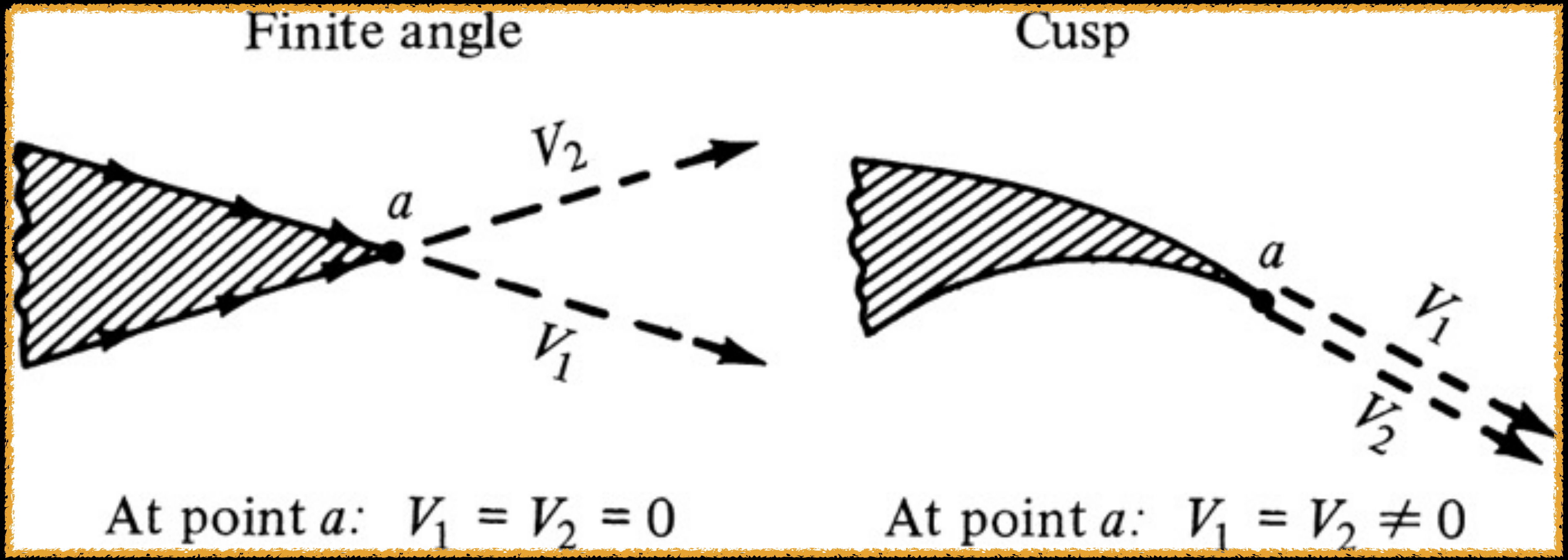
- *We know, a given angle of attack produces a single value of lift.*
- *So, which 'gamma' does nature choose?*
- *It has to fix the value of 'gamma'.*
- *To find out, let us look at an airfoil set into motion from a state of rest.*

The Kutta Condition



- *In (a), flow has just started.*
- *It tries to curl around the TE.*
- *Velocity becomes infinitely large at the sharp corner. Realistically impossible.*
- *Such flow is not tolerated very long by nature.*
- *So, flow leaves the top and bottom surfaces of the airfoil smoothly.*
- *Therefore, nature adopts that value of circulation which results in a smooth flow at the trailing edge - Kutta condition.*

The Kutta Condition

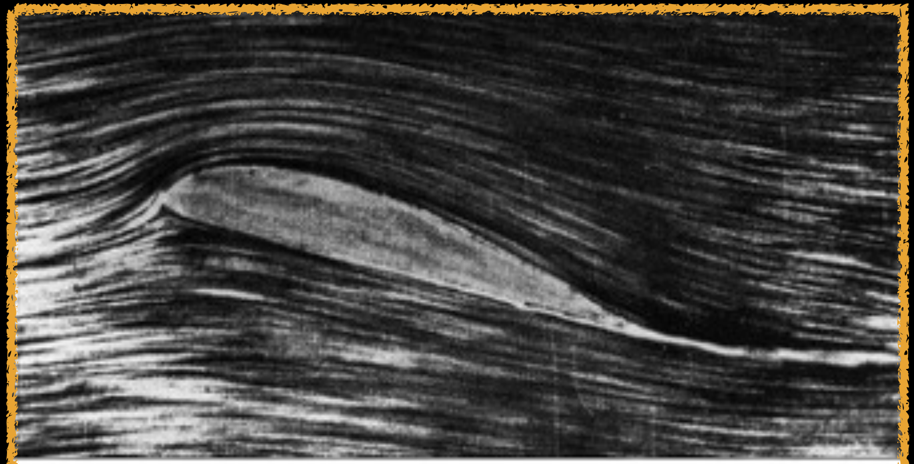


- *If the trailing edge angle is finite, then it is a stagnation point.*
- *If the trailing edge is a cusp, velocities leaving the top and bottom are finite and equal in magnitude an direction.*
- *In terms of vortex sheet, at the trailing edge:*

$$\gamma = \gamma(a) = V_1 - V_2 = 0$$

Lift Without Friction?

- *We know that lift on a airfoil is primarily due to surface pressure distribution (acts normal) and not due to shear stress (tangential).*
- *However, in a perfectly inviscid world, an airfoil would not produce lift. Sounds contradictory!!*
- *In reality, nature enforces the Kutta condition - i.e. the viscous boundary layer remains attached all the way to the TE.*

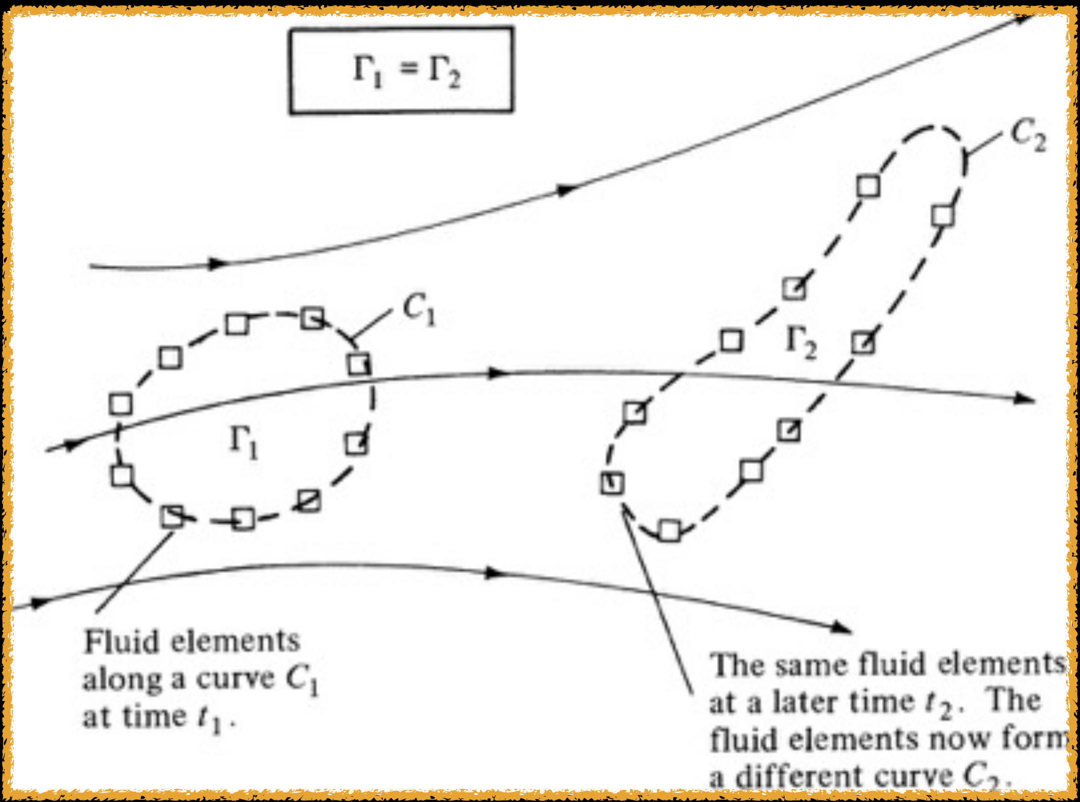


(c)
Source: Prandtl and Tietjens, Reference 8.

- *Lift, which is created by surface pressure distribution (inviscid phenomenon) cannot exist in an inviscid world.*

Kelvin's Circulation Theorem

- *How does nature enforce the Kutta condition? How does it generate this circulation for a given airfoil?*



$$\Gamma_1 = -\int_{C_1} V.ds; \qquad \Gamma_2 = -\int_{C_2} V.ds$$

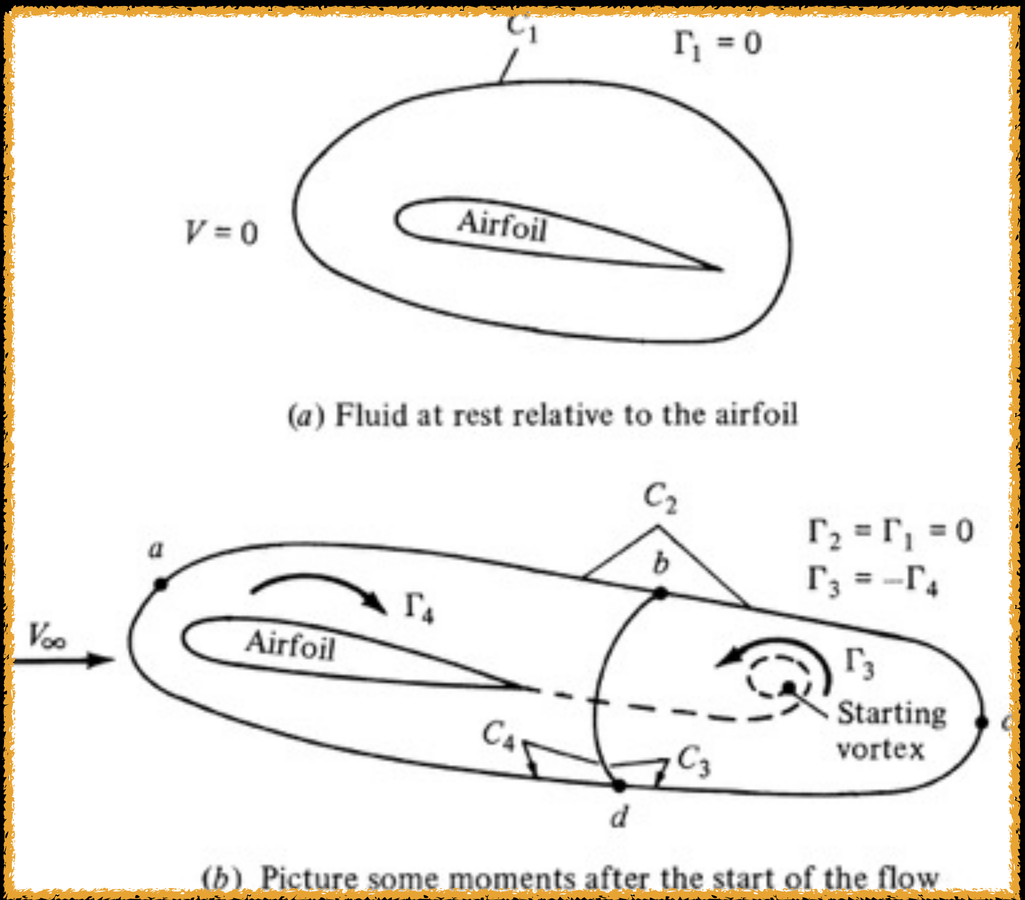
$$\Gamma_1 = \Gamma_2$$

$$\boxed{\frac{d\Gamma}{Dt} = 0}$$

- *Kelvin's Circulation Theorem: Time rate of change of circulation around a closed contour consisting of the sam fluid elements is zero.*

Kelvin's Circulation Theorem

- *So, how is circulation generated around an airfoil?*

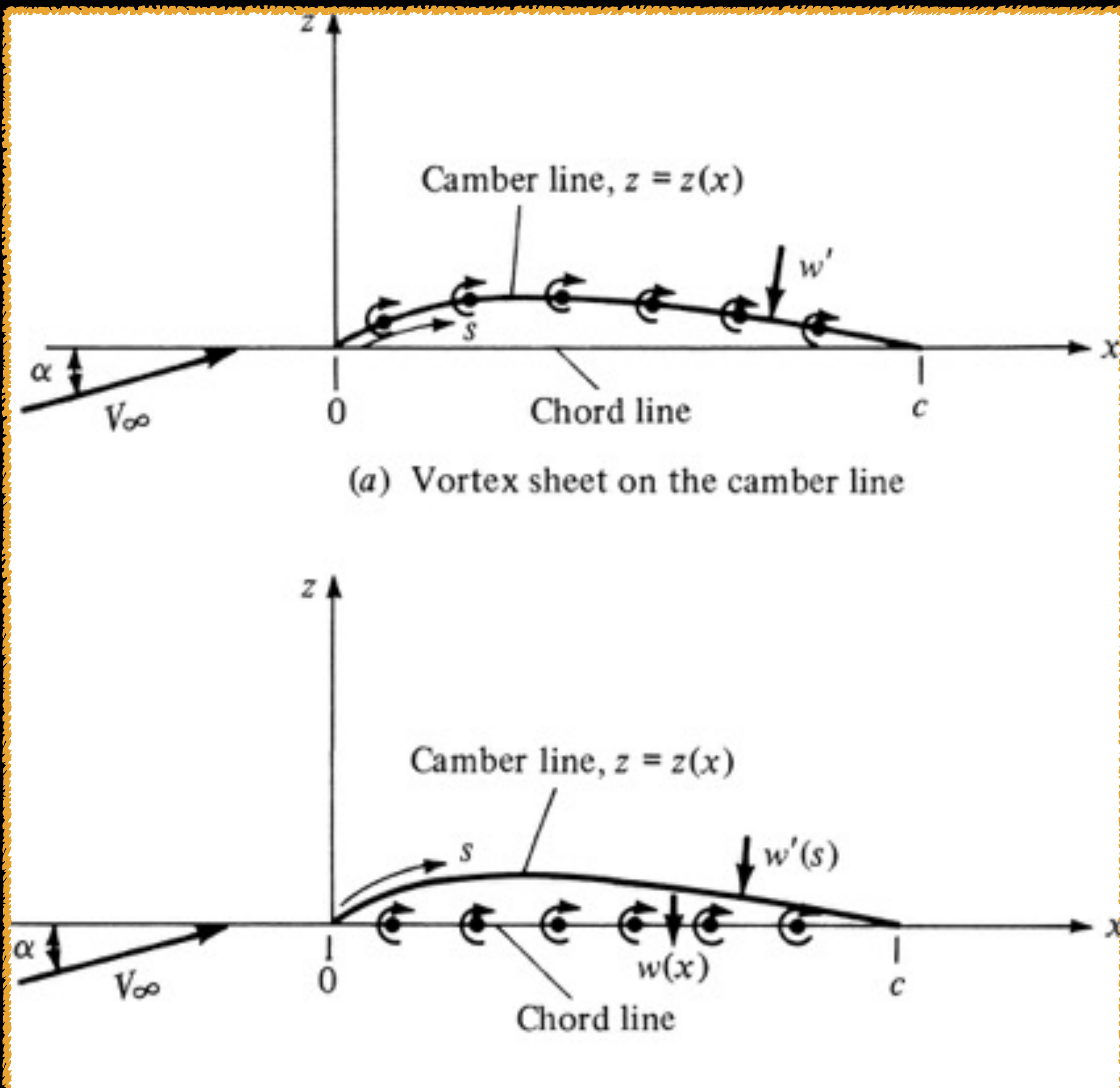


- *Let $V = 0$, so circulation = 0, around c1.*
- *As the flow is started over the airfoil, large velocity gradients at the trailing edge generate vorticity that rolls up downstream - starting vortex.*
- *Due to Kelvins theorem, the starting vortex has to induce equal and opposite circulation on the airfoil.*
- *Ideally, the starting vortex remains forever downstream. Realistically, it dissipates due to viscous action.*
- *Example!!*

The Thin Airfoil Theory

- *Thin Airfoil*
  - *A vortex sheet placed along the camber line*
  - *Our purpose is to calculate the variation in vorticity such that the camber line becomes a streamline.*
  - *Kutta condition at the trailing edge should be satisfied.*
  - *Once variation of vorticity that satisfies these conditions is found, total circulation is calculated by integrating vorticity from LE to TE.*
  - *Lift can then be calculated using the Kutta-Joukowski theorem.*

The Thin Airfoil Theory



(a) Vortex sheet on the camber line

(b) Vortex sheet on the chord line

- *Assumptions*
  - *Thin airfoil, i.e. camber line is close to the chord line.*
  - *Vortex sheet falls approximately on the chord line*

$$\gamma = \gamma(x)$$

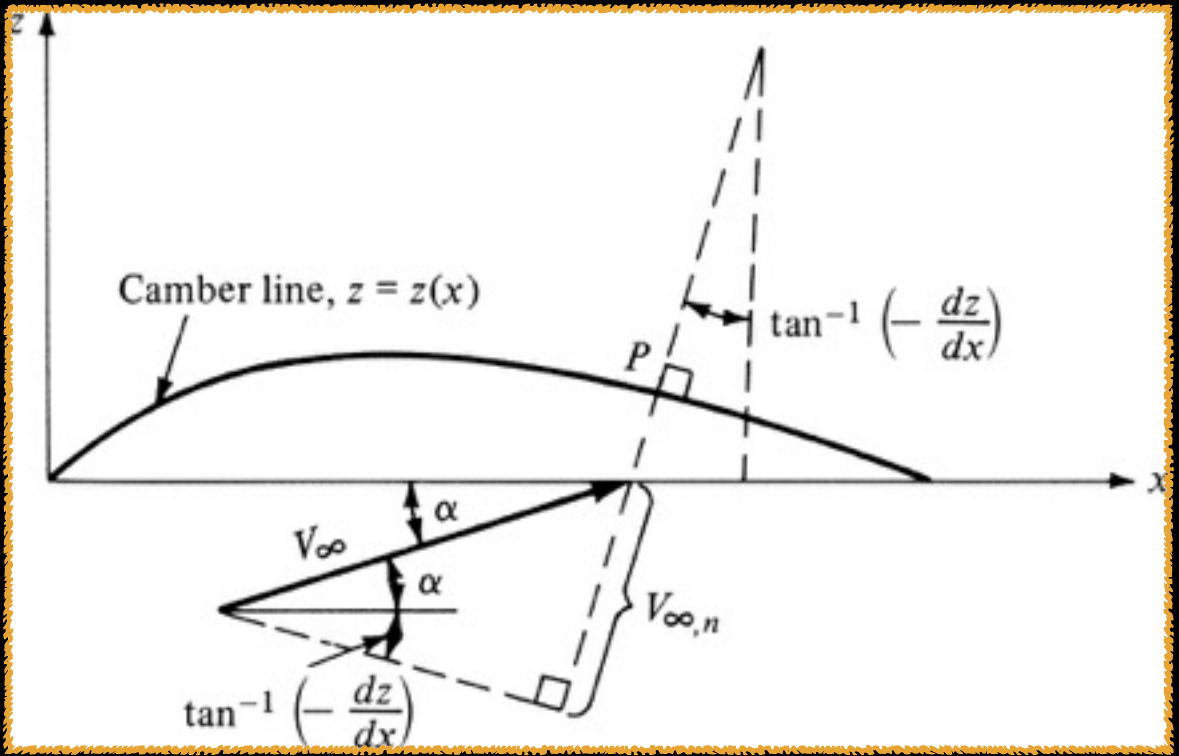
$$\gamma(c) = 0$$

- *The strength of the vortex sheet on the chord line must be determined such that the camber line (not the chord line) is a streamline.*

The Thin Airfoil Theory

- *For the camber line to be a streamline, the component of velocity normal to the camber line must be zero.*

$$V_{\infty,n} + w'(s) = 0$$



$$V_{\infty,n} = V_\infty sin\left[\alpha + tan^{-1}\left(-\frac{dz}{dx}\right)\right]$$

For small angles of attack,

$$sin\theta = tan\theta = \theta$$

$$V_{\infty,n} = V_\infty\left[\alpha - \left(\frac{dz}{dx}\right)\right]$$

The Thin Airfoil Theory

$$\int_0^c \frac{\gamma(\xi)d\xi}{(x-\xi)} = V_\infty \left[\alpha - \left(\frac{dz}{dx}\right)\right]$$

fundamental equation of thin airfoil theory

- *In the above equation,*
  - *A vortex sheet placed along the camber line*
  - *Our purpose is to calculate the variation in vortex strength such that the camber line becomes a streamline.*
  - *The central challenge is to calculate the vortex strength variation subject to the Kutta condition, i.e.*

$$\gamma(c) = 0$$

The Thin Airfoil Theory

- *Consider a symmetric airfoil*
  - *No camber.*
  - *Camber line is coincident with the chord line (dz/dx = 0). Therefore,*

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)d\xi}{x-\xi} = V_\infty \alpha$$

$$\xi \rightarrow \theta : \quad \xi = \frac{c}{2}(1 - cos\theta)$$

$$x \rightarrow \theta_0 : \quad x = \frac{c}{2}(1 - cos\theta_0)$$

$$\boxed{\text{solving, } \gamma(\theta) = 2\alpha V_\infty \frac{1 + cos\theta}{sin\theta}}$$

The Thin Airfoil Theory

- *Now, total circulation around the airfoil is given by:*

$$\Gamma = \int_0^c \gamma(\xi)d\xi = \frac{c}{2}\int_0^\pi \gamma(\theta)sin\theta d\theta$$

- *Simplifying,*

$$\Gamma = \pi\alpha c V_\infty$$

- *Lift can now be calculated using the Kutta-Jouski theorem:*

$$L = \rho_\infty V_\infty \Gamma = \pi\alpha c \rho_\infty V_\infty^2$$

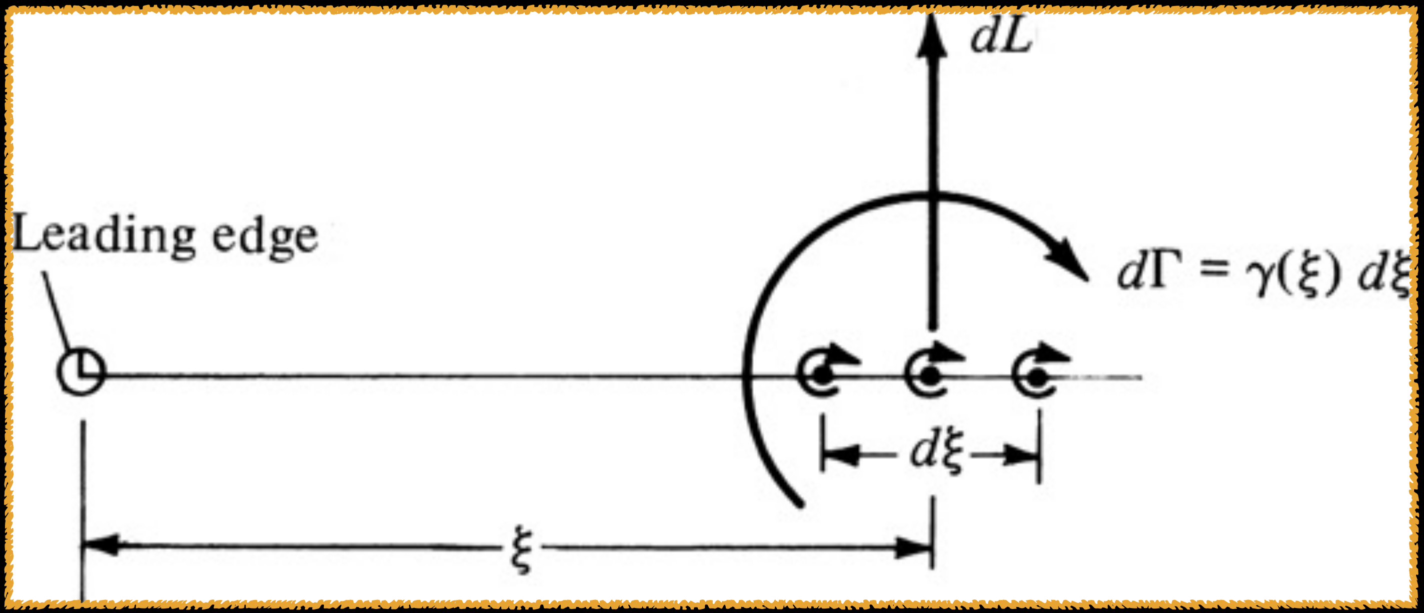
- *Lift Coefficient is then:*

$$c_l = \frac{L'}{q_\infty S} = 2\pi\alpha$$

Lift Coefficient is proportional to the angle of attack.

The Thin Airfoil Theory

- *Now, let us calculate moment about the leading edge:*



$$dL = \rho_\infty V_\infty d\Gamma$$

$$dM = -\xi(dL)$$

(about LE)

- *Total moment about the LE due to the entire vortex sheet is:*

$$M'_{LE} = -\int_0^c \xi(dL) = -\rho_\infty V_\infty \int_0^c \xi\gamma(\xi)d\xi = -q_\infty c^2 \frac{\pi\alpha}{2}$$

The Thin Airfoil Theory

- *The moment coefficient about the leading edge:*

$$c_{m,le} = \frac{M'_{LE}}{q_\infty S c} = -\frac{\pi\alpha}{2} = -\frac{c_l}{4}$$

- *Moment coefficient about the quarter-chord point is:*

$$c_{m,c/4} = c_{m,le} + \frac{c_l}{4} = 0$$

- *Center of pressure: Moments are zero*
- *Aerodynamic Center: Moments are independent of angle of attack.*
- *For a symmetrical airfoil, the quarter-chord location is both the center of pressure and the aerodynamic center.*

The Cambered Airfoil

$$\int_0^c \frac{\gamma(\xi)d\xi}{(x-\xi)} = V_\infty \left[\alpha - \left(\frac{dz}{dx}\right)\right]$$

- *For a cambered airfoil, dz/dx is finite.*

$$\frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)sin\theta d\theta}{cos\theta - cos\theta_0} = V_\infty \left(\alpha - \frac{dz}{dx}\right)$$

- *Solving, we obtain:*

$$\gamma(\theta) = 2V_\infty \left(A_0 \frac{1+cos\theta}{sin\theta} + \sum_{n=1}^{\infty} A_n sin(n\theta)\right)$$

- *A_0 and A_n are Fourier coefficients that depend on shape of the camber line and angle of attack.*

The Cambered Airfoil

- *The Fourier coefficients are:*

$$A_n = \alpha - \frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx} d\theta_0$$

$$A_n = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx} cos(n\theta_0) d\theta_0$$

- *Now, circulation due to the entire vortex sheet from LE to TE is:*

$$\Gamma = \int_0^{c} \gamma(\xi) d\xi = cV_\infty \left(\pi A_0 + \frac{\pi}{2} A_1\right)$$

- *The lift coefficient is then given by:*

$$c_l = 2\pi \left[\alpha + \frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}(cos\theta_0 - 1) d\theta_0\right]$$

The Cambered Airfoil

- *Similarly, the moment coefficient about the LE is given by:*

$$c_{m,le} = -\left[\frac{c_l}{4} + \frac{\pi}{4}(A_1 - A_2)\right]$$

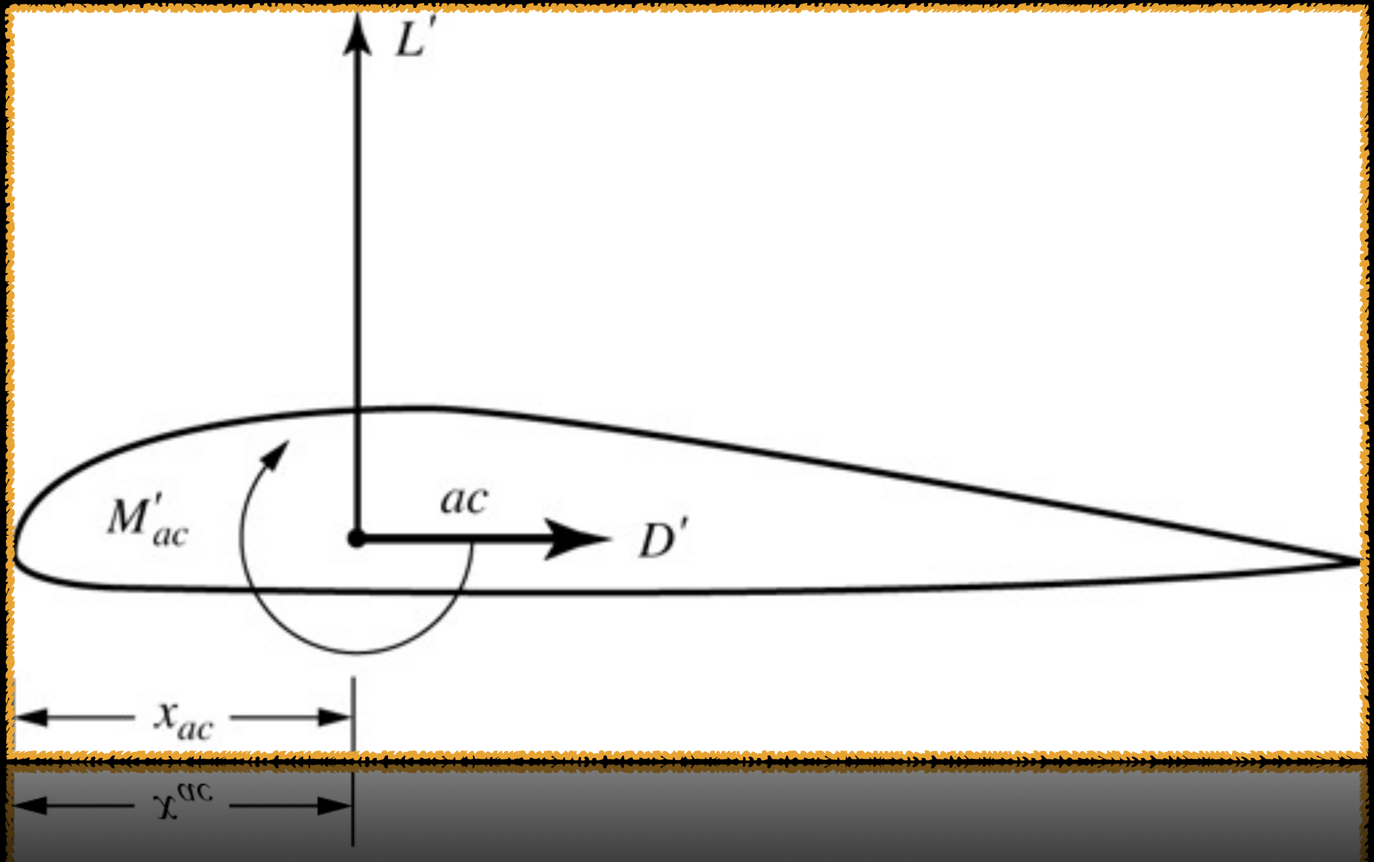
- *The moment coefficient about the quarter-chord location is given by:*

$$c_{m,c/4} = -\left[\frac{\pi}{4}(A_2 - A_1)\right]$$

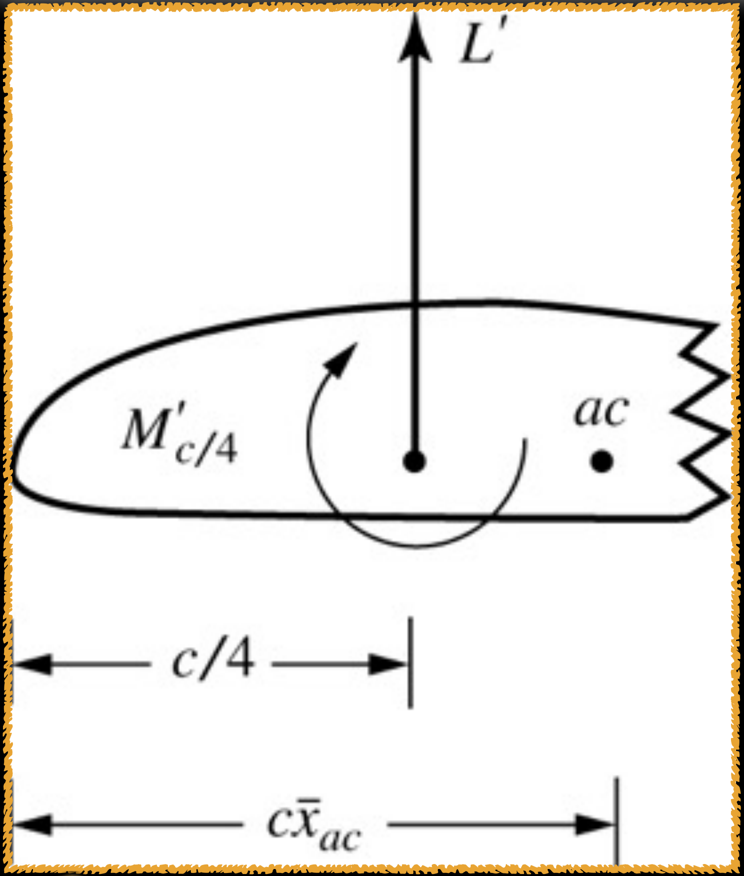
- *From the above equation, we can see that:*
    - *The quarter-chord is not the center of pressure for a cambered airfoil.*
    - *However, since the moment is independent of the angle of attack, the quarter-chord is the theoretical aerodynamic center for a cambered airfoil.*

The Aerodynamic Center

- *The aerodynamic center is that point on a body about which the aerodynamically generated moment is independent of the angle of attack.*
- *For most airfoils, it is close to, but not exactly at the quarter-chord location.*
- *So, how do we calculate its location?*



The Aerodynamic Center



- *Taking moments about the aerodynamic center, we get:*

$$M'_{ac} = L'(c\bar{x}_{ac} - c/4) + M'_{c/4}$$

$$\text{or, } c_{m,ac} = c_l(\bar{x}_{ac} - 0.25) + c_{m,c/4}$$

- *Rearranging the above equation and recognizing that the slopes of lift and moment coefficients are constants before stall, we have:*

$$\bar{x}_{ac} = -\frac{m_0}{a_0} + 0.25 \qquad \text{where, } \frac{dc_l}{d\alpha} = a_0; \quad \frac{dc_{m,c/4}}{d\alpha} = m_0$$

Modern Low Speed Airfoils

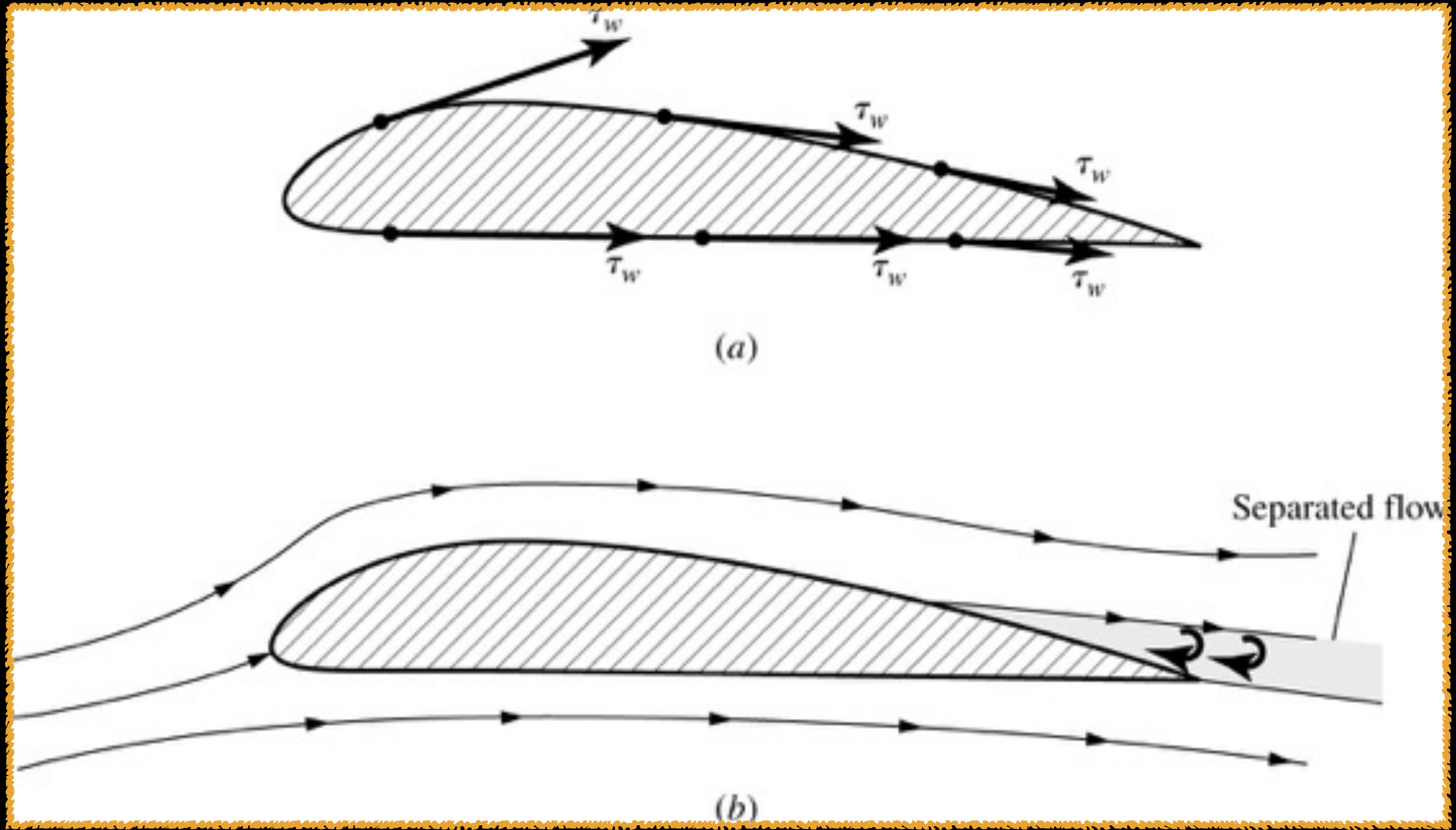
- *The standard NACA airfoils were based on experimental data in the 1930's and 1940's.*
- *New NASA airfoils were designed using source and vortex panel methods along with numerical prediction of viscous flow behavior. E.g. GA(W)-1 airfoil.*

- *Large leading edge to flatten the the pressure coefficient peak.*
- *The trailing edge is cusped to increase the camber and loading.*
- *The design discourages flow separation over the top surface leading to high lift coefficient.*

Viscous Flow: Airfoil Drag

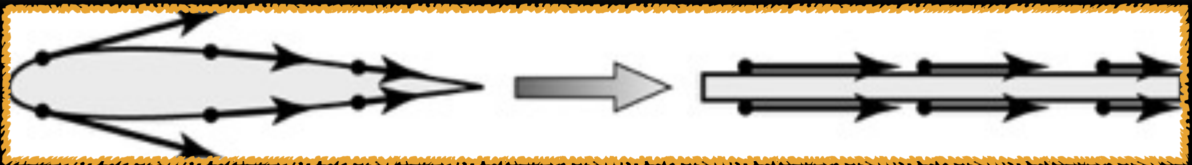
- *Lift: Primarily due to pressure distribution on airfoil surface.*
  - *Shear stress distribution in the lift direction is generally very small.*
  - *Lift can therefore be accurately calculated assuming inviscid flow in conjunction with Kutta condition at the TE.*
- *Drag: Predicting drag using an inviscid approach results in zero drag (d'Alembert's paradox).*
  - *However, when friction is included, this paradox is immediately removed.*

Viscous Flow: Airfoil Drag



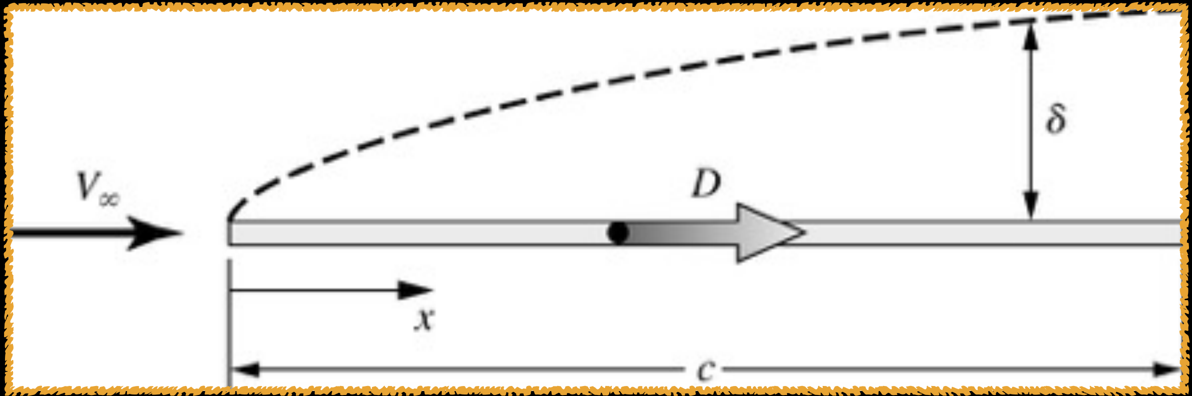
- *Skin Friction Drag*
  - *Due to shear stress acting on the surface.*
- *Pressure Drag (form drag)*
  - *Due to flow separation.*

Skin-Friction Drag: Laminar Flow



Assume that skin-friction for airfoil is same as that for a flat plate.

- *The above assumption becomes more accurate for a thinner airfoil and small angles of attack.*



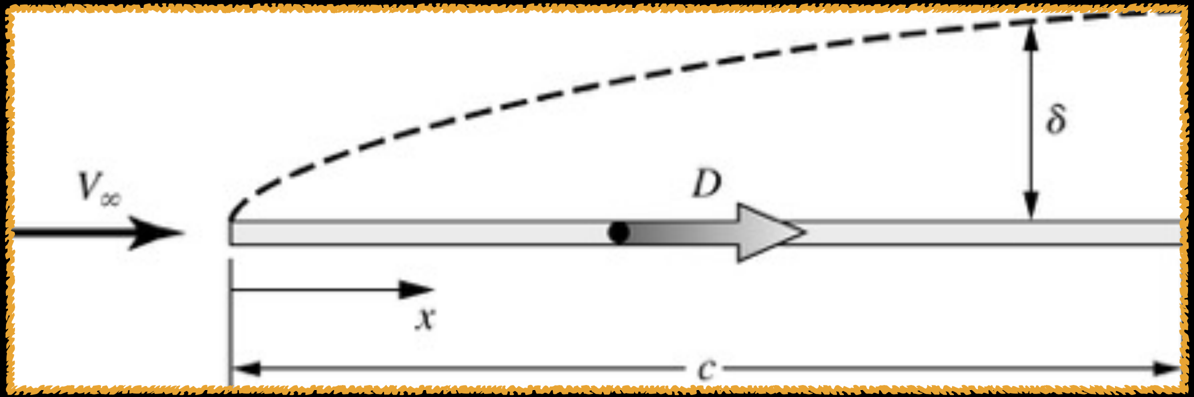
$$\delta = \frac{5.0x}{\sqrt{Re_x}} \qquad Re_x = \frac{\rho_e V_\infty x}{\mu_\infty}$$

- *The total skin-friction drag is given by:*

$$D_f = 2D_{f,top} = 2D_{f,bottom}$$

$$\text{where } c_f \equiv \frac{D_{f,top}}{q_\infty S} = \frac{D_{f,bottom}}{q_\infty S} = \frac{1.328}{\sqrt{Re_c}}$$

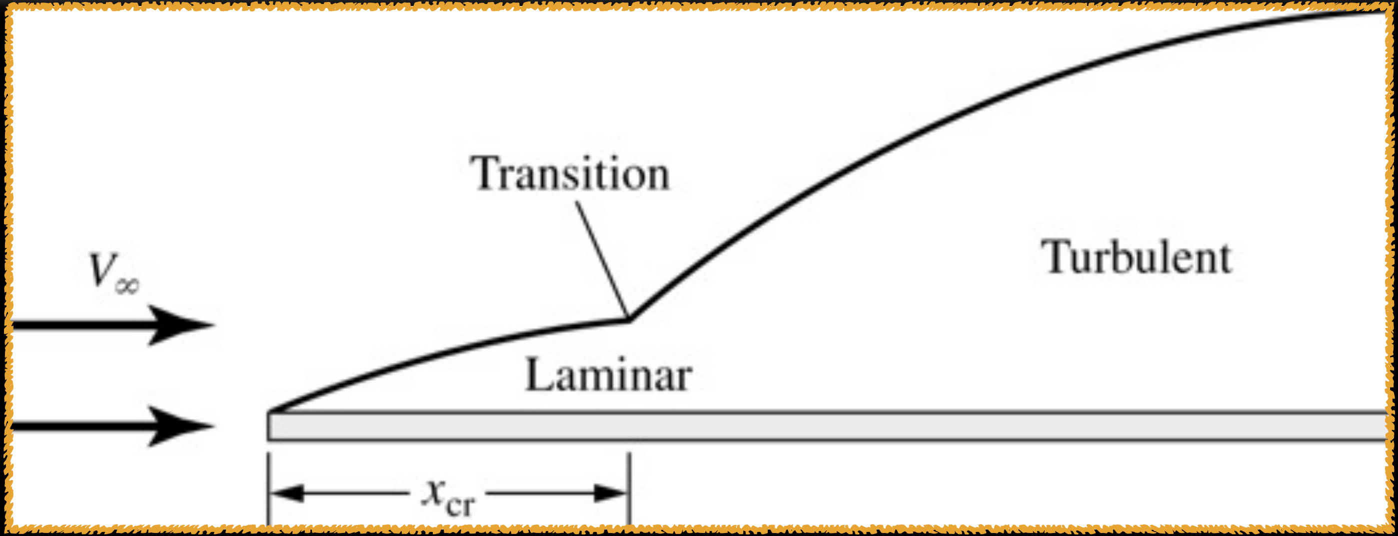
Skin-Friction Drag: Turbulent Flow



- *In contrast to laminar flow, there are no analytical solution for turbulent flow.*
- *All analyses of turbulent flow are approximate.*

$$\delta = \frac{0.37x}{Re_x^{1/5}} \qquad C_f = \frac{0.074}{Re_c^{1/5}}$$

Skin-Friction Drag: Transition Flow

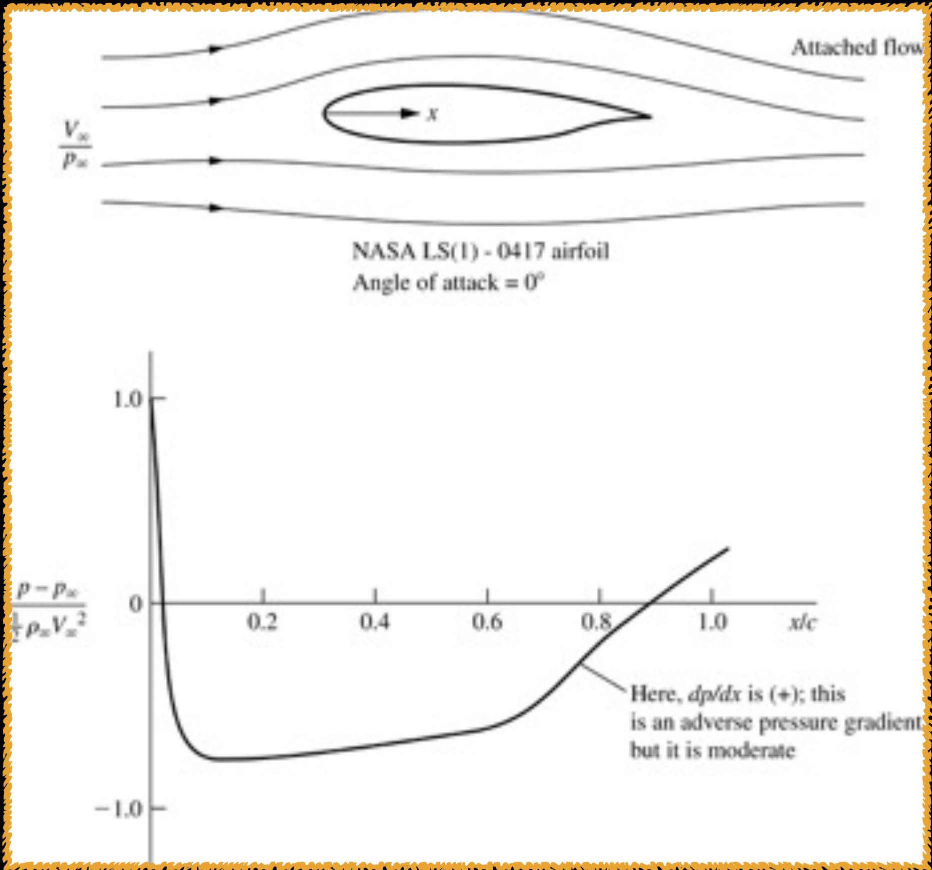


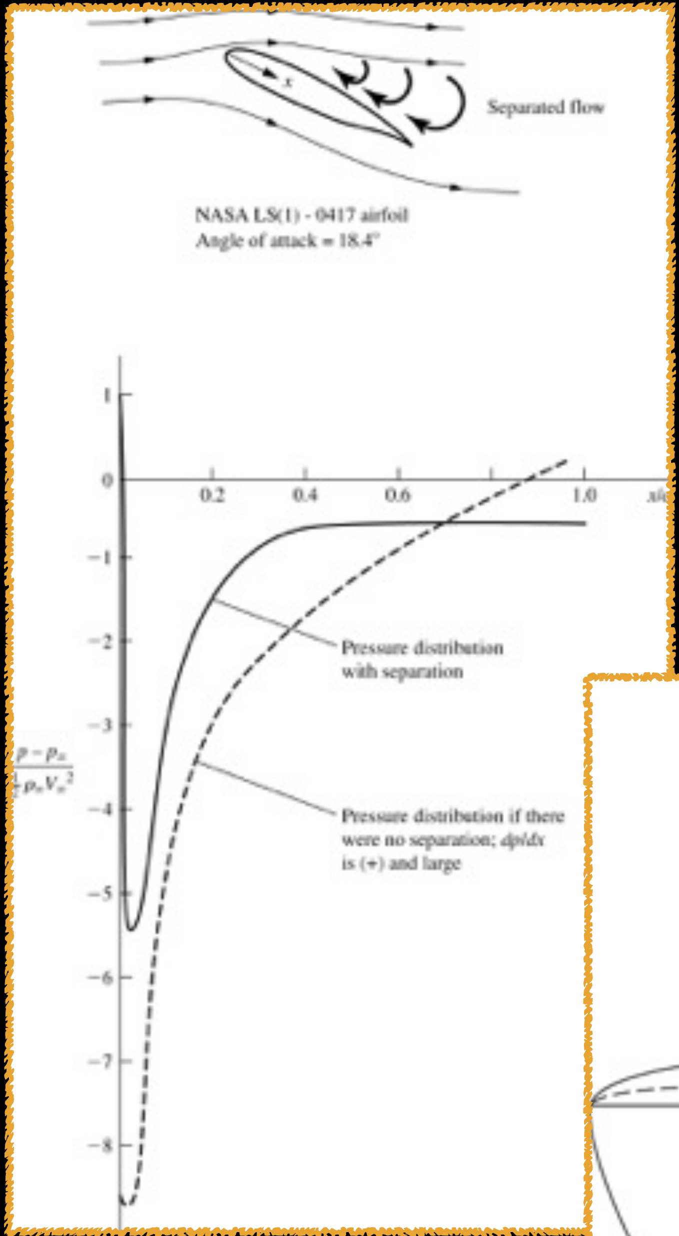
- *Flow always starts out as laminar at the leading edge, then becomes unstable and transitions into a turbulent flow.*
- *The value of x where transition takes place is the critical value* x_{cr}.

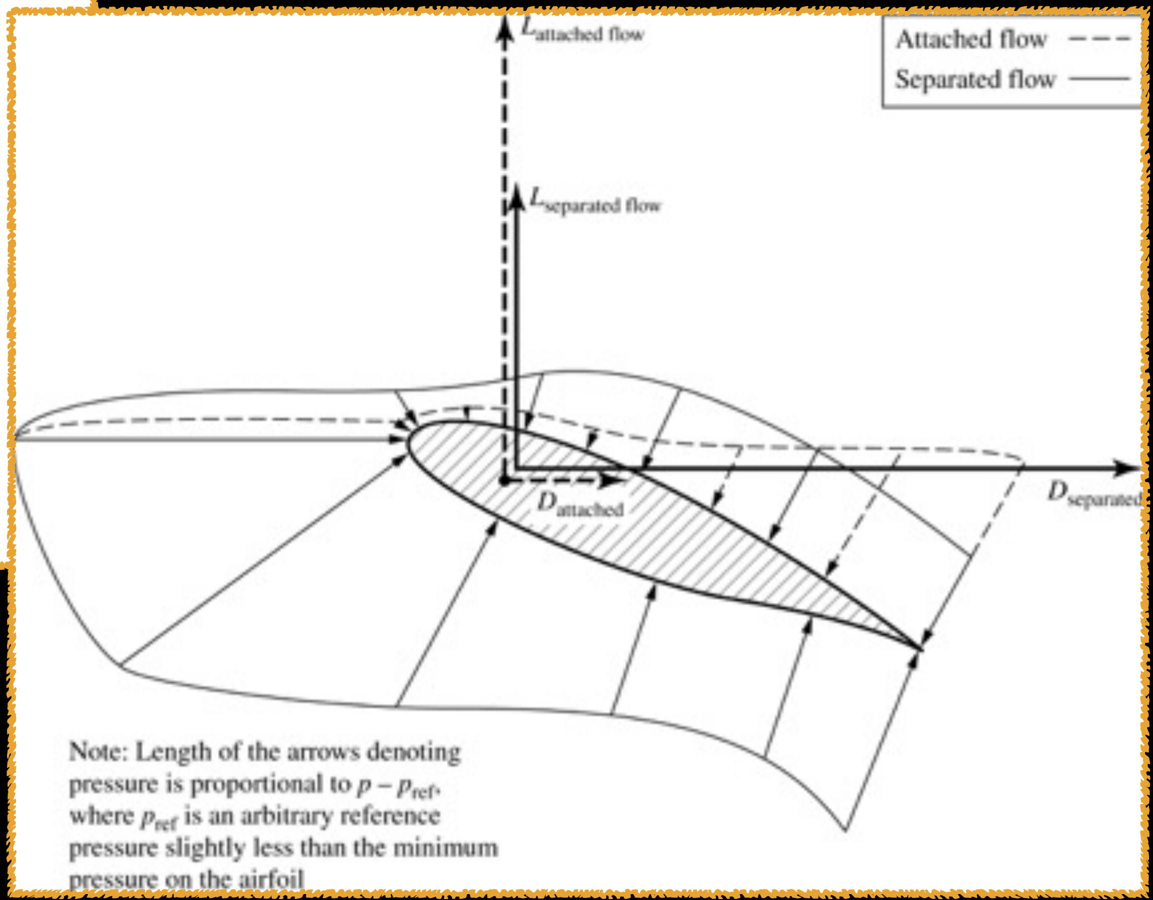
$$Re_{x_{cr}} = \frac{\rho_\infty V_\infty x_{cr}}{\mu_\infty}$$

Flow Separation

- *Pressure drag is caused by flow separation.*

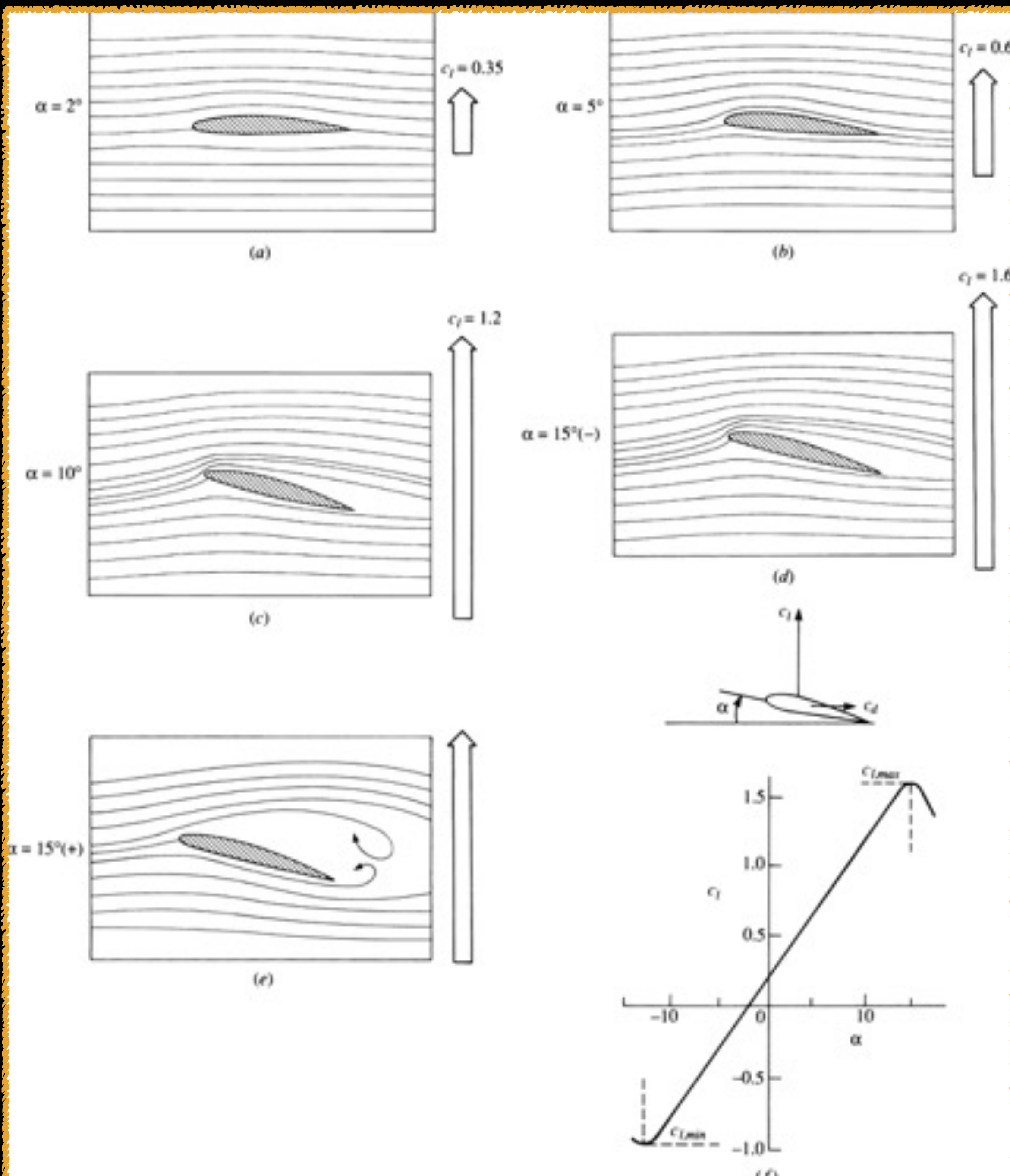






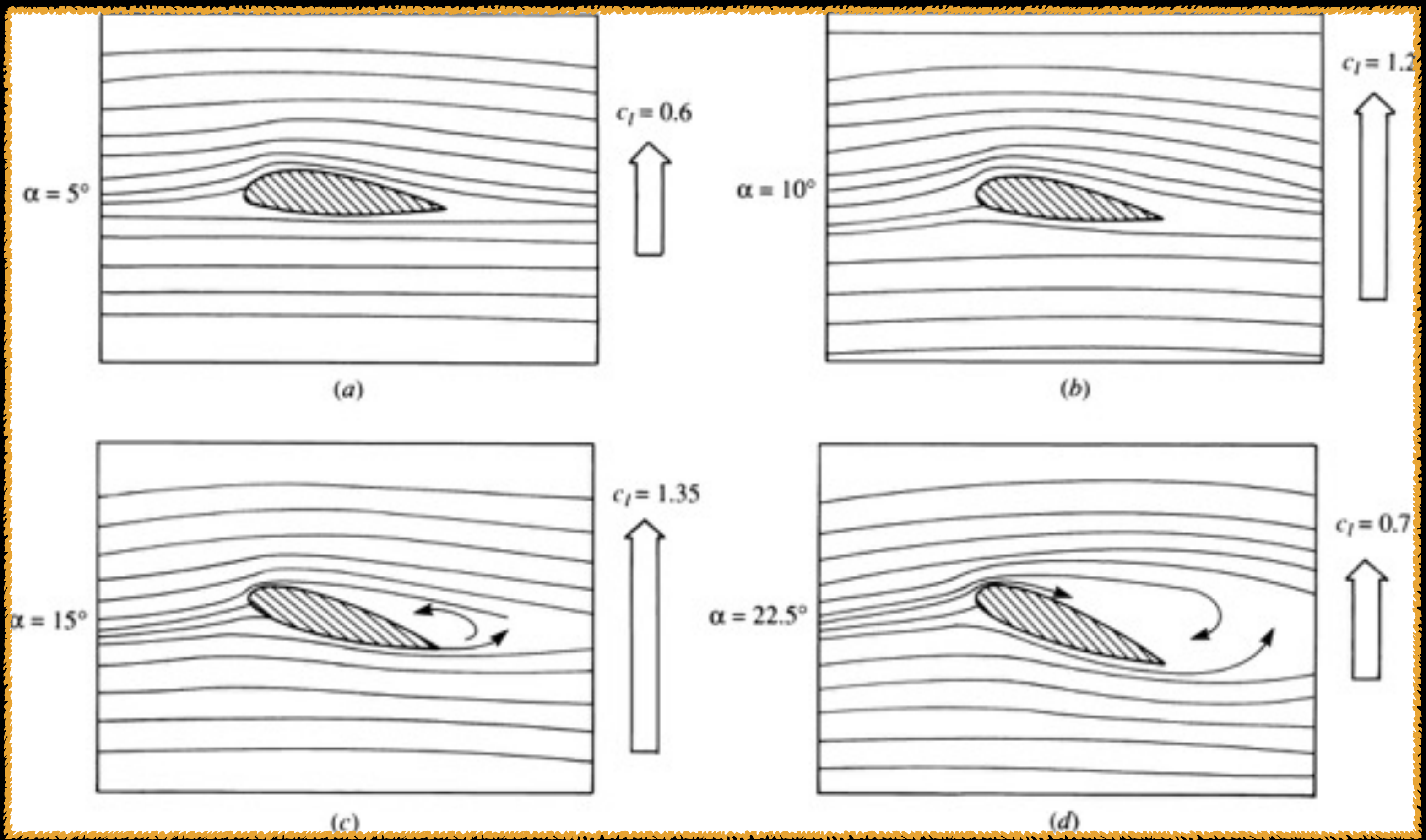
Flow Over An Airfoil - The Real Case

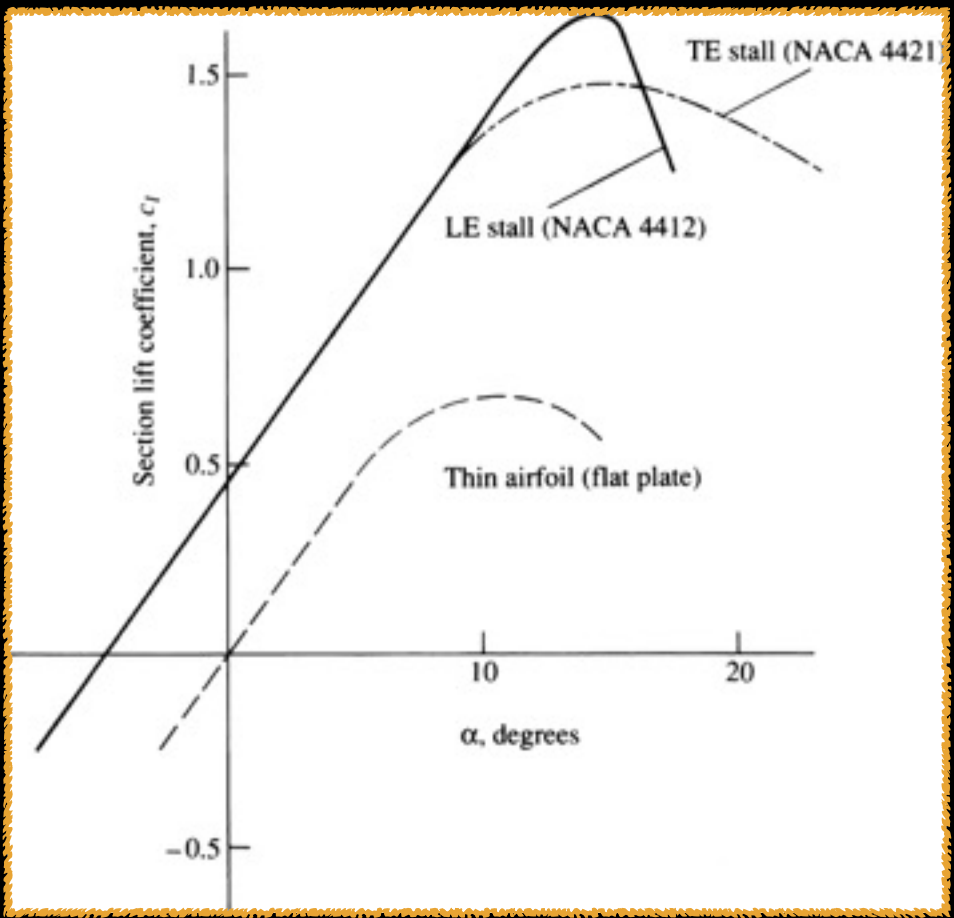
- *In the real case, flow separation occurs over the top surface of the airfoil when the angle of attack exceeds the stall angle.*



- *Leading-Edge stall*
  - *Characteristic of relatively thin airfoils.*
  - *Thickness-to-chord ratios usually between 10% - 16% of the chord length.*

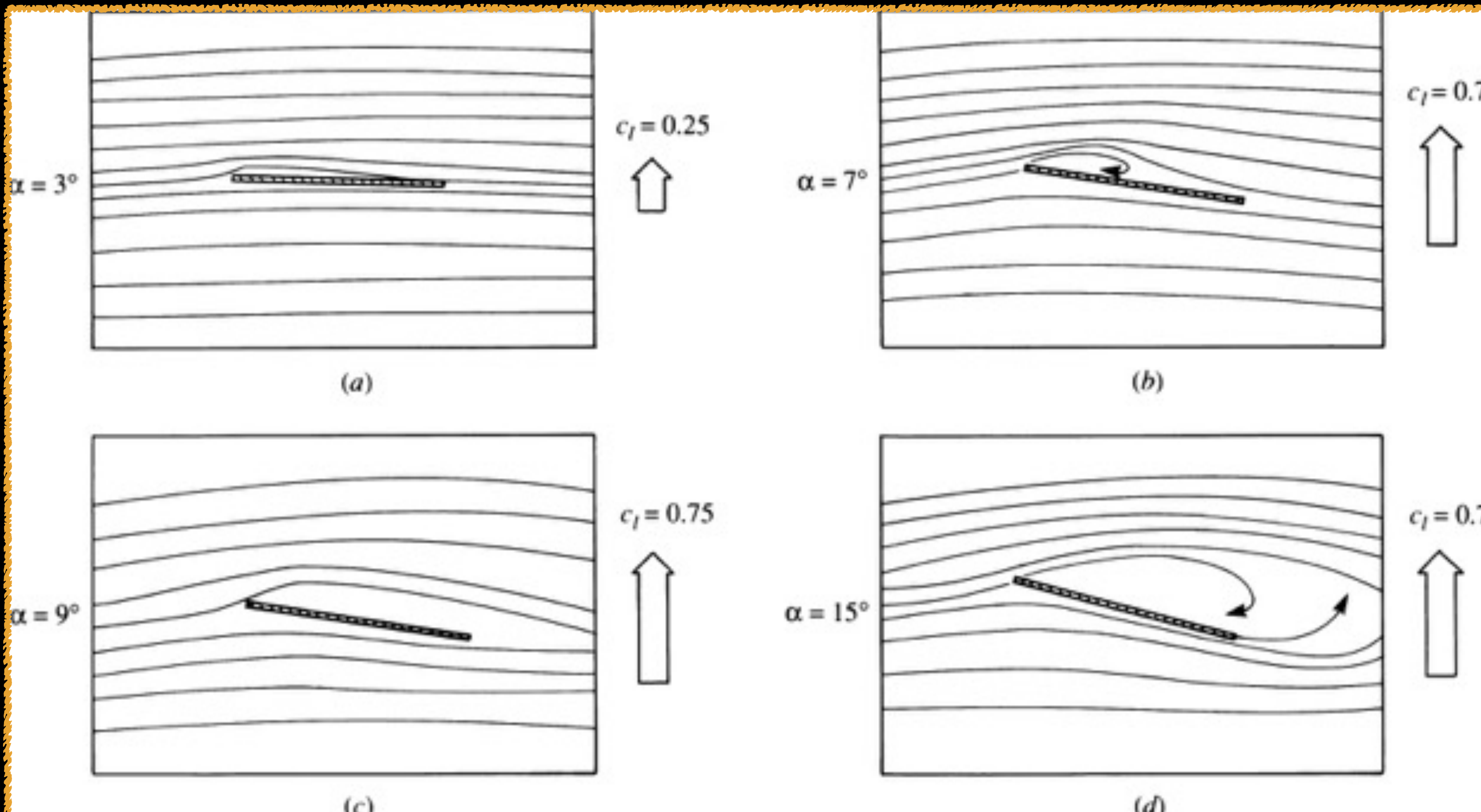
Flow Over An Airfoil - The Real Case

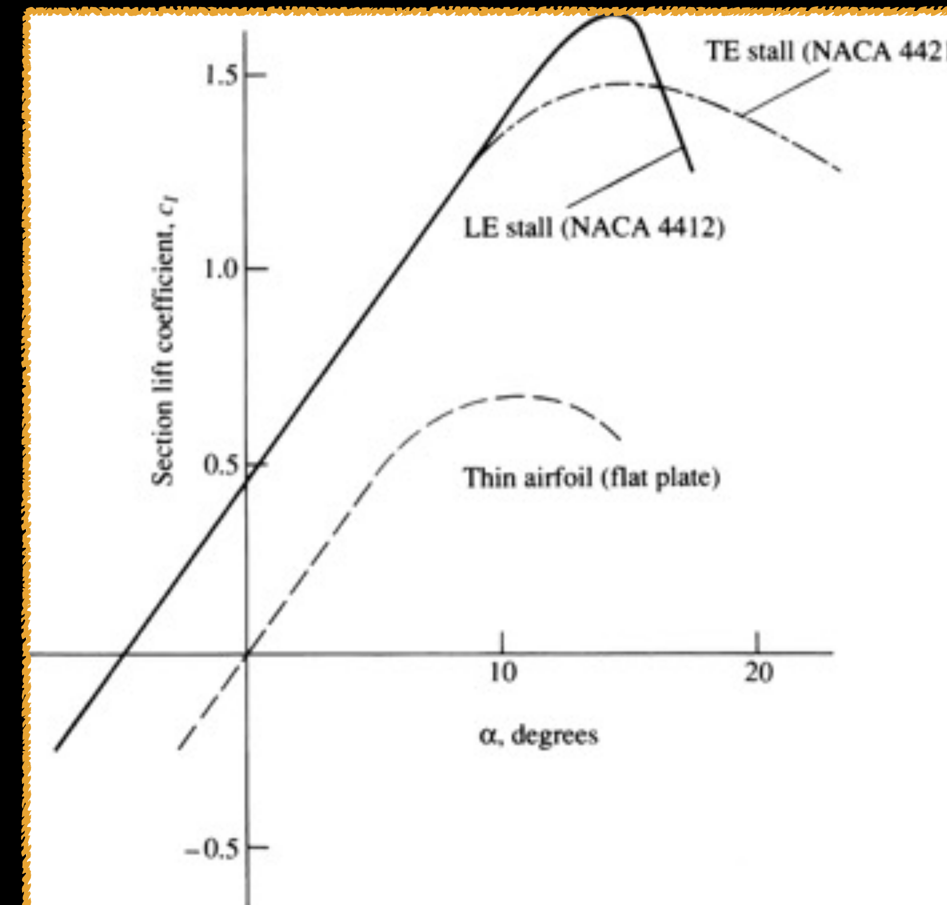


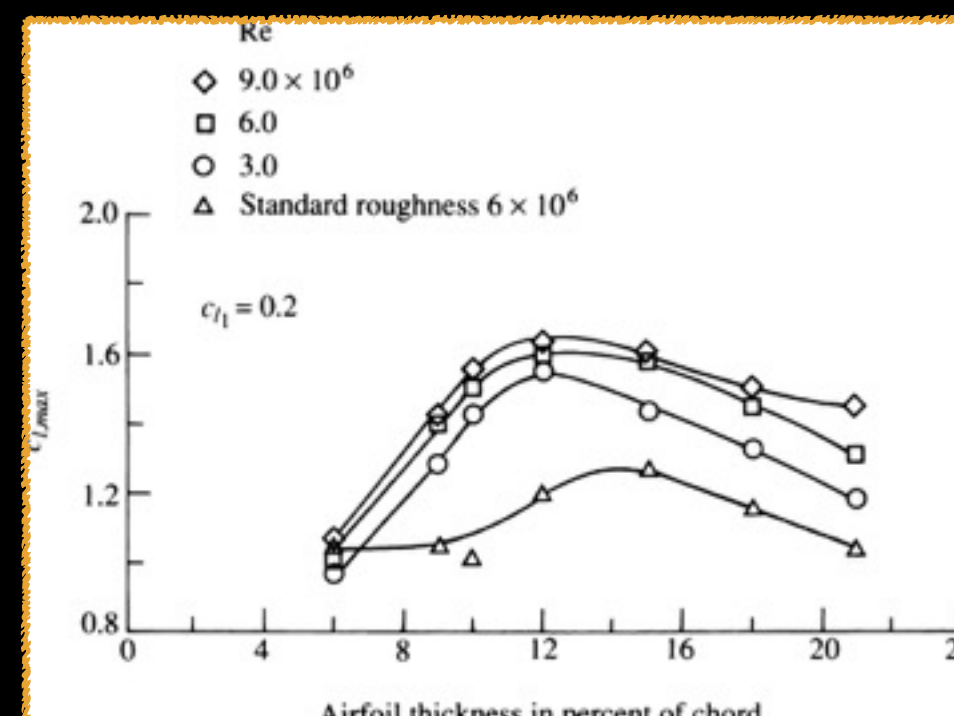


- *Trailing-Edge stall*
  - *Characteristic of thicker airfoils.*
  - *Progressive and gradual movement of separation from TE to LE as angle of attack is increased.*

Flow Over An Airfoil - The Real Case







- *Thin-Airfoil Stall (flat plate)*
  - *This type of stall is associated with the extreme thinness of the airfoil.*
  - *The thickness is about 2% of the chord length.*

Other Airfoil Aerodynamics

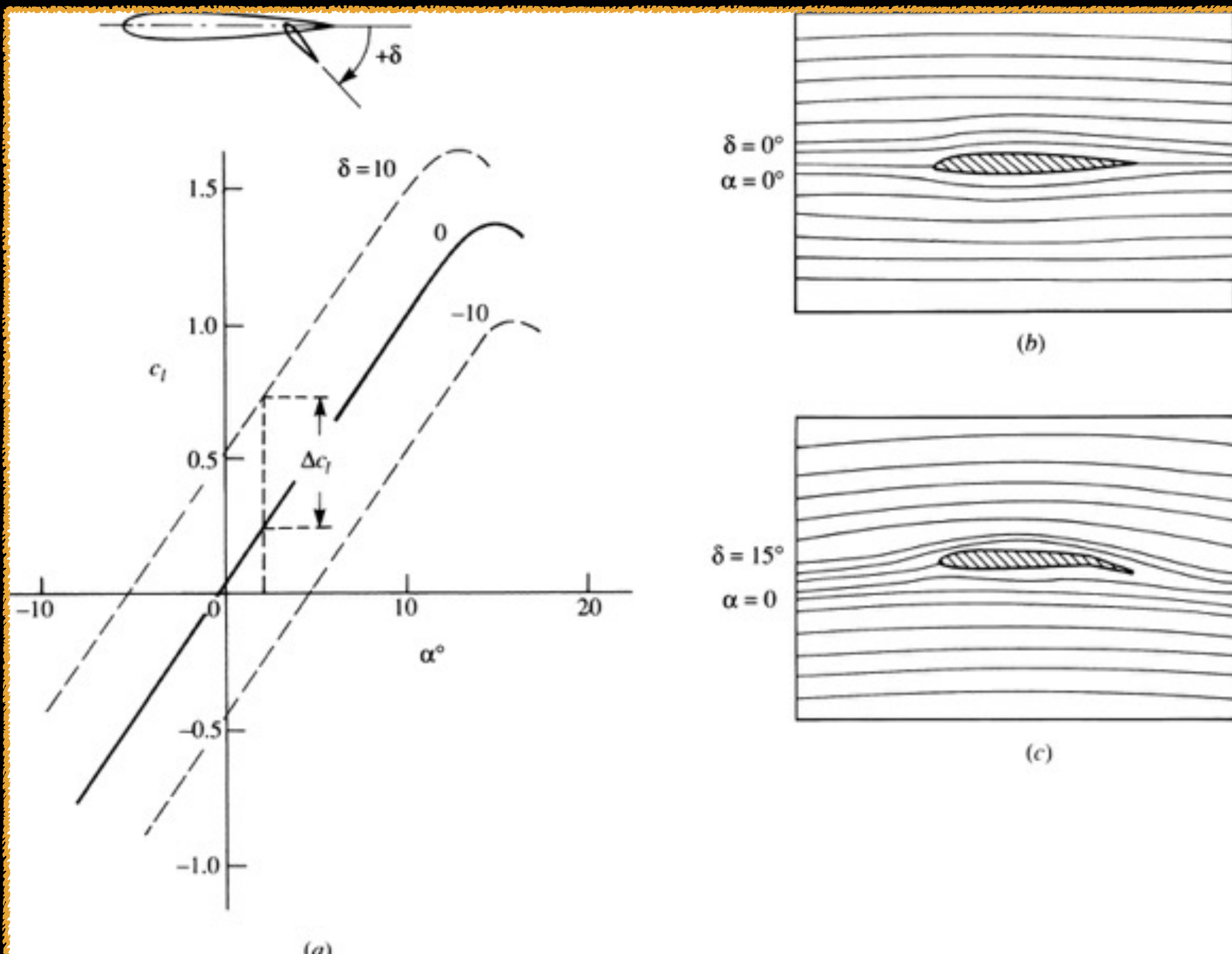
- *Two figures of merit that are primarily used to judge the quality of a given airfoil are:*
  - *L/D ratio*
  - *Maximum lift coefficient.*

$$V_{stall} = \sqrt{\frac{2W}{\rho_\infty S C_{L,max}}}$$

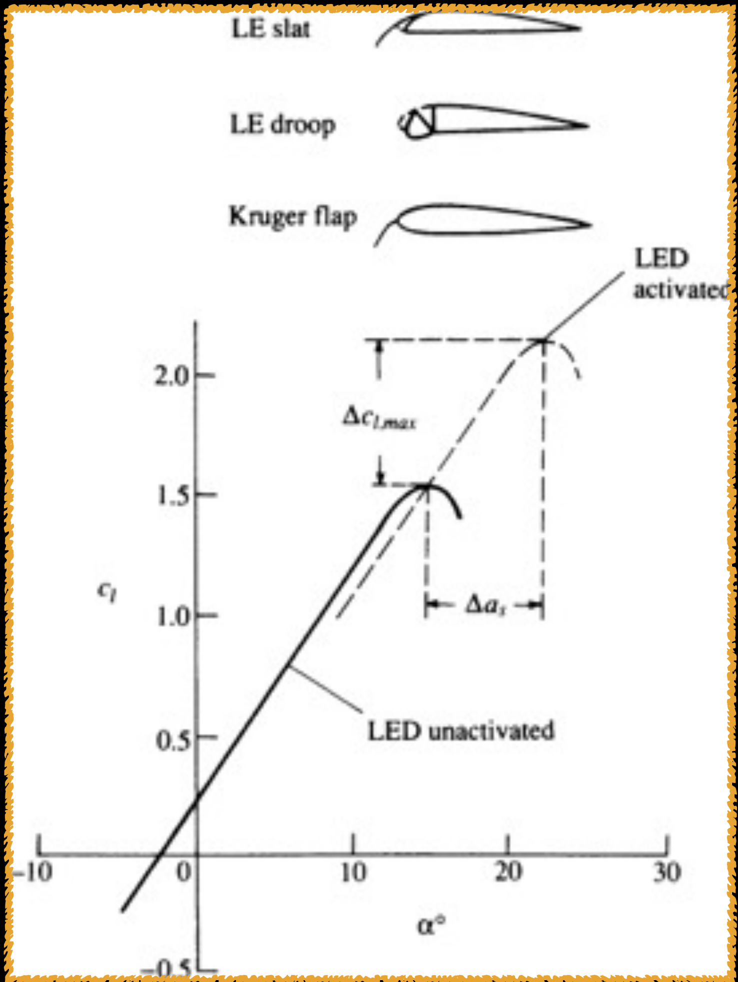
- *Tremendous incentive exists to increase the maximum lift coefficient.*
  - *Lower stalling speeds or higher payload capacity.*
  - *Maneuverability of an airplane depends on high value of $C_{L,max}$.*

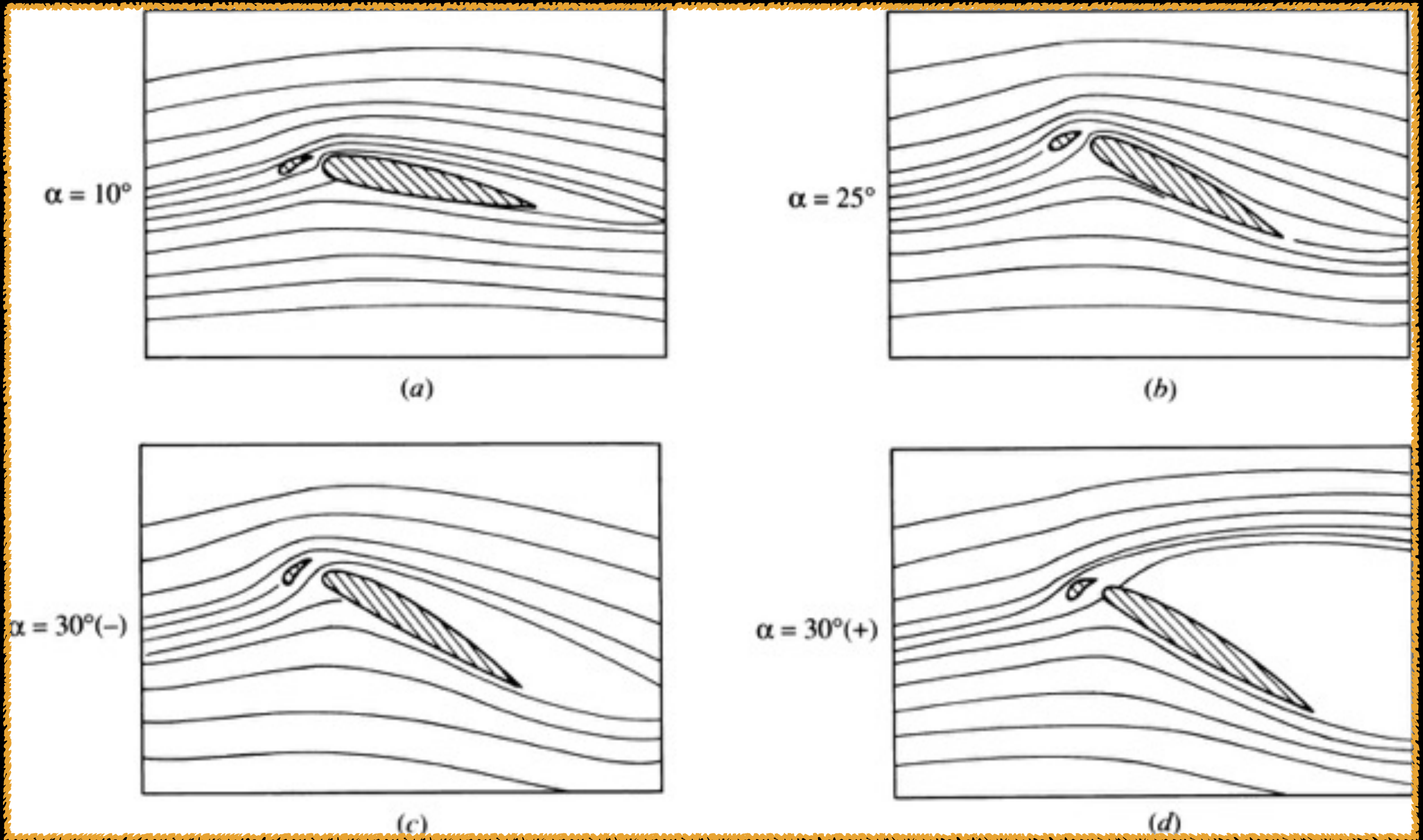
- *For an airfoil at a given Re, $C_{L,max}$ depends primarily on its shape.*
  - *To increase $C_{L,max}$ further, special measures have to be carried out.*
  - *Measures include use of flaps, and/or LE slats - high lift devices.*

High Lift Devices - TE Flaps



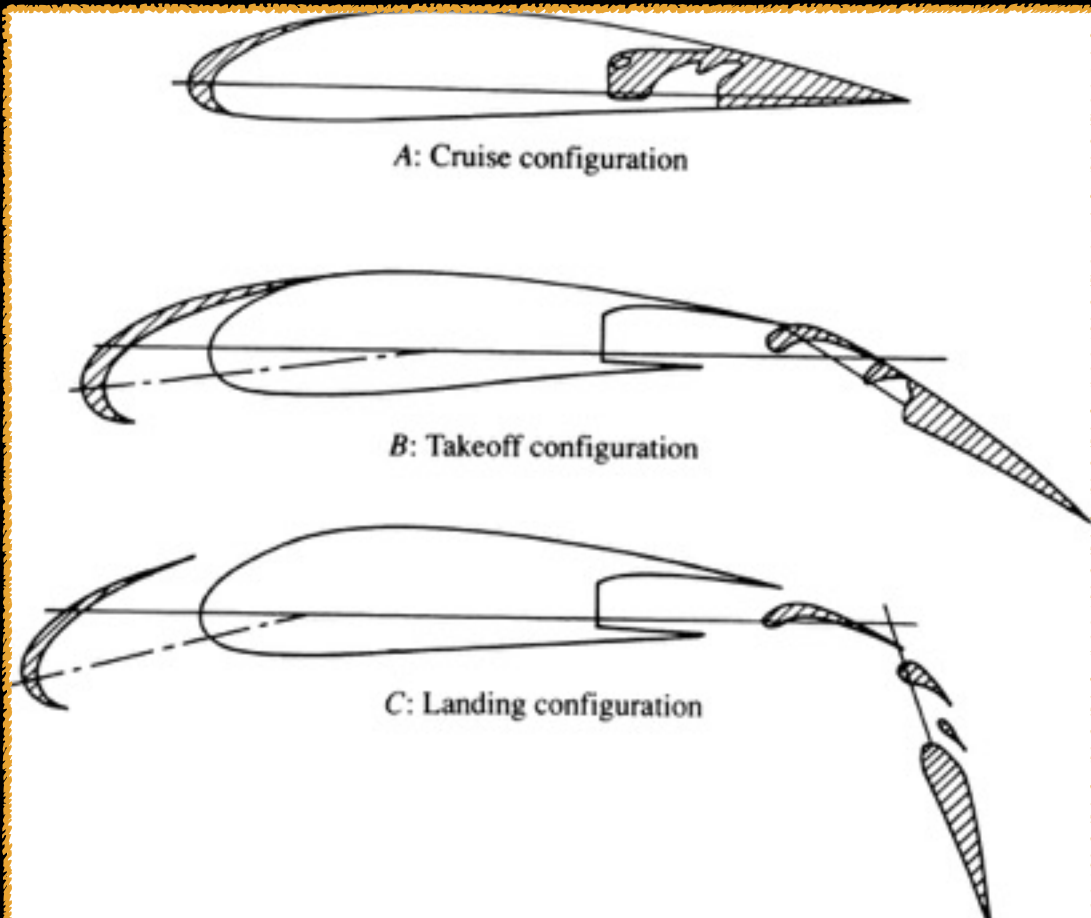
High Lift Devices - LE Slats

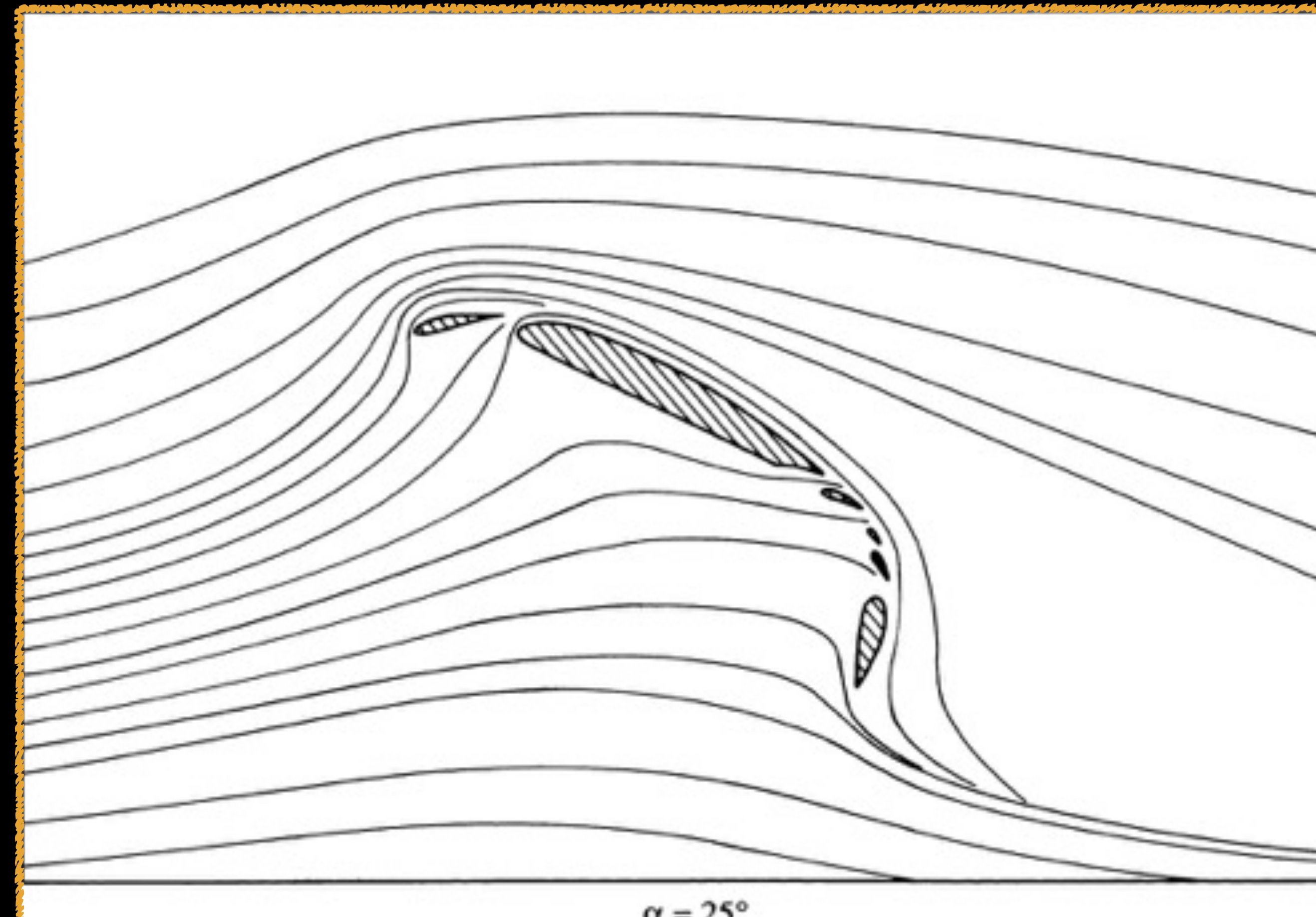




- *The adverse pressure gradient on the top surface is mitigated delaying flow separation.*
  - *Stall angle and maximum lift coefficient increased.*
- *There is no change in the zero-lift angle, but the lift curve is extended to a higher stalling angle of attack.*

High Lift Devices





- *The main flow over the top surface of the airfoil is essentially separated.*
- *The local flow through the gaps in the multi element flap is locally attached to the top surface of the flap.*
- *Because of this locally attached flow, the lift coefficient is still quit high, around 4.5.*