Fundamentals of Inviscid, Incompressible Flow

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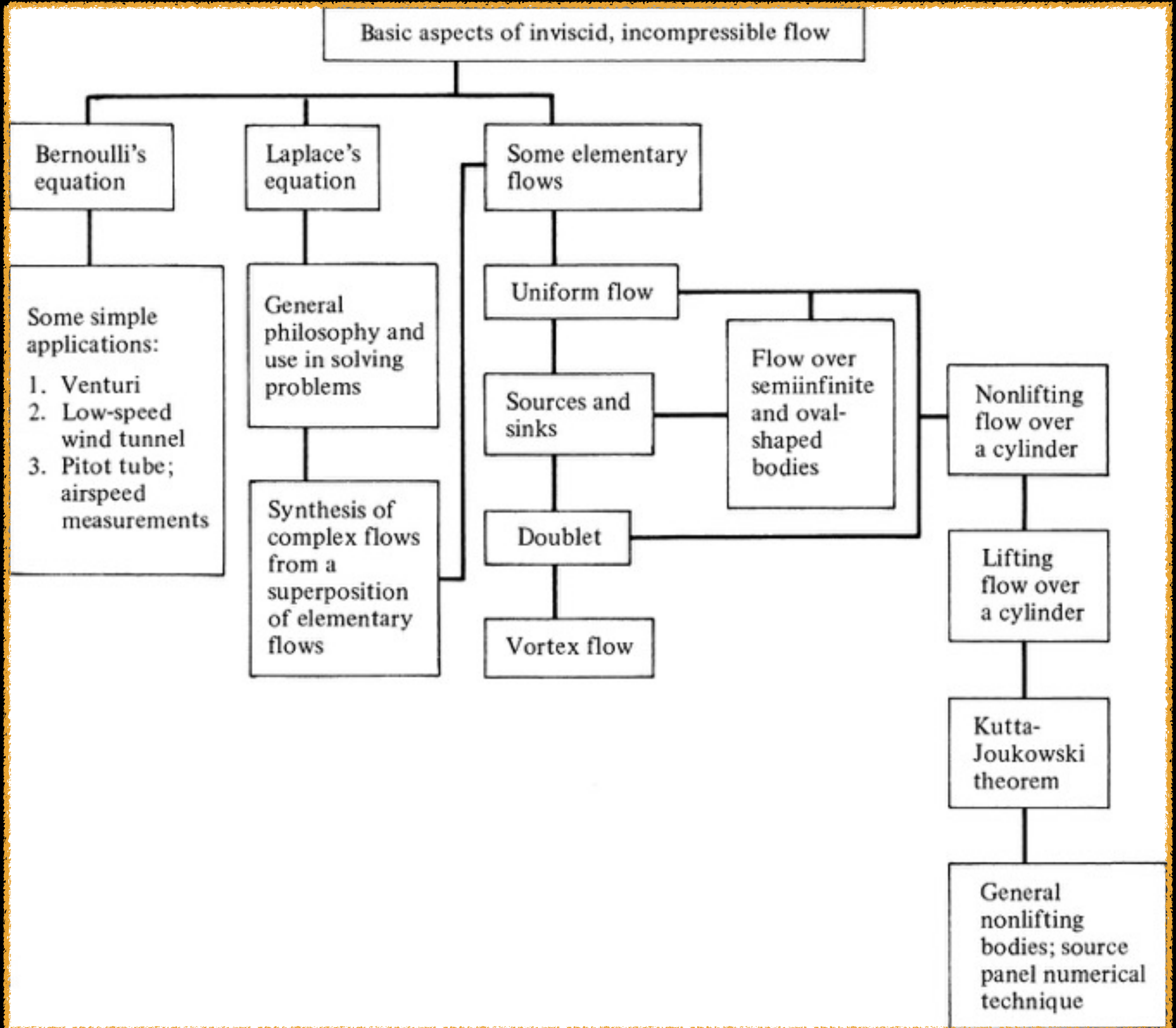
Introduction

- *We will focus on fundamentals of inviscid, incompressible flow.*
    - How low-speed tunnels work
    - Low-speed velocity measurement
    - Flow over streamlined bodies

Roadmap


Basic aspects of inviscid, incompressible flow
Bernoulli's equation
Laplace's equation
Some elementary flows
Some simple applications:
1. Venturi
2. Low-speed wind tunnel
3. Pitot tube; airspeed measurements
General philosophy and use in solving problems
Synthesis of complex flows from a superposition of elementary flows
Uniform flow
Sources and sinks
Doublet
Vortex flow
Flow over semiinfinite and oval-shaped bodies
Nonlifting flow over a cylinder
Lifting flow over a cylinder
Kutta-Joukowski theorem
General nonlifting bodies; source panel numerical technique

Introduction

$$V_{aircraft} \approx 30mi/hr$$

$$V_{aircraft} \approx 300mi/hr$$

Courtesy of the U.S. Air Force

Bernoulli's Equation

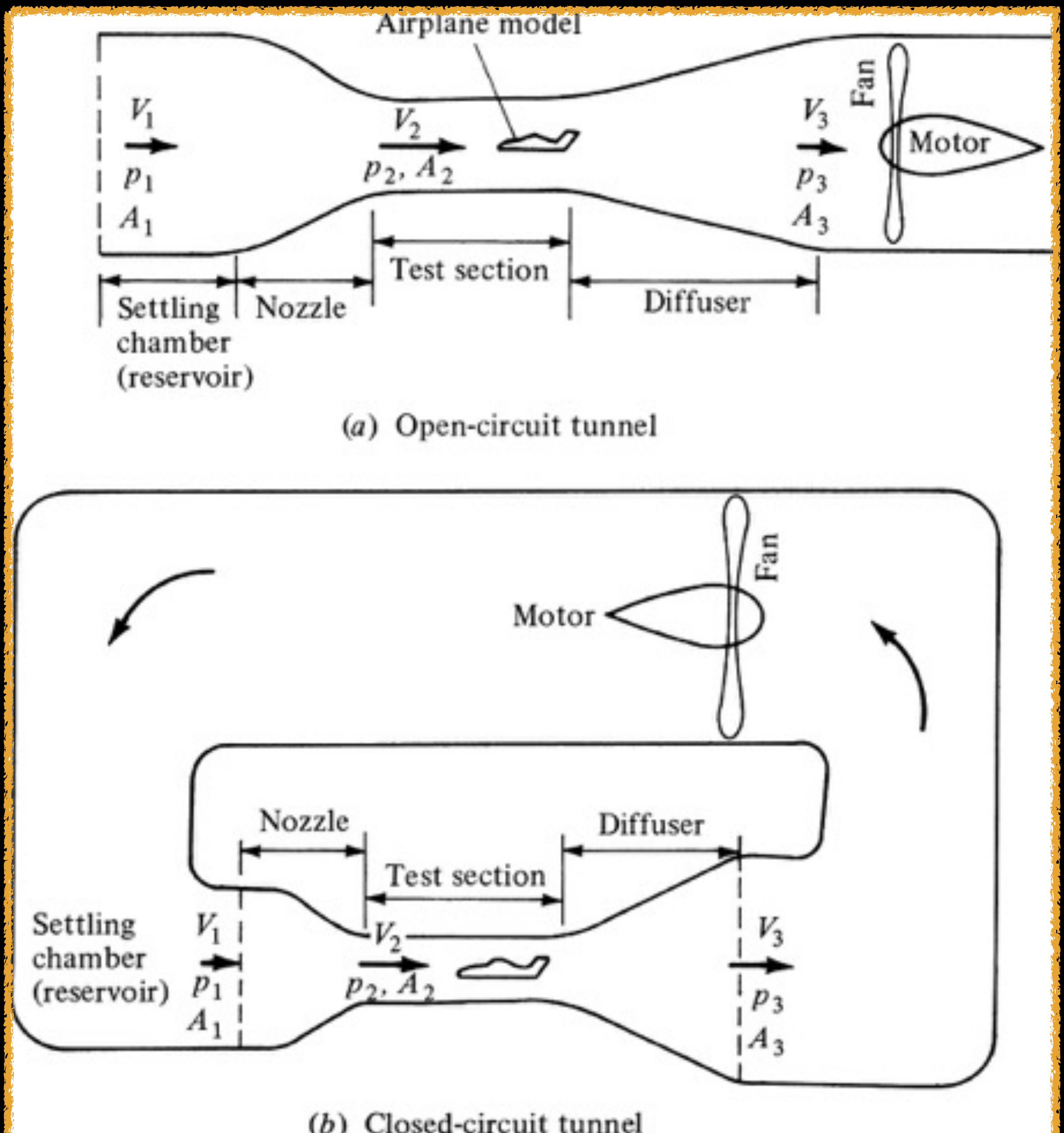
- *Relates pressure and velocity in inviscid and incompressible flow.*

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\right]=-\frac{\partial p}{\partial x}$$

Euler's Equation: $dp = -\rho V\,dV$

$$p+\frac{1}{2}\rho V^2=\text{constant}$$

Incompressible Flow in a Duct



$$\frac{\partial}{\partial t}\int_V + \int_S \rho V.dS = 0$$

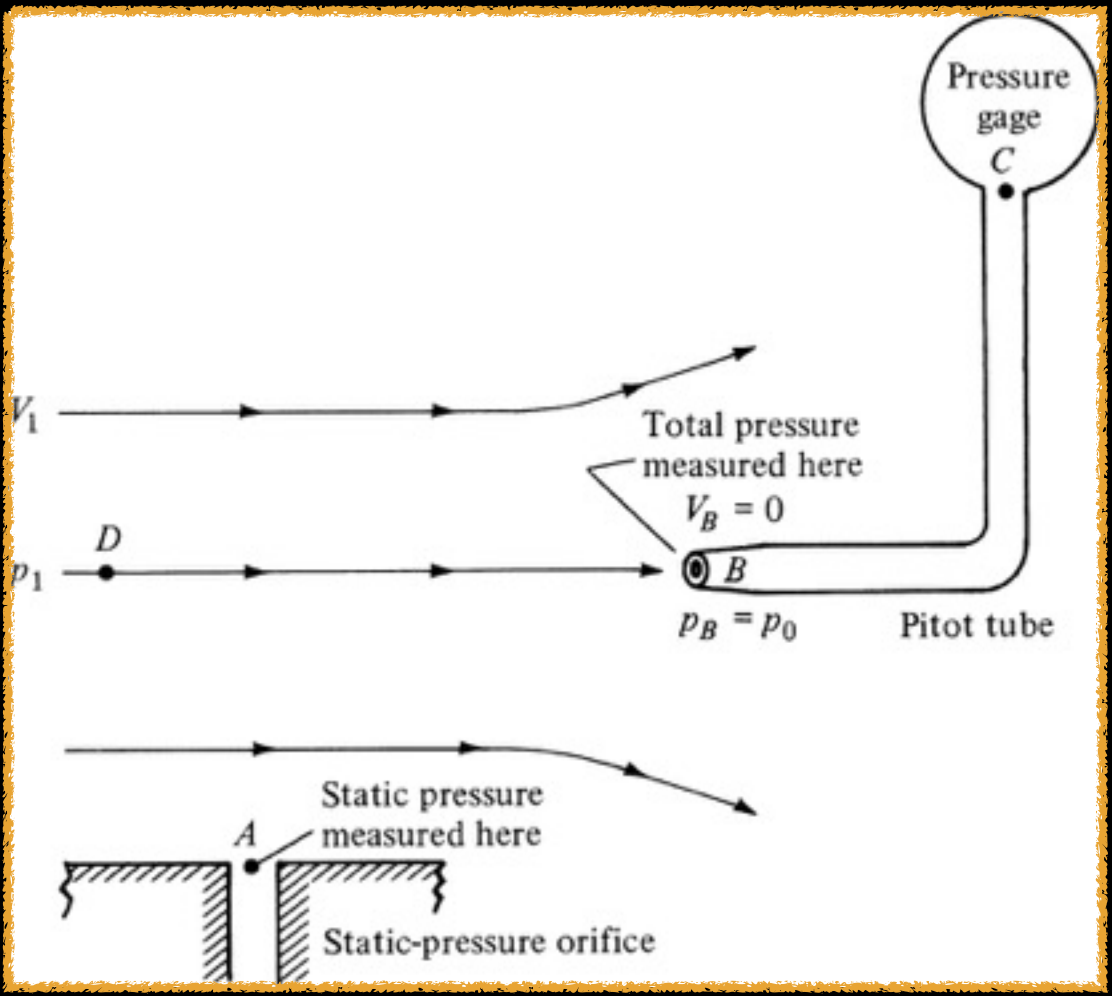
$$\frac{\partial}{\partial t}\int_V \rho d\nu + \int_S \rho V.dS = 0$$

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho[1-(A_2/A_1)^2]}}$$

$$V_2 = \sqrt{\frac{2\rho g \Delta h}{\rho[1-(A_2/A_1)^2]}}$$

Measurement of Airspeed

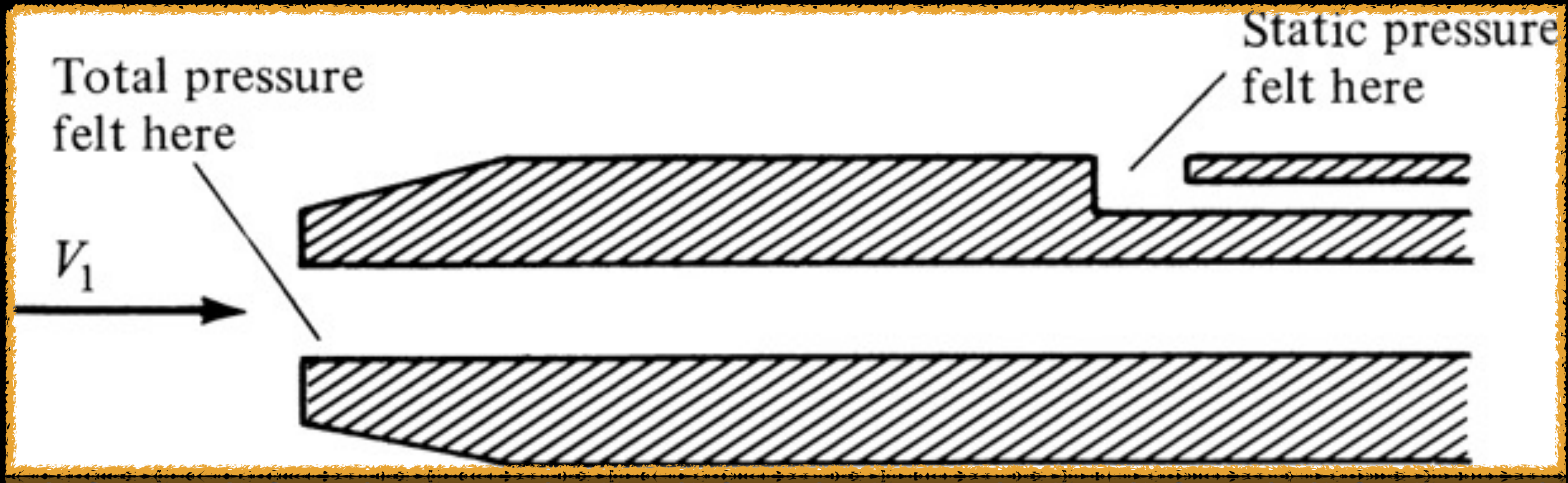
- *The most common device for measuring airspeed is the pitot tube.*



- *Static Pressure*
  - Pressure is associated with rate of change of momentum of gas molecules impacting/crossing a surface.
  - Pressure clearly related to the random motion of the molecules.
  - *Static pressure is a measure of the purely random motion of molecules in a Lagrangian frame of reference*

- *When flow moves over point A, the pressure felt is due only to the random motion of the particles - Static pressure.*

The Pitot-Static Probe

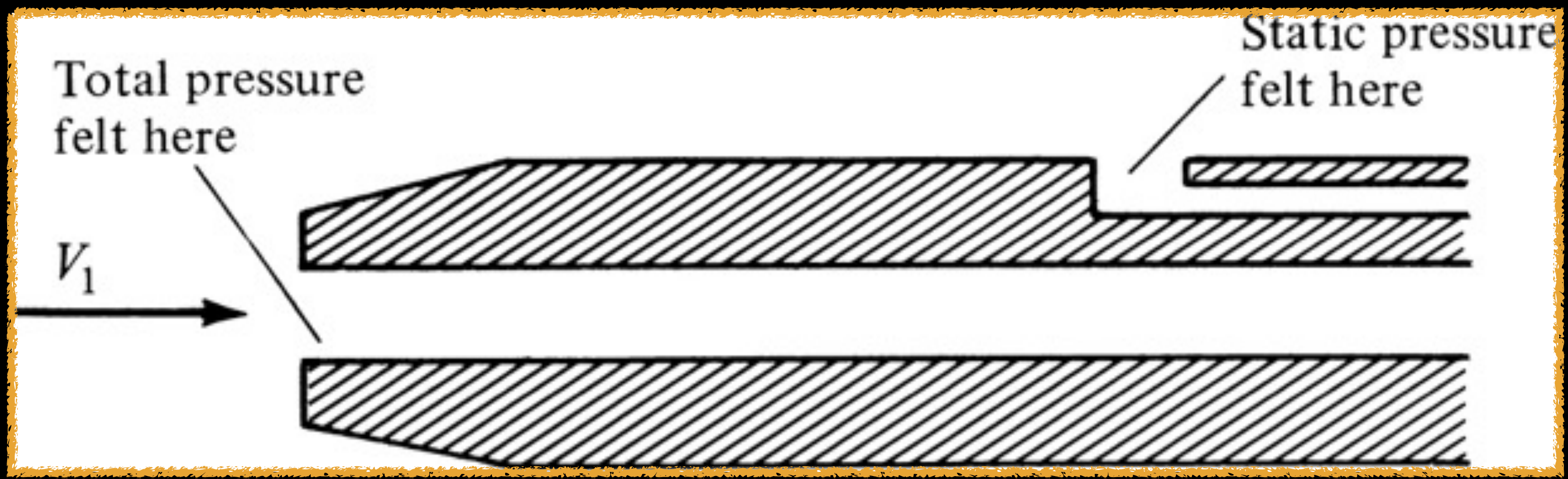


$$V_1 = \sqrt{\frac{2(p_0 - p_1)}{\rho}}$$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_0$$

Static pressure *Dynamic pressure* *Total pressure*

The Pitot-Static Probe



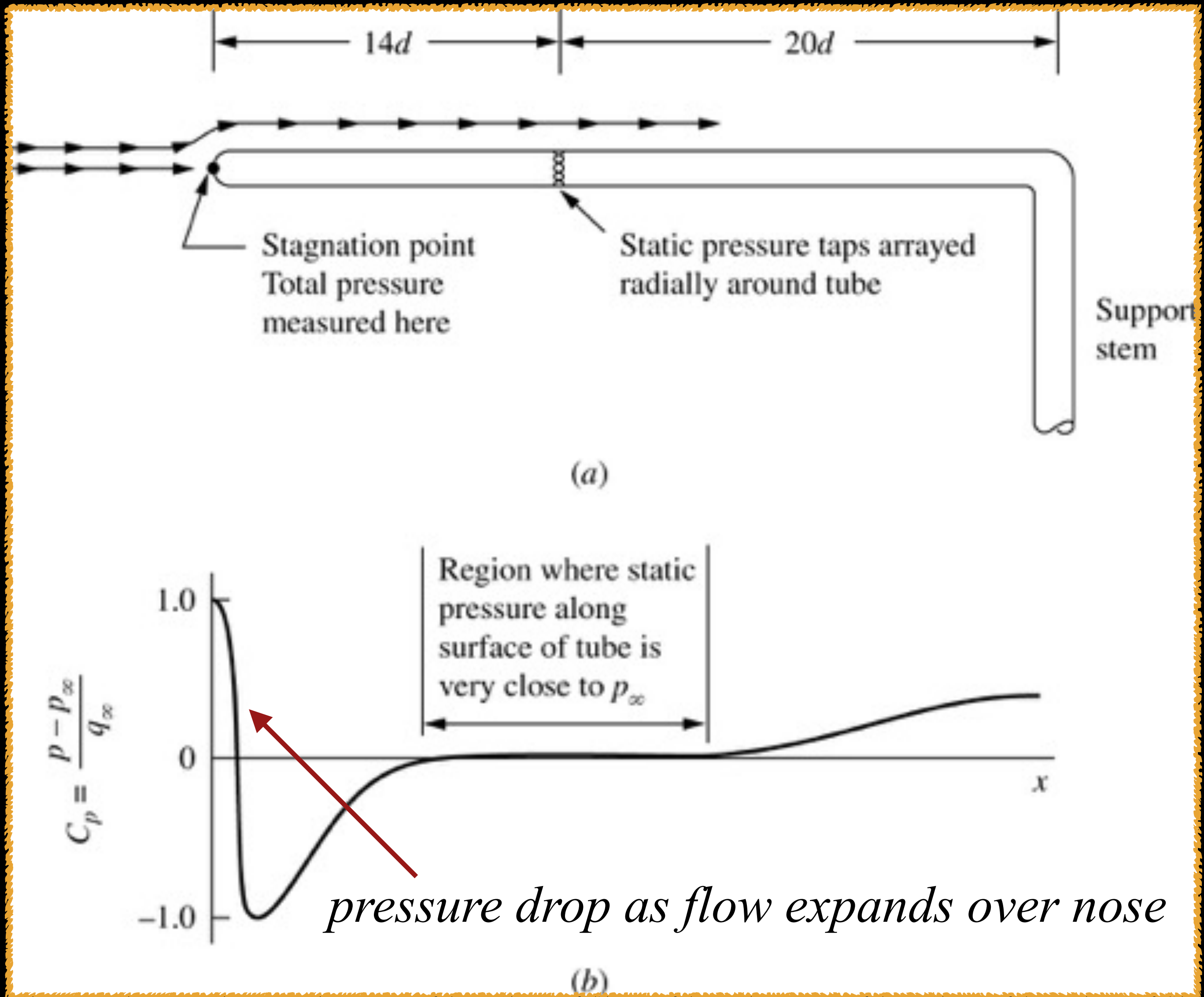
$$V_1 = \sqrt{\frac{2(p_0 - p_1)}{\rho}}$$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_0$$

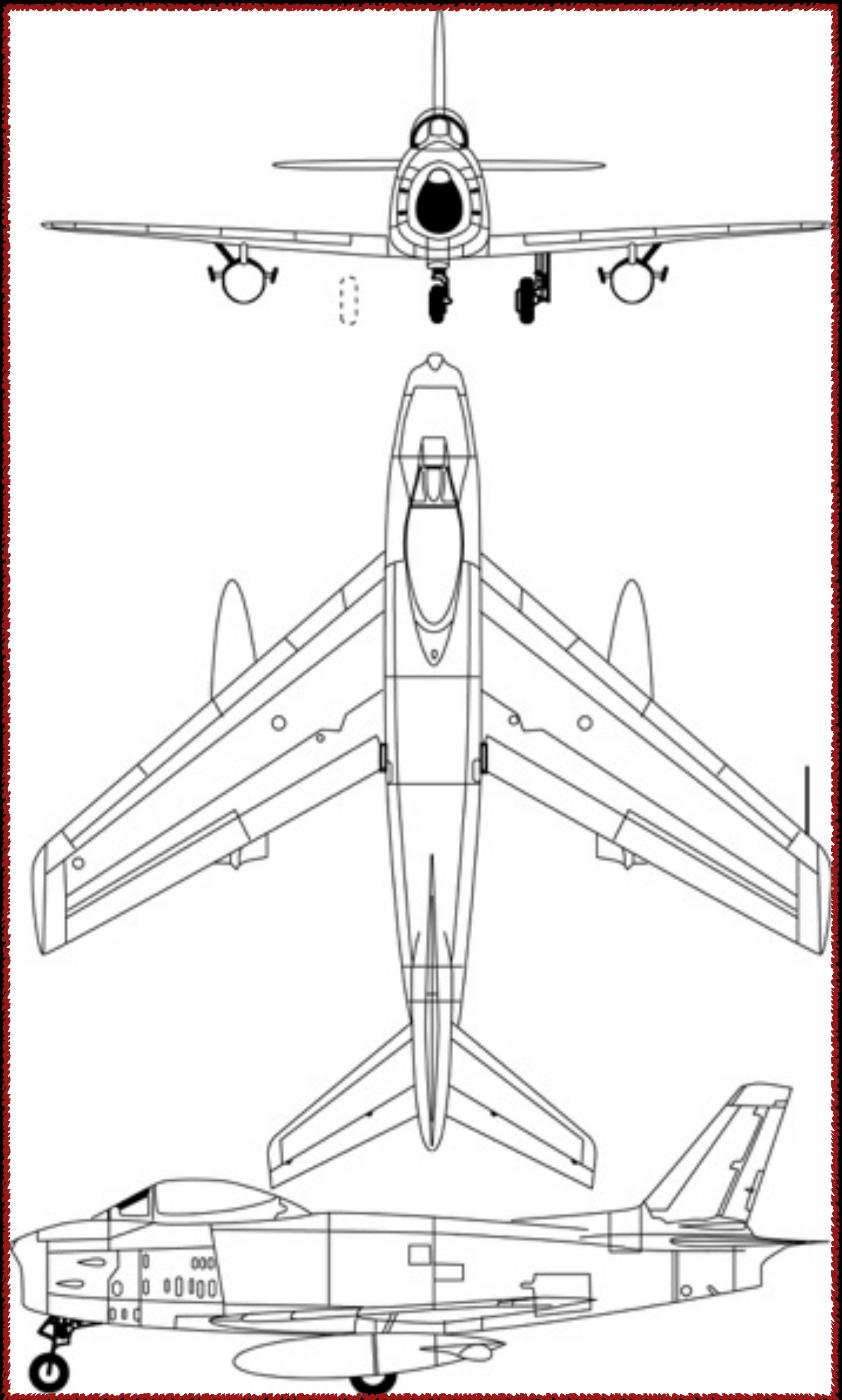
Static pressure *Dynamic pressure* *Total pressure*

Pitot-Static Probe Design

- *The probe should be long, streamlined such that the static pressure over a substantial portion of the probe is equal to free stream static pressure*



Pitot-Static Probe Design



- *Pitot-static probes should be located in a position away from influences of local flow field.*
- *On aircraft, static pressure is generally obtained by a properly placed static pressure tap on the fuselage.*

Pressure Coefficient

- Pressure coefficient C_p *- a similarity parameter* used through out aerodynamics, from incompressible to hypersonic flow.

$$C_p = \frac{p - p_\infty}{q_\infty} \qquad q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$$

$$\text{For incompressible flow, } C_p = 1 - \left(\frac{V}{V_\infty}\right)^2$$

- For incompressible flow, *pressure coefficient at stagnation point is always 1.0* (can be greater than 1.0 for compressible flow).
- In regions of flow where local velocity is less than freestream velocity, pressure coefficient will be negative (local pressure is less than freestream pressure).

Pressure Coefficient contd.

- *From the worked out example C_p is obviously unchanged as the velocity is increased.*
  - From dimensional analysis, *C_p depends on Mach number, Reynolds number, shape, orientation of body and location on body..*
  - In this example, *M* and *Re* and not in picture (why?). *C_p only depends on location, shape and orientation of the body.*
  - *Hence, C_p will not change with free stream velocity or density as long as the flow is incompressible and inviscid.*
- *For such flows, if C_p distribution can be determined by some means beforehand, it will remain the same for all freestream values of velocity and density.*

Pressure Coefficient contd.

- *The velocity when C_p = -5.3 was calculated to be 753ft/s. Is this correct?*

$$M = \frac{753}{1117} \approx 0.7$$

- *Here, the flow has expanded locally into the compressible flow regime.*
- *Therefore, use of incompressible Bernoulli's equation to calculate C_p is incorrect.*
- *Even if the model is being tested in a low-speed wind tunnel, the flow can some times accelerate to achieve high Mach numbers that it must be considered compressible.*

Governing Equations

Incompressible flow: $\nabla . V = 0$

$V = \nabla \Phi$

$\Rightarrow \ \nabla . (\nabla \Phi) = 0$

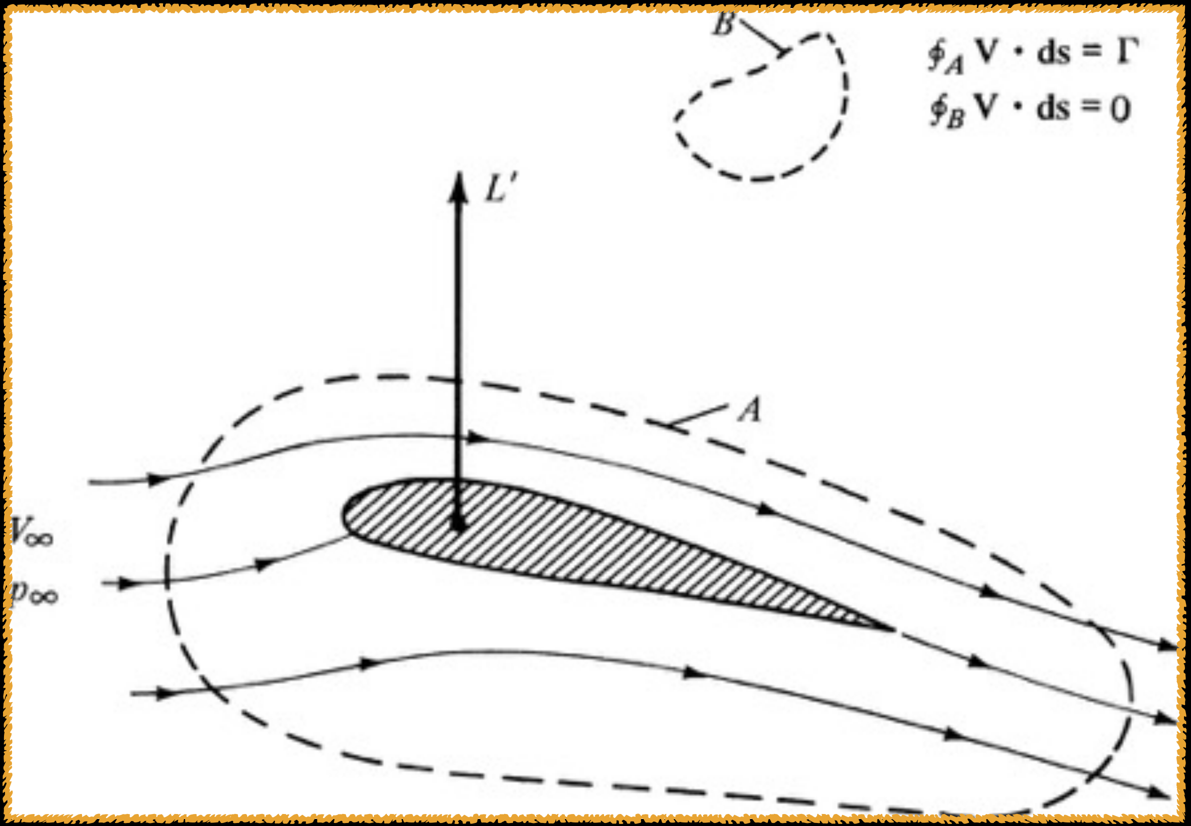
$\Rightarrow \ \nabla^2 \Phi = 0$ and $\nabla^2 \psi = 0$

- *The above equation is the Laplace equation, the solutions of which are called the harmonic functions.*
- *Incompressible flows can be described by the solutions of the Laplace equation.*

Governing Equations

- *Irrotational, incompressible flows are governed by Laplace equation which is linear.*
- *Linearity implies, a complicated flow pattern can be constructed by adding together various elementary flows which also are linear.*
- *Different elementary flows*
  - Uniform flow
  - Source/Sink flow
  - Doublet
  - Vortex flow
  - Combination of the above

The Kutta-Joukowski Theorem



$$\Gamma = \oint_A V.ds$$

$$L' = \rho_\infty V_\infty \Gamma$$

The Kutta-Joukowski Theorem

- *Lifting flow over a cylinder = uniform flow + doublet + vortex*
  - All three are irrotational except for vortex
  - Vortex has infinite vorticity at the origin
  - So, lifting flow over the cylinder is irrotational at every point except origin.

$$\Gamma = \oint V.ds = 0$$

not including origin

- *Similar arguments for flow over airfoil*
  - Flow outside the airfoil is irrotational
  - Circulation by not including the airfoil is zero
  - Kutta-Joukowski theorem requires inclusion of airfoil body to obtain finite circulation (*sum of all vortex's strengths that constitue the airfoil*)

Real Flow over a Circular Cylinder