MECE 4333/6399: Homework
Aerodynamics: Introduction

Isaac Choutapalli

Read all of the following information before starting the homework:

- ⇛ *Before you begin the solution, always state what is given, asked and then present your solution.* Your homework must have the following three parts:
  - ⇛ Given:
  - ⇛ Asked:
  - ⇛ Solution:
- ⇛ Your homework must be very neat and presentable. I will assign a zero grade if I cannot understand what you wrote, if your hand writing is illegible or if your homework consists of couple of pieces of paper put together hurriedly.
- ⇛ Show ALL steps, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- ⇛ If you are using a MATLAB code, you will also need to include the code as part of your HW.
- ⇛ Homework will be due at the beginning of the class indicated by the instructor. ***Late homework will not be accepted under any circumstances***.

Name: $\underbrace{\hspace{6cm}}_{\text{by writing my name, I swear by the bronc honor code}}$

1.1 For most gases at standard or near standard conditions, the relationship among pressure, density, and temperature is given by the perfect gas equation of state: $p = \rho RT$, where R is the specific gas constant. For air at near standard conditions, $R = 287$ J/(kg · K) in the International System of Units and $R = 1716$ ft · lb/(slug · °R) in the English Engineering System of Units. (More details on the perfect gas equation of state are given in Chapter 7.) Using the above information, consider the following two cases:

a. At a given point on the wing of a Boeing 727, the pressure and temperature of the air are 1.9×10^4N/m^2 and 203 K, respectively. Calculate the density at this point.

b. At a point in the test section of a supersonic wind tunnel, the pressure and density of the air are 1058 lb/ft^2 and 1.23×10^{-3} slug/ft^3, respectively. Calculate the temperature at this point.

1.2 Starting with Equations (1.7), (1.8), and (1.11), derive in detail Equations (1.15), (1.16), and (1.17).

1.3 Consider an infinitely thin flat plate of chord c at an angle of attack α in a supersonic flow. The pressures on the upper and lower surfaces are different but constant over each surface; that is, $p_u(s) = c_1$ and $p_l(s) = c_2$, where c_1 and c_2 are constants and $c_2 > c_1$. Ignoring the shear stress, calculate the location of the center of pressure.

1.4 Consider an infinitely thin flat plate with a 1 m chord at an angle of attack of 10° in a supersonic flow. The pressure and shear stress distributions on the upper and lower surfaces are given by $p_u = 4 \times 10^4(x-1)^2 + 5.4 \times 10^4$, $p_l = 2 \times 10^4(x-1)^2 + 1.73 \times 10^5$, $\tau_u = 288x^{-0.2}$, and $\tau_l = 731x^{-0.2}$, respectively, where x is the distance from the leading edge in meters and p and τ are in newtons per square meter. Calculate the normal and axial forces, the lift and drag, moments about the leading edge, and moments about the quarter chord, all per unit span. Also, calculate the location of the center of pressure.

1.5 Consider an airfoil at 12° angle of attack. The normal and axial force coefficients are 1.2 and 0.03, respectively. Calculate the lift and drag coefficients.

1.6 Consider an NACA 2412 airfoil (the meaning of the number designations for standard NACA airfoil shapes is discussed in Chapter 4). The following is a tabulation of the lift, drag, and moment coefficients about the quarter chord for this airfoil, as a function of angle of attack.

α (degrees)	c_l	c_d	$c_{m,c/4}$
−2.0	0.05	0.006	−0.042
0	0.25	0.006	−0.040
2.0	0.44	0.006	−0.038
4.0	0.64	0.007	−0.036
6.0	0.85	0.0075	−0.036
8.0	1.08	0.0092	−0.036
10.0	1.26	0.0115	−0.034
12.0	1.43	0.0150	−0.030
14.0	1.56	0.0186	−0.025

From this table, plot on graph paper the variation of x_{cp}/c as a function of α.

1.7 The drag on the hull of a ship depends in part on the height of the water waves produced by the hull. The potential energy associated with these waves therefore depends on the acceleration of gravity g. Hence, we can state that the wave drag on the hull is $D = f(\rho_\infty, V_\infty, c, g)$ where c is a length scale associated with the hull, say, the maximum width of the hull. Define the drag coefficient as $C_D \equiv D/q_\infty c^2$. Also, define a similarity parameter called the *Froude number,* $\mathrm{Fr} = V/\sqrt{gc}$. Using Buckingham's pi theorem, prove that $C_D = f(\mathrm{Fr})$.

1.8 The shock waves on a vehicle in supersonic flight cause a component of drag called supersonic wave drag D_w. Define the wave-drag coefficient as $C_{D,w} = D_w/q_\infty S$, where S is a suitable reference area for the body. In supersonic flight, the flow is governed in part by its thermodynamic properties, given by the specific heats at constant pressure c_p and at constant volume c_v. Define the ratio $c_p/c_v \equiv \gamma$. Using Buckingham's pi theorem, show that $C_{D,w} = f(M_\infty, \gamma)$. Neglect the influence of friction.

1.9 Consider two different flows over geometrically similar airfoil shapes, one airfoil being twice the size of the other. The flow over the smaller airfoil has freestream properties given by $T_\infty = 200$ K, $\rho_\infty = 1.23$ kg/m^3, and $V_\infty = 100$ m/s. The flow over the larger airfoil is described by $T_\infty = 800$ K, $\rho_\infty = 1.739$ kg/m^3, and $V_\infty = 200$ m/s. Assume that both μ and a are proportional to $T^{1/2}$. Are the two flows dynamically similar?

1.10 Consider a Lear jet flying at a velocity of 250 m/s at an altitude of 10 km, where the density and temperature are 0.414 kg/m^3 and 223 K, respectively. Consider also a one-fifth scale model of the Lear jet being tested in a wind tunnel in the laboratory. The pressure in the test section of the wind tunnel is 1 atm $= 1.01 \times 10^5$ N/m^2. Calculate the necessary velocity, temperature, and density of the airflow in the wind-tunnel test section such that the lift and drag coefficients are the same for the wind-tunnel model and the actual airplane in flight. *Note:* The relation among pressure, density, and temperature is given by the equation of state described in Problem 1.1.

1.11 A U-tube mercury manometer is used to measure the pressure at a point on the wing of a wind-tunnel model. One side of the manometer is connected to the model, and the other side is open to the atmosphere. Atmospheric pressure and the density of liquid mercury are 1.01×10^5 N/m^2 and 1.36×10^4 kg/m^3, respectively. When the displacement of the two columns of mercury is 20 cm, with the high column on the model side, what is the pressure on the wing?

1.12 The German Zeppelins of World War I were dirigibles with the following typical characteristics: volume $= 15{,}000$ m^3 and maximum diameter $=$ 14.0 m. Consider a Zeppelin flying at a velocity of 30 m/s at a standard altitude of 1000 m (look up the corresponding density in Appendix D). The Zeppelin is at a small angle of attack such that its lift coefficient is 0.05 (based on the maximum cross-sectional area). The Zeppelin is flying in straight-and-level flight with no acceleration. Calculate the total weight of the Zeppelin.

1.13 Consider a circular cylinder in a hypersonic flow, with its axis perpendicular to the flow. Let ϕ be the angle measured between radii drawn to the leading edge (the stagnation point) and to any arbitrary point on the cylinder. The pressure coefficient distribution along the cylindrical surface is given by $C_p = 2\cos^2\phi$ for $0 \leq \phi \leq \pi/2$ and $3\pi/2 \leq \phi \leq 2\pi$ and $C_p = 0$ for $\pi/2 \leq \phi \leq 3\pi/2$. Calculate the drag coefficient for the cylinder, based on projected frontal area of the cylinder.

1.14 Derive Archimedes' principle using a body of general shape.

1.15 Consider a light, single-engine, propeller-driven airplane similar to a Cessna Skylane. The airplane weight is 2950 lb and the wing reference area is 174 ft^2. The drag coefficient of the airplane C_D is a function of the lift coefficient C_L for reasons that are given in Chapter 5; this function for the given airplane is $C_D = 0.025 + 0.054C_L^2$.

a. For steady, level flight at sea level, where the ambient atmospheric density is 0.002377 $slug/ft^3$, plot on a graph the variation of C_L, C_D, and the lift-to-drag ratio L/D with flight velocity ranging between 70 ft/s and 250 ft/s.

b. Make some observations about the variation of these quantities with velocity.

1.16 Consider a flat plate at zero angle of attack in a hypersonic flow at Mach 10 at standard sea level conditions. At a point 0.5 m downstream from the leading edge, the local shear stress at the wall is 282 N/m^2. The gas temperature at the wall is equal to standard sea level temperature. At this point, calculate the velocity gradient at the wall normal to the wall.

1.17 Consider the Space Shuttle during its atmospheric entry at the end of a mission in space. At the altitude where the Shuttle has slowed to Mach 9, the local heat transfer at a given point on the lower surface of the wing is 0.03 MW/m^2. Calculate the normal temperature gradient in the air at this point on the wall, assuming the gas temperature at the wall is equal to the standard sea-level temperature.

1.18 The purpose of this problem is to give you a feel for the magnitude of Reynolds number appropriate to real airplanes in actual flight.

a. Consider the DC-3 shown in Figure 1.1. The wing root chord length (distance from the front to the back of the wing where the wing joins the fuselage) is 14.25 ft. Consider the DC-3 flying at 200 miles per hour at sea level. Calculate the Reynolds number for the flow over the wing root chord. (This is an important number, because as we will see later, it governs the skin-friction drag over that portion of the wing.)

b. Consider the F-22 shown in Figure 1.5, and also gracing the cover of this book. The chord length where the wing joins the center body is 21.5 ft. Consider the airplane making a high-speed pass at a velocity of 1320 ft/s at sea level (Mach 1.2). Calculate the Reynolds number at the wing root.

1.19 For the design of their gliders in 1900 and 1901, the Wright brothers used the Lilienthal Table given in Figure 1.65 for their aerodynamic data. Based on these data, they chose a design angle of attack of 3 degrees, and made all their calculations of size, weight, etc., based on this design angle of attack. Why do you think they chose three degrees?

Hint: From the table, calculate the ratio of lift to drag, *L/D*, at 3 degrees angle of attack, and compare this with the lift-to-drag ratio at other angles of attack. You might want to review the design box at the end of Section 1.8, especially Figure 1.36, for the importance of *L/D*.