Strain Measurements

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MECE 3320: Measurements & Instrumentation Lab Exercise

I. Introduction

Measurement of stress in a mechanical component is important when assessing whether or not the component is subjected to safe load levels. Stress and strain measurements can also be used to indirectly measure other physical quantities such as force (by measuring force in a flexural element), pressure (by measuring strain in a diaphragm), temperature (by measuring thermal expansion of a material). The most common transducer to measure strain is the electrical resistance strain gage. Stress values can then be determined from strain measurements using the principles of solid mechanics.

II. Stress and Strain Relations

When a cylindrical rod is loaded axially, it will lengthen by an amount ΔL and deform radially by an amount ΔD. The axial strain (ε_{axial}) is defined by the change in length per unit length:

$$\varepsilon_{axial} = \frac{\Delta L}{L} \tag{1}$$

Note that strain is a dimensionless quantity. The axial stress (σ_{axial}) is related to axial strain through Hooke's law, which states that for a uniaxially loaded linear elastic material the axial stress is directly proportional to the axial strain:

$$\sigma_{axial} = E\varepsilon_{axial} \tag{2}$$

where E is the constant of proportionality called the modulus of elasticity or Young's modulus. The axial stress in the rod is:

$$\sigma_{axial} = \frac{F}{A} \tag{3}$$

where F is the axial force and A is the cross-sectional area of the rod. Therefore, the axial strain is related to the axial stress and load:

$$\varepsilon_{axial} = \frac{\sigma_{axial}}{E} = \frac{F/A}{E} \tag{4}$$

The transverse strain is defined as the change in width divided by the original width:

$$\varepsilon_{transverse} = \frac{\Delta D}{D} \tag{5}$$

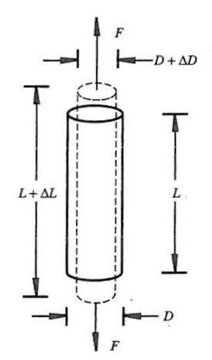


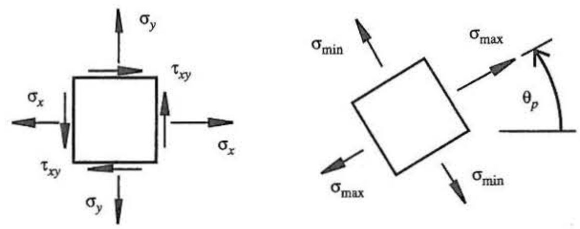
Figure 1. Axial and transverse deformation of a cylindrical bar.

The ratio of the transverse and axial strain is defined as *Poisson's ratio* (ν):

$$\nu = \frac{\varepsilon_{transverse}}{\varepsilon_{axial}} \tag{6}$$

Note that for axial elongation ($\varepsilon_{axial} > 0$), $\varepsilon_{transverse}$, and therefore ΔD are negative, implying contraction in the transverse radial direction. Poisson's ratio for most metals is approximately 0.3, implying the transverse strain is -30% of the axial strain.

A general state of planar stress at a point, acting on an infinitesimal square element, is illustrated in the figure. It includes two normal stress components (σ_x and σ_y) and a shear stress component (τ_{xy}) whose values depend on the orientation of the element. At any point, there is always an orientation of theelement that results in the maximum normal stress magnitude and zero shear stress ($\tau_{xy} = 0$). The two orthogonal normal stress directions corresponding to this orientation are called the *principal axes*, and the normal stress magnitudes are referred to as the *principal stresses* (σ_{max} and σ_{max}). The figure above illustrates this orientation and its corresponding state of stress. The magnitude and direction of the principal stresses are related to the stresses in any other orientation by



(a) general state of stress (b) principal stresses

Figure 2. General and principle stresses.

$$\sigma_{max} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{7}$$

$$\sigma_{min} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{8}$$

$$tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{9}$$

where θ_p is the angle from σ_x to σ_{max}, measured counterclockwise.

The principal stresses are important quantities when determining if a material will yield or fail when loaded because they determine the maximum values of stress, which can be compared to the yield strength of the material. The maximum shear stress is also important when assessing failure and is given by

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_{max} - \sigma_{min}}{2} \tag{10}$$

This relation can be used to rewrite equations 7 and 8 as

$$\sigma_{max} = \sigma_{avg} + \tau_{max} \tag{11}$$

$$\sigma_{min} = \sigma_{avg} - \tau_{max} \tag{12}$$

where

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \tag{13}$$

The orientation of the element that results in τ_{max} is given by

$$tan(2\theta_s) = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \tag{14}$$

As with θ_p, θ_s is measured counterclockwise from the direction of σ_x. For the cylindrical bar shown in figure 1, with an element oriented in the axial (y) direction, $\sigma_{max} = \sigma_y = F/A$, $\sigma_x = 0$, and $\theta_p = 0$ because the element is aligned in the direction of the principal stress. Also, $\theta_s = 45^o$ and $\tau_{max} = \sigma_y/2 = F/2A$.

III. Procedure

A. Part 1: Strain Gages and the Wheatstone Bridge

The metal foil strain gages used in this lab are resistors with a nominal (unstrained) resistance of 120 ohms. As they are put in tension, their resistance increases; as they are compressed, their resistance decreases. The Wheatstone bridge provides a way to convert these changes in resistance to changes in voltage, which are easy to work with. These voltages can be conditioned, transmitted, or stored digitally.

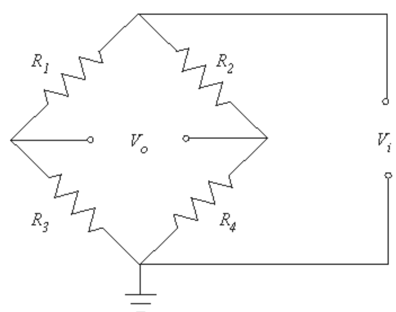


Figure 3. Wheatstone Bridge Circuit

Figure 3 shows a Wheatstone bridge configuration.

- ➩ Four resistors are connected in an end-to-end fashion.
- ➩ The input or excitation voltage is connected to the bridge between top and bottom nodes of the circuit.
- ➩ The output is the difference between the voltage at the left node and the voltage at the right node.
- ➩ An excitation voltage is required to convert the change in resistance (in the legs of the bridge) to a change in voltage at the output of the bridge.

For the bridge shown, the output voltage is expressed as

$$V_0 = \left[\frac{R_3}{R_1 + R3} - \frac{R_4}{R_2 + R_4}\right] V_i \tag{15}$$

When building a Wheatstone bridge with strain gages, all four resistors have the same nominal value. Bridges can be built in the following configurations:

- ➩ Quarter Bridge-One strain gage and three fixed resistors
- ➩ Half Bridge- Two strain gages and two fixed resistors
- ➩ Full Bridge- Four strain gages

1. Quarter Bridges

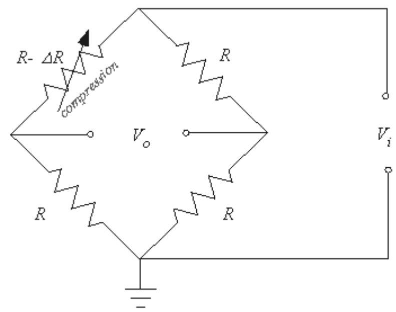


Figure 4. Quarter Bridge Configuration

Figure 4 illustrates a quarter bridge configuration. The quarter bridge has one active leg, i.e., one leg with a changing resistance. From equation (1) above we can derive an expression for the output voltage as a function of the resistance change ΔR:

$$\begin{aligned} V_0 &= \left[\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right] V_i = \left[\frac{R}{(R-\Delta R)+R} - \frac{R}{R+R}\right] V_i \\ &= \left[\frac{R}{2R-\Delta R} - \frac{1}{2}\right] V_i = \left[\frac{2R-(2R-\Delta R)}{2(2R-\Delta R)}\right] V_i \\ &= \left[\frac{\Delta R/R}{2(2-\Delta R/R)}\right] V_i \\ &\approx \frac{\Delta R}{4R} V_i \end{aligned} \tag{16}$$

2. Half and Full Bridges

Figures 5 and 6 show half-bridge and full-bridge configurations respectively.

- *Half bridge*: two active legs, one in tension and one in compression. These legs are adjacent legs in the bridge.
- *Full bridge*: four active legs, two in tension and two in compression. The gages in tension are on opposite legs of the bridge.

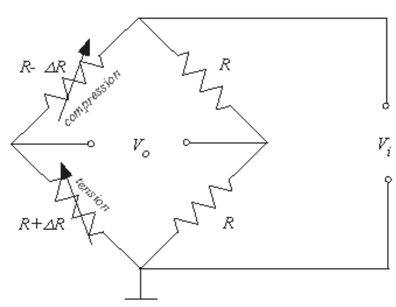


Figure 5. Half Bridge Configuration

Using equation 1 and Figure 1 as a guide, derive expressions for the output voltage of the half-bridge and full-bridge circuits.

Thought Question: A half bridge could be made with two gages in tension on opposite legs. When would this be useful? What would be the main problem with doing this?

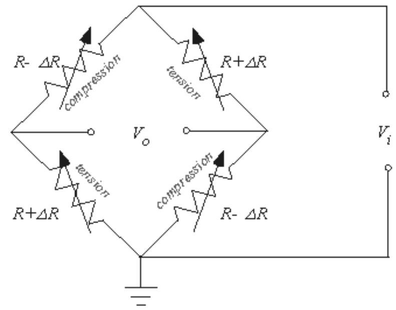


Figure 6. Full Bridge Configuration

B. Part 2: Calibration of the Strain-gaged Cantilever Beam

Your TA will provide an aluminum beam instrumented with strain gages.

1. Check the connections from the beam to the bridge module as well as to the amplifier.
2. Hang weights from the end of the beam.
3. Record the voltage measured with LabVIEW in your lab book.
4. Repeat the above steps for several different weights.
5. In Matlab, plot the voltage (input) versus weight (output).
6. Find the best fit linear relationship for the data. The resulting equation can be used to calibrate the voltage output of the strain gages.
7. Apply several loads not used for calibration to test the validity of the linear curve fit.

C. Part 3: Comparison of Theoretical Strain to Measured Strain

In this step, we will compare a theoretically based estimate of strain for a given load to that which was measured earlier. First we will determine the strain corresponding to the voltage measurements of step 2.

1. Measure the excitation voltage for bridge circuit, V_i.

The circuit analyses from step one showed that the bridge output V_0 is related to the excitation voltage by the following relationship:

$$V_0 = K \frac{\Delta R}{R} V_i \tag{17}$$

where K equals 1/4 for a quarter bridge, 1/2 for a half bridge, and 1 for a full bridge. The voltage measured in LabVIEW is related to the bridge output by

$$V_{display} = K_{amp} V_0 + V_{offset} \tag{18}$$

where K_{amp} is the gain of the signal conditioning board. Because the bridge resistances are not balanced exactly and the weight of the beam itself produces some strain, you will observe a nonzero output voltage when there is no load applied.

2. Measure the output voltage with no load. Call this voltage V_{offset}.

3. To determine the strain induced by the applied loads, measure the changes in the display voltages relative to this offset voltage ($V_{display} - V_{offset}$).

For a strain gage, the gage factor is defined as

$$GF = \frac{1}{\varepsilon}\frac{\Delta R}{R} \tag{19}$$

where ε is the strain experienced by the gage. The gages used in this lab have a gage factor of $2.12 \pm 0.8\%$.

4. Derive an theoretical expression for the strain in the beam using equations. Your empirical strain estimate should be in the range of 0 to 2000 microstrains.

The stress on the surface of a beam in bending is given by

$$\sigma = \frac{My}{I} \tag{20}$$

where M is the applied moment at the location of interest, y is the distance from the neutral axis (in this case, the half height of the beam cross section), I is the area moment of inertia of the cross section with respect to the neutral axis. Recall that for a rectangular cross section,

$$I = \frac{1}{12}bh^3 \tag{21}$$

Recall also that stress and strain are related by Youngs modulus:

$$\sigma = E\varepsilon \tag{22}$$

For aluminum, $E = 10.4 \times 10^6 psi$.

5. Using equations, theoretically estimate the strain where the gages are bonded to the beam. How does the theoretically obtained strain compare to the value determined from measurements? If they are different, what are some possible reasons?

D. Part 4: Measurement of an Unknown Load

Based upon the calibration determined in step 2, use your beam to determine the weight of an arbitrary object. Measure the actual weight using a precision scale. How does the weight determined with your beam compare to the object's true weight? How certain is your measurement? What are some possible sources of uncertainty?

E. Part 5: Dynamic Characteristics

Up to this part of the lab, you have examined the static characteristics of the strain-gage bridge. Now you will measure the dynamic characteristics of a signal from the strain-gage bridge. You will excite the dynamics of the cantilever beam by plucking it. You will use the LabView program to display the response of the vibrating beam.

1. Differential Model

The first mode of vibration of the cantilever beam can be modeled using a simple mass-spring-damper model. This model results in a second-order differential equation that describes the dynamics of the system. You will observe that the response of the strain gages to an initial deflection is a damped sinusoid. This is the expected response for a second-order system. Using acquired data; you will compute and display values for the damping ratio and the natural frequency of the first vibrational mode of the beam.

2. Damping Ratio and Damped Natural Frequency

Figure 7 shows the response of a second-order system to an initial condition. This response plot will be used to define the damping ratio and the natural frequency for this system. From the amplitudes x_1 and x_n, the damping ratio can be calculated using the following expression:

$$\zeta = \frac{\frac{1}{n-1} ln\left(\frac{x_1}{x_n}\right)}{\sqrt{4\pi^2 + \left[\frac{1}{n-1} ln\left(\frac{x_1}{x_n}\right)\right]^2}} \tag{23}$$

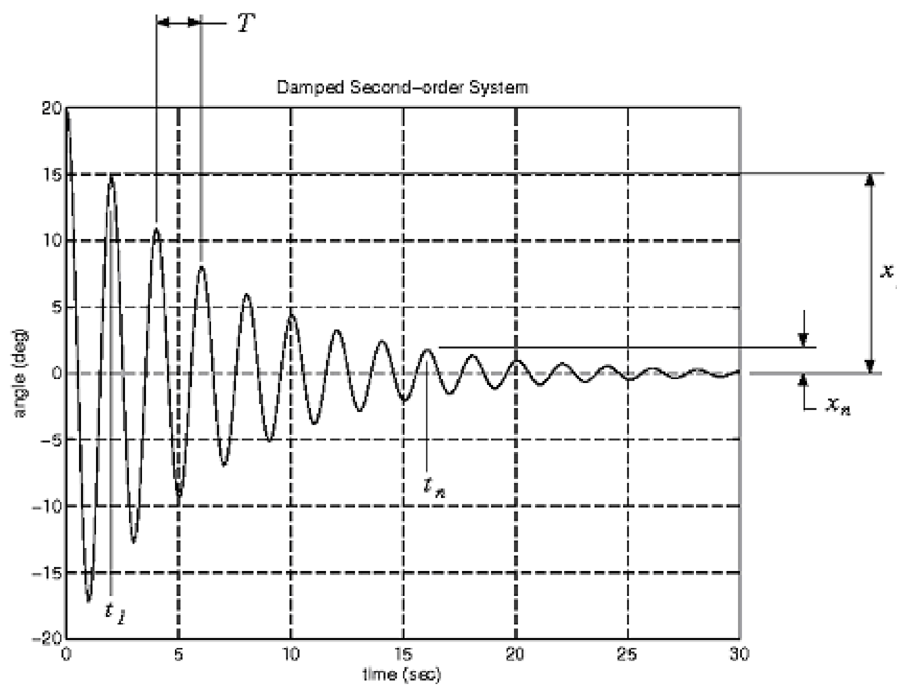


Figure 7. Second-order system Response

The damped natural frequency can be determined by measuring the period of the damped oscillations. An accurate measurement of the period T is obtained by considering several periods:

$$T = \frac{t_n - t_1}{n-1} \omega_d \quad = \frac{2\pi}{T} \tag{24}$$

Once the above quantities are known, the natural frequency of oscillation can be calculated by

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} \tag{25}$$

IV. Lab Report

For this lab you will write the Results and Discussion of Results sections of a full report. As you perform the lab, think about what data should be saved or recorded for presentation and why these data are important. For some of your data, tabulation is sufficient (e.g., calculated strain vs. measured strain). Other data should be recorded using LabVIEW (e.g., dynamic response of strain when the beam is plucked). Make sure that you include your thoughts about the results you obtained and why they are important. Discuss their agreement with your expectations.

References

[1]Luke Graham, Strain Gages and Force Measurement. OpenStax CNX. August 15, 2006 http://cnx.org/contents/da31d8d7-0eb6-4673-94e3-e448c3f59779@1@1.

[2]D. G. Alciatore and M. B. Histand, Introduction to Mechatronics and Measurement Systems, Fourth Edition, McGraw Hill, 2011.