Pressure concepts

- Pressure is force per unit area.
- Pressure definitions

Absolute pressure = Gauge pressure + Atmospheric pressure
EXAMPLE 9.1

Determine the absolute and gauge pressures and the equivalent pressure head at a depth of 10 m below the free surface of a pool of water at 20°C.

KNOWN

- $h = 10 \text{ m}$, where $h = h_0 = 0$ is the free surface
- $T = 20^\circ \text{C}$
- $\rho_{\text{H}_2\text{O}} = 998.207 \text{ kg/m}^3$
- Specific gravity of mercury, $S_{\text{Hg}} = 13.57$.

ASSUMPTIONS  Water density constant

- $p_0(h_0) = 1.0132 \times 10^5 \text{ Pa abs}$

FIND  $p_{\text{abs}}, p_{\text{gauge}}$, and $h$

SOLUTION  The absolute pressure can be determined directly from equation (9.2). Using the pressure at the free surface as the reference pressure and the datum line for $h_0$, the absolute pressure must be

$$p_{\text{abs}} = 1.0132 \times 10^5 \text{ N/m}^2 + \frac{(997.4 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(10 \text{ m})}{1 \text{ kg - m/N - s}^2}$$

$$= 1.9906 \times 10^5 \text{ N/m}^2 \text{ abs}$$
This is equivalent to 199.06 kPa abs or 1.96 atm abs or 28.80 lb/in.\(^2\) abs or 1.99 bar abs. The gauge pressure is found from equation (9.1) to be

\[ p_g = p_{\text{abs}} - p_0 = \gamma h \]
\[ = 9.7745 \times 10^4 \text{ N/m}^2 \]

which is equivalent to 97.7 kPa or 0.96 atm or 14.1 lb/in\(^2\) or 0.98 bar. In terms of equivalent head, the pressure is stated from equation (9.3):

\[ h_{\text{abs}} = \frac{p_{\text{abs}}}{\rho g} = \frac{1.9906 \times 10^5 \text{ N/m}^2}{(997.4 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} \]
\[ = 20.36 \text{ m H}_2\text{O abs} = 1.50 \text{ m Hg abs} \]

or in terms of gauge pressure relative to 760 mm Hg abs:

\[ h_g = \frac{p_{\text{abs}} - p_g}{\rho g} = \frac{(1.9906 \times 10^5) - (1.0132 \times 10^5) \text{ N/m}^2}{(998.2 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1\text{N-s}^2/\text{kg-m})} \]
\[ = 10 \text{ m H}_2\text{O} = 0.73 \text{ m Hg} \]
Barometer: Measures atmospheric pressure

Figure 9.4 Fortin barometer.
**Manometer**: Measures differential pressure

\[ p_1 - p_2 = (\gamma_m - \gamma)H \]

*Static sensitivity of the U-tube Manometer* = \( K = 1/(\gamma_m - \gamma) \)

**Figure 9.5** U-tube manometer.
EXAMPLE 9.2

An inclined manometer with indicating leg at 30° is to be used at 20°C to measure an air pressure of nominal magnitude of 100 N/m² relative to ambient. “Unity” oil ($S = 1$) is to be used. The specific weight of the oil is $9770 \pm 0.5\% \text{ N/m}^2$ (95%) at 20°C, the angle of inclination can be set to within 1° using a bubble level, and the manometer resolution is 1 mm with a manometer zero error equal to its interpolation error. Estimate the uncertainty in indicated differential pressure at the design stage.

**KNOWN**

\[ p = 100 \text{ N/m}^2 \text{ (nominal)} \]

Manometer

Resolution : 1 mm

Zero error : 0.5 mm

\[ \theta = 30 \pm 1^\circ \text{ (95% assumed)} \]

\[ \gamma_m = 9770 \pm 0.5\% \text{ N/m}^3 \text{ (95%)} \]
ASSUMPTIONS Temperature and capillary effects in manometer and gravity error in the specific weights of the fluids are negligible.

FIND

\( u_d \)

SOLUTION The relation between pressure and manometer deflection is given by equation (9.5) with \( H = L \sin \theta \):

\[
\Delta p = p_1 - p_2 = L(\gamma_m - \gamma) \sin \theta
\]

where \( p_2 \) is ambient pressure so that \( \Delta p \) is the nominal pressure relative to ambient. For a nominal \( \Delta p = 100 \text{ N/m}^2 \), the nominal manometer rise \( L \) would be

\[
L = \frac{\Delta p}{(\gamma_m - \gamma) \sin \theta} \approx \frac{\Delta p}{\gamma_m \sin \theta} = 21 \text{ mm}
\]

where \( \gamma_m \gg \gamma \) and the value for \( \gamma \) and its uncertainty are neglected. For the design stage analysis, \( p = f(\gamma_m, L, \theta) \), so that the uncertainty in pressure, \( \Delta p \), is estimated by

\[
(u_d)_p = \pm \sqrt{\left[ \frac{\partial \Delta p}{\partial \gamma_m} (u_d)_{\gamma_m} \right]^2 + \left[ \frac{\partial \Delta p}{\partial L} (u_d)_L \right]^2 + \left[ \frac{\partial \Delta p}{\partial \theta} (u_d)_\theta \right]^2}
\]
At assumed 95% confidence levels, the manometer specific weight uncertainty and angle uncertainty are estimated from the problem as

$$(u_d)_{\gamma_m} = (9770 \text{ N/m}^3)(0.005) \approx 49 \text{ N/m}^3$$

$$(u_d)_{\theta} = 1^\circ = 0.0175 \text{ rad}$$

The uncertainty in estimating the pressure from the indicated deflection is due both to the manometer resolution, $u_o$, and the zero point offset error, $u_c$. Using the uncertainties associated with these errors,

$$(u_d)_L = \sqrt{u_o^2 + u_c^2} = \sqrt{(0.5 \text{ mm})^2 + (0.5 \text{ mm})^2} = 0.7 \text{ mm}$$

Evaluating the derivatives and substituting values gives a design-stage uncertainty in $\Delta p$ of

$$(u_d)_{\Delta p} = \pm \sqrt{(0.26)^2 + (3.42)^2 + (3.10)^2} = \pm 4.6 \text{ N/m}^2 \ (95\%)$$
Deadweight Testers: Used as a laboratory standard for the calibration of pressure measuring devices.

**Figure 9.8** Deadweight tester.

\[ p = \frac{F}{A_e} + \sum \text{errors} = p_i (1 + e_1 + e_2) \]

\[ e_2 = -\frac{\gamma_{\text{air}}}{\gamma_{\text{masses}}} \]

Errors: air buoyancy effects, local gravity variations, uncertainty in mass of piston and added masses, shear effects, thermal expansion, elastic deformation of piston.
EXAMPLE 9.3

A deadweight tester indicates 100.00 lb/in.² (i.e., 100.00 psi), at 70°F in Clemson, SC (φ = 34°, z = 841 ft). Manufacturer specifications for the effective piston area were stated at 72°F so that thermal expansion effects remain negligible. Take γ_{air} = 0.076 lb/ft³ and γ_{masses} = 496 lb/ft³. Correct the indicated reading for known errors.

**KNOWN**

\[ p_i = 100.00 \text{ psi} \]
\[ z = 841 \text{ ft} \]
\[ φ = 34° \]

**ASSUMPTION** Systematic error corrections for altitude and latitude apply.

**FIND**

\[ p \]

**SOLUTION** The corrected pressure is found by equation (9.8). From equation (9.9), the correction for buoyancy effects is

\[ e_2 = \frac{γ_{air}}{γ_{masses}} = \frac{-0.076}{496} = -0.000154 \]

The correction for gravity effects is from equation (9.6a)

\[ e_1 = -(2.637 \times 10^{-3} \cos 2φ + 9.6 \times 10^{-8} z + 5 \times 10^{-5}) \]
\[ = -(0.0010 + 8 \times 10^{-5} + 5 \times 10^{-5}) = 0.001119 \]

From equation (9.8), the corrected pressure becomes

\[ p = 100.00 \times (1 - 0.000154 - 0.001119) \text{ lb/in}^2 = 99.87 \text{ lb/in}^2 \]

**COMMENT** This amounts to correcting an indicated signal for known systematic errors. Here that correction is ≈ 0.13%.
A pressure transducer converts a measured pressure into a mechanical or electrical signal.

**Figure 9.9** Elastic elements used as pressure sensors.
Calibration is the process of relating the output of a transducer to the input parameter.

Calibration requires that the input magnitude is known very accurately
- A well known standard
- Examined against a secondary source with known accuracy

The equation/relation obtained from calibration is inverted to relate an unknown input parameter based on the transducer/measurement system output magnitude.
Figure 9.17 Streamline flow over a bluff body.

\[ p_2 = p_1 + \frac{1}{2} \rho U_1^2 \]

- **Total pr.**
- **Static pr.**
- **Dynamic pr.**
- Pitot probe relatively insensitive to misalignment within ±7°.

**Figure 9.18** Total pressure measurement devices. (a) Impact cylinder. (b) Pitot tube. (c) Kiel probe.
Since $\partial p/ \partial n \approx 0$, the static pressure can be measured by sensing the pressure in a direction normal to the flow.

In ducted flows, static pressure is measured by drilling walls taps into the duct wall perpendicular to flow direction.

Tap hole diameter is typically between 1% - 10% of pipe diameter (smaller the better).

The wall taps should not disturb the flow since that would change the streamline curvature, hence changing the local pressure.
Static Pressure Measurement

- Static pressure probe can be inserted into flow to measure local pressure.
- Should be a streamlined design to minimize flow disturbance.
- Frontal area of the probe should not increase 5% of the pipe size (minimizes local flow velocity increase).
- The static pressure port should be located well downstream of the leading edge to allow streamlines to realign themselves parallel with the probe (Prandtl tube).

![Diagram of Improved Prandtl tube for static pressure measurement](image)

*Figure 9.20* Improved Prandtl tube for static pressure. (a) Design. (b) Relative static error along tube length.
The size of the pressure tap diameter and tubing length between the transducer and the tap can have a dynamic response that could be very different from the transducer itself.

The response behavior of the tubing will dominate the system output from the transducer.
Gases:

**Dynamic response when \( \forall_i << \forall \)**

\[ \omega_n = \frac{d \sqrt{\pi a^2 / L \forall}}{2} \]
\[ \zeta = \frac{32 \mu \sqrt{\forall L / \pi}}{a \rho d^3} \]

**Dynamic response when \( \forall_i << \forall \)**

\[ \omega_n = \frac{a}{L (0.5 + 4 \forall / \forall_i)} \]
\[ \zeta = \frac{16 \mu L \sqrt{0.5 + 4 \forall / \forall_i}}{a \rho d^2} \text{ where } a = \sqrt{kRT} \]
Liquids:

- In liquids, pressure forces are transmitted more readily (why?)
- A momentum correction factor is introduced to account for inertial effects
- This in effect increases the inertial force by 1.33

\[ \omega_n = \frac{d \sqrt{3 \pi E_m / \rho L}}{4} \]
\[ \zeta = \frac{16 \mu \sqrt{3 \gamma L / \pi \rho E_m}}{d^3} \]
Heavily Damped Systems (damping ratio > 1.5):

- A transducer has a rated compliance, $C_{vp}$, which is a measure of the transducer volume change relative to an applied pressure change.

- The response of this first order system is indicated through its time constant given by

  \[ \tau = \frac{128 \mu L C_{vp}}{\pi d^4} \]

- The time constant is proportional to $(L/d)^2/vol.$, i.e. long and small diameter connecting tubes will result in relatively sluggish measurement response to changes in pressure.
Fluid Velocity Measuring Systems

The Pitot-Static Pressure Probe:

- Pitot-static probe relatively insensitive to misalignment over the yaw angle range of ±15°.
- Probes have lower velocity limits due to strong viscous effects in the entry regions of the pressure ports. $Re_r$ should be greater than 500.
- If $Re_r < 500$, a viscous correction factor is applied, $p_v = C_v p_i$.
  \[ C_v = 1 + \left( \frac{4}{Re_r} \right) \]
- Even with the applied correction, the measured dynamic pressure, $p_i$, can have a systematic uncertainty of about 40% at $Re_r \sim 10$ decreasing to 1% for $Re_r > 500$.

\[ p_t = p_x + \frac{1}{2} \rho U_x^2 \]

(Incompressible flow)
The Pitot-Static Pressure Probe:
Subsonic Compressible Flow

\[ M = \frac{\gamma}{a} = \sqrt{\frac{2}{\gamma - 1} \left[ \left( \frac{p_{\text{stagnation}}}{p_{\text{static}}} \right) \left( \frac{\gamma - 1}{\gamma} \right) - 1 \right]} \]

where \( a = \sqrt{\gamma RT} \)

\( \gamma = \text{Specific heat ratio} \)

\( R = \text{universal gas constant} = 287 \text{kJ/kg.K} \)

The Pitot-Static Pressure Probe:
Supersonic Compressible Flow

\[ \frac{p_{\text{stagnation}}}{p_{\text{static}}} = \frac{\gamma + 1}{2} M^2 \left[ \frac{\left( \gamma + 1 \right)^2 M^2}{4 \gamma M^2 - 2(\gamma - 1)} \right]^{\frac{1}{\gamma - 1}} \]

for \( Re > 400 \)
Rate at which energy $Q$ is transferred from a warm body at $T_s$ and a cooler fluid at $T_f$ is proportional to $(T_s - T_f)$ and thermal conductance of the heat transfer path $hA$.

Rate of heat transfer is proportional to fluid velocity – A thermal anemometer.

A thermal anemometer is usually connected to one leg of a Wheatstone bridge.
Two types of sensors:

**Hot Wire Anemometer**
- Wire length: 1 – 4mm
- Wire dia: 1.5 - 15μm
- Generally used for non-conducting fluids

**Hot Film Anemometer**
- Generally used for electrically non-conducting & conducting fluids, rugged environments.
Two modes of operation: Constant current and Constant Resistance Modes

**Constant current Mode**

- **Fixed current is passed and sensor is heated (no errors as long as \( Re_d > Gr^{1/3} \)).**
- **Sensor resistance and therefore its temperature vary with heat transfer rate between sensor and fluid.**
- **Bridge deflection voltage is measure of fluid velocity.**
Constant Resistance Mode

- Sensor resistance, and therefore temperature, is originally set by adjustment of bridge balance.
- Sensor resistance is held constant by using a differential feedback amplifier.
- Differential amplifier adjusts the bridge applied voltage, thus adjusting the sensor current to bring the sensor back to set point resistance and hence original temperature.
- The change in the applied bridge voltage can be used to measure the fluid velocity (King’s Law)

\[ E^2 = C + DU^n \]; \( n \) varies between 0.45 and 0.52