Effects of Alpha-Element Enhancement and the Thermally Pulsing-Asymptotic Giant Branch on Surface Brightness Fluctuation Magnitudes and Broadband Colors

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ABSTRACT

We investigate the effects of α-element enhancement and the thermally pulsing-asymptotic giant branch (TP-AGB) stars on the surface brightness fluctuation (SBF) magnitudes and broadband colors of simple stellar populations and compare to the empirical calibrations. We consider a broad range of ages and metallicities using the recently updated Teramo BaSTI isochrones. We find that the α-element enhanced I-band SBF magnitudes are brighter and their integrated V − I colors are redder, mostly because of oxygen enhancement effects on the upper red giant branch and asymptotic giant branch. We also demonstrate, using both the Teramo BaSTI and Padova isochrones, the acute sensitivity of SBF magnitudes to the presence of TP-AGB stars, particularly in the near-IR, but in the I-band as well. Empirical SBF trends therefore hold great promise for constraining this important but still highly uncertain stage of stellar evolution. In a similar vein, non-negligible disparities are found among several different models available in the literature due to intrinsic model uncertainties.

Subject headings: Stars: abundances — Stars: evolution — galaxies: stellar content
1. Introduction

The surface brightness fluctuation (SBF) method, which measures the intrinsic pixel-to-pixel intensity variance in a galaxy image, is widely used as one of the most powerful distance indicators as well as a useful tool for probing stellar populations in the integrated light of early-type galaxies and spiral bulges. It is now a well-known fact that SBF magnitudes vary as a function of galaxy colors (e.g., Tonry et al. 2001; Jensen et al. 2003). For instance, empirical relations show that galaxies with redder $V - I$ colors have fainter $I$-band SBF magnitudes (e.g., Tonry 1991; Tonry et al. 1997, 2001). Moreover, there have been some suggestions that bluer dwarf elliptical galaxies have a shallower slope in the $V - I$ vs. $I$-band SBF magnitude diagram compared to the redder, massive galaxies (e.g., Blakeslee et al. 2001; Mei et al. 2005; Mieske et al. 2006).

From the observational side, it is relatively well established that the red massive early-type galaxies have at least some lighter elements enhanced relative to Fe-peak elements by about 0.3 - 0.4 dex (e.g., Worthey et al. 1992; Lee & Worthey 2005). This abundance pattern may resemble that of halo $\alpha$-element enhancement, although this has not been thoroughly proven (Worthey 1998). There are several theoretical spectrophotometric studies that consider $\alpha$-enhancement in order to address those observations (e.g., Thomas et al. 2003; Lee & Worthey 2005; Coelho et al. 2007; Schiavon 2007; Lee et al. 2009). However, only solar-scaled SBF model predictions have been calculated in the past (Worthey 1993, 1994; Liu et al. 2000; Blakeslee et al. 2001; Lee 2001; Mei et al. 2001; Cantiello et al. 2003; Mouhcine et al. 2005; Raimondo et al. 2005; Marín-Franch & Aparicio 2006).

Moreover, because of the nature of the SBF method, which is far more sensitive to the brighter stars compared to the integrated photometry, thermally pulsing-asymptotic giant branch (TP-AGB) phase is predominately important (e.g., Liu et al. 2000; Mouhcine et al. 2005; Raimondo et al. 2005). The TP-AGB phase is the last stage of AGB evolution. After the high-mass main-sequence (MS) stars and supergiants fade away, it is AGB stars that dominate the integrated bolometric light until full-fledged red giant branch (RGB) stars are looming. After $t > 2$ Gyr, the RGB tip becomes nearly as bright as the AGB tip but is much more numerously populated, by nearly a factor of 10 (e.g., Ferraro et al. 2004; Mucciarelli et al. 2009). The recent development of detailed studies of TP-AGBs (e.g., Maraston 2005; Maraston et al. 2006; Lee et al. 2007b; Marigo et al. 2008) can be thoroughly inspected with observations by the SBF method. In this study, we explore the question of how the $\alpha$-elements (e.g., O, Mg, Si, S, Ca, Ti) and TP-AGBs affect the galaxy colors and SBF magnitudes.
2. Models

The present stellar population synthesis models are based upon the Teramo BaSTI isochrones\(^1\) (Pietrinferni et al. 2004, 2006; Cordier et al. 2007). The models we compute are all single starbursts, characterized by a single age and metallicity. In reality, galaxies are generally composed of mixtures of stellar populations with a range of ages and metallicities.

We have, however, deliberately employed a simplified (i.e., single burst) star formation prescription in this study so that we can explore the importance of varying one parameter, namely the \(\alpha\)-element enhancement. In this regard, our single-burst models approximately represent the luminosity weighted mean age and metallicity for the stellar systems in question. Following our previous models (Lee et al. 2007b, 2009), we employ the standard Salpeter (1955) initial mass function. The low-mass cutoff is 0.5 \(M_\odot\) as given in the Teramo BaSTI isochrones. We do not consider the stochastic nature of the TP-AGB in this study. We merely calculate the SBF magnitudes and the broadband colors using the isochrones as they are available from stellar modelers websites. It is our intention though for future study to investigate those stochastic variations.

In § 2.2, we elaborate the nature of the \(\alpha\)-enhancement in stellar models, particularly the different definitions and degrees of \(\alpha\)-element mixture by different groups and their implications.

2.1. HRDs and CMDs

Here we present, for the first time, \(\alpha\)-element enhanced SBF models and compare them with observations. Before we fully examine the model outputs and the comparisons with observations, however, we first look into the \(\alpha\)-element effects at the H-R diagrams (HRDs) and the color-magnitude diagrams (CMDs) as well as at the emergent fluxes. The reason that we primarily employ the Teramo BaSTI Isochrones in this study is because the Teramo BaSTI stellar models provide both solar-scaled and \(\alpha\)-enhanced isochrones with correct matching of stellar model atmospheres (Cassisi et al. 2004) all the way to the full TP-AGB stages that are crucial for the SBF calculations.

Figure 1 contrasts the solar-scaled standard (sss; by “standard,” we indicate that they adopt no convective-core overshooting) Teramo BaSTI isochrones with \(~0.4\) dex \(\alpha\)-element enhanced ones (aes) in the \(\log T_{\text{eff}}\) vs. \(\log L/L_\odot\) plane. At solar metallicity \((Z=0.02)\), the

\(^1\)http://193.204.1.62/index.html
Teramo BaSTI sss and aes isochrones with and without TP-AGBs are compared at three
given ages (1, 5, 13 Gyr). The sss isochrones are solid lines, while the aes ones are depicted
with dashed lines. To guide the eye, RGB tips are denoted with squares for the sss and
triangles for the aes, respectively. Bottom panels show the details in the giant branches.
In general, the aes isochrones are of slightly higher temperature in the red giant branch
(\sim 45K) as well as in the upper MS compared to the solar-scaled ones. They also show
slightly lower luminosity in the subgiant branch at 1 Gyr. These effects are a reflection of
Fe-depression (see Figure 10 of Dotter et al. 2007a) traded with \( \alpha \)-element enhancement at
fixed total metallicity, \( Z \). In the right panels of Figure 1, TP-AGBs are additionally depicted
with thicker lines. It is noted from the right panels that at younger ages (\( t < 5 \) Gyr), the
TP-AGBs go far cooler and brighter compared to their RGB tips.

\( V - I \) vs. \( I \) color-magnitude diagrams are displayed in Figure 2 in order to compare the
sss Teramo BaSTI isochrones with the aes ones. RGB tips are again denoted with squares
for the sss and triangles for the aes, respectively, to guide the eye. It is shown that the
\( \alpha \)-enhanced red giant branches (RGB), especially at the upper part (\( I < -2 \)), are relatively
redder and brighter in this \( V - I \) vs. \( I \) color-magnitude diagrams compared to the solar-
scaled ones. In right panels of Figure 2, TP-AGBs are additionally depicted with thicker
lines. The \( \alpha \)-enhanced TP-AGBs are also relatively redder and brighter in the \( V - I \) vs. \( I \)
color-magnitude diagrams compared to the solar-scaled ones. This is mainly because of the
oxygen-enhancement among \( \alpha \)-elements (see Figure 3). The next section addresses this issue
in detail.

2.2. Clarification of \( \alpha \)-element Enhancement in Stellar Models

There are now several versions of \( \alpha \)-element enhanced stellar models in the literature
(e.g., Salasnich et al. 2000; Kim et al. 2002; Vandenberg et al. 2006; Dotter et al. 2007ab,
2008) besides the Teramo BaSTI ones of Pietrinferni et al. (2006) that we employ in this
study. One should, however, carefully examine (1) whether their \( \alpha \)-element enhancement is
defined at fixed total metallicity, \( Z \) or at fixed [Fe/H] and (2) how their \( \alpha \)-element mixture
is defined for their \( \alpha \)-element enhanced stellar models. In this study, we are contrasting the
Teramo BaSTI stellar models with the Dartmouth ones (Dotter et al. 2008) and duly note
that different \( \alpha \)-element mixtures by different groups have significant differences even though
they are all commonly referred to as “\( \alpha \)-element enhanced stellar models”.

We have scrutinized, for example, the differences between Figure 1 of this paper and
the results shown by Dotter et al. (2007a). Compared to our Figure 1, Figure 11 of Dotter
et al. (2007a) shows almost no temperature changes although they are similarly \( \alpha \)-element
enhanced isochrones at fixed $Z$. The culprit is the $\alpha$-element mixture. Although the Teramo BaSTI $\alpha$-enhanced stellar models are of $[\alpha/Fe] \sim 0.4$ dex, this is an average only. The Dartmouth models' $\alpha$-enhanced mix, on the contrary, is a constant enhancement of all $\alpha$-elements with respect to solar ratios. From inspection of Table 1 of Pietrinferni et al. (2006), their oxygen abundance is very high, close to the Dartmouth models' $[\alpha/Fe] = +0.8$ dex value. With this information in mind, the behavior in the H-R diagram of Teramo BaSTI's $\alpha$-enhanced isochrones makes sense if they are compared with Figure 7 (oxygen-enhanced) in Dotter et al. (2007a). It is, in fact, the main reason why the New Standard Stellar Population Models (NSSPM) project has opened up new windows on element-by-element variations in order to decipher the rather cryptic collection of $\alpha$-element mixtures (Dotter et al. 2007a; Lee et al. 2009).

It would be useful to compare different sets of $\alpha$-enhanced stellar population models to see if they predict the same effects of $\alpha$-enhancement on SBF. Unfortunately, we do not have the luxury of investigating several different versions of $\alpha$-enhanced SBF models at the moment. Padova isochrones and stellar evolutionary tracks are perhaps more sophisticated at the TP-AGB stages compared to the Teramo BaSTI ones (e.g., Marigo et al. 2008$^2$), but the Padova ones do not yet provide the matching $\alpha$-enhanced stellar evolutionary tracks and isochrones. Dotter et al. (2008, Dartmouth stellar evolutionary models), Kim et al. (2002, Yonsei-Yale models), and Vandenberg et al. (2006, Victoria-Regina models) all do provide the $\alpha$-enhanced stellar models as well as the solar-scaled ones, but they do not provide the matching TP-AGB stages that are crucial for the SBF models. These three sets are of keen interest because they present the $\alpha$-enhanced stellar models at fixed $[Fe/H]$ instead of at fixed $Z$ as the Teramo BaSTI stellar models do. Regarding the $\alpha$-enhancement at fixed $Z$, a depressed Fe abundance preserves the total metallicity. For instance, in the Teramo BaSTI stellar models there is about 0.35 dex $[Fe/H]$ shift at fixed total metallicity, $Z$, between the solar-scaled (sss; solid lines) and the $\alpha$-enhanced (aes; dashed lines) models. The case of fixed $[Fe/H]$ is, alas, also less than perfectly straightforward. In that case, the enhancement of the $\alpha$-elements increases the overall metallicity, and therefore either the abundance of hydrogen or helium (or both) must be modified in order to compensate for the increased $Z$.

Moreover, there is the stellar atmosphere (emergent flux) issue. The Teramo BaSTI $\alpha$-enhanced stellar models incorporate the matching $\alpha$-enhanced stellar model atmospheres self-consistently in order to generate the observables (magnitudes and colors) as described

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$^2$Table 1 of Marigo et al. (2008) lists the available stellar isochrones including the TP-AGB phase. Marigo et al. (2008) describe that the TP-AGB of the Teramo BaSTI isochrones by Cordier et al. (2007) was computed in a rather crude way, i.e., without considering the third dredge-up events (carbon star formation) and hot bottom burning nucleosynthesis.
in Cassisi et al. (2004). All the other $\alpha$-enhanced stellar models, however, employ the solar-scaled stellar atmosphere for their calculations of $\alpha$-enhanced model observables. It would be useful to have $\alpha$-enhanced models at fixed [Fe/H] from the Teramo BaSTI group, as well as at fixed $Z$, so that the effects of the $\alpha$-element variation could be seen more directly, instead of mixing in the effects of Fe depression for the $\alpha$-enhanced models at fixed $Z$ (Salaris et al. 1993). In the same context, we are also looking forward to implementing the Padova $\alpha$-enhanced stellar models with matching $\alpha$-enhanced stellar model atmospheres as they become available.

Figure 3 demonstrates how the enhancement of each element (carbon, nitrogen, oxygen, and iron) modifies the emergent fluxes at 4000 K and log $g = 0.5$, which is the typical temperature and surface gravity of the upper RGBs and AGBs. The lower left panel of Figure 3 illuminates that the oxygen-enhancement generates a brighter $I$-band luminosity and a redder $V - I$ color. From Table 1, it is noted that the $V - I$ color becomes 0.031 mag redder because of the 0.3 dex oxygen-enhancement. Being a dominant $\alpha$-element (e.g., O, Mg, Si, S, Ca, Ti), this behavior from the oxygen-enhanced spectrum is very useful in order to understand the $V - I$ colors and $I$-band magnitudes seen in Figure 2 as well as the model results in Figure 4 that we discuss in the following section. The upper panels illustrate the carbon- and nitrogen-enhanced spectra again at 4000 K and log $g = 0.5$. They are, in general, displaying the opposite from what we see in lower left panel of Figure 3. By comparing the C-, N-, and O-enhanced spectra, it is interesting to find that many features around the $I$-band are CN-bands and they are more sensitive to carbon abundance than to nitrogen. The oxygen-enhanced spectrum in the lower left panel of Figure 3 illustrates that increasing oxygen abundance soaks up more C into the CO molecule, decreasing C$_2$, CH, and CN feature strengths.

The lower right panel of Figure 3 shows the iron-enhanced spectrum at 4000 K and log $g = 0.5$. It is evident that many strong iron absorption line features occur around the $UBV$-bandpasses. From Table 1, it is noted that the $U - B$ color becomes 0.084 mag redder because of the 0.3 dex iron-enhancement. Moreover, it is worthwhile to emphasize that $I$-band luminosity is relatively insensitive to the iron-abundance. The lower right panel of Figure 3 is quite helpful in understanding the observed color-magnitude effects because the $\alpha$-enhanced Teramo BaSTI stellar models are essentially equivalent to Fe-depressed ones.

3. Results

Having discussed the nature of the $\alpha$-element enhancement in terms of the isochrones and the emergent fluxes, we now present the integrated $\alpha$-enhanced SBF models and broad-
band colors. Following our initial results on this topic (Lee et al. 2007a), there have been recent critical updates from the Teramo BaSTI stellar models. The \( \alpha \)-enhanced isochrones and stellar evolutionary tracks have been recomputed for \( Z > 0.001 \) after employing the low-temperature opacities by Ferguson et al. (2005)\(^3\).

Figure 4 shows our new \( I \)-band SBF model calculations (aes: \( \alpha \)-enhanced standard, sss: solar-scaled standard, where ‘standard’ again means no convective core overshooting) as a function of integrated \( V - I \) colors based upon the recently updated Teramo BaSTI isochrones, which became available after May 2008. Two observational fiducial lines (thick bent straight lines) are overlaid with our theoretical models. The line on the blue-side is \( I \)-band SBF Mag = \(-2.25 + 2.44 \times [(V - I) − 1.00]\) from Mieske, Hilker, & Infante (2006)\(^4\) for dwarf blue galaxies, while the line on the red-side is \( I \)-band SBF Mag = \(-1.68 + 4.5 \times [(V - I) − 1.15]\) from Tonry et al. (2000)\(^5\) for massive red galaxies.

Compared to our earlier results (Lee et al. 2007a), the differences between the solar-scaled and the \( \alpha \)-enhanced models are smaller because of the Teramo BaSTI isochrones updates (mostly due to the low temperature opacities by Ferguson et al. 2005 for the \( \alpha \)-enhanced stellar models). At solar metallicity (\( Z=0.02 \), filled squares), it is found from Figure 4 that the \( \alpha \)-enhanced models become redder and brighter in this integrated \( V - I \) vs. \( I \)-band SBF magnitude plane compared to the solar-scaled ones mostly because of the oxygen-enhancement as we noted in Figures 2 and 3. The right panel of Figure 4 displays the integrated \( V - I \) colors vs. \( I \)-band SBF magnitudes when the TP-AGBs are not included in the calculations. The general trend of the \( \alpha \)-enhanced models at solar metallicity (\( Z=0.02 \), filled squares) becoming redder and brighter compared to the solar-scaled ones for \( t \geq 1 \) Gyr is mostly unchanged. It is seen, however, that the \( I \)-band SBF magnitudes are much too faint without the TP-AGBs to match the observations, especially at the metal-poor end.

Figure 5 is similar to Figure 2, but here displays the comparison of the sss and the aes Teramo BaSTI isochrones in \( V - I \) vs. \( I \) color-magnitude diagrams at \( Z=0.0003 \). It is important to note here that compared to Figure 2, the \( V - I \) colors and the \( I \)-band magnitudes are hardly changed with the \( \alpha \)-enhancement on this very metal-poor side even at the upper RGB. It explains the comparably smaller effects of \( \alpha \)-enhancement on SBF

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\(^4\)In Mieske, Hilker, & Infante (2006), \( I \)-band SBF Mag = \(-2.13 (±0.17) + 2.44 (±1.94) \times [(V - I) − 1.00]\) for \( 0.8 < V - I < 1.10 \). We have shifted within the permitted errors in order to match with the empirical line by Tonry et al. (2000).

\(^5\)We have adopted 0.06 mag zero-point correction by Blakeslee et al. (2002) to match the final set of \( H_0 \) Key Project Cepheid distances from Freedman et al. (2001).
models and integrated broadband colors at $Z=0.0003$ in Figure 4. It is also interesting to see from the left panels (upper: without post-RGB, lower: with post-RGB, but without TP-AGB) that the blue horizontal-branch of 13 Gyr overlaps in $V-I$ color and $I$-band magnitude with the MS turnoff of 1 Gyr here at $Z=0.0003$. In right panel of Figure 5, TP-AGBs are additionally depicted with thicker lines. It is seen that the TP-AGBs are far redder and brighter compared to their RGB tips, particularly on this very metal-poor side (cf., right panels of Figure 2). It explains the importance of the TP-AGBs on SBF models and integrated broadband colors at $Z=0.0003$ in Figure 4 (see also Figure 14).

We can also address the convective core overshooting issue at young ages ($t<5$ Gyr). Figure 6 is similar to the left panel of Figure 4, but here integrated $V-I$ colors and $I$-band SBF magnitudes using the Teramo BaSTI aes (dashed lines; $\alpha$-enhanced and without convective core overshooting) and aeo (solid lines; $\alpha$-enhanced and with convective core overshooting) isochrones are compared at three different relatively young ages. Note that at 1 Gyr and solar metallicity (filled squares) overshooting effects make the integrated $V-I$ colors bluer and the $I$-band SBF magnitudes fainter as indicated with an arrow. At 5 Gyr, however, it is seen that the overshooting effects become negligible. Figure 7 is similar to the right panels of Figures 1 and 2, but the aes (dashed lines) and the aeo (solid lines) Teramo BaSTI isochrones are compared in the H-R diagrams and the C-M diagrams at 1 and 5 Gyr for $Z=0.02$. The RGB tips are denoted with triangles for the aes and circles for the aeo, respectively, to guide the eye. It is interesting to note that the aeo models (1) have a hotter upper MS near the turnoff compared to the aes ones and (2) do not go all the way to the RGB tip at 1 Gyr (see also Lee et al. 2007b; Yi 2003). However, the aes and aeo models become virtually identical by 5 Gyr. The behavior due to the overshooting effects on the integrated $V-I$ colors (bluer because of the bluer upper MS near the turnoff and the fainter RGB) and on the $I$-band SBF magnitudes (fainter because of the less developed upper RGB) that we described in Figure 6 can be understood from this C-M diagram.

### 3.1. Comparison with Earlier Models

Figures 8 and 9 compare our scaled-solar (sss) models with other recent models available in the literature. Figure 8 contrasts our models based on the Teramo BaSTI 'sss' isochrones with Raimondo et al. (2005), while Figure 9 does that with Marín-Franch & Aparicio (2006)\(^6\). The large filled symbols are used to indicate the solar metallicity in order to guide the eye.

\(^6\)Among three different models in Marín-Franch & Aparicio (2006), we show the one with Pietrinferni et al. (2004) isochrones.
Both Figures 8 and 9 demonstrate that there are significant differences among models. Hence, we have collected several other available models in the literature and listed them in Table 2. We have only listed their integrated $V - I$ colors and $I$-band SBF magnitudes at 5 and 13 Gyr at solar metallicity.

Table 2 tells that there are non-negligible disparities among models at the same age and metallicity. The two different Worthey (1994) models\(^7\) evidently show that the different input ingredients (i.e., isochrones) make significant ($\sim 0.1$ mag in $V - I$ and $\sim 0.55$ mag in $I$-band SBF magnitudes) differences at the same age and metallicity. Obviously, depending upon the ingredients and recipe the modelers adopt such as (1) isochrones, particularly the late-evolutionary stage evolutionary tracks such as RGB, AGB, and TP-AGB, (2) mass-loss scheme such as $\eta$ in Reimers (1975), (3) stellar library to convert the temperature and luminosity to colors and magnitudes, and (4) the IMF, one can get significantly different model outputs. It is therefore considerably important to study the ingredients and recipe of different models carefully before use. A rigorous test of the integrated photometric models is imperative.

On a positive note, however, our models agree extremely well with other very recent works. For example, our scaled-solar (sss) models and Percival et al. (2009; P09) that are based upon the most up-to-date Teramo BaSTI with $\eta = 0.4$ and Salpeter IMF agree each other within 0.003 mag in $V - I$. Moreover, our scaled-solar models that are based upon the most up-to-date Padova isochrones with Salpeter IMF and Padova SSP models\(^8\) (Marigo et al. 2008) agree each other within 0.002 mag in $V - I$. Our models based upon the different isochrones will be compared with one another in detail in the following section.

### 3.2. Comparison of Padova and Teramo/BaSTI Solar-Scaled SBF Models

Having discovered the non-negligible impacts of the input ingredients (i.e., isochrones) on the SBF magnitudes and broadband colors, here we calculate those quantities by employing stellar models from different groups but at the same solar-scaled composition. In this study, we contrast the widely-used two stellar models, the Padova\(^9\) and the Teramo BaSTI stellar models\(^10\) at the same solar-scaled chemical composition. It is worthwhile to reiterate

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\(^7\)http://astro.wsu.edu/worthey/dial/dialLaamodel.html

\(^8\)They are available from http://stev.oapd.inaf.it/cgi-bin/cmd

\(^9\)http://stev.oapd.inaf.it/cgi-bin/cmd

\(^10\)http://193.204.1.62/index.html
that $\alpha$-enhanced stellar models could be even more diverse than expected because of differing definitions of “alpha-element”.

Figure 10 is similar to Figure 4, but here we contrast the $I$-band SBF models as a function of integrated $V - I$ colors at given ages and metallicities using two different stellar models but at the same solar-scaled chemical composition with convective core overshooting. One is using the Teramo BaSTI (solid lines) and the other is using the Padova (dashed lines) stellar models. We employ their latest stellar models, which we directly download from their websites (see footnotes 1 and 9, respectively) in order to calculate the SBF predictions. We have employed the ‘sso’ (solar-scaled with convective core overshooting) Teramo BaSTI models because we note that the Padova stellar models employ the convective core overshoot as the default in their models. To guide the eye, solar metallicity models of varying ages are marked with filled squares. The two sets of models show significant differences. In general, integrated $V - I$ colors based on the Padova isochrones are comparatively redder than that from the Teramo BaSTI. It is also noted from the left panel that the $I$-band SBF magnitudes based upon the Teramo BaSTI become much fainter ($> 1$ mag) at younger ages ($t < 5$ Gyr) with $Z \geq 0.0004$ compared to Padova models. In order to ascertain whether the remarkable disparities are mainly caused by the rather poorly understood bright TP-AGB stars, we display the same models without TP-AGBs in the right panel of Figure 10. The differences using two different stellar models persist even without the TP-AGBs although they are much less compared to that with the TP-AGBs.

From Figure 10, however, it is evident that the inclusion of TP-AGB stages is indeed necessary in matching the observations, which are represented by the thick bent lines. The systematic redder integrated $V - I$ colors using the Padova stellar models compared to that using the Teramo BaSTI ones can be understood from the fact that the Padova RGBs are systematically cooler and redder than the Teramo BaSTI ones, as illustrated in Figures 11 and 12. For a clearer understanding of the cause of the dissimilarities of the stellar population model $I$-band SBF predictions as well as the integrated $V - I$ colors using the the Padova and the Teramo BaSTI stellar models, we illustrate the comparison of the Padova and the Teramo BaSTI stellar models in the H-R diagrams and the C-M diagrams in the following figures.

Figure 11 shows the comparison of the Teramo BaSTI (solid lines) and the Padova (dashed lines) isochrones at the same solar-scaled chemical composition in the HR diagrams without and with TP-AGB stages, respectively. At solar metallicity, three ages (1, 5, 13 Gyr) are compared. RGB tips are denoted with squares for Teramo BaSTI and triangles for Padova, respectively. From Figure 11, it is noted that the MSTO and RGB temperatures of the older ages ($t \geq 5$ Gyr) are generally cooler in the Padova stellar models, which cause the
redder integrated \( V - I \) colors compared to those using the Teramo BaSTI ones in Figure 10. The small RGB temperature differences may be understood depending upon the treatment of convection in the RGB stars. In the right panels of Figure 11, the TP-AGB stages are additionally depicted with thicker lines. It is noted that at younger ages \( (t < 5 \text{ Gyr}) \), the TP-AGBs go far cooler and brighter than their RGB tips.

Figure 12 displays the comparison of the Teramo BaSTI (solid lines) and the Padova (dashed lines) isochrones in \( V - I \) vs. \( I \) color-magnitude diagrams at the same solar-scaled composition. Symbols for the RGB tips are same as in Figure 11. It is noted that the Padova RGBs, especially at the older ages \( (t \geq 5 \text{ Gyr}) \), are relatively redder than the Teramo BaSTI ones in these C-M diagrams. In the right panels of Figure 12, the TP-AGBs are additionally depicted with thicker lines. In general, the Padova TP-AGBs models are more complex than the Teramo BaSTI TP-AGBs. It is further noted from the right panels of Figure 12 that there are discontinuities for the Padova stellar models at the onset of the TP-AGB stage caused by structural changes (Marigo et al. 2008). For instance, the H-exhausted core mass on the TP-AGB starts to increase (and is set to zero before the TP-AGB).

### 3.3. Near-IR SBF Models

Having found that (1) there are significant differences in the \( I \)-band SBF model predictions using the Padova and the Teramo BaSTI models and (2) the proper inclusion of TP-AGBs is crucial to match the observations, we now extend our investigation to longer wavelengths, including near-IR SBF models where the effects of the TP-AGB stage are considerably more pronounced.

Figure 13 is similar to the left panel of Figure 10, but here we contrast the \( z_{850} \)-band SBF models based on the Padova stellar models\(^\text{11}\) as a function of integrated \( g_{475} - z_{850} \) colors at given ages and metallicities. The thick curved line is an empirical relation from Blakeslee et al. (2009). The \textit{HST} ACS/WFC photometric systems\(^\text{12}\) that we present here are all AB magnitudes in order to be consistent with the observations. We have converted the Vega magnitude system to AB magnitude system by employing \( F475W (\text{AB}) = F475W (\text{Vega}) - 0.101 \) and \( F850LP (\text{AB}) = F850LP (\text{Vega}) + 0.565 \) from Sirianni et al. (2005). To guide the eye, solar metallicities are marked with filled squares and 8 and 13 Gyr are displayed in different colors.

\(^{11}\)The reason that we show the \( z_{850} \)-band SBF models only based on the Padova stellar models is because the \( z_{850} \)-band magnitude is not yet available from the BaSTI websites.

\(^{12}\)The \( F475W \) and \( F850LP \) are equivalent to \( g_{475} \) and \( z_{850} \)-bands, respectively.
The arrows in Figure 13 at 5, 8, and 13 Gyr indicate the estimated effects on the models from 0.4 dex $\alpha$-enhancement. Since the Padova group do not yet provide the matching $\alpha$-enhanced stellar models, those arrows are inferred from the $\alpha$-enhanced $I$-band SBF and integrated $V - I$ model predictions based on the Teramo BaSTI stellar models. We have used the $(V - I) = 0.603 \times (g_{475} - z_{850}) + 0.375$ and $I$-band SBF $\text{mag} = 0.930 \times z_{850}$-band SBF $\text{mag} + 0.433$. It is interesting to find that the effects of the $\alpha$-enhancement on the $z_{850}$-band SBF models alleviate the mismatches with the observations on the red-side, $g_{475} - z_{850} > 1.3$. Biscardi et al. (2008) provide another interesting comparison of $z_{850}$-band SBF observations with model predictions, but using the Teramo SPoT models\textsuperscript{13}. The mismatch that is seen on the blue-side, $g_{475} - z_{850} < 1.1$ may be relieved if we could employ the BaSTI isochrones as we saw in Figure 10 that the BaSTI isochrones generate systematically much fainter SBF magnitudes compared to Padova isochrones at younger ages ($t < 5$ Gyr) with $Z \geq 0.004$.

Figure 14 contrasts the $HST$ NICMOS F160W-band SBF models as a function of integrated $V - I$ colors using the Padova and the Teramo BaSTI stellar models at the same solar-scaled chemical composition. The thicker line is the empirical sequence from Jensen et al. (2003). The $HST$ NICMOS photometric bandpass, F160W, that we present here is Vega magnitudes in order to be consistent with the observations. We have converted $JHK$-band SBF models using the Teramo BaSTI isochrones to F160W-band SBF ones by employing equation (3) of Jensen et al. (2003). As expected, the importance of the inclusion of the TP-AGB stage is considerably greater here in the near-IR compared to Figure 10. For instance, at 1 Gyr and solar metallicity, the F160W-band SBF magnitude predictions from the Teramo BaSTI models become $\sim$1.8 mag fainter without TP-AGBs. Also, it is noted that the trend between F160W-band SBF models and the integrated $V - I$ colors reverses, in the sense that the near-IR SBF magnitudes become brighter with bluer colors when the TP-AGB is included, but fainter at bluer colors when it is omitted. The significant differences between the models based on the Teramo BaSTI and the Padova models are still noticeable even without TP-AGBs from the right panel of Figure 14 at the same solar-scaled chemical composition.

Figure 15 is similar to the left panel of Figure 14, but displays the $\alpha$-element enhancement effects for the $HST$ NICMOS F160W-band SBF models based on the Teramo BaSTI. The thick observational fiducial line is the same as in Figure 14. The F160W-band SBF magnitude differences between the solar-scaled and the $\alpha$-enhanced models using the Teramo BaSTI stellar models are relatively small, less than 0.2 mag. The reason for the mismatches with the data at the very red end remains to be resolved. Compared to the observations,

\textsuperscript{13}http://193.204.1.79:21075/models.html
the left panel of Figure 4 showed that the $I$-band SBF models are a bit fainter, but here in Figure 15, F160W-band SBF models are brighter. Figure 1 of Blakeslee et al. (2009), however, shows that the very red galaxies become progressively fainter in $z$-band SBF causing the non-linear relation between SBF magnitudes and the integrated colors as we have seen in Figure 13.

4. Summary and Discussion

We have presented for the first time the effects of $\alpha$-element enhancement on SBF models and have compared these models with observations. For this purpose, we have employed the Teramo BaSTI Isochrones in this study. In general, we find that the $\alpha$-element enhanced $I$-band SBF magnitudes are brighter and their integrated $V - I$ colors are redder mostly because of the oxygen enhancement effects on the upper RGBs and AGBs. Moreover, the importance of the TP-AGB stages for the $I$-band and near-IR band SBF magnitudes is illustrated and it becomes clear that TP-AGBs are the indispensable component in order to match the theoretical predictions with the observations. It is also noted that the TP-AGBs are more important in the metal-poor regime as shown in Figures 4, 5, 10, and 14. Furthermore, our results illuminate the significant impact of the convective core overshooting on the predicted broadband colors for models with ages near 1 Gyr, confirming the earlier findings by Lee et al. (2007b).

We have further shown that the disparity in the SBF model and integrated $V - I$ color predictions is non-negligible when different stellar models are employed as input ingredients even at the same solar-scaled chemical composition. It is our understanding that what causes the disagreement in the integrated $V - I$ colors and $I$-band SBF models using different stellar models are (1) giant branch temperature differences, which occur depending upon the treatment of convection and (2) dissimilar stellar model atmospheres that are employed in order to convert from theoretical planes to observables. Further comparison of SBF model predictions with multi-band observations can help illuminate many of the remaining problems in the evolution of bright stars relevant to population synthesis modeling. Table 3 lists our findings at $Z = 0.02$.

Our results indicate that $\alpha$-enhanced models can ameliorate the match with the data for red massive galaxies, while solar-scaled models are fine for blue dwarf galaxies. The $I$-band SBF models explored here are, however, slightly fainter than the observations at the red end ($V - I > 1.15$), as shown in Figure 4. A partial solution may come from the non-linear behavior between galaxy colors in $g_{175} - z_{850}$ and the $z_{850}$-band SBF magnitudes as illuminated by Blakeslee et al. (2009) in the sense that the red-end galaxies become rapidly fainter in
\(z_{850}\)-band SBF magnitudes. In this context, the \(\alpha\)-enhanced SBF models help to reduce the disagreement by making the theoretical \(I\)-band and \(z\)-band SBF brighter. Also, rigorous statistical investigations of model isochrones with TP-AGBs that are poorly populated yet very salient because of their prominent luminosity should help alleviate the discrepancies between the observations and the theoretical predictions as suggested by Cerviño et al. (2008).

It is evident from the smaller scatter in the observations compared to the theoretical predictions that there is generally less variation among the AGB phases in actual galaxies than may be expected based on the models. The observed relations between SBF magnitudes and integrated colors are very tight, at least for evolved galaxies. Moreover, \(\alpha\)-enhanced SBF studies in various bandpasses should provide powerful constraints on the fine details of the calibration in the near future.

We thank Michele Cantiello, Santi Cassisi, Aaron Dotter, Leo Girardi, Paola Marigo, Reynier Peletier, and Gabriella Raimondo for very insightful discussions. We are also grateful to the anonymous referee for her/his constructive report that improves our presentation. Support for this work was provided by the NSF through grant AST-0307487, the New Standard Stellar Population Models (NSSPM) project and by the NASA through grant HST-GO-11083.
REFERENCES


Raimondo, G., Brocato, E., Cantiello, M, & Capaccioli, M. 2005, AJ, 130, 2625 (R05 = SPoT)
Table 1. Color Behavior with Elemental Enhancements for a Star with $T_{\text{eff}} = 4000$ K, $\log g = 0.5$

<table>
<thead>
<tr>
<th>Color</th>
<th>C</th>
<th>N</th>
<th>O</th>
<th>Mg</th>
<th>Si</th>
<th>S</th>
<th>Ca</th>
<th>Ti</th>
<th>Fe</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U - B$</td>
<td>30</td>
<td>26</td>
<td>-21</td>
<td>34</td>
<td>-43</td>
<td>0</td>
<td>16</td>
<td>24</td>
<td>84</td>
</tr>
<tr>
<td>$B - V$</td>
<td>16</td>
<td>-2</td>
<td>-13</td>
<td>-35</td>
<td>18</td>
<td>0</td>
<td>7</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>$V - I$</td>
<td>-30</td>
<td>-28</td>
<td>31</td>
<td>12</td>
<td>0</td>
<td>0</td>
<td>-4</td>
<td>2</td>
<td>16</td>
</tr>
</tbody>
</table>

Note. — 1. All elements scaled individually by +0.3 dex, except C, which is increased by +0.15 dex. 2. Numbers are in milli-magnitude.
Table 2. Integrated $V - I$ and $I$-band SBF Mag from Different SSP Models

<table>
<thead>
<tr>
<th>Models</th>
<th>$V - I$ (5 Gyr)</th>
<th>$V - I$ (13 Gyr)</th>
<th>$\bar{M}_I$ (5 Gyr)</th>
<th>$\bar{M}_I$ (13 Gyr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W94, Sal</td>
<td>1.186</td>
<td>1.312</td>
<td>-1.680</td>
<td>-1.255</td>
</tr>
<tr>
<td>W94, Sal, B94</td>
<td>1.148</td>
<td>1.210</td>
<td>-1.129</td>
<td>-0.699</td>
</tr>
<tr>
<td>B01, Sal, G00</td>
<td>1.140</td>
<td>1.240</td>
<td>-1.410</td>
<td>-0.960</td>
</tr>
<tr>
<td>BC03, Cha, B94</td>
<td>1.124</td>
<td>1.227</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>BC03, Sal, B94</td>
<td>1.141</td>
<td>1.255</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>M05, Kro</td>
<td>1.088</td>
<td>1.176</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>M05, Sal</td>
<td>1.099</td>
<td>1.198</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>R05, Sca, P04</td>
<td>1.174</td>
<td>1.234</td>
<td>-1.548</td>
<td>-1.225</td>
</tr>
<tr>
<td>MA06, Kro, B94</td>
<td>1.167</td>
<td>1.282</td>
<td>-1.097</td>
<td>-0.664</td>
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<tr>
<td>MA06, Kro, G00</td>
<td>1.163</td>
<td>1.278</td>
<td>-1.290</td>
<td>-0.909</td>
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<tr>
<td>MA06, Kro, P04</td>
<td>1.124</td>
<td>1.243</td>
<td>-1.663</td>
<td>-1.104</td>
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<tr>
<td>Pad_SSP, Cha</td>
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<td>...</td>
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<tr>
<td>Pad_SSP, Sal</td>
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<td>1.261</td>
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<td>...</td>
</tr>
<tr>
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<td>1.185</td>
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<td>...</td>
</tr>
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<td>P09, sss, 0.4</td>
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<td>1.177</td>
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<td>...</td>
</tr>
<tr>
<td>P09, aes, 0.2</td>
<td>1.121</td>
<td>1.203</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>P09, aes, 0.4</td>
<td>1.117</td>
<td>1.193</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>This Work, sss</td>
<td>1.086</td>
<td>1.178</td>
<td>-1.235</td>
<td>-0.853</td>
</tr>
<tr>
<td>This Work, aes</td>
<td>1.111</td>
<td>1.192</td>
<td>-1.567</td>
<td>-1.217</td>
</tr>
<tr>
<td>This Work, Pad</td>
<td>1.173</td>
<td>1.263</td>
<td>-1.373</td>
<td>-0.831</td>
</tr>
</tbody>
</table>

Note. — 1. W94 (Worthey 1994) is from http://astro.wsu.edu/worthey/dial/dial\_model.html ‘Sal’ is Salpeter IMF and ‘B94’ is Bertelli et al. (1994). 2. B01 (Blakeslee, Vazdekis, & Ajhar 2001) are 5 and 12.6 Gyr values from their Table 2. 3. BC03 (Bruzual & Charlot 2003) is from http://www2.iap.fr/users/charlot/\_bc2003/ ‘Cha’ is Chabrier (2003) IMF. 4. M05 (Maraston 2005) is from http://www.icg.port.ac.uk/\~maraston/SSPn/colors/SSPcolours\_Mar05\_JohnsonCousins.tab ‘Kro’ is Kroupa (2001) IMF. 5. R05 (Raimondo et al. 2005) is from http://193.204.1.79:21075/models.html ‘Sca’ is Scalo (1998) IMF and ‘P04’ is Pietrinferni et al. (2004). 6. MA06 (Marin-Franch & Aparicio 2006) used Kroupa IMF (Kroupa et al. 2003). 7. Pad\_SSP (Marigo et al. 2008) is from http://stev.oapd.inaf.it/cgi-bin/cmd 8. P09 (Percival et al. 2009) is from http://193.204.1.62/index.html ‘sss’ is scaled-solar and ‘aes’ is α-enhanced. Two mass-loss schemes are used; one with $\eta = 0.2$ and the other with $\eta = 0.4$. 9. This Work, sss and aes adopt the $\eta = 0.4$. This Work, Pad is the SSP based on Marigo et al. (2008).
Table 3. SBF Magnitude And Integrated $V - I$ Color Differences At $Z = 0.02$

<table>
<thead>
<tr>
<th>Age (Gyr)</th>
<th>$\Delta I$-band SBF (mag)</th>
<th>$\Delta F160W$-band SBF (mag)</th>
<th>$\Delta V - I$ (mag)</th>
</tr>
</thead>
</table>

Effects of 0.4 dex $\alpha$-enhancement using Teramo BaSTI: aes – sss

0.5  -0.380  0.168  0.058
1    -0.242  0.152  0.029
5    -0.332  0.080  0.025
13   -0.364  0.066  0.014

Effects of TP-AGB (using Teramo BaSTI sss): with TP-AGB – without TP-AGB

0.5  -1.084  -4.571  0.068
1    -0.286  -1.880  0.057
5    -0.068  -0.472  0.015
13   -0.046  -0.207  0.011

Effects of convective core overshooting (using Teramo BaSTI): aeo – aes

0.5  -0.094  \cdots  \cdots  -0.098
1    0.180  \cdots  \cdots  -0.050
5    -0.003  \cdots  \cdots  0.004

Effects of stellar models at solar-scaled with TP-AGB: Padova – Teramo BaSTI

0.5  -1.444  -0.736  0.041
1    -0.743  -0.945  -0.036
5    -0.138  -0.546  0.087
13   0.022   0.006   0.085

Note. — 1. The negative numbers in columns 2 and 3 indicate that the effects make the SBF magnitude brighter and vice versa. 2. The positive numbers in column 4 indicate that the effects make the integrated $V - I$ color redder and vice versa.
Fig. 1.— Comparison of solar-scaled (sss; solid lines) and α-enhanced (aes) Teramo BaSTI isochrones in the HR diagram without convective core overshooting. At solar metallicity ($Z=0.02$), three ages (1, 5, 13 Gyr) are compared. Left panels display isochrones without thermally pulsing-asymptotic giant branches (TP-AGB), while right panels show isochrones with TP-AGBs (thicker lines). Red giant branch tips (RGB tips) are depicted with squares for the sss and triangles for the aes, respectively. Bottom panels show the details in the giant branches. The RGB temperatures are generally slightly warmer ($\sim$45K) with α-enhancement at fixed total metallicity, partly reflecting the depression of iron abundance. Note also from the right panels that at younger ages ($t < 5$ Gyr), the TP-AGBs go far cooler and brighter compared to their RGB tips.
Fig. 2.— Similar to Figure 1, but here we display the comparison of the sss and the aes Teramo BaSTI isochrones in the $V-I$ vs. $I$ color-magnitude diagrams. Symbols for the RGB tips and the TP-AGBs are same as in Figure 1. Note from the left panels that the $\alpha$-enhanced red giant branches (RGBs), especially at the upper part ($I < -2$), are relatively brighter and redder in this $V-I$ vs. $I$ C-M diagrams compared to that of the solar-scaled. Similarly, it is noted from the right panels that the $\alpha$-enhanced TP-AGBs are also comparatively brighter and redder than the solar-scaled models. Moreover, it is seen from the right panels that at younger ages ($t < 5$ Gyr), the TP-AGBs go far redder compared to their RGB tips.
Fig. 3.— At 4000 K, log $g = 0.5$, and [Fe/H] = 0, the solar-scaled spectrum is divided by the 0.15 dex carbon-enhanced spectrum (upper left), the 0.3 dex nitrogen-enhanced spectrum (upper right), the 0.3 dex oxygen-enhanced spectrum (lower left), and the 0.3 dex iron-enhanced spectrum (lower right), respectively. $UBVRI$-band filter locations are indicated. It is seen from the lower left panel that the oxygen-enhancement makes the higher $I$-band luminosity and the redder $V-I$ color, while from the upper panels it is noted that carbon- and nitrogen-enhancement make the bluer $V-I$ color at this temperature and surface gravity of the typical upper RGB and AGB. From the comparison of C-, N-, O-enhanced cases, it is identified that many features around the $I$-band are CN-bands and that they are more sensitive to the carbon abundance than to the nitrogen. Furthermore, it is seen from the lower right panel that many strong iron absorption line features are mostly located within the $UBV$-bandpass. Note that the $I$-band luminosity is relatively insensitive to the iron-abundance.
Fig. 4.— I-band SBF magnitudes as a function of integrated $V-I$ colors are shown at 0.5, 1, 5, and 13 Gyr (left to right) for five different metallicities using the most up-to-date Teramo BaSTI isochrones available after May 2008. Solid lines with squares are solar-scaled models, while dashed lines with squares are $\alpha$-enhanced models. At given ages, I-band SBF magnitudes become fainter and $V-I$ colors become redder, in general, with increasing metallicity. To guide the eye, solar-metallicity ($Z=0.02$) is depicted with filled squares. Two observational fiducial lines (bent thick solid lines; bluer one from Mieske et al. 2006 and redder one from Tonry et al. 2000) are compared with our theoretical models. The $\alpha$-enhanced models at solar metallicity, $Z=0.02$ (filled squares), become redder and brighter compared to the solar-scaled ones mostly caused by the oxygen-enhancement as we noted in Figures 2 and 3. The models without TP-AGBs on the right panel show that the general effect of the $\alpha$-enhanced models at solar metallicity ($Z=0.02$, filled squares) becoming redder and brighter compared to the solar-scaled ones for $t \geq 1$ Gyr are mostly unchanged. It is noted, however, that the I-band SBF magnitudes are much too faint without TP-AGBs to match the observations.
Fig. 5.— Similar to Figure 2, but here we display the comparison of the solar-scaled (sss) and the α-enhanced (aes) Teramo BaSTI isochrones in the $V - I$ vs. $I$ color-magnitude diagram at $Z=0.0003$. Symbols for the RGB tips and the TP-AGBs are same as in Figure 1. From left panels, it is interesting to find that the blue horizontal-branch at 13 Gyr overlaps in $V - I$ color and $I$-band magnitude with the MS turnoff of 1 Gyr here at $Z=0.0003$. Note that compared to Figure 2, the $V - I$ colors and the $I$-band magnitudes are hardly changed with the α-enhancement at this very metal-poor regime even at the upper RGB. It explains the comparably smaller effects of α-enhancement at $Z=0.0003$ in Figure 4. Also, note from the right panel that at all ages, the TP-AGBs go far redder and brighter compared to their RGB tips, particularly at this very metal-poor regime. It explains the importance of the TP-AGBs at $Z=0.0003$ shown in Figure 4.
Fig. 6.— Similar to the left panel of Figure 4, but here I-band SBF magnitudes as a function of $V-I$ are compared from the Teramo BaSTI aes (dashed lines; without convective core overshooting) and aeo (solid lines; with convective core overshooting) isochrones at three given ages. The observational fiducial lines are same as in Figure 4. Note that at 1 Gyr and solar metallicity ($Z=0.02$, filled squares), overshooting effects make the integrated $V-I$ colors bluer and the I-band SBF magnitudes fainter as indicated with an arrow. At 5 Gyr, however, the overshooting effects become minimal.
Fig. 7.— Similar to the right panels of Figures 1 and 2, but the aes (dashed lines; without convective core overshooting) and the aeo (solid lines; with convective core overshooting) Teramo BaSTI isochrones are compared in the HR diagrams (left panels) and C-M diagrams (right panels) at 1 and 5 Gyr at $Z=0.02$. RGB tips are denoted with triangles for the aes and circles for the aeo, respectively. Bottom panels show the details in the giant branches. It is interesting to note that the aeo models (1) are hotter at the upper MS compared to the aes models and (2) do not go all the way to the RGB tip at 1 Gyr. They become virtually identical at 5 Gyr. The overshooting effects on the integrated $V-I$ colors and the $I$-band SBF magnitudes that we described in Figure 6 can be understood from this C-M diagram (see text).
Fig. 8.— Similar to the left panel of Figure 4, but here our solar-scaled (sss; without convective core overshooting) $I$-band SBF models as a function of $V - I$ are compared with the Teramo SPoT models (Raimondo et al. 2005; R05) at 1, 5, and 13 Gyr. To guide the eye, solar-metallicity ($Z=0.02$) is depicted with filled symbols.
Fig. 9.— Similar to Figure 8, but here our solar-scaled $I$-band SBF models as a function of $V-I$ are compared from that of Marín-Franch & Aparicio (2006; MA06) at 5, and 13 Gyr. To guide the eye, solar-metallicity ($Z=0.02$) is depicted with filled symbols.
Fig. 10.— Similar to Figure 4, but here we contrast the $I$-band SBF models as a function of integrated $V - I$ colors at the same solar-scaled compositions (sso; convective core overshooting is “on”) using the Padova (dashed lines) and the Teramo BaSTI (solid lines) stellar models at given ages and metallicities. To guide the eye, solar-metallicity is depicted with filled squares. The observational fiducial lines are same as in Figure 4. It is noted that two models show significant differences. In general, $V - I$ colors from the Padova isochrones are comparatively redder than that from the Teramo BaSTI. Note also from the left panel that the $I$-band SBF magnitudes based upon the Teramo BaSTI become much fainter ($>1$ mag) at younger ages ($t < 5$ Gyr) with $Z \geq 0.0004$ compared to Padova models. The model disparities based on the two different stellar models linger even without TP-AGBs as shown on the right panel. It is evident though that the inclusion of TP-AGB stages is crucial to match the observations, which are the bent thick lines.
also from the right panels that at younger ages (t < 5 Gyr), the TP-AGBs go far cooler and brighter compared to their RGB tips.

Fig. 11.— Comparison of the Teramo BaSTI (solid lines) and the Padova (dashed lines) isochrones at the same solar-scaled composition (sso; convective core overshooting is “on”) in the HR diagrams without and with TP-AGB stages. At solar metallicity, three ages (1, 5, 13 Gyr) are compared. RGB tips are marked with squares for Teramo BaSTI and triangles for Padova, respectively. Bottom panels show the details in the giant branches. Note that the RGB temperatures are generally cooler in the Padova stellar models, which causes the redder $V - I$ colors compared to those of the Teramo BaSTI models in Figure 12. Note also from the right panels that at younger ages (t < 5 Gyr), the TP-AGBs go far cooler and brighter compared to their RGB tips.
Fig. 12.— Similar to Figure 11, but here displaying the comparison of the Teramo BaSTI (solid lines) and the Padova (dashed lines) isochrones in $V - I$ vs. $I$ C-M diagrams. Symbols for the RGB tips are same as in Figure 11. Bottom panels show the details in the giant branches. Note that the Padova RGBs and AGBs, especially at the upper part, are relatively redder than the Teramo BaSTI ones in this $V - I$ vs. $I$ C-M diagrams. Note also from the right panels that there are some discontinuities for the Padova stellar models at the onset of TP-AGBs because of their structural changes (see text).
Fig. 13.— Similar to the left panel of Figure 10, but here we contrast the HST ACS/WFC $z_{850}$-band SBF models based on the Padova stellar models as a function of integrated $g_{475} - z_{850}$ colors at given ages and metallicities. The magnitudes are all in AB system. To guide the eye, solar metallicities are marked with filled squares and 8 and 13 Gyr are displayed in different colors. The thick curved line is the empirical relation from Blakeslee et al. (2009). The arrows at 5, 8, and 13 Gyr indicate the estimated 0.4 dex $\alpha$-enhancement effects (see text).
Fig. 14.— Similar to Figure 10, but here the HST NICMOS F160W-band SBF models as a function of integrated $V - I$ colors are contrasted using the Padova (dashed lines) and the Teramo BaSTI (solid lines) stellar models at the same solar-scaled compositions (sso; convective core overshooting is “on”). The thicker line is the empirical sequence from Jensen et al. (2003). The importance of the inclusion of TP-AGB stages is considerably greater here for the near-IR SBF models in order to match the observations. Note that the predicted near-IR SBF magnitudes become brighter with bluer colors, as observed, when the TP-AGB phase is included. In the right panel, the opposite dependence is seen for the models without the TP-AGB. Significant model disparities based on two different stellar models are still noticeable even without TP-AGBs from the right panel.
Fig. 15.— Similar to the left panel of Figure 14, but here the $\alpha$-element enhancement effects are shown for the HST NICMOS F160W-band SBF magnitudes as a function of integrated $V - I$ colors based on the Teramo BaSTI stellar models. Solid lines with squares are solar-scaled models, while dashed lines with squares are $\alpha$-enhanced models. The thick observational fiducial line is same as in Figure 14. The F160W-band SBF magnitude differences between the solar-scaled and the $\alpha$-enhanced models using the Teramo BaSTI stellar models are relatively small, less than 0.2 mag.