Complete N-ary Subtrees on Branching Family Tree

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Abstract

We review the existing and present new results on certain subtrees of a branching tree. Consider the rooted subtrees of a Galton-Watson family tree. For a fixed positive integer N, define a complete N-ary tree to be the tree of a deterministic N-splitting. Let the random variable $V(N)$ be the number of complete infinite and disjoined N-ary subtrees of a branching tree, rooted at the ancestor. In [3], Yanev and Mutafchiev study the distribution of $V(N)$. The event $\{V(1) > 0\}$ implies that there is at least one infinite unary subtree and thus the process would never die. The event $\{V(2) > 0\}$ can be interpreted as the set of process' trajectories when the family tree grows faster than binary splitting. Dekking [1] raised and answered the question how to compute the probability for a branching process to possess the "binary splitting property", i.e., $\Pr(V(2) > 0) > 0$. Pakes and Decking [2] study the general situation when $N \geq 2$. Mutafchiev [4] proves limit results for the survival probability of a N-ary subtree. It is surprising that the case $N \geq 2$ is studied so late, whereas the classical question for extinction of a branching process, i.e., non-existence of an infinite unary tree has been studied extensively over the past 120-150 years.

References

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