The University of Texas – Rio Grande Valley
Syllabus for Math 6387.01R/90L: Math. Modeling Summer I 2022
MAGC 2.418 MTWRF 15:00-16:30

Contact information

Instructor: Dr. Eleftherios Gkioulekas, School of Mathematical and Statistical Sciences
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Office hours: TR 16:30-17:30
Office location: EMAGC 3.214

Course information

Prerequisites: Consent of instructor.
Corequisites: None.
Course Description: This course presents the theory and application of mathematical modeling. Topics will be selected from dynamic models, stable and unstable motion, stability of linear and nonlinear systems, Liapunov functions, feedback, growth and decay, the logistic model, population models, cycles, bifurcation, catastrophe, biological and biomedical models, chaos, strange attractors, deterministic and random behavior.
Course modality: The course lectures will be live-streamed synchronously at the scheduled days and times. Some of the live streams will be conducted from the classroom, however due to the reduced sitting format, 50% to 80% of the lectures will be fully online. Attendance will be tracked via weekly submission of homework on the Discussion Forums. Exams will be given in take-home format and will be distributed and collected via Blackboard, and returned via email.

Textbook

(1) E. Gkioulekas: "Lecture Notes on Mathematical Modeling", *Online Lecture Notes on Mathematics*, Edinburg, University of Texas Pan American (2010), 206 pp.
- Open Educational Resource
- This document can be downloaded as a pdf file, at no cost, from https://faculty.utrgv.edu/eleftherios.gkioulekas/Teaching/notes.html

Other References

(1) S.H. Strogatz (2015): "Nonlinear dynamics and chaos: With Applications to Physics, Biology, Chemistry, and Engineering", 2nd edition, Addison-Wesley [course textbook]
- Commercial Textbook
- Available new for $81 from https://www.amazon.com/Nonlinear-Dynamics-Student-Solutions-Manual/dp/0813349109/

(2) S. Wiggins (2003): "Introduction to Applied Nonlinear Dynamical Systems and Chaos", 2nd edition, Springer-Verlag
- Commercial Textbook
- Available new for $81 from https://www.amazon.com/Introduction-Applied-Nonlinear-Dynamical-Mathematics/dp/0387001778

See course website for hyperlinks.

Outline of Topics

- **Autonomous dynamical systems**
  Introduction to autonomous dynamical systems
  Existence and uniqueness
  Fixed points and stability
  Lyapunov functions
- **1d autonomous dynamical systems**
  Stability analysis for 1d systems
  Potential and 1d systems
  Local bifurcations with 1D systems
  More on sufficient conditions for bifurcation events
- **Exam 1**
- **Linear autonomous systems**
  Exact Solutions
  Lyapunov function for $\dot{x} = Ax$
  The 2×2 linear autonomous system
- **Exam 2**
- **Nonlinear autonomous systems**
  Local analysis of fixed points
  Nonlinear centers – conversion to polar coordinates
  Nonlinear centers – conservative systems
  Nonlinear centers – reversible systems
  Index theory – definition
  Index theory – properties of the index
  Index theory – index of a fixed point
  Index theory – index of a curve surrounding fixed points
- **Exam 3**
- **Limit cycles**
  Ruling out closed orbits – gradient systems
  Ruling out closed orbits – Lyapunov functions
  Ruling out closed orbits – Dulac's criterion
  Definition of orbital stability
  The Poincare-Bendixson theorem
  Lienard systems
  Hopf bifurcation – Phenomenology
  Hopf bifurcation – Prototype Hopf bifurcation
  Hopf bifurcation – Classification of Hopf bifurcations
- **Exam 4**
- **Center manifold reduction**
  Center manifold reduction – Methodology
  Center manifold reduction – Inclusion of linearly unstable directions
  Center manifold reduction – Application to local bifurcations.
- **Final Exam**

Pedagogical objectives and expectations

The fundamental pedagogical objectives that students should strive for in every Mathematics course are:

(1) To understand, learn, and remember the formal and rigorous mathematical *definition* for every concept covered in the course.
(2) To understand, learn, and remember all the *theorems* and *propositions* that are applicable to previously defined concepts.
(3) To understand, learn, and practice the *methods* for applying theorems in the solution of routine problems, and to be able to creatively synthesize techniques to solve problems that are non-routine and may require creative thinking.
(4) To master *rigorous mathematical writing*, understand and use *logic and quantifier notation*, and realize and appreciate that every mathematical argument, from basic arithmetic, to advanced mathematics, with almost no exceptions, is a mathematical proof.
(5) To master the course material to a level of excellence that will ensure sustained success in more advanced mathematics courses.

To be successful in this course, it is expected that you should:

(1) Spend about 12 hours each week working homework problems, reviewing lecture notes, reading the textbook and online lecture notes, studying for exams, and seeking help from the tutors and instructor;
(2) Complete all homework problems, check the correctness of your work, and understand the methods and principles they illustrate;
(3) Master the designed course topics before each test, and if necessary, complete additional problems beyond those assigned and consult other sources if you find the assigned problems and text are insufficient;
(4) Recognize that mastery of the solution to a problem is not demonstrated by simply obtaining the correct numerical answer, but only by a clear, systematic, and detailed solution that traces the given information to the final numerical answer and that employs knowledge developed in this and previous courses;
(5) When you experience difficulty in the course, seek help from the tutors and instructor immediately;
(6) Attend class meetings regularly, pay attention, and do not hesitate to ask questions; and
(7) Write your solutions to homework, test, and quiz problems in an organized and legible way.

Grading Policies

- **Grading:** There will be 4 major exams, and a comprehensive final exam. The time and location of exams will be announced in class. Exams count for 80% and final exam for 20%of your grade. Combined, you get a numerical grade on a scale 0-20. Each exam question is graded on a 0-4 scale with $4 = A$, $3 = B$, $2 = C$, $1 = D$, $0 = F$. Combining all exams, as explained above, gives a weighted average score on a 0-20 scale. This score is then mapped to a letter grade as follows: A: 16-20; B: 12-16; C: 10-12; D; 7-10; F: 0-7. There will be no curve and no extra credit.
- **Explanation of grades:** Your exams are graded question by question on a 0-4 scale per question. Overall, if you are planning to take future Mathematics or STEM courses, I would like to see you score 3 or 4 on all questions on all major examinations. If you score less than that on any questions, it indicates weaknesses in understanding the material. You should be proactive about addressing these weaknesses.
- **Missed exams:** If a major exam is missed during an excused absence, your score for that exam will be replaced with your final exam score.
- **Homework:** Homework will be assigned and will be collected via Blackboard discussion forums on a weekly basis. It is crucial to do the homework as part of your preparation for the exams. To keep up, I recommend that **after every lecture you should solve the homework problems corresponding to the material covered on that day's lecture. Thus you need to work on**

a continuous basis! Maintain a well-organized written record of your homework solutions by **writing the statement of each problem** (so that your document is stand-alone and can be read by itself), followed by your detailed solution, and clearly indicate the problem, section, and chapter number of the question. Most homework problems require more than simply writing the answer, and so you must write all steps of your solution and provide appropriate justification, as illustrated by the instructor's solved examples, as you would on a test. Write neatly and legibly, using rigorous mathematical notation. While you are encouraged to discuss homework problems with other students, tutors, your instructor, and other faculty, the write-up of your solutions must be your own work and not simply copied from another student or another source. Use a ring binder to collect your homework, and write with a black pen, as that will help you to later scan the ring binder as a PDF file, for possible future use, and as a form of backup. This will provide you with a readily available resource to prepare for tests and quizzes, as well as providing documentation of the homework problems should you have a question about a problem and seek help from the instructor or a tutor.

- **Make-ups:** There are no make-up exams. In the case of **excused** absences the final exam will be used as a make-up exam. **Each student MUST take the final exam at the scheduled date and time.** There will be no make-ups for the final exam, after the official final exam date!
- **Regrading policy:** If you believe that a mistake in grading has been made you may request that your paper be regraded. Such request must be submitted **in writing** within one week from the day the graded test has been returned in class, and must be accompanied by the original (unaltered) paper. If you make any changes to the paper your request will be denied. Please note that if you request regrading, all problems are subject to review. Thus, your overall grade may be increased or decreased.

Other Policies

- **Calendar of Activities:** Information regarding important dates, such as, first day of classes, holidays, last day to drop a class before it appears on the transcript (the census date), last day to drop or withdraw with a DR grade, and final exam schedule are available via the academic calendar, linked from the course website. Please be advised of these important dates, and feel free to inquire with the instructor about any questions you may have with regard to the academic calendar.
- **Calculators:** The problems you will encounter in my exams will not require a calculator, and you are better served in the long-term by minimizing your dependence on calculators. Don't use the calculator to approximate roots, exponentials, logarithms, etc. Mathematical problems require **exact** answers. Approximations are reasonable **only** on application problems where the numbers given may be approximate themselves, and thus the best answer that can be deduced is unavoidably approximate.
- **Classroom Conduct:** Common courtesy requires that students arrive in class on time, and stay the entire class period. Turn your cellphones and pagers off. You are required to treat your classmate and instructor with respect and courtesy. Use of any electronic devices, except for calculators, is not allowed in class, and I reserve the option to remove you from the classroom without warning for any behaviour that I deem as disrespectful or disruptive. You agree to indemnify and hold harmless the professor with respect to all actions undertaken by the professor to enforce classroom conduct or to properly proctor exams. Taking my course implies your consent to this policy.
- **Revisions:** This syllabus may be revised at any time. The syllabus posted on the professor's course web site is the only copy guaranteed to incorporate all revisions that may be made under this policy and will thus supersede any other versions posted on other university websites.

- **Mandatory Course Evaluations:** Students are required to complete an ONLINE evaluation of this course, accessed through your UTRGV account (http://my.utrgv.edu); you will be contacted through email with further instructions. Online evaluations will be available August 6 - August 13. Students who complete their evaluations will have priority access to their grades.
- **Students with disabilities:** Students with a documented disability (physical, psychological, learning, or other disability which affects academic performance) who would like to receive academic accommodations should contact Student Accessibility Services (SAS) as soon as possible to schedule an appointment to initiate services. Accommodations can be arranged through SAS at any time, but are not retroactive. Students who experience a broken bone, severe injury, or undergo surgery during the semester are eligible for temporary services.
  - *Pregnancy, Pregnancy-related, and Parenting Accommodations:* Title IX of the Education Amendments of 1972 prohibits sex discrimination, which includes discrimination based on pregnancy, marital status, or parental status. Students seeking accommodations related to pregnancy, pregnancy-related condition, or parenting (reasonably immediate postpartum period) are encouraged to contact Student Accessibility Services for additional information and to request accommodations.
  - *Student Accessibility Services – Brownsville Campus:* Student Accessibility Services is located in 1.107 in the Music and Learning Center building (BMSLC) and can be contacted by phone at (956) 882-7374 or via email at ability@utrgv.edu.
  - *Student Accessibility Services – Edinburg Campus:* Student Accessibility Services is located in 108 University Center (EUCTR) and can be contacted by phone at (956) 665-7005 or via email at ability@utrgv.edu.
- **Scholastic dishonesty:** As members of a community dedicated to Honesty, Integrity and Respect, students are reminded that those who engage in scholastic dishonesty are subject to disciplinary penalties, including the possibility of failure in the course and expulsion from the University. Scholastic dishonesty includes but is not limited to: cheating, plagiarism (including self-plagiarism), and collusion; submission for credit of any work or materials that are attributable in whole or in part to another person; taking an examination for another person; any act designed to give unfair advantage to a student; or the attempt to commit such acts. Since scholastic dishonesty harms the individual, all students and the integrity of the University, policies on scholastic dishonesty will be strictly enforced (Board of Regents Rules and Regulations and UTRGV Academic Integrity Guidelines). All scholastic dishonesty incidents will be reported to Student Rights and Responsibilities.
- **Sexual misconduct and mandatory reporting:** In accordance with UT System regulations, your instructor is a "Responsible Employee" for reporting purposes under Title IX regulations and so must report to the Office of Institutional Equity & Diversity (oie@utrgv.edu) any instance, occurring during a student's time in college, of sexual misconduct, which includes sexual assault, stalking, dating violence, domestic violence, and sexual harassment, about which she/he becomes aware during this course through writing, discussion, or personal disclosure. More information can be found at www.utrgv.edu/equity, including confidential resources available on campus. The faculty and staff of UTRGV actively strive to provide a learning, working, and living environment that promotes personal integrity, civility, and mutual respect that is free from sexual misconduct, discrimination, and all forms of violence. If students, faculty, or staff would like confidential assistance, or have questions, they can contact OVAVP (Office for Victim Advocacy & Violence Prevention) at 665-8287, 882-8282, or OVAVP@utrgv.edu.

Technical Requirements

- **Computer Hardware:** To participate in this course, you should have easy access to a computer less than 5-years old with high-speed internet connection via cable modem, LAN or DSL. It is strongly recommended that you also have a printer (to print lecture notes) and scanner (to scan homework and take-home exams).
- **Student Technical Skills:** You are expected to be proficient with installing and using basic computer applications and have the ability to send and receive email attachments.
- **Software:**
  - Mozilla's Firefox or Google Chrome
  - Adobe Acrobat
  - Zoom
  - Media player software (e.g. Quicktime, Windows Media Player, etc.)
  - Virus protection Software
  - Microsoft Word or TeXLive
- **Blackboard Support Contact Information:** If you need Blackboard support at any time during the course or to report a problem with Blackboard you can:
  - Visit the Blackboard Student Help Site: https://help.blackboard.com/Learn/Student
  - Submit a Blackboard Help Ticket: http://utrgv.edu/coltthelp
  - Need Blackboard assistance after hours? You can call our main office numbers, 956-882-6792 or 956-665-5327, to speak with a support representative.

Document Scanning

- Take home exams and homework assignments should be submitted as ONE PDF file per submission. Name your file: **Lastname-Firstname.pdf** using your First and Last name.
- **DO NOT JUST TAKE PHOTOGRAPHS OF YOUR PAPERS WITH A PHONE!!!!! DON'T SEND ME IMAGE FILES!!! I need a PDF document that can be printed, and just taking pictures will not work.**
- **WRITE WITH A DARK PEN. DO NOT USE A PENCIL, ESPECIALLY A LIGHT ONE.** Your document may be barely readable on screen but not print well.
- **I won't be able to grade an exam that looks unreadable on paper.**
- To scan with a printer/scanner, please use the following settings:
  - 400 dpi (less than 300dpi will not look good)
  - Black and White
  - Scan as PDF file

  The printer/scanner will then create a pdf file.
- To scan with a phone **YOU MUST USE A SCANNING APP**. For Android phones, I recommend Mobile Doc Scan. For iphones, you can try CamScanner. In both apps, you take a photograph of the paper and then adjust the bounding box to the four corners of your paper. Image processing algorithms then produce an image that looks as if you fed the document through a real scanner. You need to do this for each page, so this is less convenient than an actual scanner. The apps can be used to generate a PDF file, which you can then transfer to a computer and submit by email.

Student Learning Outcomes

After completing this course students will be able to

(1) Solve problems involving optimization models, dynamic models, growth and decay, the logistic model and population models.

(2) Understand concepts of stable and unstable motion, stability of linear and nonlinear systems, Liapunov theory.
(3) Identify bifurcation and catastrophe phenomena.
(4) Recognize dynamics and bifurcation in biological and biomedical models.
(5) Distinguish "deterministic" and "random" behavior.
(6) Implement a variety of problem-solving strategies, and evaluate and transform mathematical expressions.
(7) Find and utilize mathematical models which reasonably approximate the behavior of real world problems.

About your Professor

Dr. Gkioulekas was raised in a small mining village, Stratonion, in Greece. He was inspired into a teaching career by the example of his High School Mathematics Teacher, Alexandros Pistofidis, and by independently studying all three volumes of the Feynman Lectures on Physics during the last 4 years of High School. He graduated with a B.Sc. in Applied Mathematics from the California Institute of Technology in 1997, a M.Sc. in 2000 and a Ph.D in Applied Mathematics in 2006, both from the University of Washington. Dr. Gkioulekas has conducted research and published research papers in national and international refereed research journals in Applied Mathematics and Mathematics Education in the areas of hydrodynamic and geophysical turbulence, statistical mechanics, theoretical physics, and curriculum innovations. He has also authored more than 2800 pages of online lecture notes for various undergraduate and graduate courses that are made freely available to the general public via his faculty web page, as a form of service to the world community. With the onset of the COVID-19 pandemic, he has taken a deep dive into the statistical analysis of the early outpatient COVID-19 treatment protocols pioneered by Dr. Vladimir Zelenko and Dr. Peter McCullough.