Trigonometric identities

$\boxed{a \pm b}$ $\Downarrow$

$$\left.\begin{aligned}
&\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a \\
&\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b \\
&\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b} \\
&\cot(a \pm b) = \frac{\cot a \cot b \mp 1}{\cot b \pm \cot a} \quad \textbf{(!!)}
\end{aligned}\right\} \Longrightarrow$$

$\boxed{2a}$ $\Downarrow$

$$\begin{aligned}
&\sin(2a) = 2 \sin a \cos a \\
&\cos(2a) = \cos^2 a - \sin^2 a = 2\cos^2 a - 1 = 1 - 2\sin^2 a \\
&\tan(2a) = \frac{2\tan a}{1 - \tan^2 a} \\
&\cot(2a) = \frac{\cot^2 a - 1}{2 \cot a}
\end{aligned}$$

- $\sin(a+b)\sin(a-b) = \sin^2 a - \sin^2 b$
- $\cos(a+b)\cos(a-b) = \cos^2 a - \sin^2 b$

$\boxed{3a} \Longrightarrow$ $\quad \sin(3a) = -4\sin^3 a + 3\sin a$ $\quad \cos(3a) = +4\cos^3 a - 3\cos a$ $\quad \tan(3a) = \dfrac{3\tan a - \tan^3 a}{1 - 3\tan^2 a}$

In terms of

$\boxed{\cos 2a}$ $\Downarrow$

$$\sin^2 a = \frac{1 - \cos(2a)}{2} \quad \cos^2 a = \frac{1 + \cos(2a)}{2}$$
$$\tan^2 a = \frac{1 - \cos(2a)}{1 + \cos(2a)} \quad \cot^2 a = \frac{1 + \cos(2a)}{1 - \cos(2a)}$$

$\boxed{\tan(a/2)}$ $\Downarrow$

$$\sin a = \frac{2\tan(a/2)}{1 + \tan^2(a/2)} \quad \cos a = \frac{1 - \tan^2(a/2)}{1 + \tan^2(a/2)}$$
$$\tan a = \frac{2\tan(a/2)}{1 - \tan^2(a/2)} \quad \cot a = \frac{1 - \tan^2(a/2)}{2\tan(a/2)}$$

Transformation to

$\boxed{\text{sum}}$ $\Downarrow$

$$\left.\begin{aligned}
&2\sin a \cos b = \sin(a-b) + \sin(a+b) \\
&2\cos a \cos b = \cos(a-b) + \cos(a+b) \\
&2\sin a \sin b = \cos(a-b) - \cos(a+b)
\end{aligned}\right\} \Longrightarrow$$

$\boxed{\text{product}}$ $\Downarrow$

$$\begin{aligned}
&\sin a \pm \sin b = 2\sin\frac{a \pm b}{2}\cos\frac{a \mp b}{2} \\
&\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2} \\
&\cos a - \cos b = 2\sin\frac{a+b}{2}\sin\frac{b-a}{2} \quad \textbf{(!!)} \\
&\tan a \pm \tan b = \frac{\sin(a \pm b)}{\cos a \cos b} \\
&\cot a \pm \cot b = \frac{\sin(b \mp a)}{\sin a \sin b} \quad \textbf{(!!)}
\end{aligned}$$

Also note the factorizations:

- $1 \pm \sin a = \sin(\pi/2) \pm \sin a = 2\sin\dfrac{(\pi/2) \pm a}{2}\cos\dfrac{(\pi/2) \mp a}{2}$
- $\sin a \pm \cos b = \sin a \pm \sin(\pi/2 - b) = 2\sin\dfrac{a \pm (\pi/2 - b)}{2}\cos\dfrac{a \mp (\pi/2 - b)}{2}$
- $1 + \cos a = 2\cos^2(a/2)$
- $1 - \cos a = 2\sin^2(a/2)$