Exam 3 Solution

Math 1450.01

Find all solutions to the equation $\cos(2x) - \cos(x/2) = 0$

<u>Solution</u>

$$\cos(2x) - \cos(x/2) = 0 \Leftrightarrow \cos(2x) = \cos(x/2) \Leftrightarrow$$
$$\Leftrightarrow 2x = 2k\pi + x/2 \vee 2x = 2k\pi - x/2 \Leftrightarrow$$
$$\Leftrightarrow 4x = 4k\pi + x \vee 4x = 4k\pi - x \Leftrightarrow$$
$$\Leftrightarrow 4x - x = 4k\pi \vee 4x + x = 4k\pi \Leftrightarrow$$
$$\Leftrightarrow 3x = 4k\pi \vee 5x = 4k\pi$$
$$\Leftrightarrow x = \frac{4k\pi}{3} \vee x = \frac{4k\pi}{5}$$

Find all solutions to the following equation

$$2\cos^2 x = \sqrt{2}\cos x + 2$$

Solution

Define $y = \cos x$. Then

$$2\cos^2 x = \sqrt{2}\cos x + 2 \Leftrightarrow 2y^2 = \sqrt{2}y + 2 \Leftrightarrow 2y^2 - \sqrt{2}y - 2 = 0 \qquad (1)$$

$$\Delta = (-\sqrt{2})^2 - 4\cdot 2\cdot(-2) = 2 + 16 = 18 = 9\cdot 2 \Rightarrow$$

$$\Rightarrow y_{1,2} = \frac{-(-\sqrt{2}) \pm 3\sqrt{2}}{2\cdot 2} = \frac{\sqrt{2} \pm 3\sqrt{2}}{4} = \begin{cases} 4\sqrt{2}/4 = \sqrt{2} \\ -2\sqrt{2}/4 = -\sqrt{2}/2. \end{cases}$$

and therefore

$$\text{Eq.}(1) \Leftrightarrow y = \sqrt{2} \vee y = -\sqrt{2}/2 \Leftrightarrow \cos x = \sqrt{2} \vee \cos x = -\sqrt{2}/2 \qquad (2)$$

and note that

$$\cos x = \sqrt{2} \Leftrightarrow \cos x = \cos(\operatorname{Arccos}(\sqrt{2})) \Leftrightarrow$$

$$\Leftrightarrow x = \operatorname{Arccos}(\sqrt{2}) + 2k\pi \vee x = -\operatorname{Arccos}(\sqrt{2}) + 2k\pi$$

$$\cos x = -\sqrt{2}/2 \Leftrightarrow \cos x = -\cos(\pi/4) \Leftrightarrow \cos x = \cos(\pi + \pi/4)$$

$$\Leftrightarrow x = 2k\pi + \pi + \pi/4 \vee x = 2k\pi - (\pi + \pi/4)$$

$$\Leftrightarrow x = 2k\pi + 5\pi/4 \vee x = 2k\pi - 5\pi/4$$

and therefore

$$\text{Eq}(2) \Leftrightarrow x = 2k\pi + \operatorname{Arccos}(\sqrt{2}) \vee x = 2k\pi - \operatorname{Arccos}(\sqrt{2}) \vee$$

$$\vee x = 2k\pi + 5\pi/4 \vee x = 2k\pi - 5\pi/4.$$

Find all solutions to the following equation:

$\cos(2x) + \sin^2 x = 0$

<u>Solution</u>

$\cos(2x) + \sin^2 x = 0 \Leftrightarrow 1 - 2\sin^2 x + \sin^2 x = 0 \Leftrightarrow 1 - \sin^2 x = 0$

$\Leftrightarrow (1 - \sin x)(1 + \sin x) = 0 \Leftrightarrow 1 - \sin x = 0 \vee 1 + \sin x = 0 \Leftrightarrow$

$\Leftrightarrow \sin x = 1 \vee \sin x = -1 \Leftrightarrow$

$\Leftrightarrow x = 2k\pi + \pi/2 \vee x = 2k\pi - \pi/2$

Find all solutions to the equation

$\sin x + \cos x = 1$

Solution

$$\sin x + \cos x = 1 \Leftrightarrow \sin x + \tan(\pi/4)\cos x = 1 \Leftrightarrow$$

$$\Leftrightarrow \sin x + \frac{\sin(\pi/4)}{\cos(\pi/4)}\cos x = 1 \Leftrightarrow$$

$$\Leftrightarrow \sin x \cos(\pi/4) + \sin(\pi/4)\cos x = \cos(\pi/4)$$

$$\Leftrightarrow \sin(x+\pi/4) = \cos(\pi/4) \Leftrightarrow$$

$$\Leftrightarrow \sin(x+\pi/4) = \sin(\pi/2 - \pi/4) \Leftrightarrow$$

$$\Leftrightarrow \sin(x+\pi/4) = \sin(\pi/4) \Leftrightarrow$$

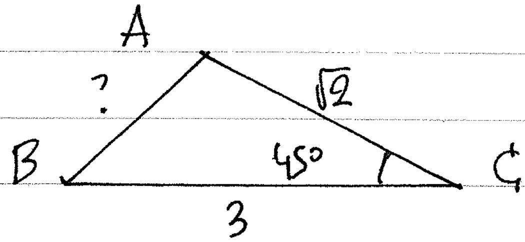
$$\Leftrightarrow x + \pi/4 = 2k\pi + \pi/4 \vee x + \pi/4 = (2k+1)\pi - \pi/4$$

$$\Leftrightarrow x = 2k\pi \vee x = (2k+1)\pi - \pi/4 - \pi/4$$

$$\Leftrightarrow x = 2k\pi \vee x = (2k+1)\pi - \pi/2.$$

Given a triangle with $a=3$, $b=\sqrt{2}$, $\hat{C}=45^\circ$, find the exact value of c.

Solution



$$c^2 = a^2+b^2-2ab\cos\hat{C} = 3^2+(\sqrt{2})^2-2\cdot 3\sqrt{2}\cos(45^\circ) =$$

$$= 9+2-6\sqrt{2}\,(\sqrt{2}/2) = 9+2-3\cdot 2 = 9+2-6 = 5$$

$$\Rightarrow c=\sqrt{5}$$

Given a triangle with $\hat{A} = 60°$, $\hat{B} = 45°$, $a = 5$; find the exact value of c.

<u>Solution</u>

$$\hat{C} = 180° - \hat{A} - \hat{B} = 180° - 60° - 45° = 75° = 45° + 30° \Rightarrow$$

$$\Rightarrow \sin\hat{C} = \sin(45° + 30°) = \sin(45°)\cos(30°) + \sin(30°)\cos(45°) =$$

$$= \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} + \frac{1}{2}\,\frac{\sqrt{2}}{2} = \frac{\sqrt{2}(\sqrt{3}+1)}{4}$$

From law of sines:

$$\frac{a}{\sin\hat{A}} = \frac{c}{\sin\hat{C}} \Rightarrow c = \frac{a\sin\hat{C}}{\sin\hat{A}} = \frac{5\sin(75°)}{\sin(60°)} =$$

$$= \frac{5 \cdot \frac{\sqrt{2}(\sqrt{3}+1)}{4}}{\frac{\sqrt{3}}{2}} = \frac{5\sqrt{2}(\sqrt{3}+1)\cdot 2}{4\sqrt{3}} =$$

$$= \frac{5\sqrt{2}(\sqrt{3}+1)}{2\sqrt{3}} = \frac{5\sqrt{2}(\sqrt{3}+1)\sqrt{3}}{2\cdot 3} =$$

$$= \frac{5\sqrt{6}(\sqrt{3}+1)}{6}$$