Analysis of a Symmetry leading to an Inertial Range Similarity Theory for Isotropic Turbulence.

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We present a theoretical attack on the classical problem of intermittency and anomalous scaling in turbulence. Our focus is on an ideal situation: high Reynolds number isotropic turbulence driven by steady large scale forcing. Moreover, the fluid is incompressible and no confining boundaries are present. We start from a good set of basis functions for the velocity field. These are real and divergence-free. To each wave-vector $\vec{k}$ in Fourier space there is one pair of basis functions with respectively left and right-handed polarity. Isotropy makes all $\vec{k}$ on the shell of constant $|\vec{k}|$ statistically equivalent. Consequently, the coefficients , χ^+ and χ^-, to the basis functions in that shell become two random variables whose joint pdf describes the statistics at scale $\ell = 2\pi / k$. Moreover, $(\chi^+)^2 + (\chi^-)^2$ becomes a random variable for the energy. Switching to polar coordinates, the joint pdf expands in azimuthal modes. We focus on the axisymmetric mode which is itself a pdf and characterized by it radial profile $P_0(r;\ell)$. Observations from both shell model and DNS data indicate that (1) the moments of $P_0(r;\ell)$ scale as power laws in ℓ, and (2) the profile obeys an affine symmetry $P_0(r;\ell) = C(\ell)\, f\left(\dfrac{\ln r - \mu(\ell)}{\sigma(\ell)}\right)$. We raise the question: What statistics agree with both observation? The answer is pleasing. We find the functions f, μ, σ and C analytically in terms of a few constants. Moreover, we obtain closed form expressions for both scaling exponents and coefficients in the power laws. A virtual origin also emerges as an intrinsic length scale ℓ_o for the inertial range.