Using Online Lecture Notes in Teaching

Eleftherios Gkioulekas

University of Texas Rio Grande Valley

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What are the Online Lecture Notes

- They are handwritten scanned lecture notes that summarize the subject matter by including:
- **Definitions:** Rigorous definitions of mathematical concepts
- **Interpretations:** As needed, explain the idea underlying a rigorous definition (e.g. the geometric interpretation of continuity, differentiability, etc)
- **Theorems:** True Statements about the defined concepts
- **Methodology:** How to use theorems to complete a standardized task (e.g. how to solve an equation of certain type, how to find the local min/max of a function, etc.)
- **Examples:** Solved problems illustrating methodology, given in full detail.
- **Exercises:** Problems for homework ranging from routine to challenging.

What is available and how good is it?

- Available at: http://faculty.utrgv.edu/eleftherios.gkioulekas/Teaching/notes.html
- More than 2500 pages of Lecture Notes currently available:

Course	Size	Status
College Algebra	298pp	excellent to average
Precalculus	190pp	very good
Calculus 1	338pp	excellent to good
Calculus 2	316pp	very good
Calculus 3	282pp	good to average
Math for EE	302pp	very good
Ordinary Differential Equations	257pp	excellent
Intro to Math Proof	172pp	excellent to very good
Linear Algebra	232pp	excellent to good
Discrete Structures	235pp	excellent
Mathematical Modelling	206pp	excellent to good

About the quality of notes

- Quality ranges because improvements are introduced chapter by chapter
- Notes written later in my career are of better quality than notes written when I was junior faculty
- Courses that I taught multiple times have had more opportunities for improvement
- Notes ready for general use: Precalculus, Calculus 1 (except for 1 chapter), Calculus 2, Ordinary Differential Equations, Discrete Structures, Mathematical Modelling.

Why did I develop these notes?

- Initially for personal use, made them available to students just because they were there.
- Found it necessary to deviate from textbooks in order to:
  1. Present concepts more rigorously
  2. Modify order of topic presentation or emphasis
  3. Emphasize correct mathematical writing of solutions
- In advanced courses, my lecture notes are assigned as the primary textbook
- While presenting topics in class, I may deviate from the notes and present the topic in a different way. That insight can be captured in revised lecture notes and be built upon in future iterations of the course.
- Lecture notes make it possible to include more examples or other materials than what I am able to cover in classe. (e.g. Frobenius method in ODEs; proofs of some theorems in calculus)

How are the notes developed

- Use high quality paper and ink to prevent ink from bleeding into paper. Write on notepad, detach, add to ring binder.
  - Liquid Flair Extra Fine Tip Felt Porous Pens, 12 Black Pens (31001BH)
  - Ampad Gold Fibre Writing Pads
  - Wanchik's Writer
- Use black ink. Scan into bitmap images at **400 dpi** using **black and white** setting. (good resolution with small filesize)
- Convert to pdf file using Adobe Acrobat. One file per chapter
- Use LaTeX to combine files to one pdf file and automatically insert page numbers.

Guidelines for making notes

- Plan for the notes to be revised. (e.g. do not handwrite page numbers, section numbers, etc)
- Begin large examples on a new page. You may want to insert more examples later, or replace them with better ones.
- Begin exercise sets on a new page.
- For courses with a required textbook, provide a bridge document linking the notes with the book such that for each topic, the document states:
  - Sections to read from the book
  - Exercises assigned from the book
  - Exercises assigned from the lecture notes
- On procedural topics summarize general methodologies whenever possible.
- On topics with a lot of theory, construct condensed one-page or two page summaries of the theory
- Include challenging exercises to hook the best students.
- Expect to devote about 3 months per new course, so plan ahead.

Focus on providing a fresh point of view.

- Demystify mathematics and go beyond mere mechanistic learning. For example, topics not explained well in standard textbooks:
  - College Algebra: Why do we get extraneous solutions in radical equations?
  - Precalculus: Systematize the solution of trigonometric equations and inequalities
  - Calculus 1: Evaluation of limits, Overuse of the 2nd derivative test
  - Calculus 2: Sequences and series
  - Calculus 3: Multidimensional limits, change of variables in integrals, optimization problems
  - ODEs: Frobenius method done efficiently, generalized functions
- Integrate principles of logic in your mathematical writing
- Logic is the natural language of Mathematics.

Logic is the natural language of Mathematics. I

- When solving an equation

$$x^2 - 1 = 0 \iff (x-1)(x+1) = 0 \iff x - 1 = 0 \vee x + 1 = 0$$
$$\iff x = 1 \vee x = -1 \iff x \in \{-1, 1\}$$

- When solving a system of equations, braces represent the logical conjunction operation between statements

$$\begin{cases} 3a - 7b = 33 \\ a - b = 19 \end{cases} \iff \begin{cases} 3a - 7b = 33 \\ -3a + 3b = -57 \end{cases} \iff \begin{cases} a - b = 19 \\ -4b = -24 \end{cases}$$
$$\iff \begin{cases} a - 6 = 19 \\ b = 6 \end{cases} \iff \begin{cases} a = 25 \\ b = 6 \end{cases}$$
$$\iff (a, b) = (25, 6)$$

- Universal quantifier: When stating properties of algebra

$$\forall a, b \in \mathbb{R} : \ (ab = 0 \iff (a = 0 \vee b = 0))$$
$$\forall a, b \in [0, +\infty) : (a = b \iff a^2 = b^2)$$

Logic is the natural language of Mathematics. II

- Existential quantifier: When solving trigonometric equations

$$\begin{aligned}\sin(2x) = \sin x &\iff \exists \kappa \in \mathbb{Z} : 2x = 2\kappa\pi + x \vee 2x = (2\kappa+1)\pi - x \\ &\iff \exists \kappa \in \mathbb{Z} : x = 2\kappa\pi \vee 3x = (2\kappa+1)\pi \\ &\iff \exists \kappa \in \mathbb{Z} : x = 2\kappa\pi \vee x = (2\kappa\pi+1)\pi/3 \\ &\iff x \in \{2\kappa\pi, (2\kappa\pi+1)\pi/3 | \kappa \in \mathbb{Z}\}\end{aligned}$$

- Quantifiers: When defining limits via Weierstrass definitions

$$\lim_{x \to x_0} f(x) = \ell \iff \forall \varepsilon \in (0, +\infty) : \ \exists \delta \in (0, +\infty) : \forall x \in A :$$
$$(0 < |x - x_0| < \delta \implies |f(x) - \ell| < \varepsilon)$$

Logic is the natural language of Mathematics. III

- Show complete arguments: *Begin at the beginning, end at the end.*
- ***Example:*** Find all $a \in \mathbb{R}$ such that

$$f(x) = \begin{cases} x - 2, & \text{if } x \in (2, +\infty) \\ x^2 - (a+1)x + (a^2 - 1), & \text{if } x \in (-\infty, 2] \end{cases}$$

  is continuous on $\mathbb{R}$
- ***Solution***
  - We note that f is continuous on $\mathbb{R} - \{2\}$
  - $\lim_{x \to 2^+} f(x) = \cdots = 0$
  - $\lim_{x \to 2^-} f(x) = \cdots = (a-1)^2 = f(2)$
  - It follows that

$$\begin{aligned} f \text{ continuous on } \mathbb{R} &\iff f \text{ continuous at } x = 2 \\ &\iff \lim_{x \to 2} f(x) = f(2) \\ &\iff \lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = f(2) \\ &\iff (a-1)^2 = 0 \iff a - 1 = 0 \iff a = 1 \end{aligned}$$

What is the Impact from the Lecture Notes?

- **You should not worry about the impact!** You should be true to yourself and your vision of the subject, and be satisfied to create an opportunity for impact.The rest of the Universe will attend to itself. That said...
- Students report studying the notes prior to coming to class.
- Former students report using the notes as a reference resource during future coursework.
- Impact is nonlinear: A handful of students will approach me about the challenging exercises. This creates opportunities for "teaching moments".
- Received some emails internationally from multiple continents (e.g. Australia, Middle East, Africa) about the Lecture notes.

Conclusion

- Using Online Lecture Notes enables faculty to reclaim their authority as subject-matter experts in the instructional process.
- Making notes available online makes it possible to have an impact beyond the University.
- **It is all about inserting a unique and personal perspective into the content.** (e.g. consider the success of Feynman Lectures on Physics)
- Patience and long-term commitment are necessary.
- A project of this magnitude is incompatible with the vision of UTRGV as an Emerging Research University. So, please forget this presentation.

Thank you!