Helicity and inertial waves in forced rotating turbulence

A. Pouquet

National Center for Atmospheric Research

NCAR is sponsored by NSF

Duane Rosenberg (NCAR), Julien Baerenzung (Grenoble) & Pablo Mininni (Buenos Aires & NCAR)

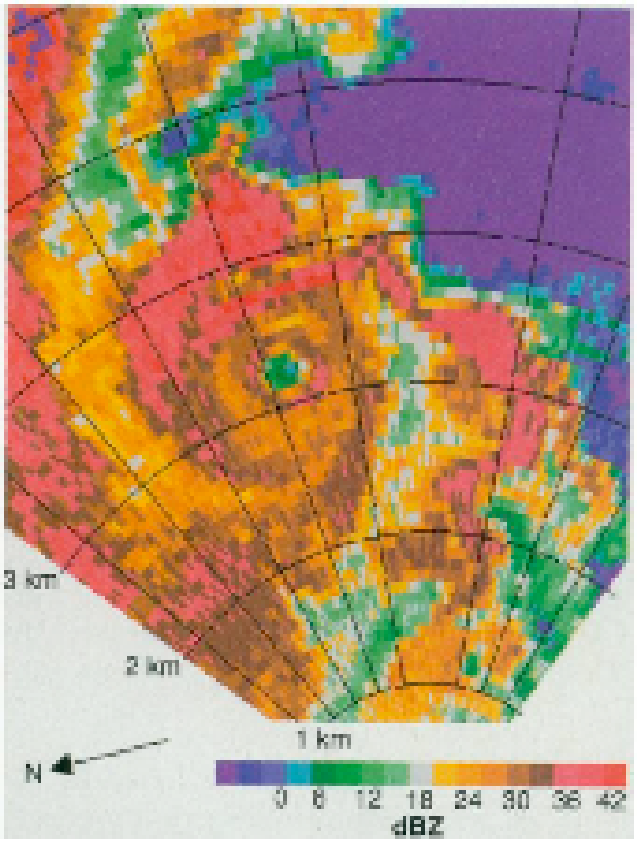
Waco (Texas), October 2009

ROTATION

- Earth's atmosphere and oceans
- Tornadoes, hurricanes
- *The planets, sun and stars, galaxies, and the origin of their magnetic fields (the dynamo)*
- Interplay between turbulent eddies & waves
- Role of symmetry breaking

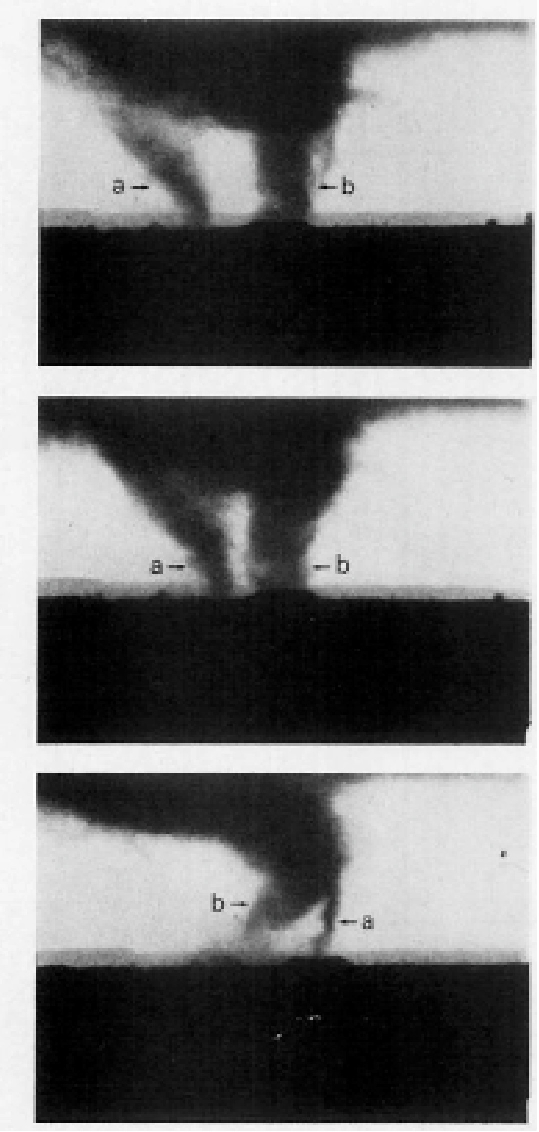
Tornadoes: VORTEX

Verification of the Origin of Rotation in Tornadoes EXp.



Radar reflectivity, range of 3 km

Wurman, Nature 1996



OUTLINE

- *The helical but non-rotating (ABC)*
- *The rotating but non-helical (Taylor-Green)*

* Helical and rotating (ABC forcing and Coriolis force)

    - Self-similarity of the energy cascade to small scales
    - A new spectral law for fluid turbulence in the presence of both helicity and strong rotation: lack of universality?
    - The domain of validity of this new law, using LES modeling

- Discussion

- Helicity H= <**U**.∇x**U**> is an ideal invariant *(Moreau, 1961; Moffatt, 1969),*

as well as energy E, with H(k) ≤ kE(k)

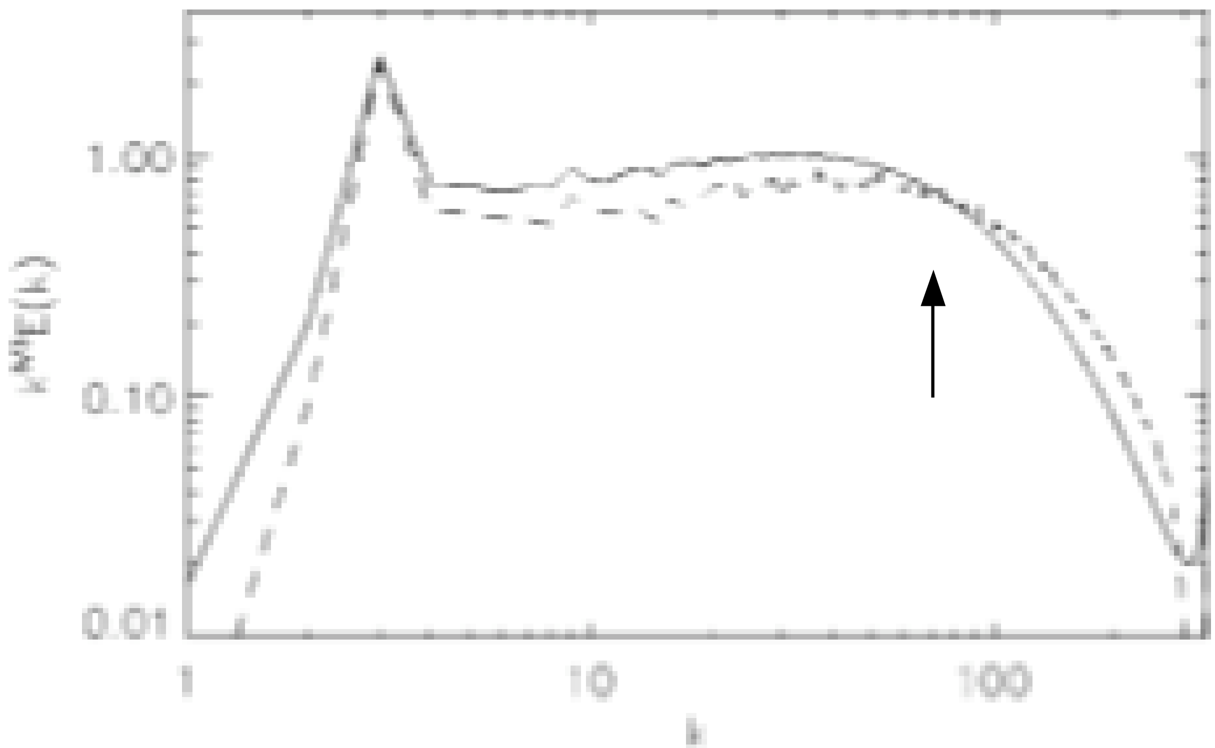
- Kraichnan, 1973: Absolute equilibria in the helical non-rotating case

$$E(k) = \frac{k^2}{\alpha} \frac{4\pi}{1-\beta^2 k^2/\alpha^2}, \quad H(k) = \frac{k^4 \beta}{\alpha^2} \frac{8\pi}{1-\beta^2 k^2/\alpha^2}.$$

→ No tendency for an inverse cascade of energy, unlike the two-dimensional case

No rotation, $H \neq 0$

Spectra of energy (solid)& helicity (dash) both compensated by a Kolmogorov 5/3 law



$E(k) \sim \varepsilon_E^{2/3}\, k^{-5/3}$
$H(k) \sim \varepsilon_E^{-1/3}\, \varepsilon_H k^{-5/3}$

Rates of transfer: $\varepsilon_E = dE/dt$, $\varepsilon_H = dH/dt$

(two-point closures, André & Lesieur, 1977; and numerous direct numerical simulations, e.g. Chen et al. 2003)

1024^3 DNS
$H = u.\omega$ with $\omega = \nabla x u$

Both E & H dissipate at the same wavenumber

Dynamical equations

$$\frac{\partial \mathbf{u}}{\partial t} + \boldsymbol{\omega} \times \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} = -\nabla P + \nu \nabla^2 \mathbf{u} + \mathbf{F}$$

$\nabla . \mathbf{u} = 0$

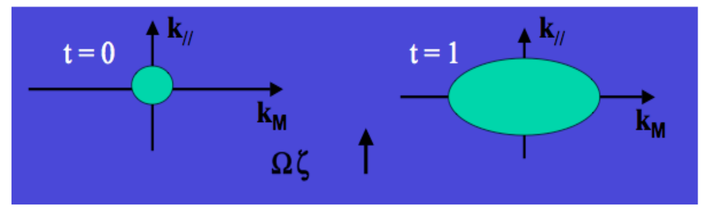
$Re = UL_0/\nu$; $1/Ro = 2\Omega L_0/U$; $1/Ek = Re/Ro = 2\Omega L_0^2 / \nu$

Reynolds nb.; Swirl (inverse of Rossby nb.); inverse Ekmann nb. or vortex Reynolds nb.

Frequency of inertial waves: $\omega_k = \pm\, k_{//}\, \Omega / k \sim \Omega$

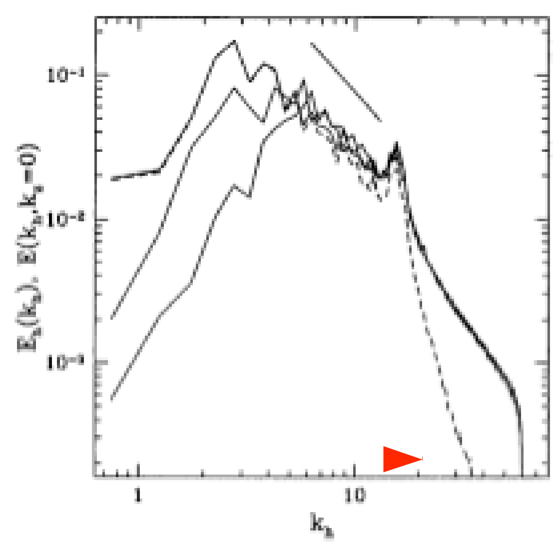
Numerous previous investigations

- Anisotropy of the resulting flow and tubular structures



- Weak turbulence of inertial waves *(Galtier, 2003)*

- Both a direct an inverse cascade of energy can coexist with three-dimensional forcing

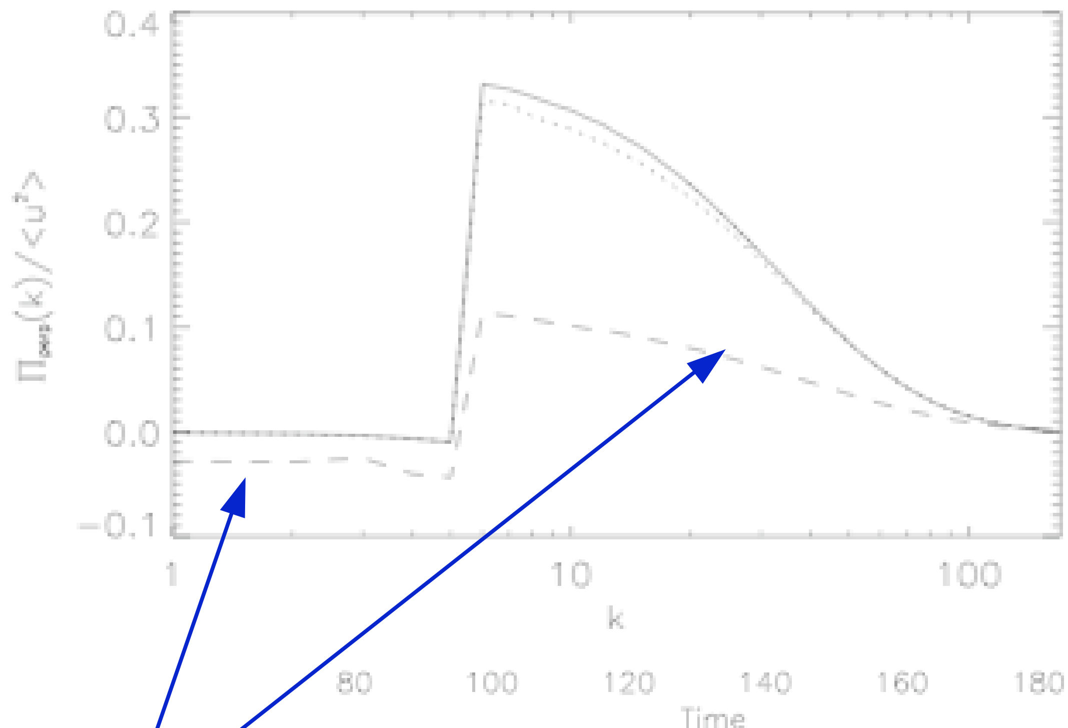


Smith et al., 1996

Energy flux **normalized** by U_{rms} for three Rossby numbers

512^3, k_0=4

Solid: Ro= 1.4
Dots: Ro= 0.35
Dash: Ro= 0.07



Weaker direct cascade of energy at lower Rossby number
together with an inverse cascade

Mininni, Alexakis & AP, Phys. Fluids, **21**, *2009*

Taylor-Green forcing

Phenomenology of turbulence with waves:

- Small parameter: τ_W / τ_{NL} ; transfer time τ_{tr} evaluated as:

$$\tau_{tr} = \tau_{NL}^* (\tau_{NL}/\tau_W) \qquad \text{with } \tau_{NL}=l/u_l \text{ and } \tau_{wave}= 1/\Omega$$

- Constant energy flux: ε = DE/Dt ~ k*E(k) / τ_{tr}

⟶ $E(k) \sim [\varepsilon\Omega]^{1/2} k^{-2}$ *(Dubrulle & Valdetarro, 1992; Zhou, 1995)*

Structure functions: $<\delta u(l)^p> \sim l^{\zeta p}$, ζ_p =p/2

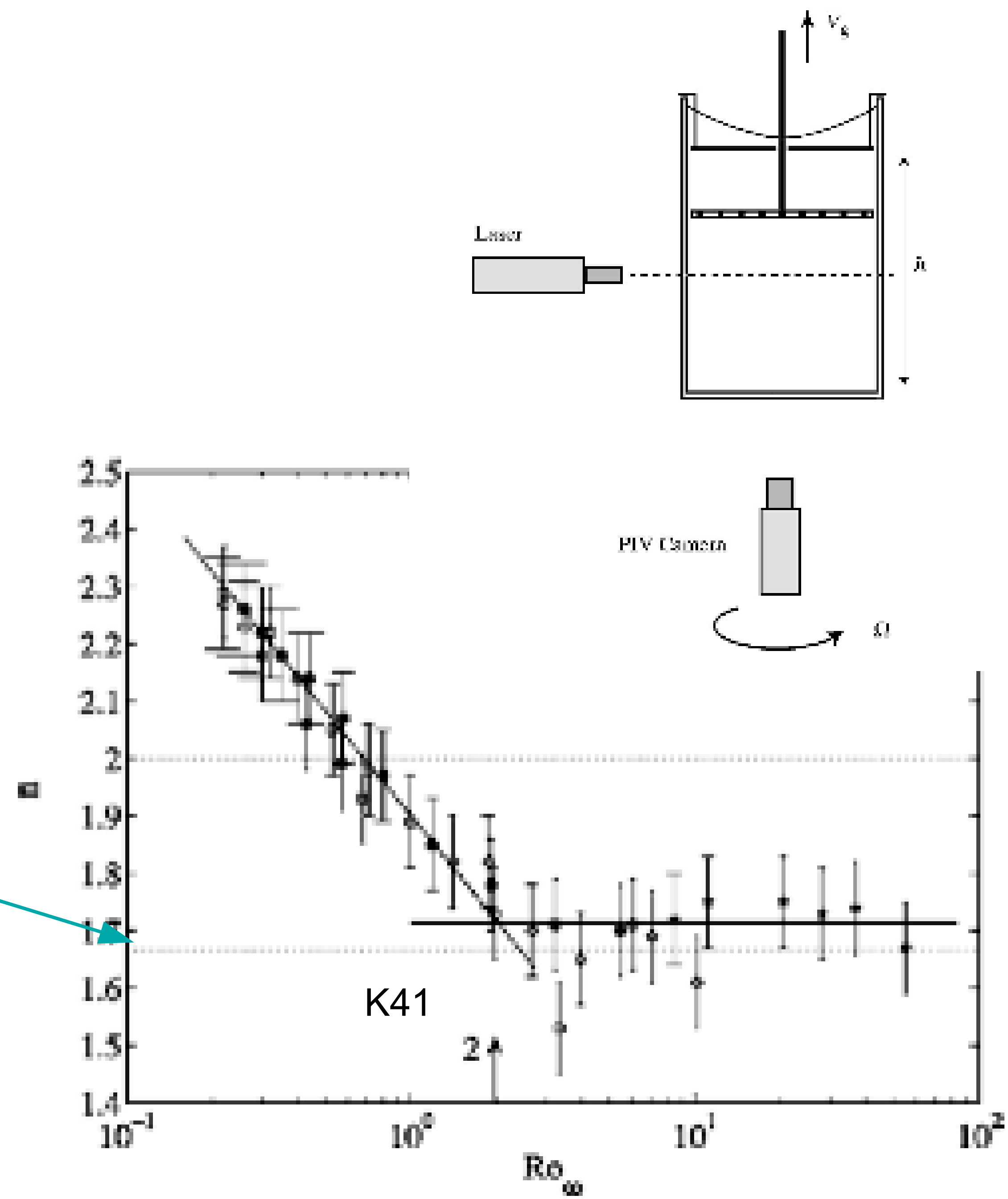
- At dissipation wavenumber k_d, $\tau_{diss} =[\nu k_d^2]^{-1} = \tau_{tr}$; this leads to

$$k_d/ k_0 \sim \varepsilon/[\nu^2\Omega]^{1/2} \sim Re * Ro$$ *(Canuto & Dubovikov, 1997)*

The scaling of the energy spectrum can differ from the classical Kolmogorov spectrum

e.g. E(k) $\neq$ *k* $^{-5/3}$,

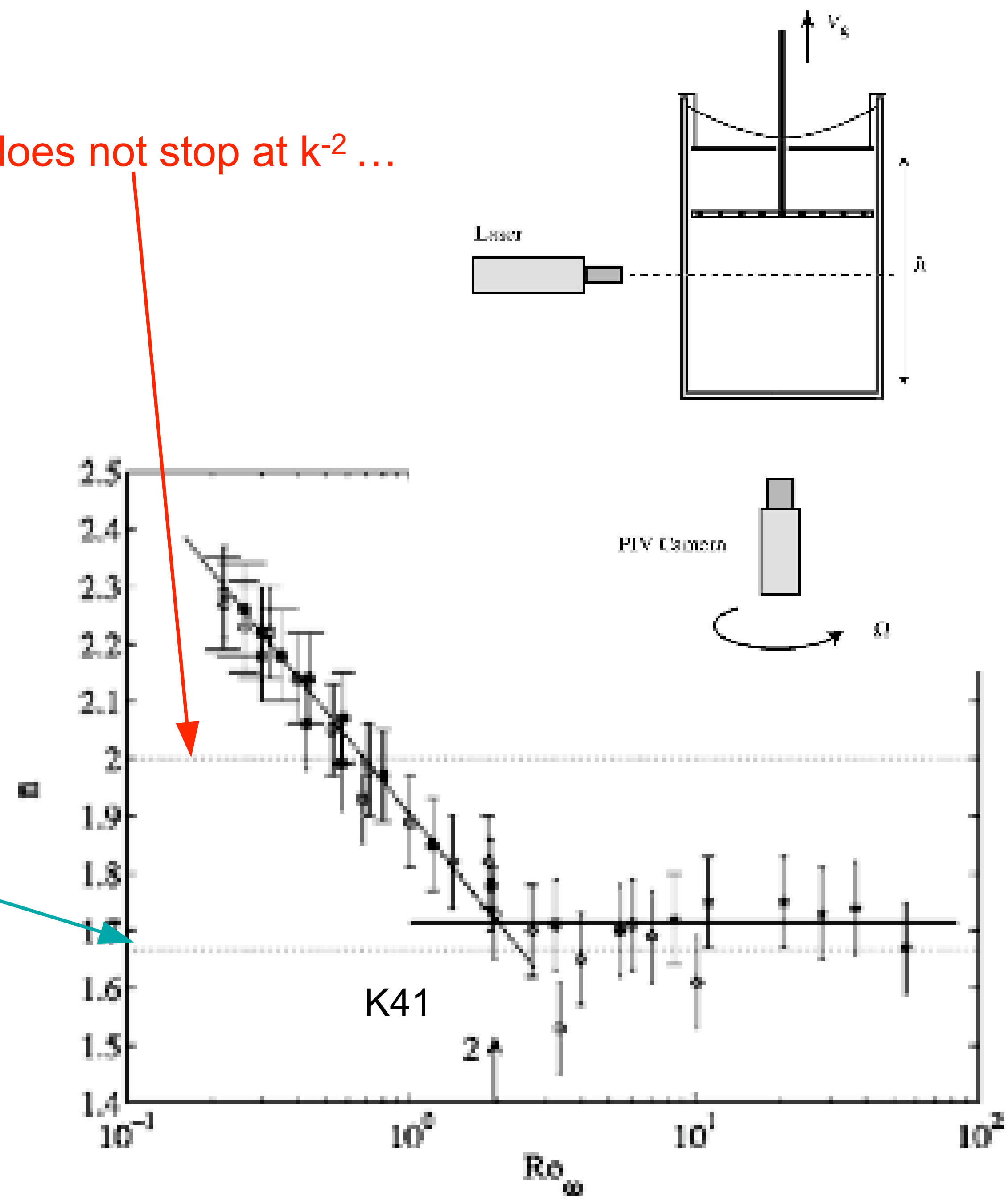
at high enough rotation rate



(Morize et al., 2005)

The scaling of the energy spectrum can differ from the classical Kolmogorov spectrum *e.g. $E(k) \neq k^{-5/3}$*,

at high enough rotation rate



(Morize et al., 2005)

From the Taylor-Green forcing

(globally non helical)

to

the ABC forcing

(Beltrami flow, fully helical)

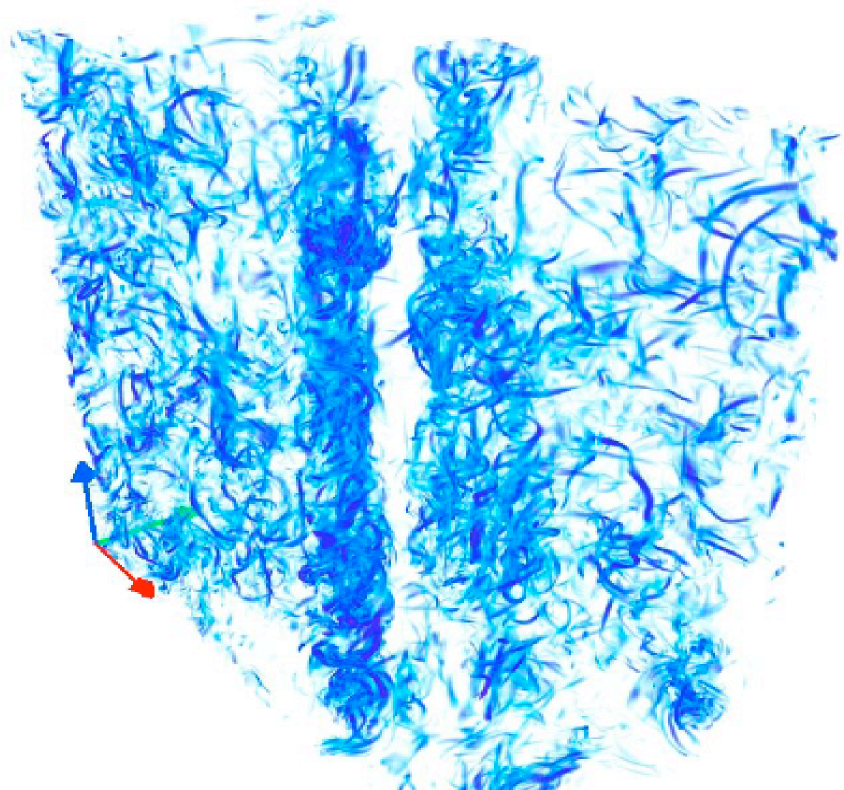
for rotating flows

Structures

Top view

and

side view



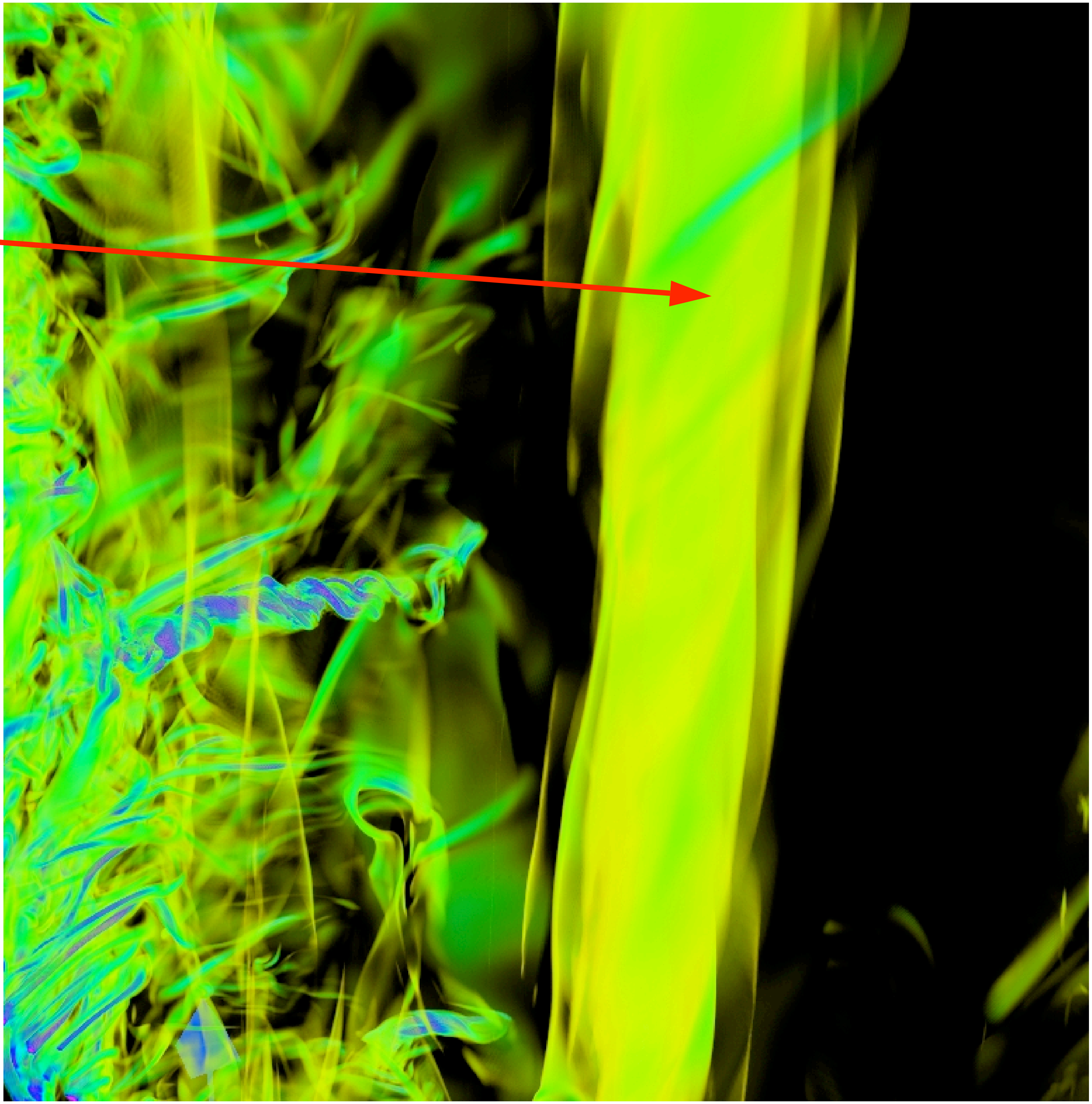
of **vorticity**,

when large

No helicity, Taylor-Green forcing, $k_0=4$, 512^3, *Ro=0.35*

ZOOM on Vorticity:

Beltrami core vortices



Helical forcing at $k_F=7$

DNS on 1536^3 grid points,
Re=5100
Ro=0.06

Mininni & AP,
arXiv:0909.1272
and 1275

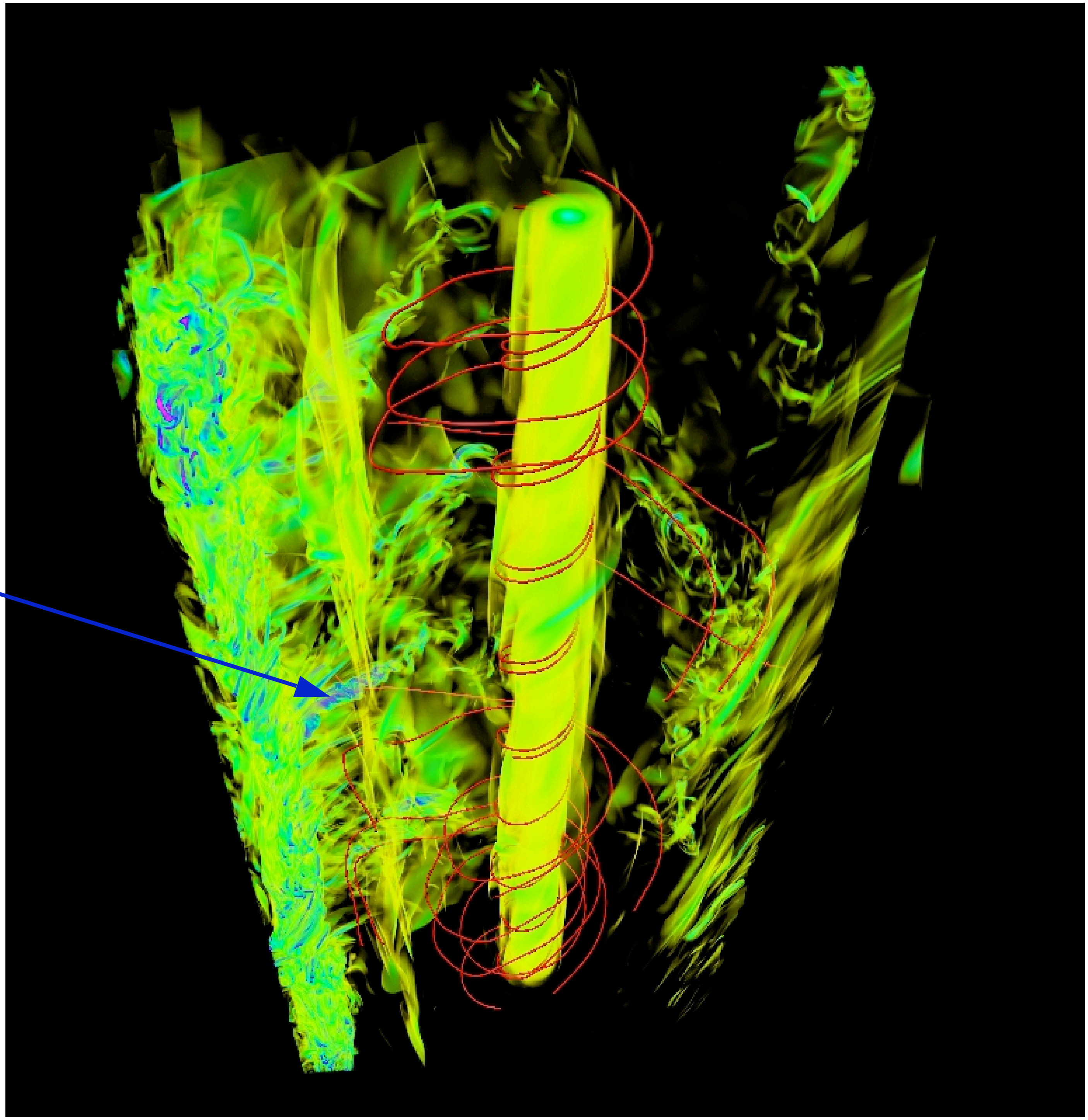
Mininni & AP,
arXiv:0909.1272
and 1275

Beltrami core vortex

amidst a tangle of smaller-scale vortex filaments

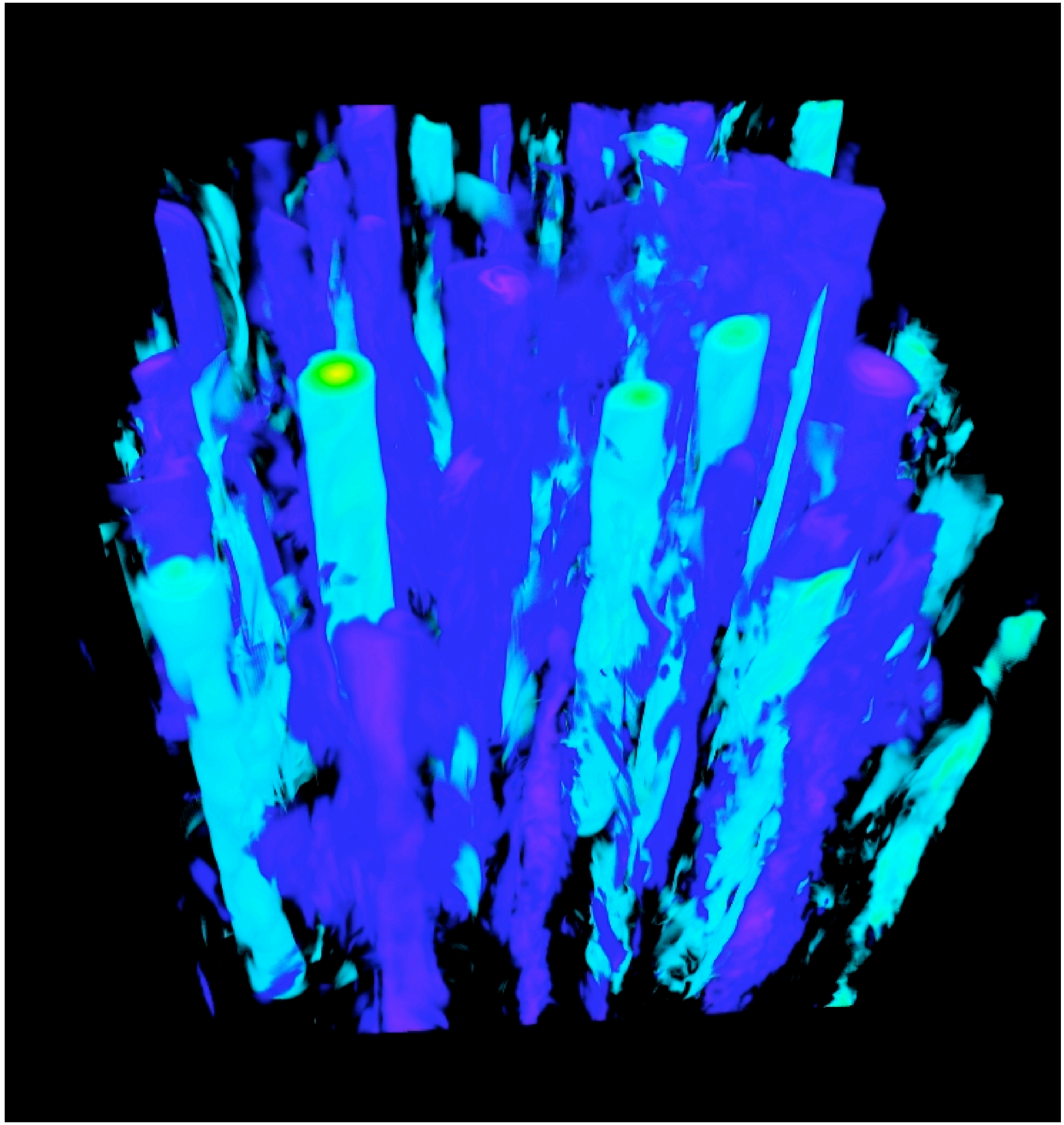
Together with particle trajectories

1536^3 grid, k_F=7,
Re=5100,
Ro=0.06



Visualizations with VAPOR freeware (NCAR)

*Clyne et al., New J. Phys. **9**, 2008*



Vertical velocity V_z, global view

1536^3 grid, $k_F=7$, Re=5100, Ro=0.06

With helicity, strong coherent structures form that are laminar, fully helical & at relatively small scale: Beltrami core vortices

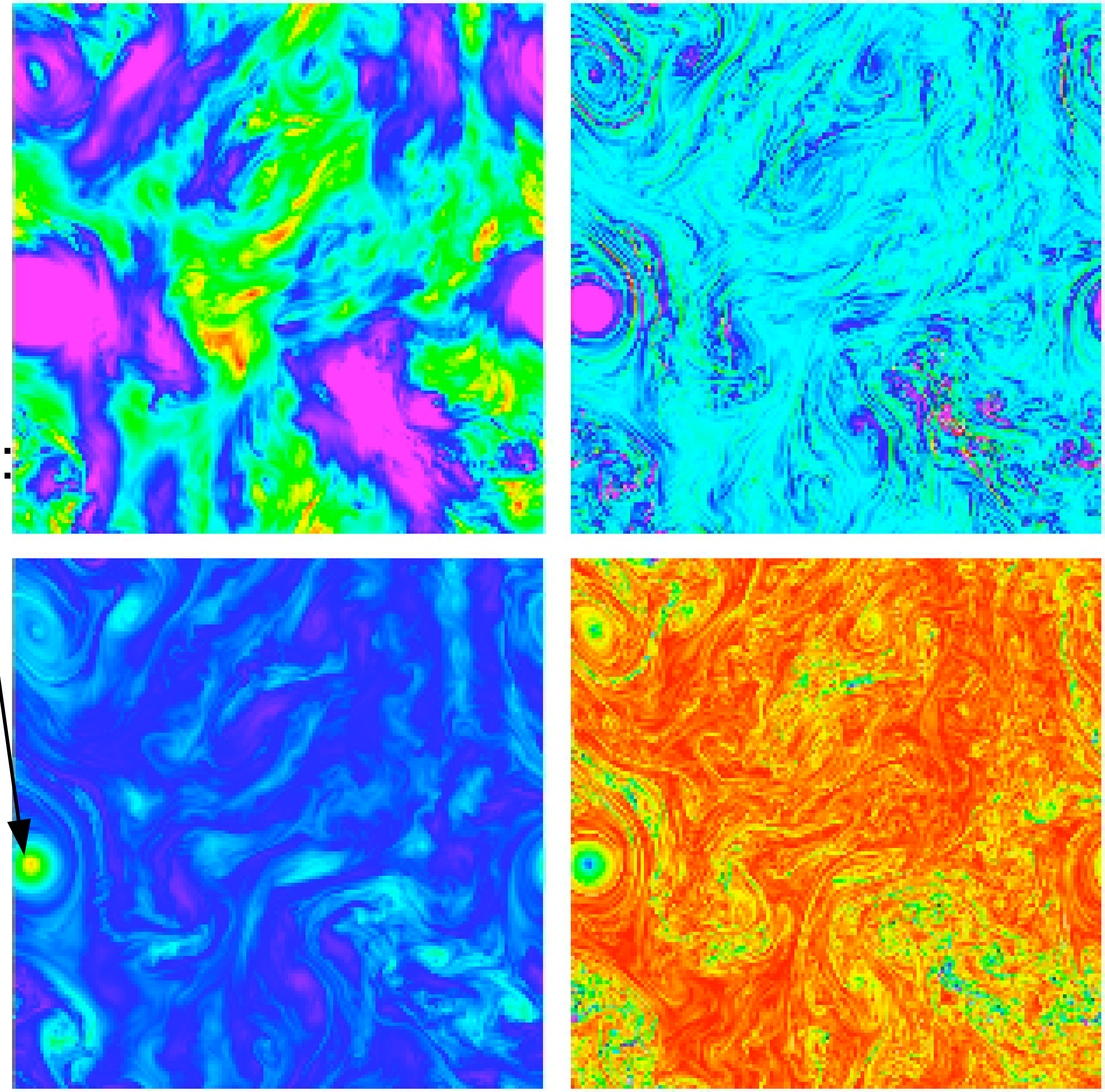


FIG. 9: From top to bottom and from left to right, slices of the energy density, vorticity intensity, z component of the velocity, and helicity density, in run B at $t \approx 30$.

They are embedded in a complex tangle of vorticity, with also a large-scale component due to the inverse cascade

1536^3 grid, k_F=7, Re=5100, Ro=0.06

With helicity, strong coherent structures form that are laminar (Beltrami Core Vortices)

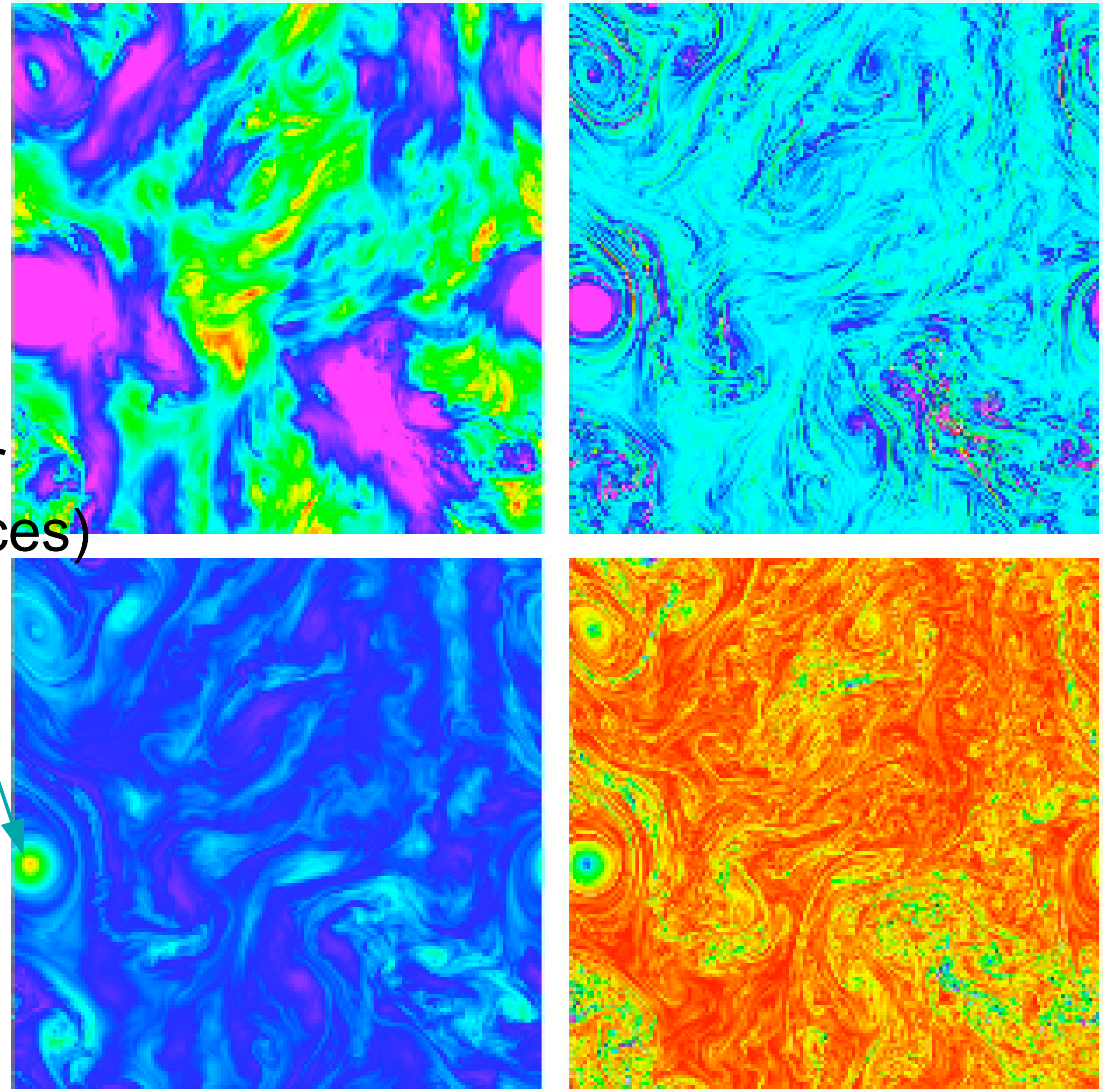
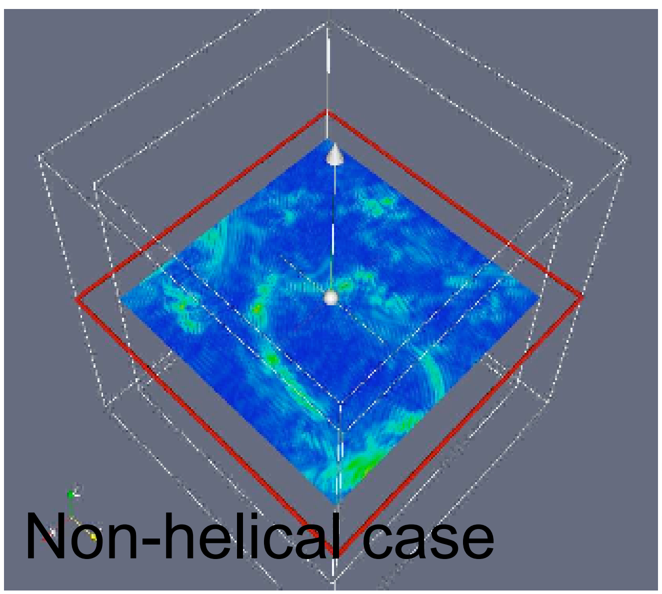
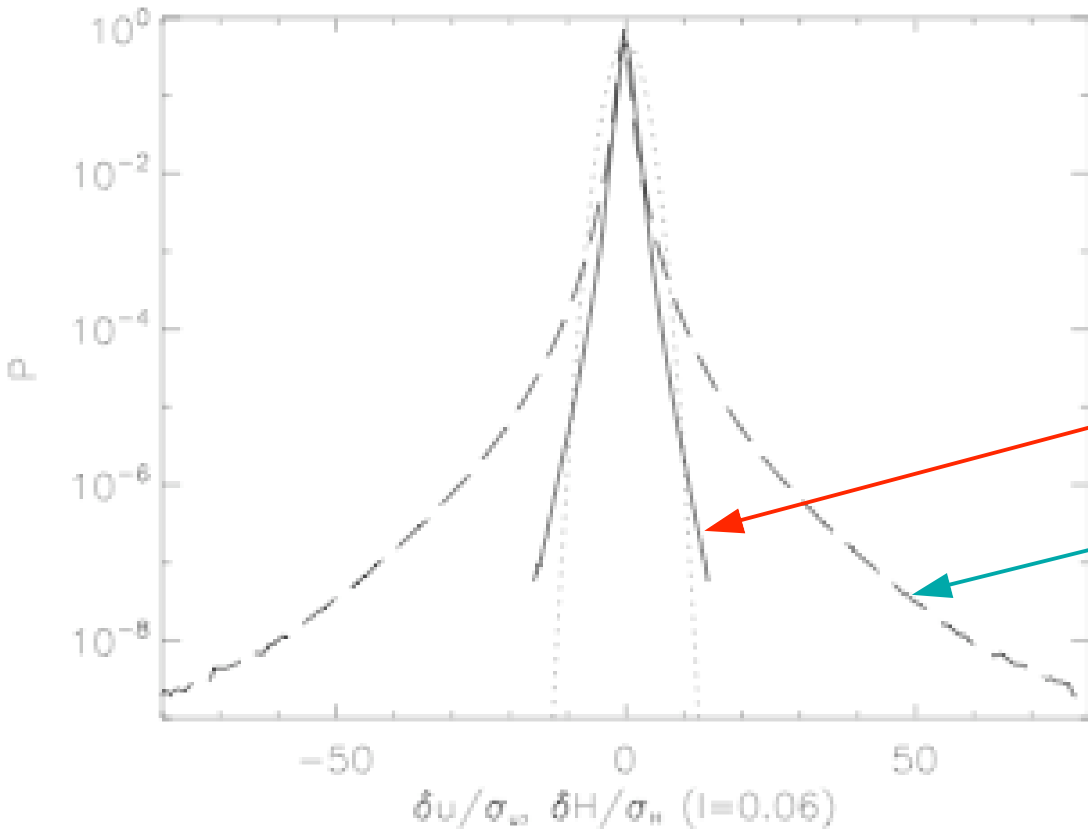


FIG. 9: From top to bottom and from left to right, slices of the energy density, vorticity intensity, z component of the velocity, and helicity density, in run B at $t \approx 30$.



Lack of intermittency of the **direct** energy cascade



PdF of normalized

velocity gradients (solid line)

& helicity gradients (dash)

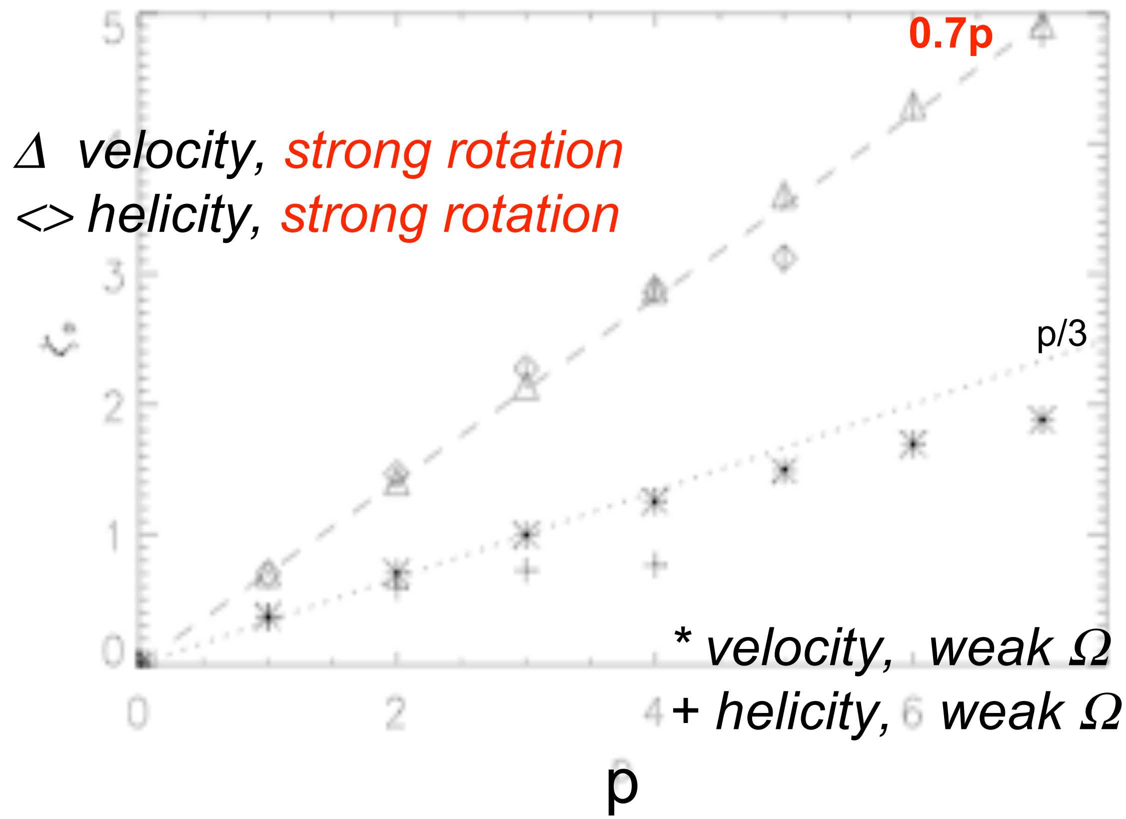
at small scale

(dots: Gaussian)

Scaling exponents of structure functions

$<\delta f=f(x+r)-f(x)]^p> \sim r^{\zeta_p}$

of velocity
and helicity



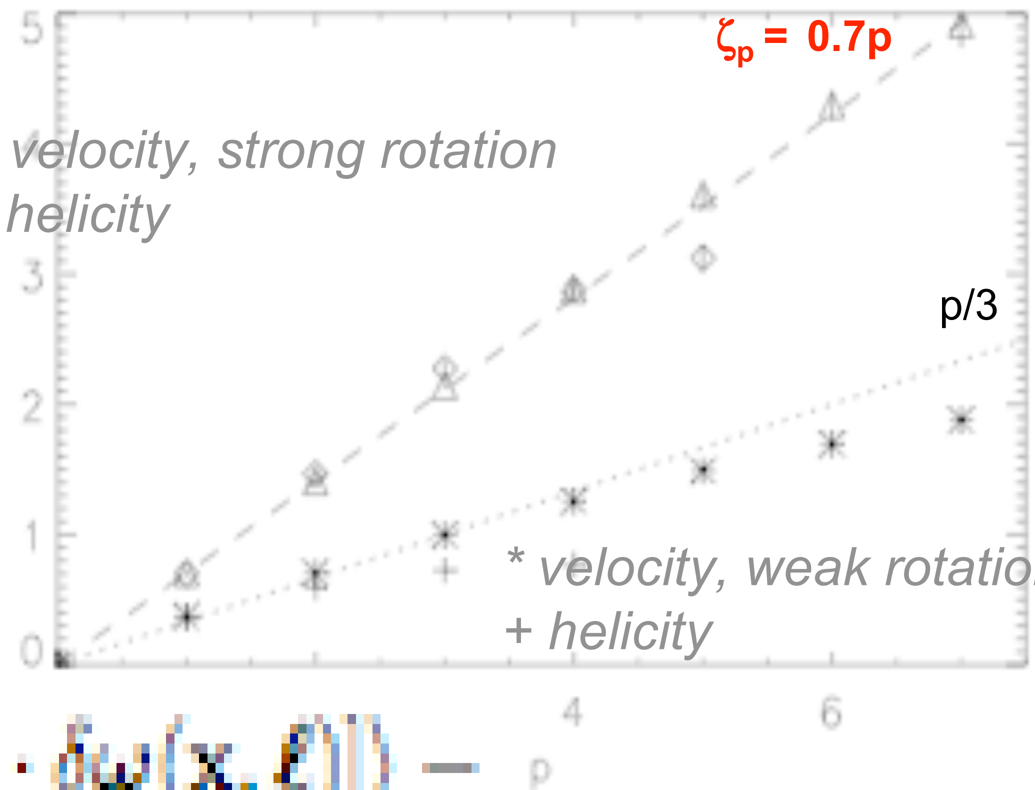
The velocity in the <u>direct cascade is **self-similar**</u> for strong rotation

whereas helicity displays some modicum of intermittency

ζ_p = p/2 for the non-helical case (Simand et al., 2000; Baroud et al., 2002; Mininni+AP, 2009)

Scaling exponents of structure functions

$< \delta f = f(x+r)-f(x)]^p > \sim r^{\zeta p}$
of velocity & helicity



$$\langle \delta \mathbf{u}(\mathbf{x},\ell)\,[\delta \mathbf{u}(\mathbf{x},\ell)\cdot \delta\boldsymbol{\omega}(\mathbf{x},\ell)]\rangle - \frac{1}{2}\left\langle \delta\boldsymbol{\omega}(\mathbf{x},\ell)\,(\delta \mathbf{u}(\mathbf{x},\ell))^2\right\rangle = -\frac{4}{3}\bar{\epsilon}\ell$$

$$\mathcal{H}_p(\ell) = \langle [\delta \mathbf{u}(\mathbf{x},\ell)\cdot \delta\boldsymbol{\omega}(\mathbf{x},\ell)]^p\rangle$$

New spectral law for energy and helicity at high rotation

- Consider the case of the cascade to small scales **dominated** by the flux ε_H of helicity H

  ε_H =dH/dt ~ kH(k) /τ_{tr} ~ constant, with τ_{tr} the transfer time

When assuming $\tau_{tr} = \tau_{NL}$, $E(k) \sim k^{-e}$, $H(k) \sim k^{-h}$ --> e+2h=5 *(Brissaud et al., '73)*

New spectral law for energy and helicity at high rotation

- Consider the case of the cascade to small scales **dominated** by the flux ε_H of helicity H

  $\varepsilon_H = dH/dt \sim kH(k) / \tau_{tr} \sim$ constant, with τ_{tr} the transfer time

When assuming $\tau_{tr} = \tau_{NL}$, $E(k) \sim k^{-e}$, $H(k) \sim k^{-h}$ --> $e+2h=5$ *(Brissaud et al., '73)*

^ Assume instead a slowing down of transfer to small scales because of wave interactions *à la Iroshnikov-Kraichnan in MHD:*

$$\tau_{tr} = \tau_{NL} * (\tau_{NL}/\tau_W)$$

with $\tau_{NL} = l/u_l$ *and* $\tau_W = 1/\Omega$

- Hence, $\underline{\varepsilon_H} \sim k\,H(k) * k^3 E(k) / \Omega$ = *constant, or*

<u>e + h = 4</u> in the helical case with rotation *(Mininni & AP, PRE **79**)*

Is e=h=2 the only solution *(thereby recovering the non-helical case)?*

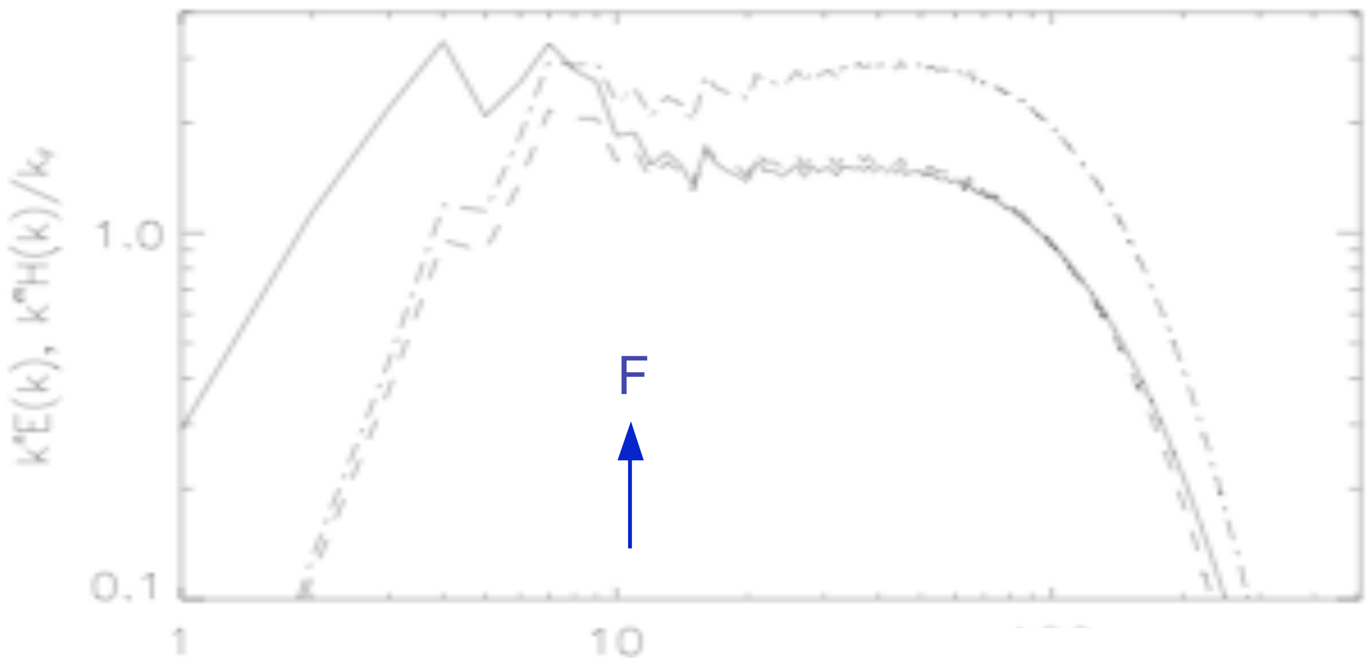
k^x - Compensated spectra for energy (x=e) & helicity (x=h)

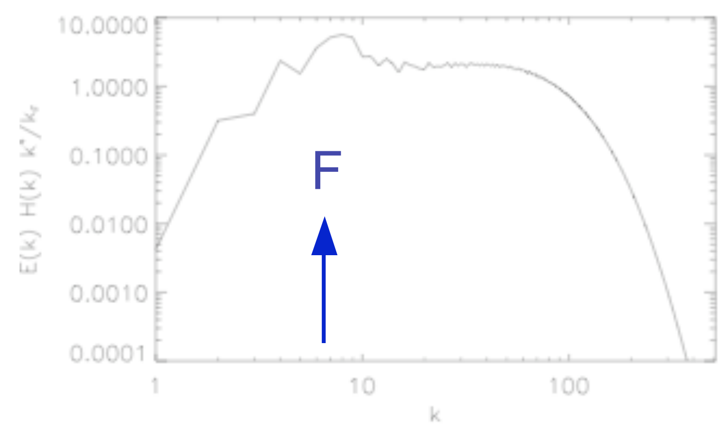
1536^3 run

$k_F=7$
Ro=0.06

Solid: e=2.1
Dash: h=1.9

Dash-dot:
h=2.1 (=e)





Mininni & AP,
arXiv:0909.1272
and 1275

Compensated spectra for the new spectral law:
$k_\perp^4 E(k)*H(k)/k_F$

Going beyond, using models of turbulence

- Are spectral indices universal or do they change
  - with Rossby number, *at fixed Reynolds number?*
  - with Reynolds number, *at fixed Rossby number?*

Large Eddy Simulation (LES) with spectral modeling of turbulent flows *(Chollet & Lesieur, 1981)* but implementing:

- A dynamical fit to the computed energy spectrum instead of imposing Kolmogorov law
- Inclusion of helicity in both the eddy viscosity and the eddy noise
- (somewhat phase-preserving) eddy noise reconstruction

Numerical modeling

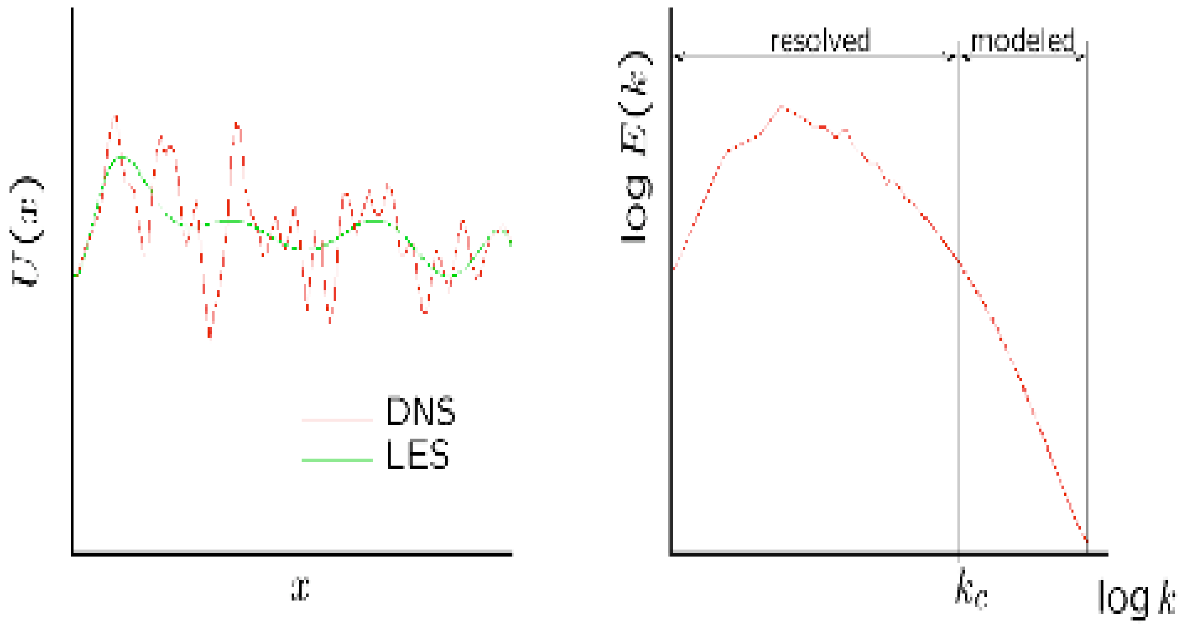
Direct Numerical Simulations
(DNS)

versus

Large Eddy Simulations
(LES)

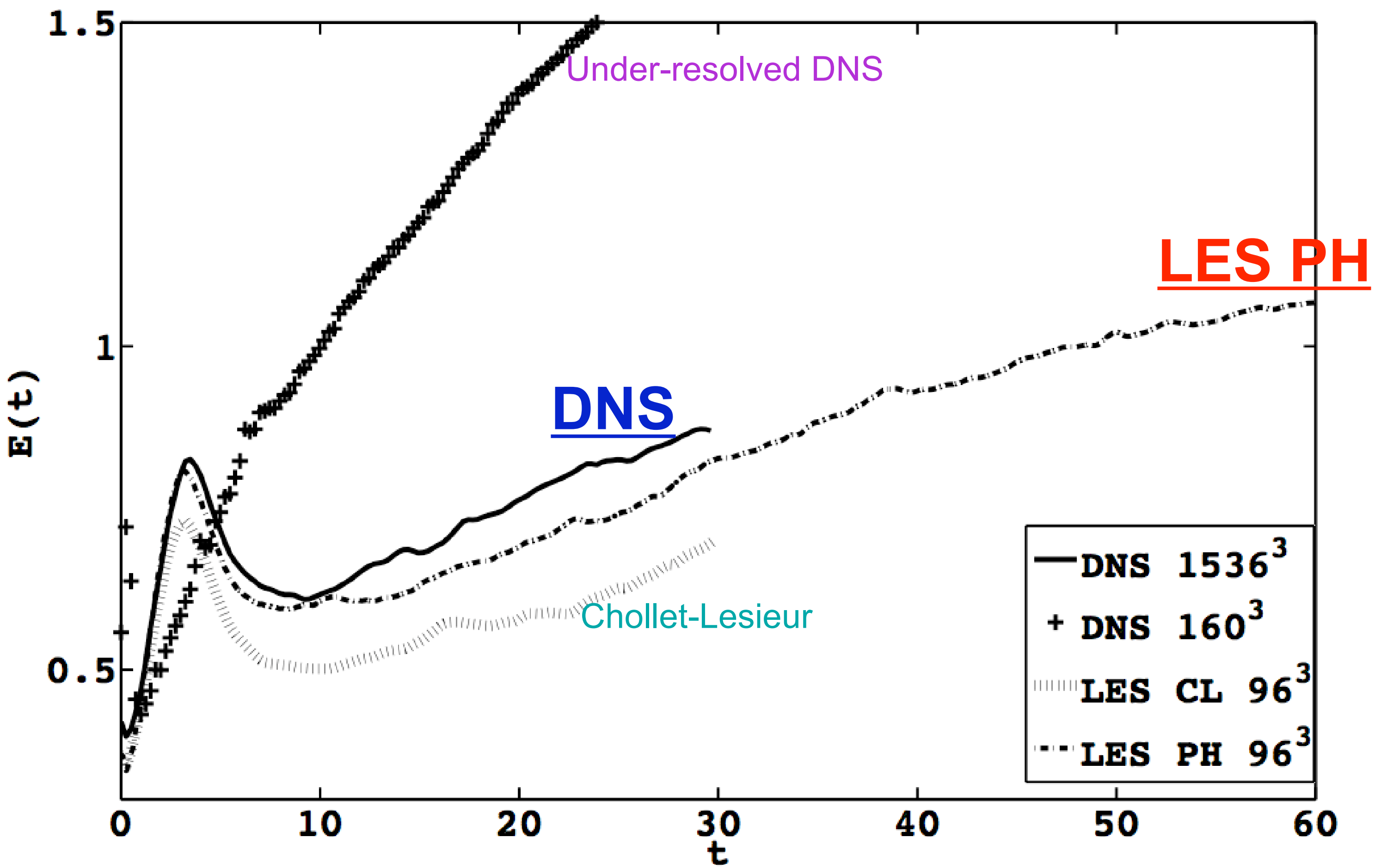
* Resolve all scales
vs.
* Model (many) small scales

3D space & Spectral space



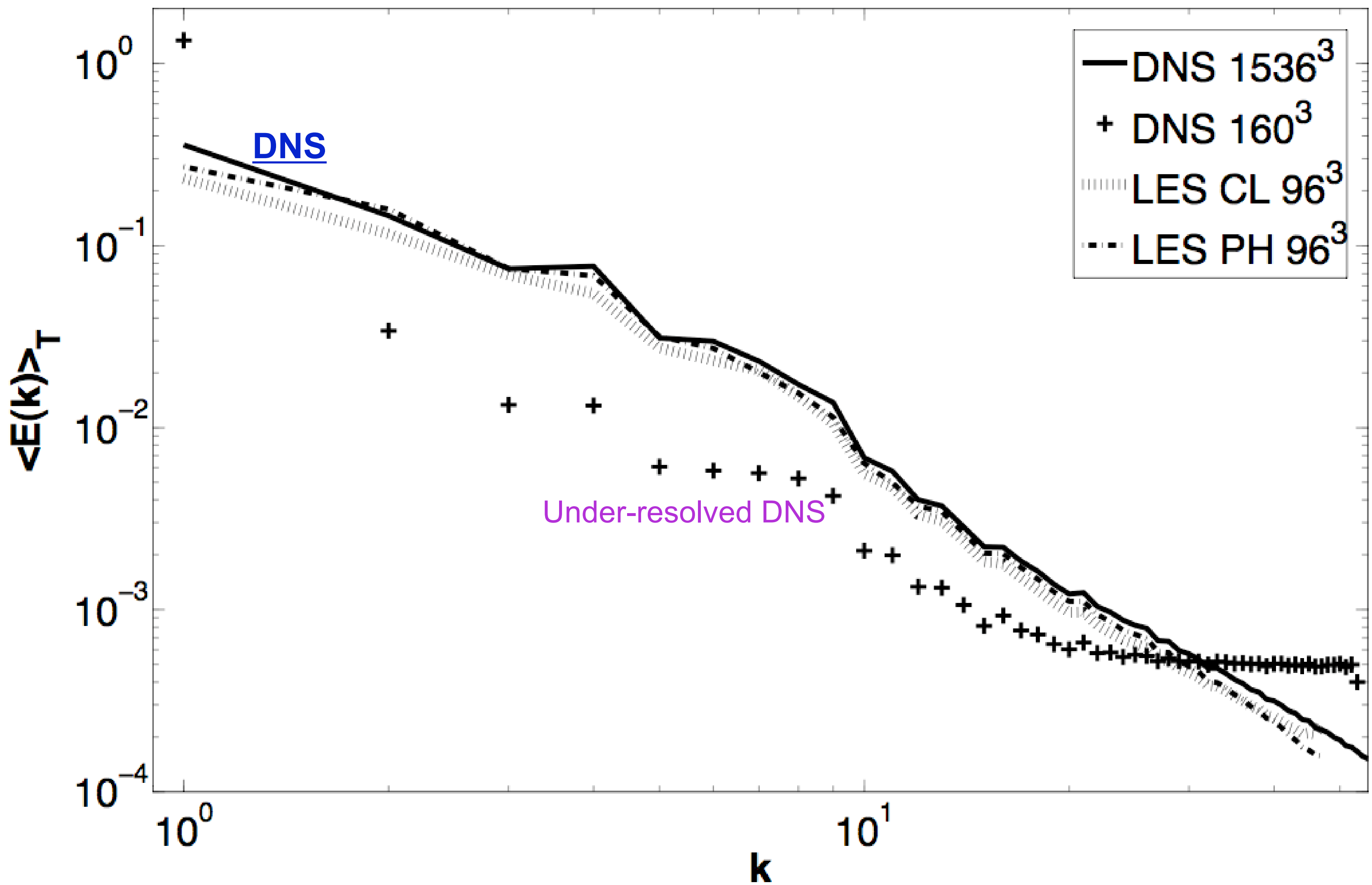
Slide from Comte, Cargese Summer school on turbulence, July 2007

Validation of LES: temporal evolution of total energy



Savings in CPU : 0.5*[1536/96]^4 ~ 30,000 *(also for memory)*

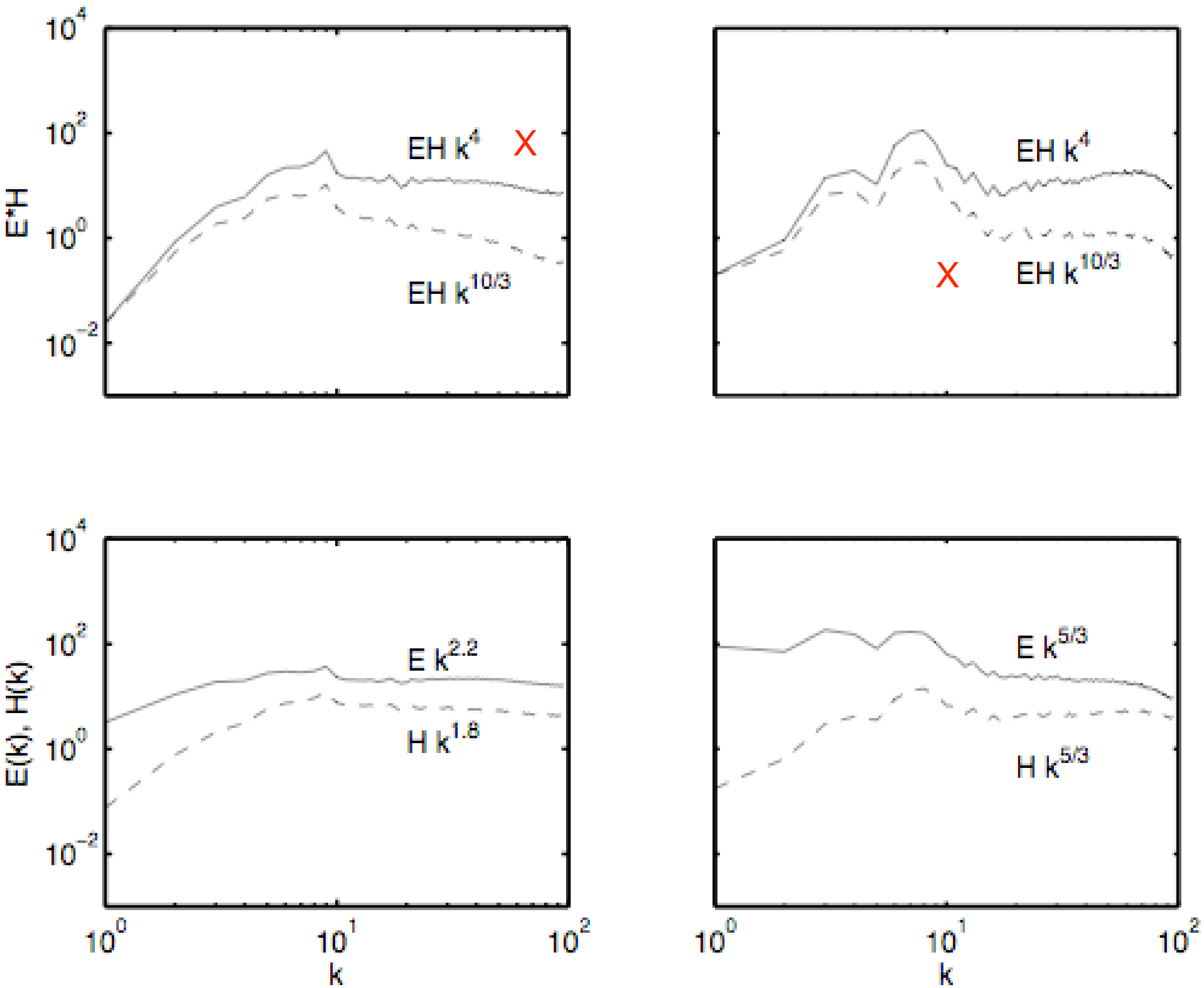
Validation of LES, spectral space



Parametric study using LES

Run	Res.	$10^5 \nu$	Ω	ω_{rms}	T_l	$10^2 Ro$	$10^{-3} Re$	U_{rms}	$L_{\Omega,I}$	$L_{\Omega,F}$	e	$e+h$	$\Pi_H/[k_F \Pi_E]$
R1	192	25.0	4.5	18	17	4.6	8.3	0.9	2.24	3.04	2.03	3.92	1.5
R2	192	25.0	9.0	20	8	2.9	9.3	1.1	2.10	4.30	2.22	4.20	1.7
R3†	192	16.0	1.8	20	14	24.9	4.3	0.8	0.88	0.99	1.52	2.69	1.1
R4	192	16.0	4.5	19	14	6.1	9.7	0.9	1.67	3.79	1.88	3.60	1.3
R5F*	1536	16.0	9.0	34	9	2.9	14.7	1.1	2.14	4.32	2.04	3.89	1.9
R5	192	16.0	9.0	20	8	3.3	13.3	1.1	1.88	4.55	2.10	3.90	1.8
R5a	96	16.0	9.0	12	10	2.7	15.3	1.1	2.23	5.21	2.31	4.03	1.9
R5z†	192	16.0	0.1	23	7	1165.8	3.6	0.9	0.64	0.64	1.39	2.22	1.1
R6	192	16.0	18.0	19	10	1.6	16.7	1.2	2.16	4.58	2.12	3.89	2.5
R7	192	16.0	36.0	18	10	0.8	19.4	1.3	2.37	4.50	2.13	3.91	2.7
R8	192	16.0	117.0	19	8	0.3	14.9	1.3	1.77	2.99	2.53	4.68	2.3
R9	192	11.9	18.0	19	10	1.4	25.0	1.2	2.41	4.76	2.18	3.98	2.3
R10	192	10.2	42.4	21	10	0.6	32.0	1.3	2.46	3.93	2.13	3.86	2.6
R11	192	8.0	9.0	23	8	3.8	24.9	1.2	1.72	3.79	1.93	3.42	1.7
R11a	96	8.0	9.0	13	14	1.9	44.9	1.1	3.22	5.87	2.37	3.98	1.9
R12	192	8.0	18.0	21	10	1.3	43.3	1.3	2.75	4.70	2.03	3.60	2.5
R13	192	8.0	36.0	21	10	0.6	47.5	1.3	2.87	4.08	2.10	3.81	2.7
R14	384	8.0	36.0	30	10	0.9	36.6	1.4	2.14	4.18	2.14	3.96	3.2
R15	384	5.3	72.0	31	6	0.6	42.3	1.4	1.58	3.54	2.17	3.91	3.0
R16	192	5.0	18.0	21	9	1.5	62.6	1.3	2.44	4.08	2.02	3.55	2.5
R17	192	4.5	36.0	22	6	1.0	53.8	1.4	1.79	3.91	2.04	3.54	2.8
R18	192	2.5	36.0	22	8	0.9	108.8	1.3	2.02	3.79	1.96	3.36	3.4
R19	512	1.6	9.0	40	8	3.7	125.5	1.2	1.74	4.16	1.77	3.43	2.1

Parametric study using LES



Spectra product compensated by

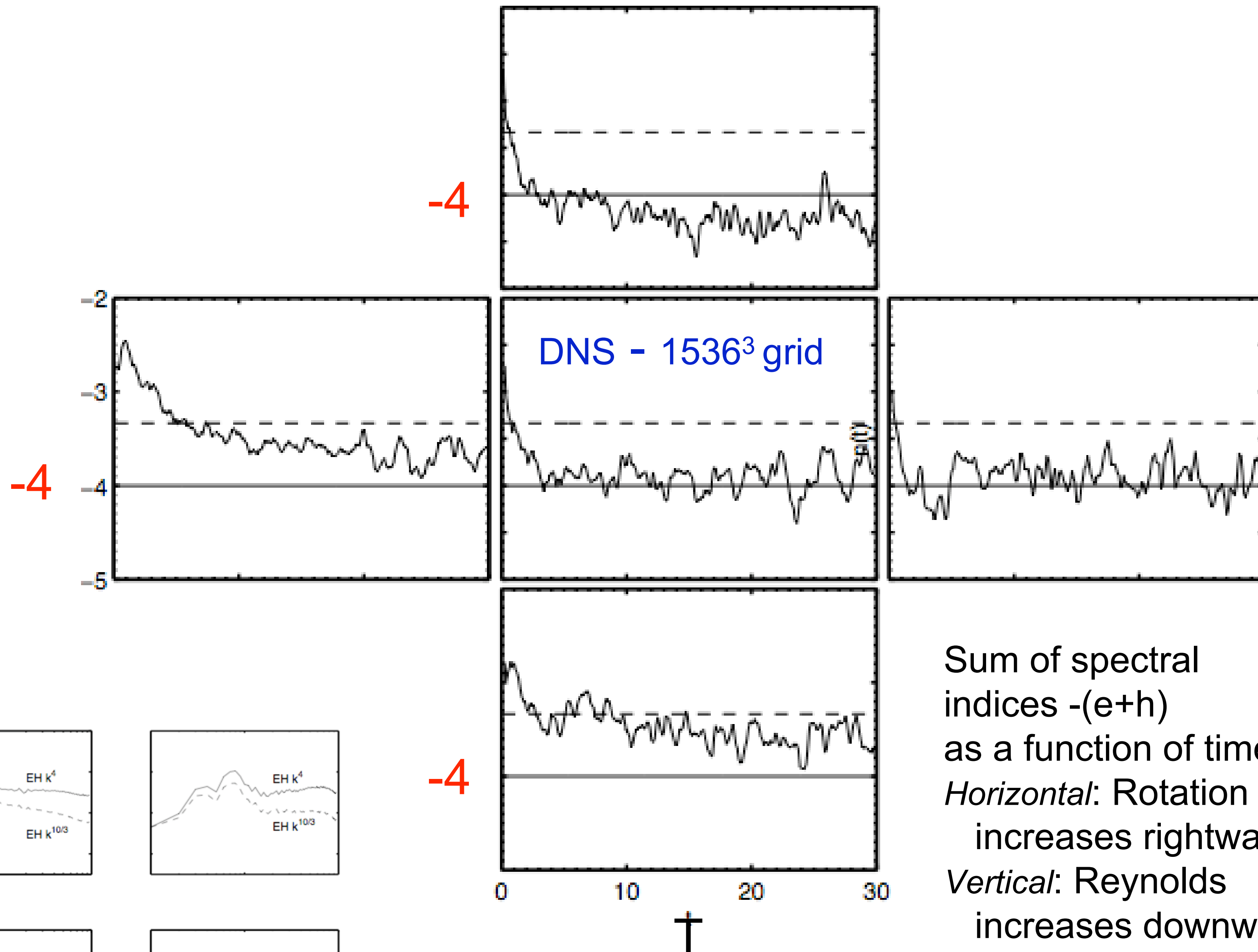
solid line: k^4

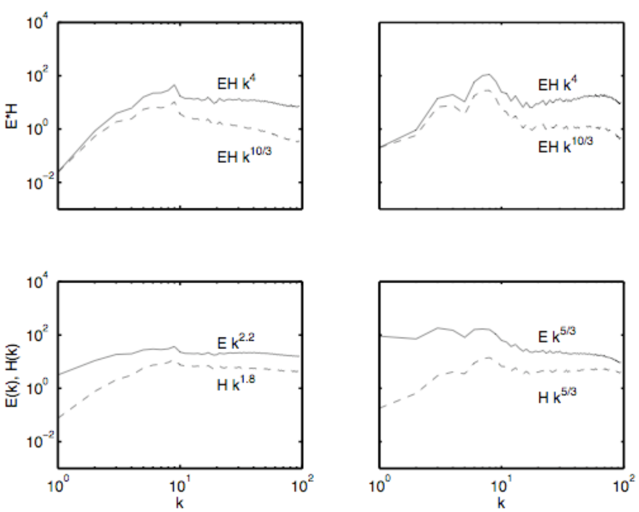
dash line: $k^{10/3}$

Compensated individual spectra

solid line: energy

dash line: helicity





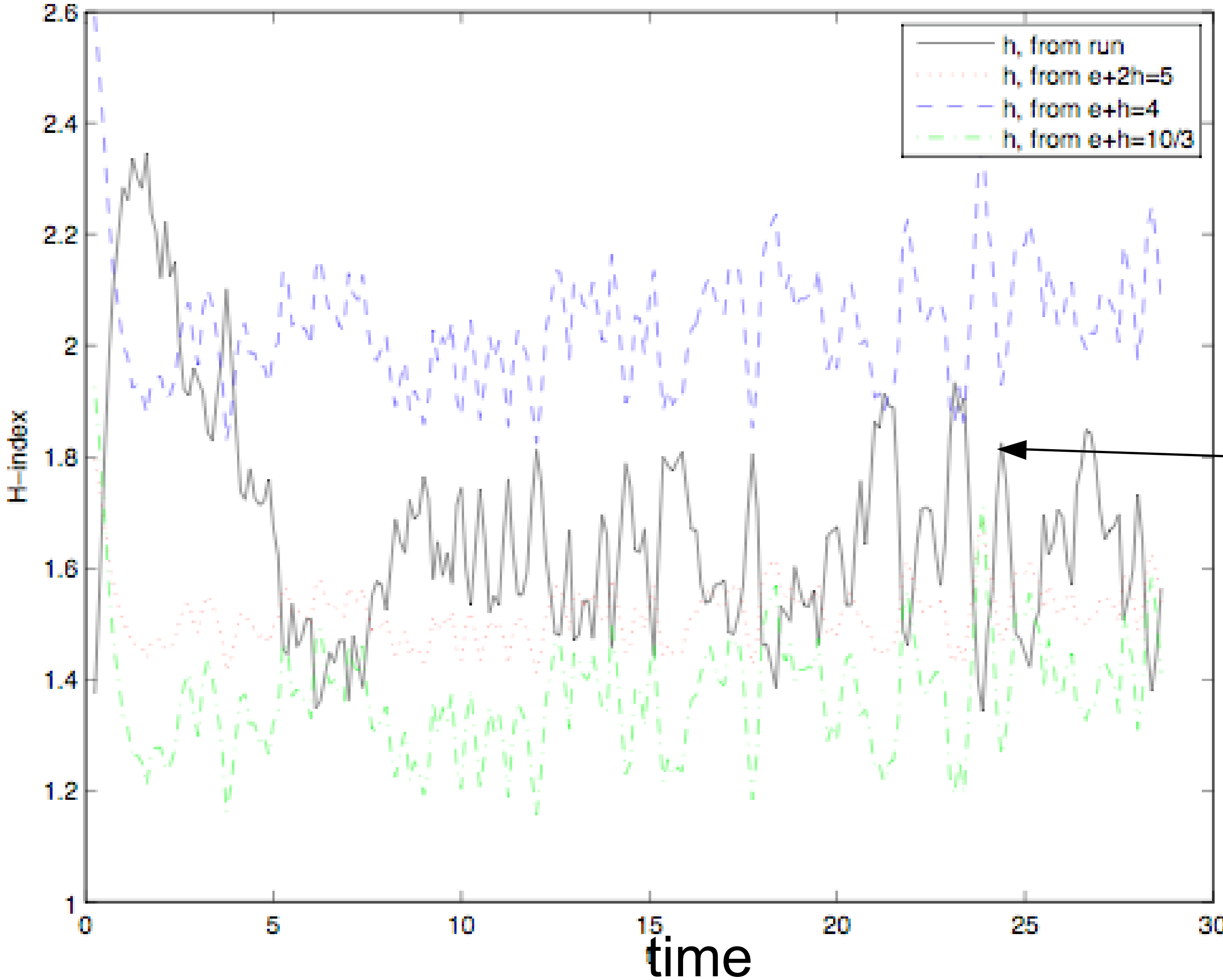
Sum of spectral indices -(e+h) as a function of time.

Horizontal: Rotation increases rightward

Vertical: Reynolds increases downward

Solid line: - 4

Dash line: - 10/3



Given e(t), what would h(t) be

IF

e+h=4 *(2009)*

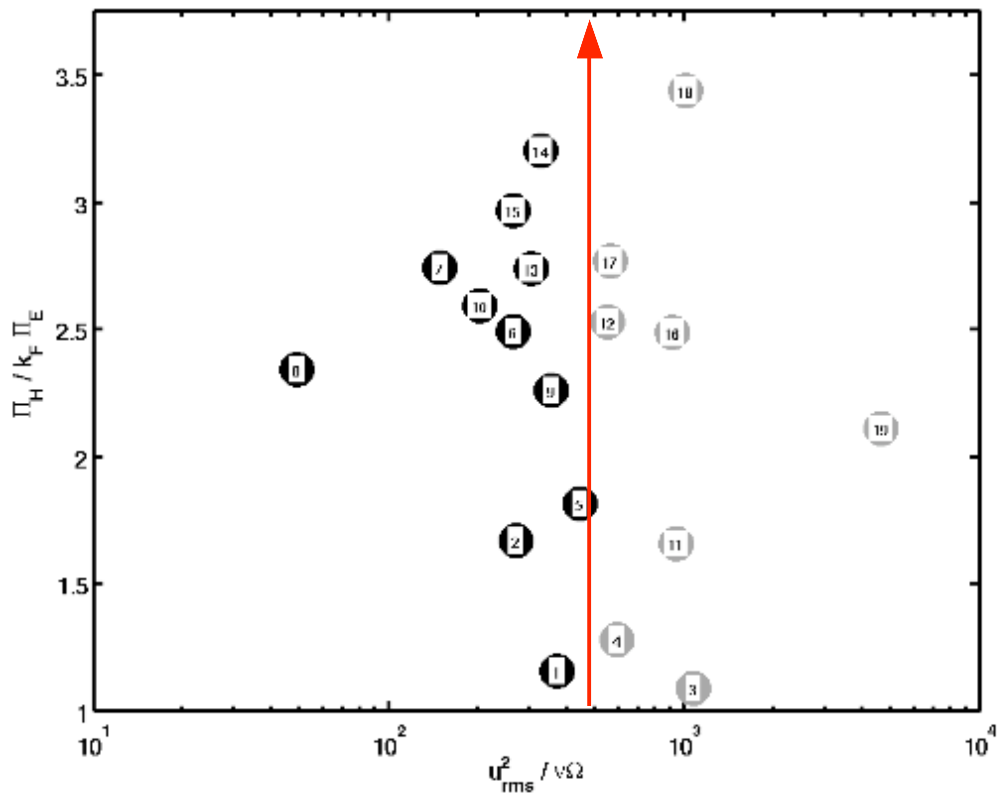
h(t) data

e+2h=5 *(1973)*

e=h=5/3 *(1941)*

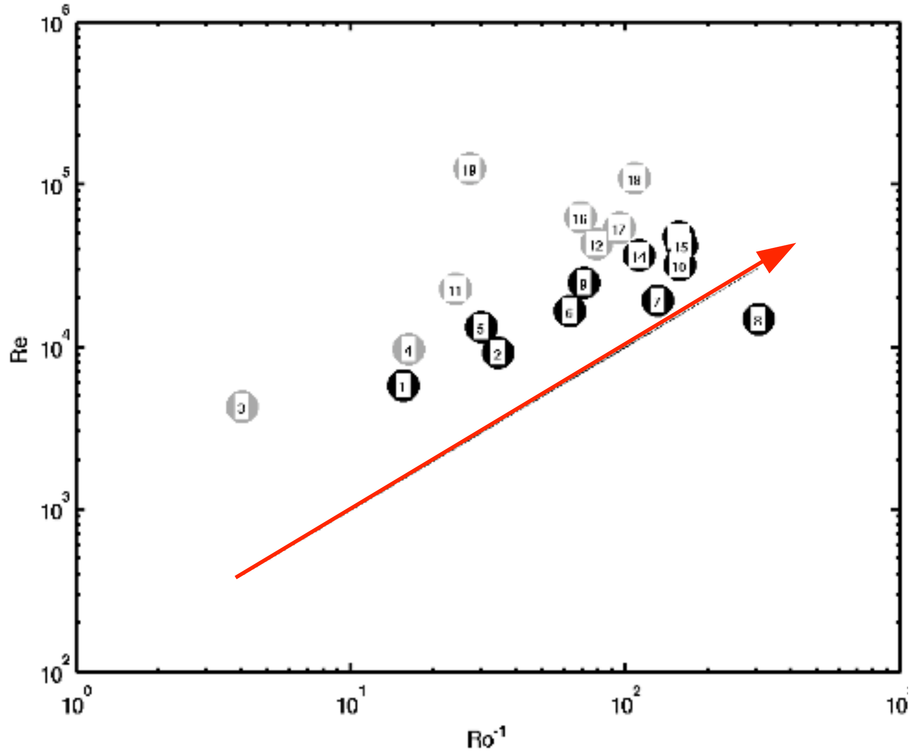
Black: e+h=4
Grey dots otherwise

Flux ratio $\varepsilon_H/[k_F\varepsilon_E]$



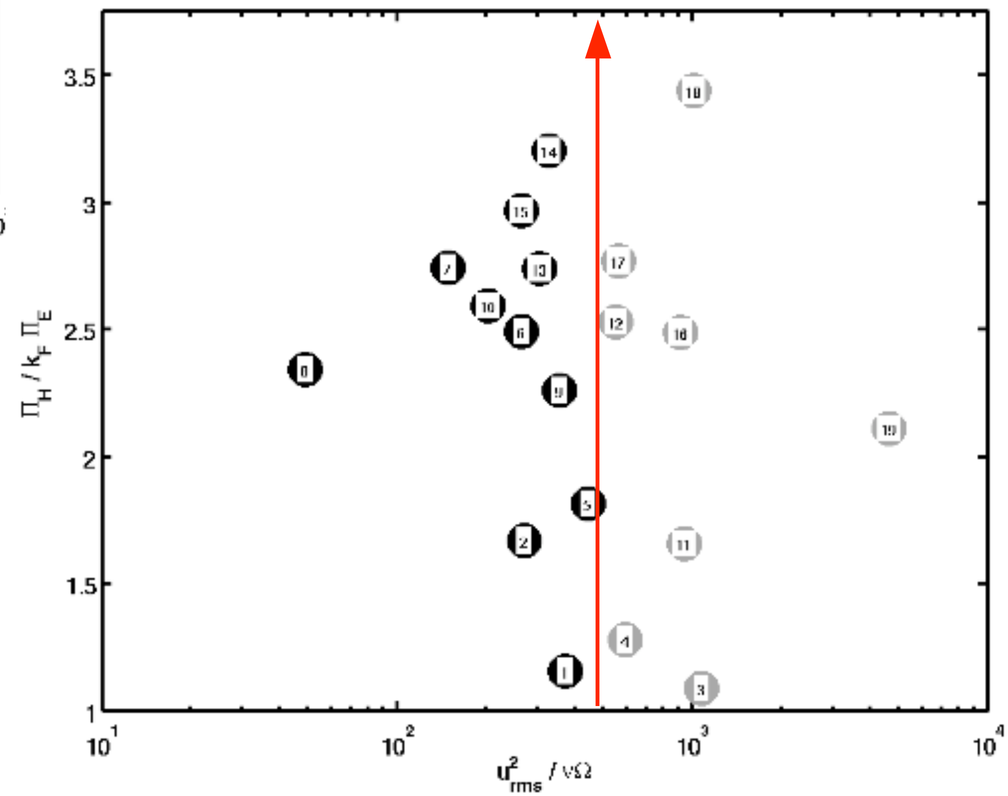
Reynolds* Rossby=$U_{rms}^2/\nu\Omega$

Scatter plot (Re, Ω) plane



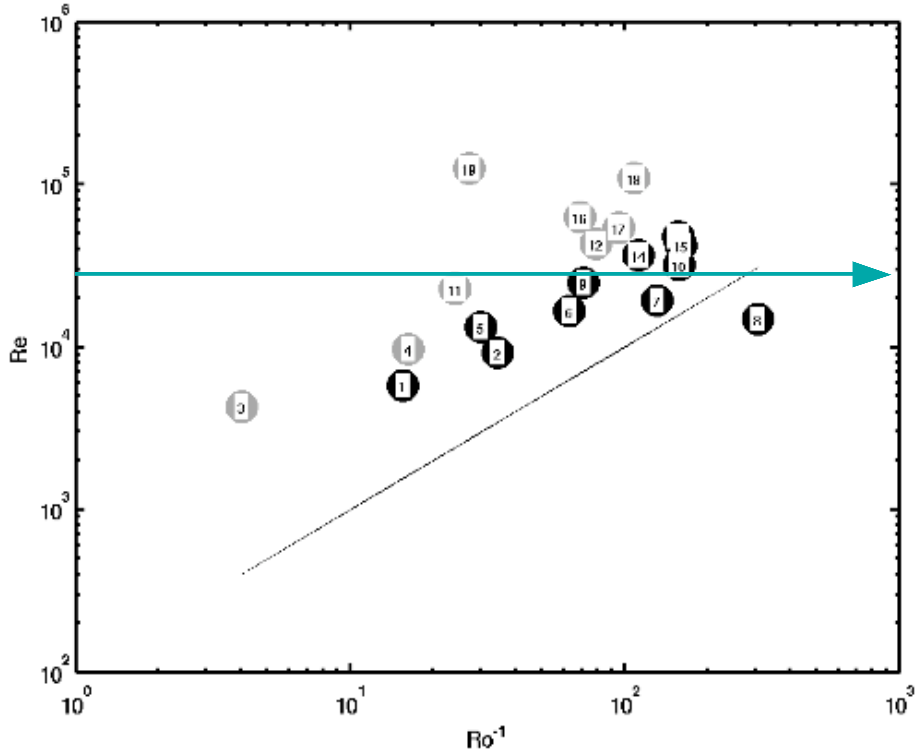
Black: e+h=4
Grey dots otherwise

Flux ratio $\varepsilon_H/[k_F\varepsilon_E]$



Reynolds* Rossby=$U_{rms}^2/\nu\Omega$

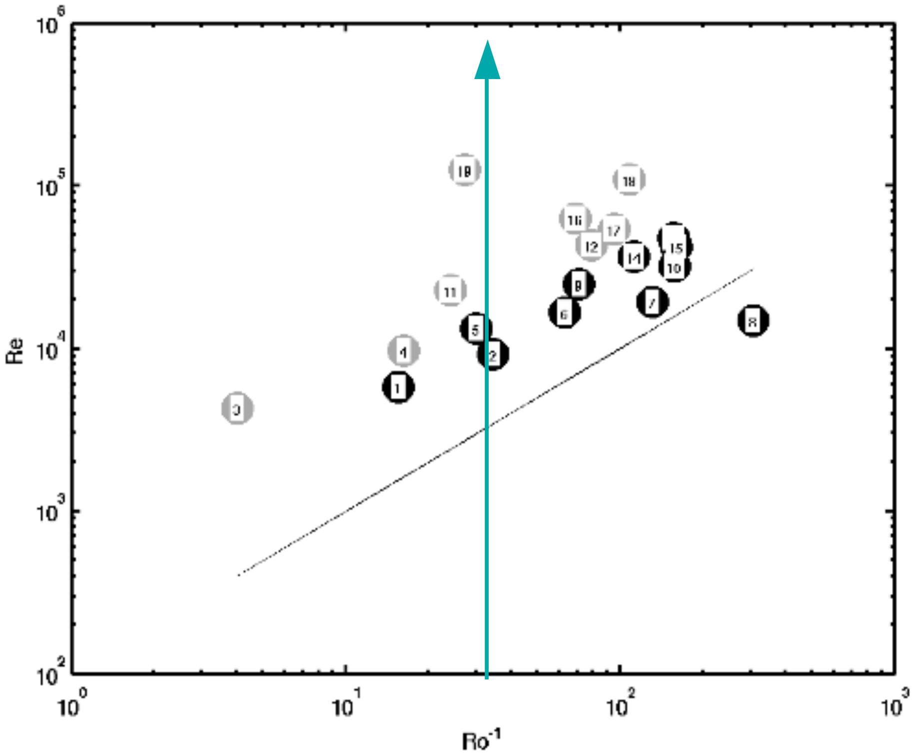
Scatter plot (Re, Ω) plane



Black: e+h=4

Grey dots otherwise

Scatter plot (Re, Ω) plane



Black: e+h=4

Grey dots otherwise

Summary of results

- In the presence of helicity and rotation, **the direct transfer to small scales is dominated by the helicity cascade** and the energy cascade to small scales is (strongly) quenched because of inverse cascade

- This provides a **small parameter for the problem** (the normalized ratio of energy to helicity fluxes), besides the Rossby number
- The **energy cascade to small scale is non-intermittent**, a result that differs from the known self-similarity of the inverse cascade of energy to large scales

- This leads to a **change of inertial index in the small scales** from a Kolmogorov law to a law steeper than the non-helical model predicts, and to a breaking of universality

- The flow produces **strong laminar long-lived columnar structures, Beltrami Core Vortices,** at scales smaller than the injection scale, structures that are fully helical *(on top of the structures that form at scales larger than the forcing and different from the early Taylor columns)*

Questions and future directions

- Can helicity help in interpreting laboratory experiments or atmospheric data?
- Can there be experimental evidence for this $e+h=4$ law?
- Same as above for $e+2h=5$, at higher Reynolds number, e.g., in the atmosphere?

- How does the dynamics change in terms of relative helicity $\rho(k)=H(k)/kE(k)$?

- *Does the organization of the force at large scale play a role (random vs. deterministic forcing)? 2D vs 3D forcing?*

- What happens locally in space? What are the Beltrami Core Vortex structures? How do they evolve and interact to lead to both a direct and inverse cascade *(A. Fournier, local -wavelet- analysis, in progress) ?*

- How does the helicity cascade behave in non-helical rotating flows?

- Universality?

Thank you for your attention

* Mininni *et al.,*``Scale interactions and scaling laws in rotating flows at moderate Rossby numbers and large Reynolds numbers," **Phys. Fluids, 21, 015108, 2009**

• Mininni & Pouquet, ``Helicity cascades in rotating turbulence," **Phys. Rev. E 79, 026304, 2009**

• Baerenzung *et al.,* ``Spectral Modeling of Rotating Turbulent Flows," submitted to Phys. Rev. E. See also arXiv:**0812.1821**

* Mininni & Pouquet, ``Persistent cyclonic structures in self-similar turbulent flows," submitted to Phys. Rev. E, see also arXiv:**0903.2294**

• Pouquet *et al.,* , ``Modeling of turbulent flows in the presence of magnetic fields or rotation,"TI2009 Conference (Ste Luce), to appear, Notes on Numerical Fluid Mechanics and Multidisciplinary Design, Springer Verlag, Michel Deville, Jean-Pierre Sagaut and Thien Hiep Eds. (2009). See also arXiv:**0904.4860**

• Baerenzung et al., ``Where we observe that helical turbulence prevails over inertial waves in forced rotating flows at high Reynolds and low Rossby numbers,'' **. . .** *almost submitted*

Data & code available, just come and visit us :)

New spectral laws for energy & helicity at high rotation,
using a well-known model of transfer in the presence of waves

- Consider the case of the cascade to small scales dominated by the flux Σ of helicity H (τ_{tr} is the transfer time):

  $\Sigma = dH/dt \sim H/\tau_{tr} \sim$ constant , $\tau_{tr} = \tau_{NL}^2 / \tau_W$

  and assume $\Sigma = k_F\, \varepsilon$; using dimensional analysis with ε, one gets:

$E(k) \sim \varepsilon^a\, \Omega^b\, \mathbf{k}^{-e}$ with $2a=3-e$, $2b=3e-5$

$H(k) \sim \Sigma^c\, \Omega^{1-b}\, \mathbf{k}^{e-4}$ with $2c=e-1$

Note that:

- *Positivity of b implies $5/3 < e < 7/3$, $5/3<h<7/3$ (it also fulfills $a>0$, $c>0$)*
- *The helicity and energy fluxes to the small scales are equally strong, <u>in terms of rotation rate</u> Ω, for $b=d$ and thus for $e = h = 2$*

  together with $a=b=c=1/2$

Je pense que cette page est inutile (incompatibilite des hypotheses)

Micro Rossby number

$R_\omega = \omega_{rms}/\Omega$

DNS 512^3, k_0=4

Ro= 0.03

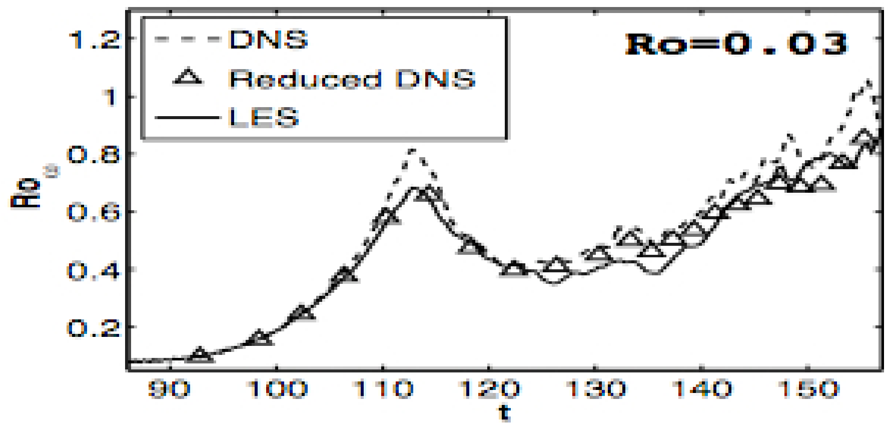
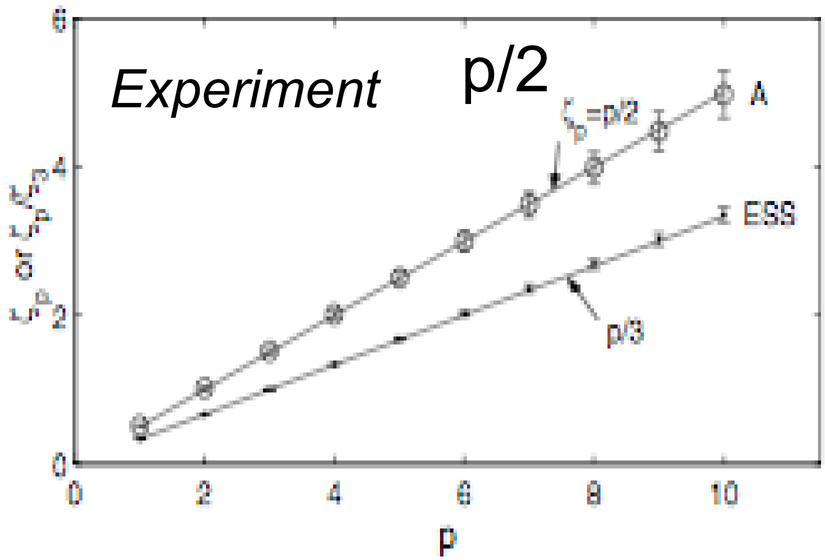
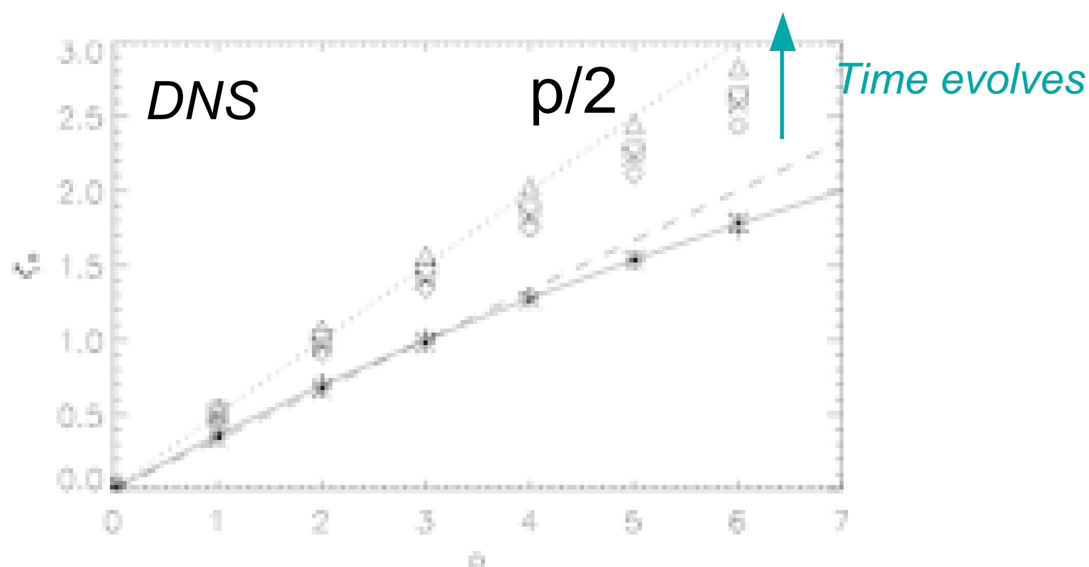


FIG. 6: Temporal evolution of the micro Rossby number at $Ro = 0.35$ (top) and $Ro = 0.03$ (bottom) for DNS (dash line) and LES (solid line). Note again the different scale on the axes for the lower Rossby number run.

Taylor-Green forcing





ζ_p = p/2 for the non-helical case *(Simand et al., '00; Baroud et al., '02; Mininni+AP, PRE **79** '09)*

* Absolute equilibria in the helical non-rotating case *(Kraichnan, 1973)*

- Dual cascade of energy and helicity with zero energy flux may have different scaling laws *(Brissaud et al., 1973)*

$$E(k) \sim k^{-e},\ H(k) \sim k^{-h},\ \ e+2h=5$$

- Simultaneous e=h=5/3 cascade *(two-point closure, André & Lesieur, 1977)*

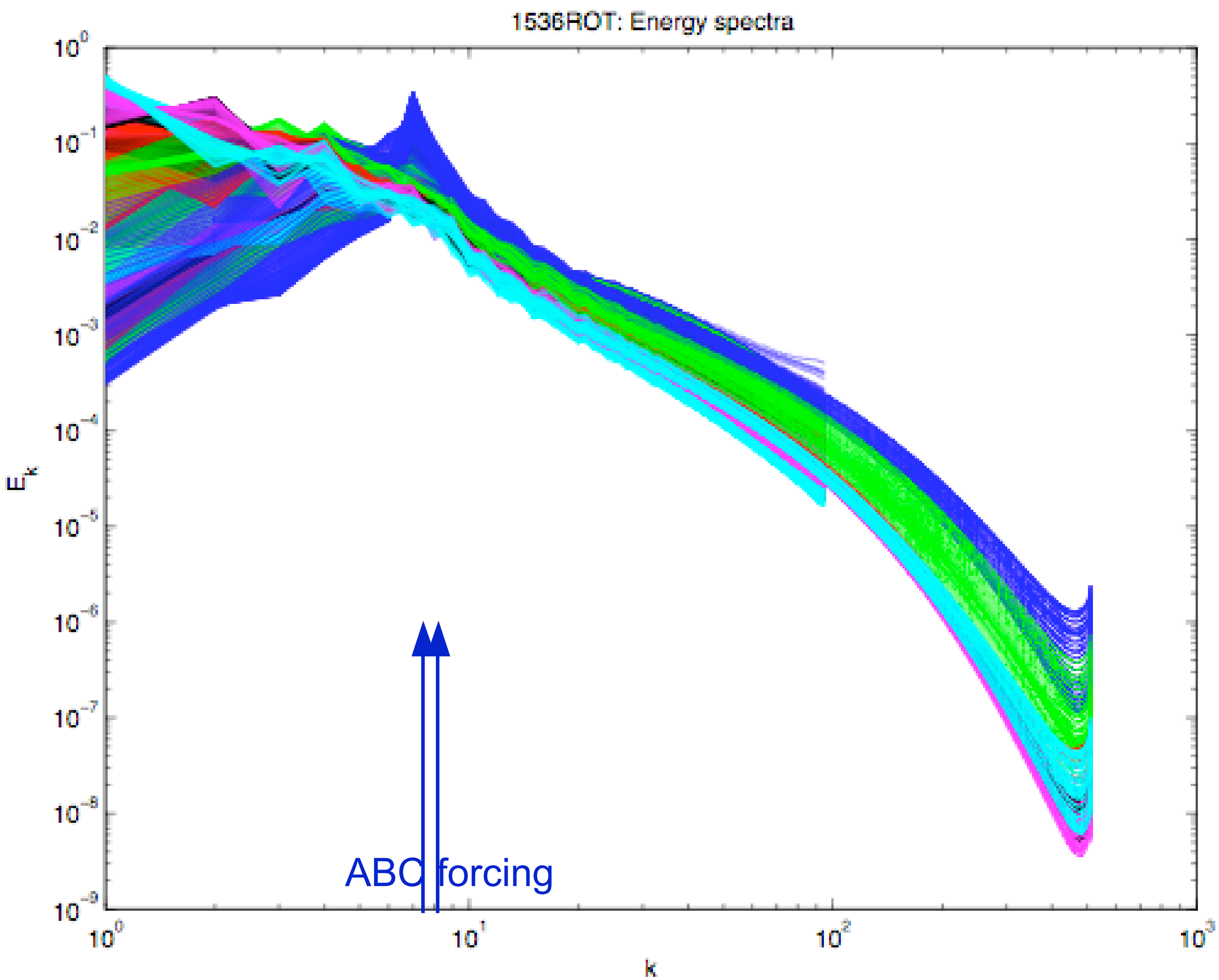
- Decomposition into helical waves *(Craya 1958, Herring 1974, Waleffe 1992, …)*

$$E(k) = E^{+}(k) + E^{-}(k)\quad ,\ H(k) = H^{+}(k) - H^{-}(k)\quad ,\ H^{\pm}(k)=kE^{\pm}(k)$$

^ $H^{\pm}$ flux cancellations *(Q. Chen et al. 2003) (Ditlevsen & Giuliani 2001)*

^ Different spectra & dissipation scales for $H^{\pm}_{k}$ and for H_{k} *(id.)*

** Effects of inhomogeneity *(Frisch et al. 1987; Yokoi and Yoshizawa 1993, …)*



Initial conditions: fully developed non rotating Kolmogorov flow, 1536^3 grid

T=0 to T=30 *, going through dark blue, green, mauve, red, pink, pale blue*

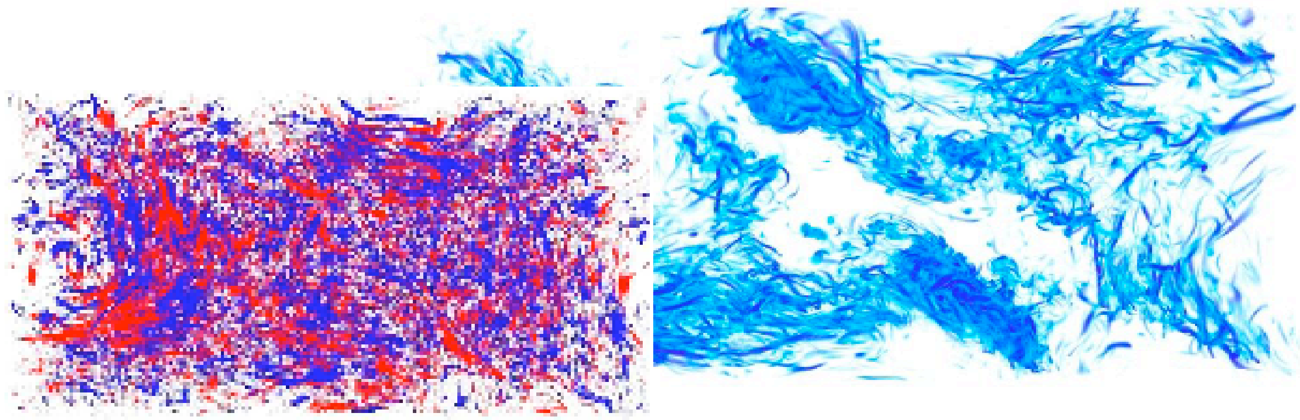
Top view and side view of

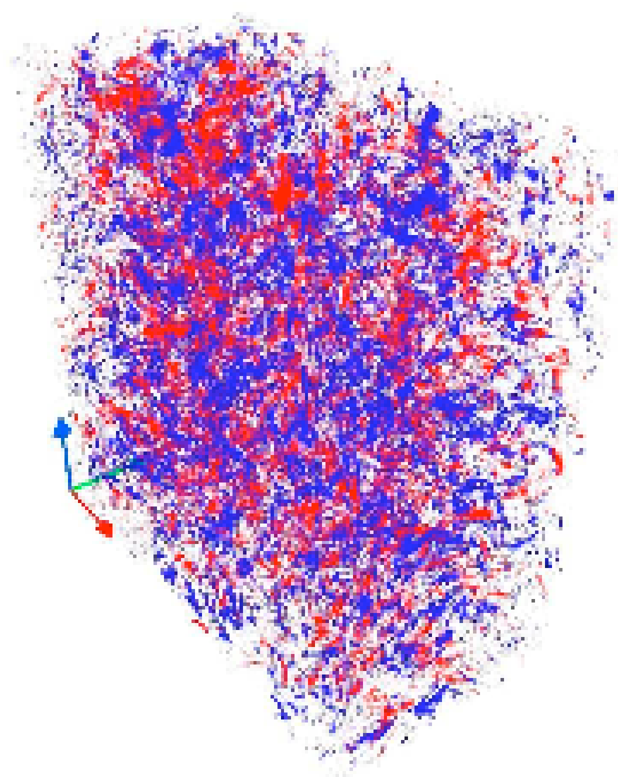
relative helicity

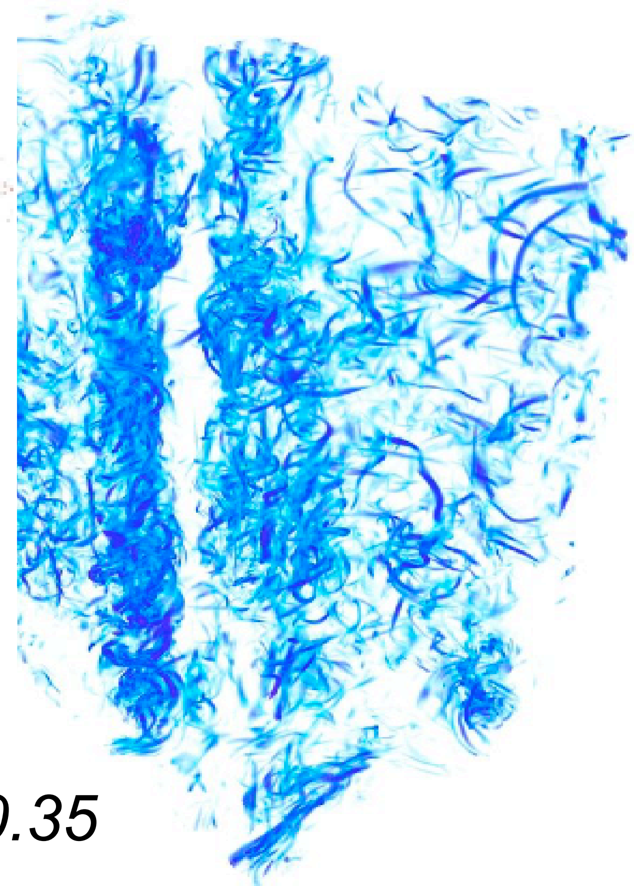
$\cos(v,\omega)$

when large

(positive or negative)

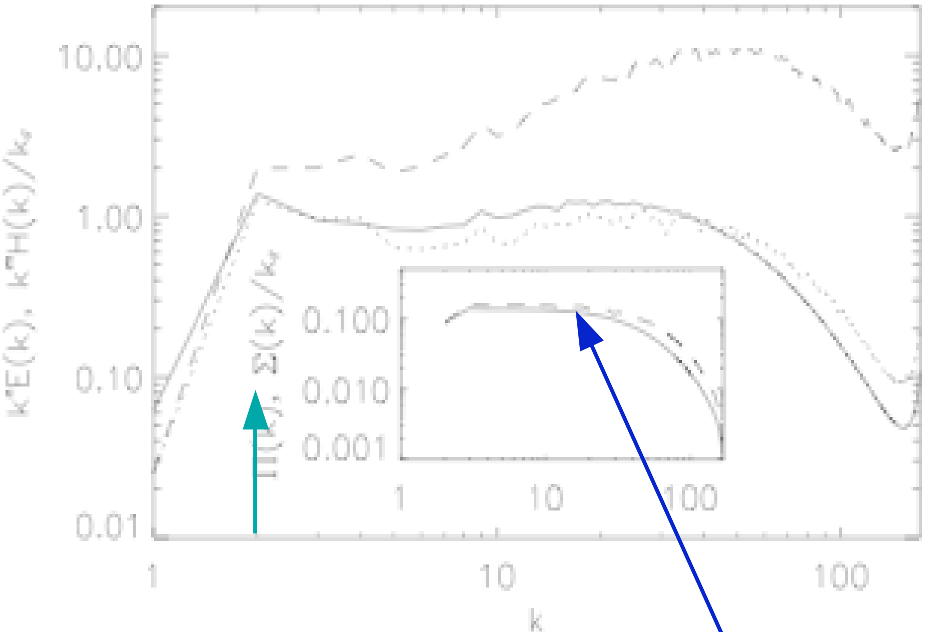






Taylor-Green forcing, k_0=4, 512^3, Ro=0.35

k^{x} - compensated spectra for energy (***x=e***) and helicity (***x=h***)



Low rotation, 512^3, k_F=2

Solid: e = 5/3

Dots: h = e

Dash: h = e - 4 = 7/3

Dash: h = e - 4 = 7/3

$$E(k) \sim k^{-5/3} \sim H(k)$$

Inserts: energy flux (solid) and helicity flux (dash)

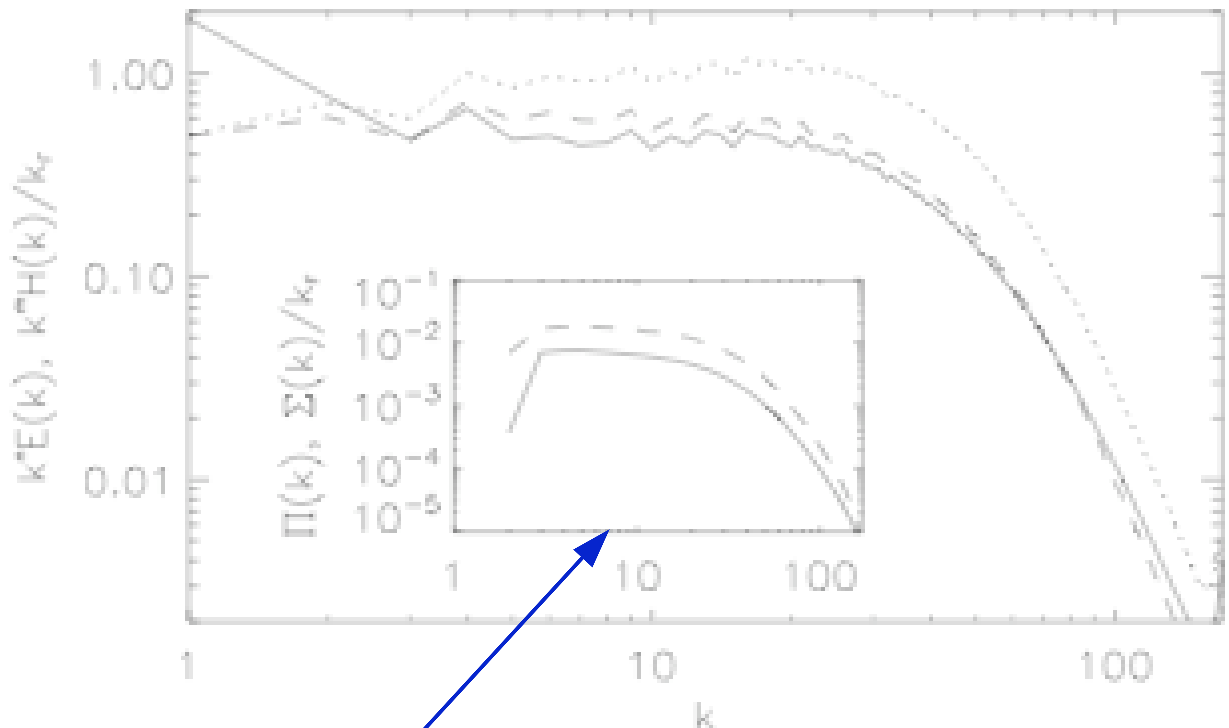
k^x - compensated spectra for energy (*x=e*) and helicity (*x=h*)

High rotation

Solid: e = **2.15**

Dots: e = h

Dash: h = e - 4 = **1.85**



512^3, k_F=3

$E(k) \sim k^{-2.15}$ and $H(k) \sim k^{-1.85}$

$H(k)/ kE(k) \sim k^{-0.7}$

Inserts: energy flux (solid) and helicity flux (dash)

- Helicity H= <**U**.∇x**U**> is an ideal invariant *(Moreau, 1961; Moffatt, 1969)*
- Absolute equilibria in the helical non-rotating case *(Kraichnan, 1973)*:

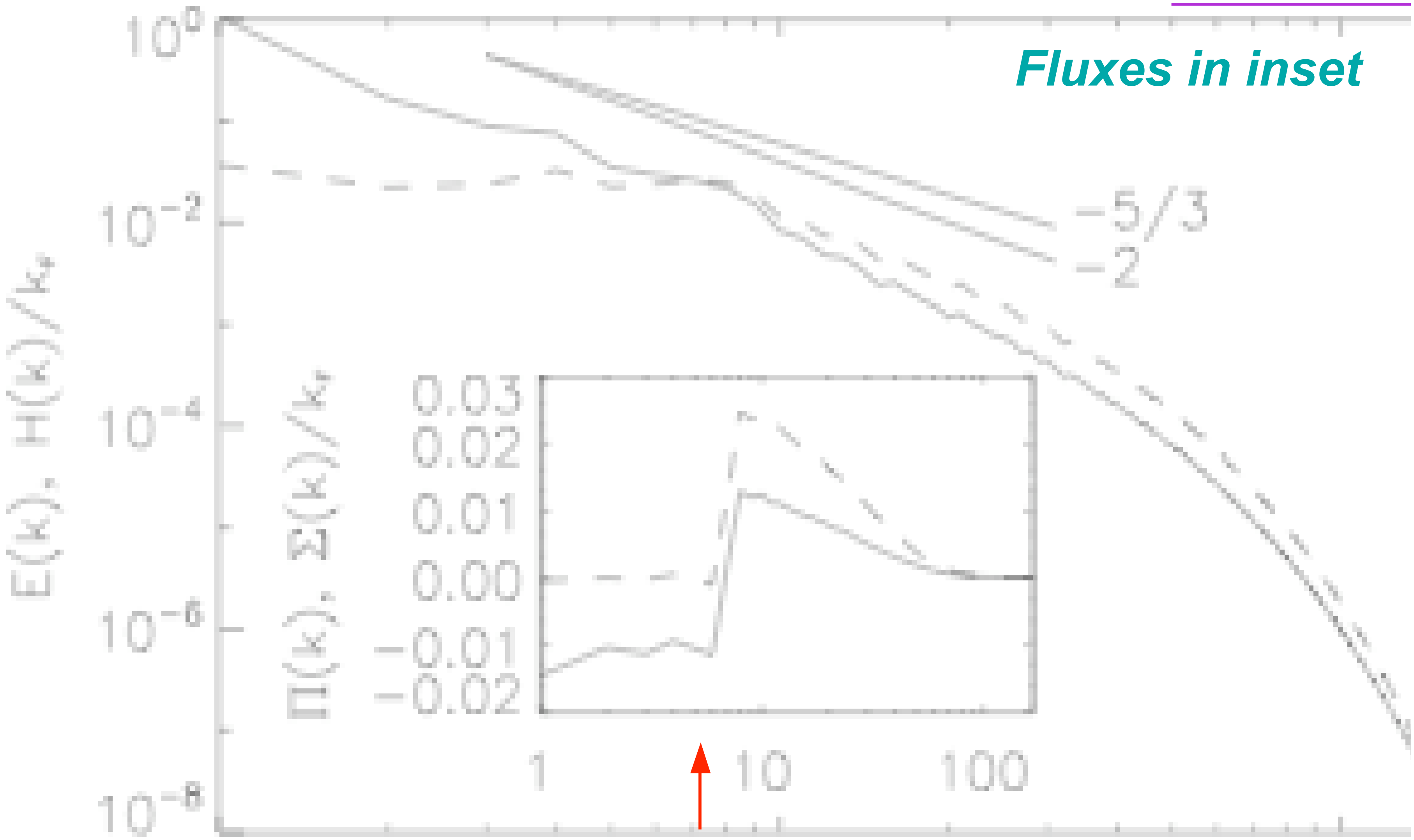
- Simultaneous e=h=5/3 cascade *(two-point closures, André & Lesieur, 1977; and numerous direct numerical simulations, e.g. Chen et al. 2003)*

- *Dual cascade of energy and helicity with zero energy flux may have different scaling laws (Brissaud et al., 1973)*

$E(k) \sim k^{-e}$, $H(k) \sim k^{-h}$, $e+2h=5$ *not observed*

ABC RUNS WITH **HELICITY**

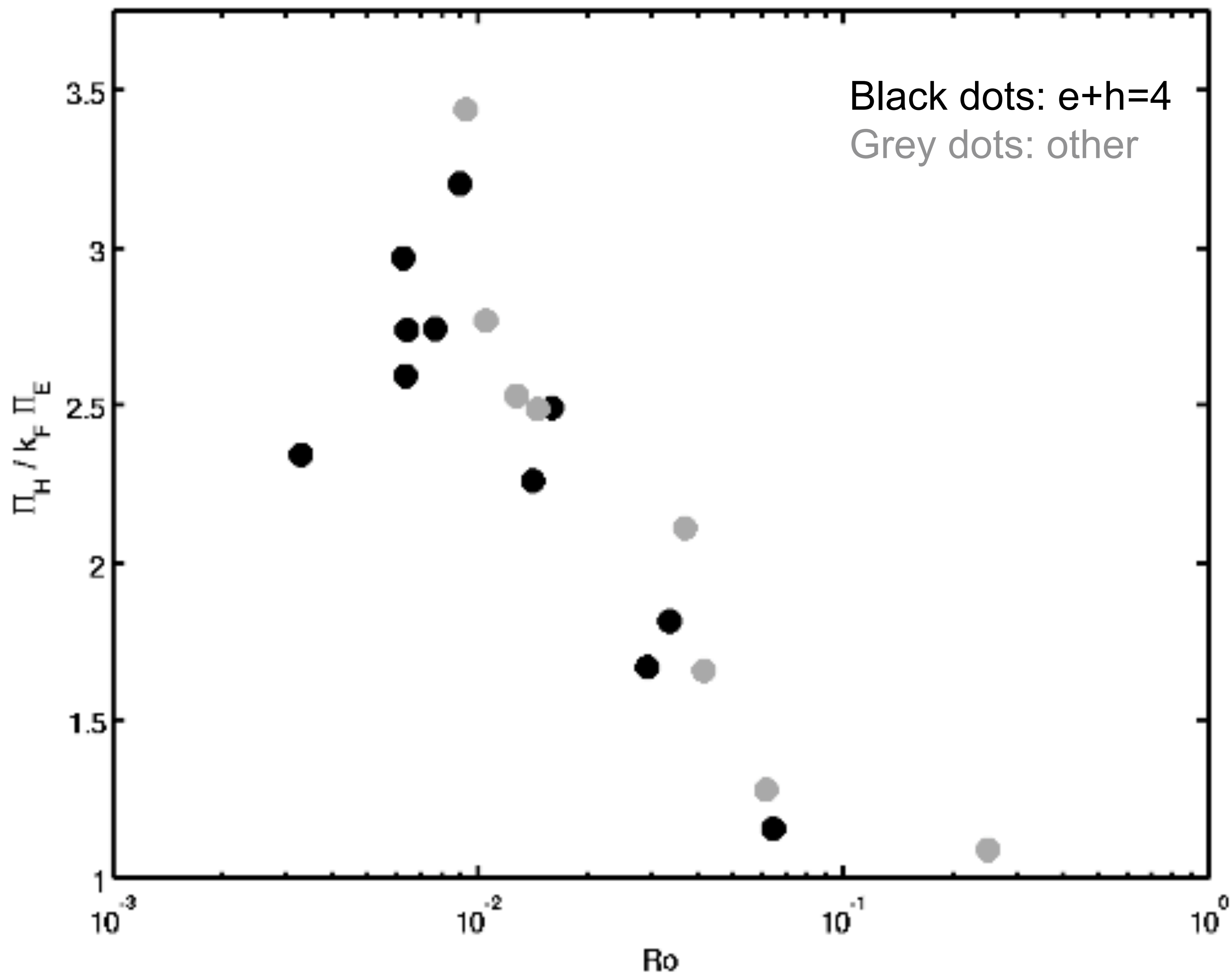
Fluxes in inset



Energy (solid): direct & inverse cascades

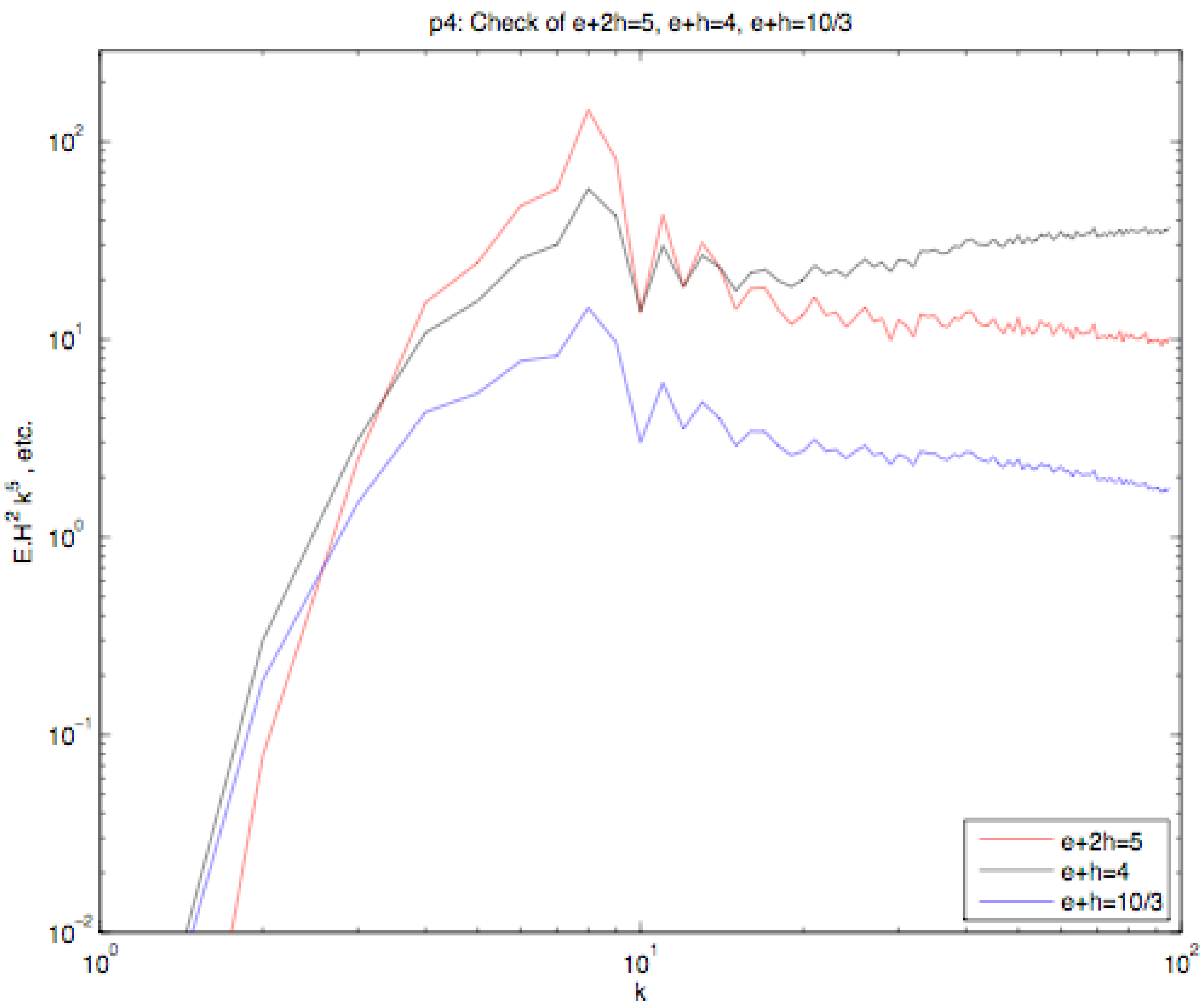
Helicity (- - -): direct cascade only

1536^3, k_F=7, Re=5100, Ro=0.06



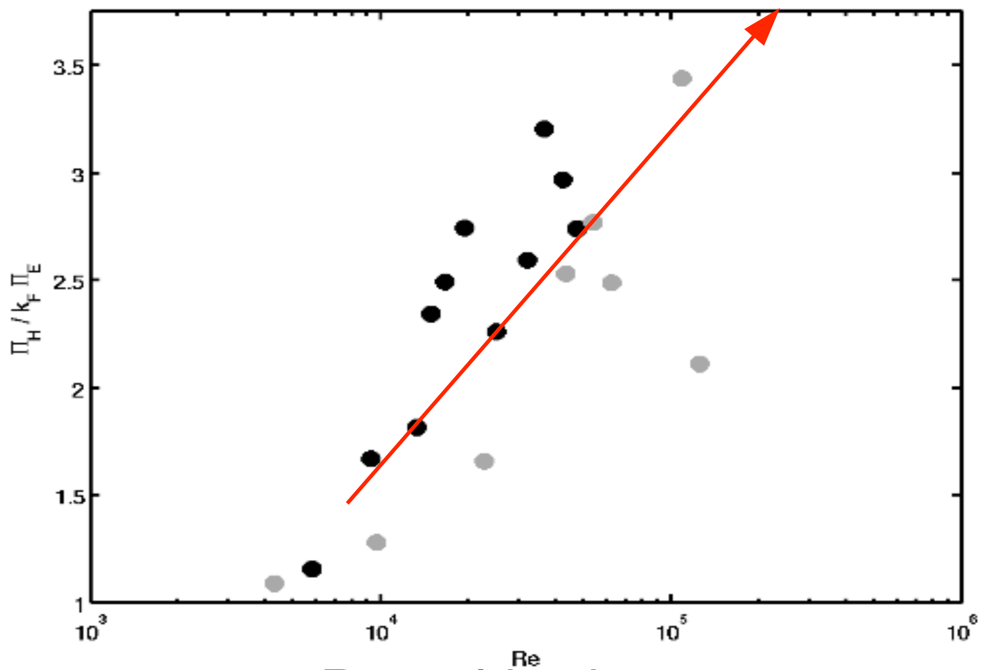
Ratio of helicity to energy flux, normalized by $k_F = f(\text{Rossby})$

The Brissaud law of helical flows?

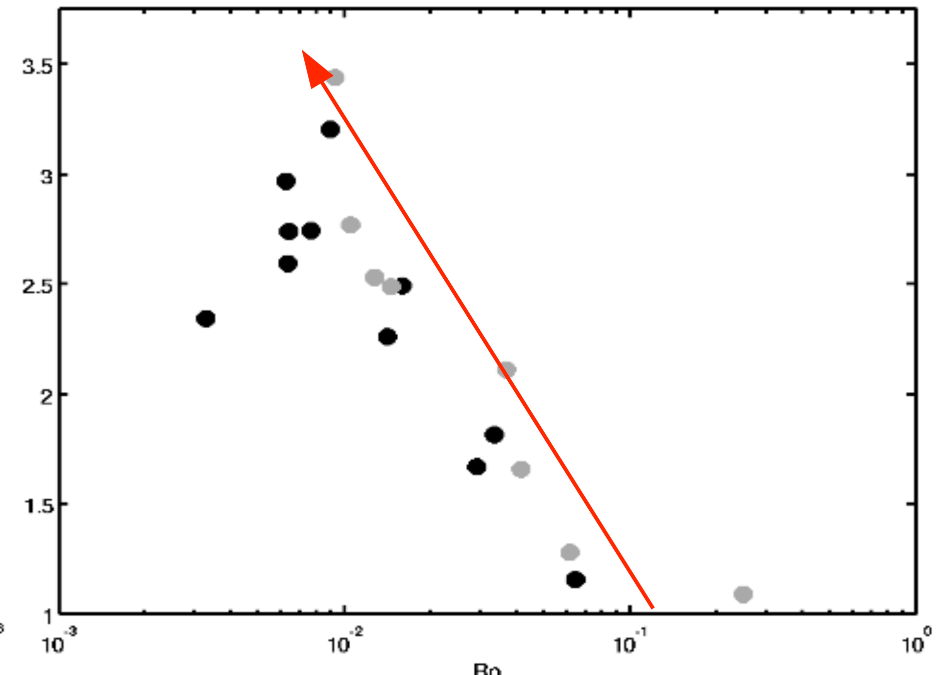


It assumes zero energy flux whereas here the helicity to energy flux ratio is ~ 3 so it is not really different from a dual Kolmogorov cascade

Ratio of helicity to energy flux normalized by k_F



Reynolds nb.



Rossby nb.

Black dots: e+h=4

Grey dots: other