The University of Texas – Rio Grande Valley
Syllabus for Math 6360.01: Ordinary Differential Equations Fall 2024
ELABS 161 MW 15:30–16:45

Contact information

>**Instructor:** Dr. Eleftherios Gkioulekas, School of Mathematical and Statistical Sciences
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>**Web:** http://faculty.utrgv.edu/eleftherios.gkioulekas/
>**Office hours:** MW 13:30–14:30
>**Office location:** EMAGC 3.214

Course information

>**Prerequisites:** Consent of instructor.
>**Corequisites:** None.
>**Course Description:** The topics in this course include existence and uniqueness theorems; methods for calculating solutions to systems of nonlinear ordinary differential equations (ODEs) and dynamical systems (DSs) relevant to applications in various areas; iterative methods for numerical solutions of ODES and DSs; and finite element methods combined with machine learning and deep learning.
>**Course modality:** Students enrolled in traditional face-to-face courses will attend class in person, on campus, and on set schedules - the traditional way.

Textbook

(1) E. Gkioulekas: "Lecture Notes on Advanced Ordinary Differential Equations", *Online Lecture Notes on Mathematics*, Edinburg, University of Texas Pan American (2010), 206 pp.
   - Open Educational Resource
   - Under construction. Will be made available online.

Other References

The following books include further discussion of some of the topics covered in this class. They could be helpful in terms of finding an interesting course project.

(1) C.M. Bender and S.A. Orzag (1978): "Advanced mathematical methods for scientists and engineers", McGraw-Hill
(2) R.P. Agarwal and D. O'Regan (2008): "An Introduction to Ordinary Differential Equations", Springer
(3) S.H. Strogatz (2015): "Nonlinear dynamics and chaos: With Applications to Physics, Biology, Chemistry, and Engineering", 2nd edition, Addison-Wesley [course textbook]
(4) S. Wiggins (2003): "Introduction to Applied Nonlinear Dynamical Systems and Chaos", 2nd edition, Springer-Verlag

Outline of Topics

We will try to cover the following topics. Additional topics could be included if necessary.
- **Introduction to Ordinary Differential Equations**
- **First-order ODES**
  Separable ODEs

Homogeneous ODEs
Integrating Factors method
- **Linear Differential Equations**
  Basic definitions
  Function operators and linear operators
  Homogeneous linear differential equations: The null space
  Homogeneous linear differential equations: The initial value problem
  Homogeneous linear differential equations: The Wronskian and it's properties
  Solving homogeneous differential equations
  Solving inhomogeneous linear differential equations
- **Series solution of linear ODEs**
  The Gamma function
  Review of power series
  Series solution of 2nd-order linear ODEs: Regular linear ODEs
- **Asymptotic methods for linear ODEs**
  Asymptotic series
  Method of dominant balance
  Generalized Schrodinger equation
- **Autonomous dynamical systems**
  Introduction to autonomous dynamical systems
  Existence and uniqueness
  Fixed points and stability
  Lyapunov functions
- **1d autonomous dynamical systems**
  Stability analysis for 1d systems
  Potential and 1d systems
  Local bifurcations with 1D systems
  More on sufficient conditions for bifurcation events
- **Linear autonomous systems**
  Exact Solutions
  Lyapunov function for $\dot{x} = Ax$
  The 2×2 linear autonomous system
- **Nonlinear autonomous systems**
  Local analysis of fixed points
  Nonlinear centers – conversion to polar coordinates
  Nonlinear centers – conservative systems
  Nonlinear centers – reversible systems
- **Center manifold reduction**
  Center manifold reduction – Methodology
  Center manifold reduction – Inclusion of linearly unstable directions
  Center manifold reduction – Application to local bifurcations.
- **Miscellaneous topics**
  Boundary value problems
  Floquet theory
- **Numerical Methods**
  Linear multistep formulas
  Runge-Kutta methods

About the enumerated course policies and procedures

- Any section or item beginning with "UTRGV" is an official UTRGV policy or statement that is required on all faculty syllabi, while those without such a designation are the instructor's own policies or statements.
- **UTRGV Statement:** *We value a positive and supportive learning environment, and for us to thrive together, we must recognize that our responsibilities, actions, and contributions can impact and transform our learning. The course policies listed below are created to ensure your success by fulfilling course expectations while remaining flexible to account for unexpected events.*

Grading Policies

- **Grading:** Periodic homework assignments will be given via Blackboard and will be expected to be returned for grading using Blackboard. Each homework question will be graded on a scale from 0 to 4 with $4 = A$, $3 = B$, $2 = C$, $1 = D$, $0 = F$. A written final project (term paper) related to the course topics is due at the end of the semester. Homeworks count for 60% of the grade and the final project counts for 40% of the grade. combining all homeworks and final project gives a weighted average score on a 0-20 scale. This score is then mapped to a letter grade as follows: A: 16-20; B: 12-16; C: 10-12; D; 7-10; F: 0-7.. there will be no curve and no extra credit.
- **Regrading policy:** If you believe that a mistake in grading has been made you may request that your paper be regraded. Such request must be submitted **in writing** within one week from the day the graded test has been returned in class, and must be accompanied by the original (unaltered) paper. If you make any changes to the paper your request will be denied. Please note that if you request regrading, all problems are subject to review. Thus, your overall grade may be increased or decreased.

Final term paper guidelines

In lieu of a final exam, you will be expected to develop a course project and write a final term paper. A 1-2 page project proposal should be submitted by Week 7, but earlier is better than later. The term paper is due on the last day of class. If your paper happens to be original research, I will be happy to help you develop it further and publish it as a co-authored paper, with you as the primary author, after the class is over. You are also welcome to submit it as a single-author paper on your own initiative, and encourage you to do so, if you are already comfortable with the peer-review process. However, it is not required for this course to write a publishable paper.

It is recommended that the proposal and the paper be typeset with LaTeX. Both should include an abstract and references, like a regular research paper. The project should be related to the topics covered in class and be a problem of interest to you and one that is fun for you to investigate. It is alright to choose a rather "easy" topic in order to be able to present it in a concise and self-contained paper. The paper should be as long as is necessary to develop your topic thoroughly. A normal paper is usually about 6-12 pages, and a long paper can go up to 20 pages.

You must edit your paper. Print your first version, go to a coffeeshop (optional), read it, mark changes on the paper. Make the changes, print again, and repeat. Remember that if this were a publishable paper, simple errors would be there to haunt you for eternity. Usual corrections are typos, equation errors, rephrasing, adding sentences/paragraphs, sometimes shuffling sentences/paragraphs around, etc. Be sure to punctuate your equations.

There are three possible types of papers you may write. Examples of these three types of papers will be posted to the course website. I do recommend that you look at them for ideas on how to express yourself in a tone that is appropriate for an academic publication.

(1) **Application:** Use the concepts covered in class to study an application in science, engineering, or any other real-world problem. The problem may involve algebraic calculations, or computer calculations. It may also be a combination of both. Include computer source codes in an appendix, if applicable.

(2) **Theory:** A theoretical paper extending the theory learned in this class. Do not simply regurgitate someone else's explanation of your topic. Work out all the calculations step by step with your own hands, and strive to find a clear, detailed, elegant, and better way to present your topic.

(3) **Review:** The paper reviews a body of literature on a given topic. The idea is to combine a number of papers into a coherent story, one that emerges only when combining the papers together, and tell us the story. You will find that papers tend to lead you to other papers and help you via their introduction to piece together your narrative. Publishable review papers may review about 100-300 references. A minimum of 10 references is expected for your term paper.

The usual structure of a paper is as follows:

(1) **Title:** Your project should have a compelling title

(2) **Abstract:** This is a short overview of the paper, a miniature version of about 100-200 words or so. Someone reading the abstract should get a good idea of what problem has been tackled, what types of techniques were used to solve it, and what sort of solution was found, and whether to actually.

(3) **Introduction:** Review the relevant literature, present the problem, explain why it is interesting, and outline the argument of your paper. Go over what will be covered in the remaining sections of your paper.

(4) **Preliminaries:** Introduce notations, definitions, and review prior relevant results that you wish to use. For application papers, explain the applivation topic and formulate the mathematical questions that it reduces to that you are going to address in your paper. For theoretical papers, announce your main results. For review paper give a general introduction of what the topic is all about that is accessible to a general audience. Briefly cite previous reviews or textbooks.

(5) **Results:** Establish your main results. Discuss the techniques used. For application papers, solve the mathematical problems stated in your setup algebraicly or numerical. For theoreticsl papers, state and prove your main results. For review papers, explain what has been done in the literature. Remember the fundamental law of story telling: every story or substory has a beginning, a middle, and an end.

(6) **Discussion:** Summarize your main results, assumptions made. How sensitive are your results if any of the assumptions are violated? Discuss if your results are realistic. If not, then why? How could the work be inproved. (may be combined with **Conclusion** as one section)

(7) **Conclusion:** Summarize what you have done and what you have learned.

(8) **Appendices:** Use these to include source code, or detailed tedious calculations.

(9) **References:** Bibliography. Refer to books or peer-reviewed articles only. LATEXcan automate the creation of the bibliography for you.

In short, **take pride in your work** and have fun!

Evaluation of final term paper

The course project's final term paper will be evaluated on the following scale (out of 20):

20	Excellent
17	Above Satisfsctory
13	Satisfactory
8	Below Satisfactory
4	Unsatisfactory
0	Not Submitted

The resulting score will be combined with the grades of the midterm exams to calculate your overall grade for the course.

Attendance Policy

- **Attendance Policy:** Attendance will be taken during most class meetings. **The instructor has the prerogative to drop any student with four (4) or more unexcused absences.** Two (2) tardies will count as one (1) unexcused absence. A tardy is defined as entering the class late or leaving the class early. **If you miss any major exam, you will be dropped from the course.**
- **How to Excuse an Absence:** To excuse an absence, you must notify the instructor **in writing** and attach **documentation**, before the date you will be absent, or no more than **three (3)** bussiness days after the date. UTRGV's attendance policy excuses students from attending class if they are participating in officially sponsored university activities, such as athletics; for observance of religious holy days; or for military service.

Other Policies

- **Calendar of Activities:** Information regarding important dates, such as, first day of classes, holidays, last day to drop a class before it appears on the transcript (the census date), last day to drop or withdraw with a DR grade, and final exam schedule are available via the academic calendar, linked from the course website. Please be advised of these important dates, and feel free to inquire with the instructor about any questions you may have with regard to the academic calendar.
- **Calculator Policy:** Calculators are not allowed in this class.
- **Classroom Conduct:** Common courtesy requires that students arrive in class on time, and stay the entire class period. Turn your cellphones and pagers off. You are required to treat your classmate and instructor with respect and courtesy. Use of any electronic devices, except for calculators, is not allowed in class, and I reserve the option to remove you from the classroom without warning for any behaviour that I deem as disrespectful or disruptive. You agree to indemnify and hold harmless the professor with respect to all actions undertaken by the professor to enforce classroom conduct or to properly proctor exams. Taking my course implies your consent to this policy.
- **Revisions:** This syllabus may be revised at any time. The syllabus posted on the professor's course web site is the only copy guaranteed to incorporate all revisions that may be made under this policy and will thus supersede any other versions posted on other university websites.
- **UTRGV Mandatory Course Evaluation Period:** Students have the opportunity to complete an ONLINE evaluation of this course, accessed through your UTRGV account (http://my.utrgv.edu). Course evaluations are used by the instructor to better understand the student experience in the course, which can inform revisions of the course to ensure student success. Additionally, course evaluations are also used by the instructor for

annual performance review and promotion applications, teaching award applications, among others. For these reasons, your feedback, reflections, and insights on your experience in the course are invaluable to ensure student success and a quality education for all. You will be contacted through email with further instructions. Students who complete their evaluations will have priority access to their grades.

- **Scholastic dishonesty:** As members of a community dedicated to Honesty, Integrity and Respect, students are reminded that those who engage in scholastic dishonesty are subject to disciplinary penalties, including the possibility of failure in the course and expulsion from the University. Scholastic dishonesty includes but is not limited to: cheating, plagiarism (including self-plagiarism), and collusion; submission for credit of any work or materials that are attributable in whole or in part to another person; taking an examination for another person; any act designed to give unfair advantage to a student; or the attempt to commit such acts. Since scholastic dishonesty harms the individual, all students and the integrity of the University, policies on scholastic dishonesty will be strictly enforced.

- **UTRGV Academic Integrity Policy:** Members of the UTRGV community uphold the Vaquero Honor Code's shared values of honesty, integrity and mutual respect in our interactions and relationships. In this regard, academic integrity is fundamental in our actions, as any act of dishonesty conflicts as much with academic achievement as with the values of honesty and integrity. The Writing Center is an excellent resource to assist in learning about and avoiding plagiarism in writing. Violations of academic integrity include, but are not limited to: cheating, plagiarism (including self-plagiarism), and collusion; submission for credit of any work or materials that are attributable in whole or in part to another person; taking an examination for another person; any act designed to give unfair advantage to a student; or the attempt to commit such acts (Board of Regents Rules and Regulations, STU 02-100, and UTRGV Academic Integrity Guidelines). All violations of Academic Integrity will be reported to Student Rights and Responsibilities through Vaqueros Report It at https://www.utrgv.edu/en-us/student-experience/report-it/.

Student Support Resources

- **UTRGV Statement:** We are committed to your personal, academic, and professional success; please know you can reach out to me for questions and/or I can help you identify the resources you need. UTRGV offers student support resources designed to contribute to your well-being and academic excellence.

- **UTRGV University Resources:** Students seeking academic help in their studies can use university resources in addition to an instructor's office hours. University Resources include the Advising Center, Career Center, Counseling Center, Learning Center, and Writing Center. These centers provide services such as tutoring, writing help, counseling services, critical thinking, study skills, degree planning, and connections to student employment (through Handshake and HR Student Employment. In addition services, such as the Food Pantry, are also provided. Locations are listed below.

Center Name	Brownsville Campus	Edinburg Campus
Advising center AcademicAdvising@utrgv.edu	BMAIN 1.400 (956) 665-7120	EITTB 1.000 (956) 665-7120
Career center CareerCenter@utrgv.edu	BINAB 1.105 (956) 882-5627	ESTAC 2.101 (956) 665-2243

Counseling Center	BSTUN 2.10	EUCTR 109
Counseling@utrgv.edu	(956) 882-3897	(956) 665-2574
Food Pantry	BCAVL 101 & 102	EUCTR 114
FoodPantry@utrgv.edu	(956) 882-7126	(956) 665-3663
Learning center	BMSLC 2.118	ELCTR 100
LearningCenter@utrgv.edu	(956) 882-8208	(956) 665-2585
University Library	BLIBR	ELIBR
circulation@utrgv.edu	(956) 882-8221	(956) 665-2005
www.utrgv.edu/library		
Writing center	BLIBR 3.206	ESTAC 3.119
WC@utrgv.edu	(956) 882-7065	(956) 665-2538

- **UTRGV Financial Need:** Students who demonstrate financial need have a variety of options when it comes to paying for college costs, such as scholarships, grants, loans and work-study. Students should visit the Student Services Center (U Central) for additional information. U Central is located in BMAIN 1.100 (Brownsville) or ESSBL 1.145 (Edinburg) or can be reached by email (ucentral@utrgv.edu) or telephone: (956) 882-4026. In addition to financial aid, U Central can assist students with registration and admissions.

- **UTRGV Blackboard Support:** If you need assistance with course technology at any time, please contact the Center for Online Learning and Teaching Technology (COLTT).

Campus:	Brownsville	Edinburg
Location:	Casa Bella (BCASA) 613	Marialice Shary Shivers (EMASS) 3.142
Phone:	956-882-6792	956-665-5327
Toll Free:	1-866-654-4555	

  ▷ **Support Tickets:** Submit a Support Case via our Ask COLTT Portal at
    https://utrgv.edusupportcenter.com/
  ▷ **Online Support:** Chat with a Support Specialist online at
    https://www.utrgv.edu/online/getting-support/chat/index.htm
  ▷ **24/7 Support:** Need Blackboard assistance after hours? You can call our main office numbers, 956-882-6792 or 956-665-5327, to speak with a support representative.

- **UTRGV Electronic Communication Policy:** Because of the Family Educational Rights and Privacy Act (FERPA) and other statutes, University policy requires all electronic communication between the University and students be conducted through official University supplied systems, such as your UTRGV account. Therefore, please use your UTRGV account for all future correspondence with faculty and staff.

University Policy Statements

- **UTRGV Statement:** *We care about creating a safe and supportive learning environment for all students. The University policy statements below are intended to create transparency for your rights and responsibilities as students. We each contribute to ensuring a safe and positive*

environment through our actions and conduct, and students are encouraged to advocate for their needs.

- **UTRGV Student Accessibility Services:** Student Accessibility Services staff can be contacted at either campus to learn about and explore accessibility services.

Campus:	Brownsville	Edinburg
Location:	Music and Learning Center	University Center
Room:	BMSLC 1.107	EUCTR 108
Phone:	956-882-7374	956-665-7005
E-mail:	ability@utrgv.edu	

- **UTRGV Students with Disabilities Policy:** Students with a documented disability (physical, psy- chological, learning, or other disability which affects academic performance) who would like to receive reasonable academic accommodations should contact Student Accessibility Services (SAS) for additional information. In order for accommodation requests to be considered for approval, the student must apply using the mySAS portal located at www.utrgv.edu/mySAS and is responsible for providing sufficient documentation of the disability to SAS. Students are required to participate in an interactive discussion, or an intake appointment, with SAS staff. Accommodations may be requested at any time but are not retroactive, meaning they are valid once approved by SAS. Please contact SAS early in the semester/module for guidance. Students who experience a broken bone, severe injury, or undergo surgery may also be eligible for temporary accommodations.

- **UTRGV Pregnancy, Pregnancy-related, and Parenting Accommodations:** Title IX of the Education Amendments of 1972 prohibits sex discrimination, which includes discrimination based on pregnancy, marital status, or parental status. Students seeking accommodations related to pregnancy, pregnancy-related condition, or parenting should submit the request using the form found at Pregnancy and Parenting UTRGV with the link below.
  https://www.utrgv.edu/accessibility/pregnancy-parenting/index.htm

- **UTRGV Sexual Misconduct and Mandatory Reporting Policy:** In accordance with UT System regulations, your instructor is a "Responsible Employee" for reporting purposes under Title IX regulations and so must report to the Office of Institutional Equity & Diversity (OIED@utrgv.edu) any instance, occurring during a student's time in college, of sexual misconduct, which includes sexual assault, stalking, dating violence, domestic violence, and sexual harassment, about which she/he becomes aware during this course through writing, discussion, or personal disclosure. More information can be found through the Office of Institutional Equity and Diversity at www.utrgv.edu/equity, including confidential resources available on campus. The faculty and staff of UTRGV actively strive to provide a learning, working, and living environment that promotes personal integrity, civility, and mutual respect that is free from sexual misconduct, discrimination, and all forms of violence. If students, faculty, or staff would like confidential assistance, or have questions, they can contact OVAVP (Office for Victim Advocacy & Violence Prevention) at (956) 665-8287, (956) 882-8282, or OVAVP@utrgv.edu.

- **UTRGV Dean of Students Resources:** The Dean of Students office assists students when they experience a challenge with an administrative process, unexpected situation such as an illness, accident, or family situation, and aids in resolving complaints. Additionally, the office facilitates student academic-related requests for religious accommodations, support students formerly in foster care, helps to advocate on behalf

of students and inform them about their rights and responsibilities, and serves as a resource and support for faculty and campus departments. Vaqueros Report It allows students, staff and faculty a way to report concern about the well-being of a student, seek assistance in resolving a complaint, or report allegations of behaviors contrary to community standards or campus policies. The Dean of Students can be reached by email dos@utrgv.edu, phone (956-665-2260), or by visiting one of the office locations: Cavalry (BCAVL) 204 or University Center (EUCTR) 323.

Student Learning Outcomes

After completing this course students will be able to
(1) Solve separable and homogeneous ordinary differential equations, as well as differential equations solvable with the integration factors method.
(2) Understand the general theory of linear differential equations of order greater than one, linear independence, and results related to the Wronskian.
(3) Solve certain types (constant coefficients and equidimensional) of linear differential equations of order greater than one.
(4) Solve second-order linear differential equations using convergent and asymptotic series techniques around points that are regular, regular-singular, or irregular-singular.
(5) Analyze the fixed point stability and bifurcations in one-dimensional autonomous differential equations
(6) Classify the fixed points of nonlinear systems of autonomous differential equations and construct face portrait diagrams.
(7) Use the center-manifold technique to study the stability of non-hyperbolic fixed points and categorize bifurcations in systems of autonomous differential equations.
(8) Other topics may be included as time allows