## The University of Texas - Rio Grande Valley

Syllabus for Math 2414.01: Calculus II Summer I 2020
Online MTWRF 09:20-11:20

## Contact information

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## Course information

Prerequisites: MATH 2413 (or MATH 2487) with a grade of 'C' or better Corequisites: None.
Course Description: Topics include derivatives and integrals of transcendental functions, methods of integration, parameterized curves, integration in polar coordinates, and infinite sequences and series.

## Textbook

(1) J. Stewart, "Essential Calculus" (early transcendentals), 2nd edition, Thomson Brooks/Cole, 2013

- Commercial Textbook.
- ISBN-10: 1133112285
- ISBN-13: 978-1133112280
- Amazon price for a new copy is listed as $\$ 94.12$ as of $03 / 11 / 2020$. Used copies or rentals can be used in this course and are available at a reduced price.

Other References
(1) E. Gkioulekas: "Lecture Notes on Calculus 2", Online Lecture Notes on Mathematics, Edinburg, University of Texas Pan American (2013), 316 pp.

- Open Educational Resource.
- This document can be downloaded at no cost from:
https://faculty.utrgv.edu/eleftherios.gkioulekas/Teaching/notes.html
(2) Z. Hannan: "wxMaxima for Calculus I", Solano Community College (2015), 158pp
- Open Educational Resource.
- This document can be downloaded at no cost from: https://wxmaximafor.wordpress.com/
(3) Z. Hannan: "wxMaxima for Calculus II", Solano Community College (2015), 176pp
- Open Educational Resource.
- This document can be downloaded at no cost from: https://wxmaximafor.wordpress.com/


## Outline of Topics

- Integration by parts

General remarks
Forms 1-6
Miscellaneous integrals
Evaluating integrals by recursion

- Integrals of rational functions

General Remarks
Case 1: Denominator with linear factors

Case 2: Denominator with linear and repeated factors
Case 3: Denominator with linear factors and an irreducible quadratic
Case 4: method of undetermined coefficients
Case 5 : Functions requiring long division
Integrals that reduce to rational function integrals
Exam 1

- Trigonometric Integrals

Forms $f(\sin x) \cos x$ and $f(\cos x) \sin x$
Forms $f(\tan x) / \cos ^{2} x$ and $f(\cot x) / \sin ^{2} x$
Forms $\tan ^{a} x / \cos ^{b} x$ and $\cot ^{a} x / \cot ^{b} x$
Form: Products of trig functions
Form: Squares of trig functions
Method of desperation

- Rationalizing substitutions

Form $\sqrt{a^{2}-b^{2} x^{2}}$
Form $\sqrt{a^{2} x^{2} \pm b^{2}}$
Form $\sqrt{a^{2} x^{2}+b x+c}$ with $a>0$
Form $\sqrt{a^{2} x^{2}+b x+c}$ with $a<0$

- Improper Integrals

Improper Integrals of the first kind
Improper Integrals of the second kind
Exam 2
Convergence criteria for improper integrals

- Sequences

Convergent sequences
Divergent sequences
Bounded sequences
Recursive sequences and monotonicity
Convergence and order

- Series

Series convergence
Integral test
Comparison test
Limit comparison test
Ratio test
Root test
Alternating series test
Absolute convergence test
Conditional convergence

- Series approximation of functions

Power Series
Uniform convergence of power series
Properties of power series
Taylor expansion with term by term differentiation
Taylor expansion via the convergence theorem
Taylor expansion via the binomial series
Taylor expansion via product of series
Exam 3

- Applications of integrals

Parametric Curves
Calculus on parametric evaluations
Arclength of parametric curve
Arclength of polar curve

## Pedagogical objectives and expectations

The fundamental pedagogical objectives that students should strive for in every Mathematics course are:
(1) To understand, learn, and remember the formal and rigorous mathematical definition for every concept covered in the course.
(2) To understand, learn, and remember all the theorems and propositions that are applicable to previously defined concepts.
(3) To understand, learn, and practice the methods for applying theorems in the solution of routine problems, and to be able to creatively synthesize techniques to solve problems that are non-routine and may require creative thinking.
(4) To master rigorous mathematical writing, understand and use logic and quantifier notation, and realize and appreciate that every mathematical argument, from basic arithmetic, to advanced mathematics, with almost no exceptions, is a mathematical proof.
(5) To master the course material to a level of excellence that will ensure sustained success in more advanced mathematics courses.
To be successful in this course, it is expected that you should:
(1) Spend about 12 hours each week working homework problems, reviewing lecture notes, reading the textbook and online lecture notes, studying for exams, and seeking help from the tutors and instructor;
(2) Complete all homework problems, check the correctness of your work, and understand the methods and principles they illustrate;
(3) Master the designed course topics before each test, and if necessary, complete additional problems beyond those assigned and consult other sources if you find the assigned problems and text are insufficient;
(4) Recognize that mastery of the solution to a problem is not demonstrated by simply obtaining the correct numerical answer, but only by a clear, systematic, and detailed solution that traces the given information to the final numerical answer and that employs knowledge developed in this and previous courses;
(5) When you experience difficulty in the course, seek help from the tutors and instructor immediately;
(6) Attend class meetings regularly, pay attention, and do not hesitate to ask questions; and
(7) Write your solutions to homework, test, and quiz problems in an organized and legible way.

## Grading Policies

- Grading: There will be 3 major exams, and a comprehensive final exam. The time and location of exams will be announced in class. Exams count for $75 \%$ and final exam for $25 \%$ of your grade. Combined, you get a numerical grade on a scale 0-20. Each exam question is graded on a $0-4$ scale with $4=A, 3=B$, $2=C, 1=D, 0=F$. Combining all exams, as explained above, gives a weighted average score on a $0-20$ scale. This score is then mapped to a letter grade as follows: A: 16-20; B: 12-16; C:10-12; D; 7-10; F: 0-7. There will be no curve and no extra credit.
- Explanation of grades: Your exams are graded question by question on a $0-4$ scale per question. Overall, if you are planning to take future Mathematics or STEM courses, I would like to see you score 3 or 4 on all questions on all major examinations. If you score less than that on any questions, it indicates weaknesses in understanding the material. You should be proactive about addressing these weaknesses.
- Homework: Homework will be assigned, but will not be collected or graded. Nevertheless, it is crucial to do the homework as part of your preparation for the exams. To keep up, I recommend that after every lecture you should solve the homework problems corresponding to the material covered on that day's lecture. Thus you need to work on a continuous basis! Maintain a well-organized written record of your homework solutions by writing the statement of each problem (so that your document is stand-alone and can be read by itself), followed by your detailed solution, and clearly indicate the problem, section, and chapter number of the question. Most homework problems require more than simply writing the answer, and so you must write all steps of your solution and provide appropriate justification, as illustrated by the instructor's solved examples, as you would on a test. Write neatly and legibly, using rigorous mathematical notation. While you are encouraged to discuss homework problems with other students, tutors, your instructor, and other faculty, the write-up of your solutions must be your own work and not simply copied from another student or another source. Use a ring binder to collect your homework, and write with a black pen, as that will help you to later scan the ring binder as a PDF file, for possible future use, and as a form of backup. This will provide you with a readily available resource to prepare for tests and quizzes, as well as providing documentation of the homework problems should you have a question about a problem and seek help from the instructor or a tutor.
- Attendance Policy: Attendance will be taken during most class meetings. It is important to sign in the sign-up sheet if you are present. The instructor has the prerogative to drop any student with four (4) or more unexcused absences. Two (2) tardies will count as one (1) unexcused absence. A tardy is defined as entering the class late or leaving the class early. If you miss any major exam, you will be dropped from the course.
- How to Excuse an Absence: To excuse an absence, you must submit in writing the "Notification of a Scheduled Excused Absence form" stapled with documentation, before the date you will be absent, or no more than three (3) bussiness days after the date. The form should be submitted in person in class, during office hours, or via the Mathematics Department secretary. The form can be downloaded from the course web page. If a major exam is missed during an excused absence, your score for that exam will be replaced with your final exam score. UTRGV's attendance policy excuses students from attending class if they are participating in officially sponsored university activities, such as athletics; for observance of religious holy days; or for military service.
- Make-ups: There are no make-up exams. In the case of excused absences the final exam will be used as a make-up exam. Each student MUST take the final exam at the scheduled date and time. There will be no make-ups for the final exam, after the official final exam date!
- Regrading policy: If you believe that a mistake in grading has been made you may request that your paper be regraded. Such request must be submitted in writing within one week from the day the graded test has been returned in class, and must be accompanied by the original (unaltered) paper. If you make any changes to the paper your request will be denied. Please note that if you request regrading, all problems are subject to review. Thus, your overall grade may be increased or decreased.


## Other Policies

- Calendar of Activities: Information regarding important dates, such as, first day of classes, holidays, last day to drop a class before it appears on the transcript (the census date), last day to drop or withdraw with a DR grade, and final exam schedule are available via the academic calendar, linked from the course website. Please be advised of these important dates, and feel free to inquire with the instructor about any questions you may have with regard to the academic calendar.
- Extra Help: You are strongly encouraged to form a study group with two or three of your classmates. The group should have no more than 4 students. The group will serve to help each other in doing homework, studying for tests, and whenever possible, teaching each other. The idea is to help each other keep up with the class and hopefully, be successful.
(1) Contact your instructor during their office hours.
(2) Get free Math tutoring from Learning Assistance Center (LAC) building in Room 114 phone \# 6652532. (Edinburg Campus)
(3) Get free Math tutoring from Math Lab in Math building (MAGC) in room MAGC 1.106 (Edinburg Campus)
(4) Visit the Math Tutoring Lab at SETB 1.408 (Brownsville Campus)
(5) Visit the Math and Natural Sciences Learning Center at Cavalry Hall; Phone number: (956) 882-7058, (956) 882-8208 (Brownsville Campus)
- Calculators: The problems you will encounter in my exams will not require a calculator, and you are better served in the long-term by minimizing your dependence on calculators. Don't use the calculator to approximate roots, exponentials, logarithms, etc. Mathematical problems require exact answers. Approximations are reasonable only on application problems where the numbers given may be approximate themselves, and thus the best answer that can be deduced is unavoidably approximate.
- Classroom Conduct: Common courtesy requires that students arrive in class on time, and stay the entire class period. Turn your cellphones and pagers off. You are required to treat your classmate and instructor with respect and courtesy. Use of any electronic devices, except for calculators, is not allowed in class, and I reserve the option to remove you from the classroom without warning for any behaviour that I deem as disrespectful or disruptive. You agree to indemnify and hold harmless the professor with respect to all actions undertaken by the professor to enforce classroom conduct or to properly proctor exams. Taking my course implies your consent to this policy.
- Revisions: This syllabus may be revised at any time. The syllabus posted on the professor's course web site is the only copy guaranteed to incorporate all revisions that may be made under this policy and will thus supersede any other versions posted on other university websites.
- Mandatory Course Evaluations: Students are required to complete an ONLINE evaluation of this course, accessed through your UTRGV account (http://my.utrgv.edu); you will be contacted through email with further instructions. Students who complete their evaluations will have priority access to their grades.
- Students with disabilities: Students with a documented disability (physical, psychological, learning, or other disability which affects academic performance) who would like to receive academic accommodations should contact Student Accessibility Services (SAS) as soon as possible to schedule an appointment to initiate services. Accommodations can be arranged through SAS at any time, but are not retroactive. Students who experience a broken bone, severe injury, or undergo surgery during the semester are eligible for temporary services.
- Pregnancy, Pregnancy-related, and Parenting Accommodations: Title IX of the Education Amendments of 1972 prohibits sex discrimination, which includes discrimination based on pregnancy, marital status, or parental status. Students seeking accommodations related to pregnancy, pregnancy-related condition, or parenting (reasonably immediate postpartum period) are encouraged to contact Student Accessibility Services for additional information and to request accommodations.
- Student Accessibility Services - Brownsville Campus: Student Accessibility Services is located in 1.107 in the Music and Learning Center building (BMSLC) and can be contacted by phone at (956) 882-7374 or via email at ability@utrgv.edu.
- Student Accessibility Services - Edinburg Campus: Student Accessibility Services is located in 108 University Center (EUCTR) and can be contacted by phone at (956) 665-7005 or via email at ability@utrgv.edu.
- Scholastic dishonesty: As members of a community dedicated to Honesty, Integrity and Respect, students are reminded that those who engage in scholastic dishonesty are subject to disciplinary penalties, including the possibility of failure in the course and expulsion from the University. Scholastic dishonesty includes but is not limited to: cheating, plagiarism (including self-plagiarism), and collusion; submission for credit of any work or materials that are attributable in whole or in part to another person; taking an examination for another person; any act designed to give unfair advantage to a student; or the attempt to commit such acts. Since scholastic dishonesty harms the individual, all students and the integrity of the University, policies on scholastic dishonesty will be strictly enforced (Board of Regents Rules and Regulations and UTRGV Academic Integrity Guidelines). All scholastic dishonesty incidents will be reported to Student Rights and Responsibilities.
- Sexual misconduct and mandatory reporting: In accordance with UT System regulations, your instructor is a "Responsible Employee" for reporting purposes under Title IX regulations and so must report to the Office of Institutional Equity \& Diversity (oie@utrgv.edu) any instance, occurring during a student's time in college, of sexual misconduct, which includes sexual assault, stalking, dating violence, domestic violence, and sexual harassment, about which she/he becomes aware during this course through writing, discussion, or personal disclosure. More information can be found at www.utrgv.edu/equity, including confidential resources available on campus. The faculty and staff of UTRGV actively strive to provide a learning, working, and living environment that promotes personal integrity, civility, and mutual respect that is free from sexual misconduct, discrimination, and all forms of violence. If students, faculty, or staff would like confidential assistance, or have questions, they can contact OVAVP (Office for Victim Advocacy \& Violence Prevention) at 665-8287, 882-8282, or OVAVP@utrgv.edu.


## Student Learning Outcomes

(1) Evaluate integrals of various types using integration by parts.
(2) Evaluate integrals of rational functions using partial fraction decomposition.
(3) Evaluate various types of trigonometric integrals using substitution, trigonometric identities, or conversation to a rational function integral.
(4) Evaluating integrals involving the square root of a quadratic using rationalizing substitutions.
(5) Understand the definition of improper integrals of the first and second kind, and prove their convergence or divergence using the using the comparison test, the ratio test, or by employing absolute convergence.
(6) Understand the definition and properties of series convergence, and prove the convergence or divergence of a series using the integral test, comparison test, limit comparison test, ratio test, root test, alternating series test (by Leibnitz), absolute convergence test, the concept of conditional convergence, and the counterintuitive absolute ratio test and absolute root test.
(7) Understand the definitions and properties regarding the convergence, uniform convergence, and absolute convergence of power series, the radius of convergence, and results concerning the differentiation and integration of power series.
(8) Construct and establish the convergence of the Taylor expansion of a function given in closed form, using the factorial and/or double factorial notation, when necessary, by leveraging (a) integrals and derivatives of the geometric power series; (b) the Taylor convergence theorem; (c) the binomial series; (d) the product of power series (Merten's theorem).
(9) Calculate the cartesian derivative, arclength, and area under parametrically defined cartesian or polar curves.

## CALCULUS 2 HOMEWORK

- This homework is based on: J. Stewart, "Essential Calculus" (early transcendentals), Cengage, 2010
- CAL2.1, etc. refer to the problems given in the online lecture notes. These notes are available at the course website.
- Problems indicated "for fun" are for math majors.
- It is crucial to do the homework as part of your preparation for the exams. To keep up, I recommend that after every lecture you should solve the homework problems corresponding to the material covered on that day's lecture. Thus you need to work on a continuous basis! Maintain a well-organized written record of your homework solutions by writing the statement of each problem (so that your document is standalone and can be read by itself), followed by your detailed solution, and clearly indicate the problem, section, and chapter number of the question. Most homework problems require more than simply writing the answer, and so you must write all steps of your solution and provide appropriate justification, as illustrated by the instructor's solved examples, as you would on a test or quiz. Write neatly and legibly, using rigorous mathematical notation. While you are encouraged to discuss homework problems with other students, tutors, your instructor, and other faculty, the write-up of your solutions must be your own work and not simply copied from another student or another source. Use a ring binder to collect your homework, and write with a black pen, as that will help you to later scan the ring binder as a PDF file, for possible future use, and as a form of backup. This will provide you with a readily available resource to prepare for tests and quizzes, as well as providing documentation of the homework problems should you have a question about a problem and seek help from the instructor or a tutor.


## Integration by parts

- General remarks

Read: Lecture Notes
CAL 2.2: 1, 2, 3

- Forms 1-6

Read §6.1
CAL 2.2: 4,5,6
§6.1: 3-10, 13, 14, 16-19

- Miscellaneous integrals

CAL 2.2: 7, 8
§6.1: 11, 12, 15, 20, 22, 24, 25, 26

- Evaluating integrals by recursion CAL 2.2: 9-12
CAL 2.2: 13, 14 (for fun!!)
§6.1: 44, 45, 46 (for fun!!)


## Integrals of rational functions

- General Remarks

Read §6.3
CAL 2.2: 15
§6.3: 11, 27, 28 ,

- Case 1: Denominator with linear factors
CAL 2.2: 16, 17
§6.3: 9, 10, 12, 13, 14, 17, 18
- Case 2: Denominator with linear and repeated factors
CAL 2.2: 18
§6.3: 19, 20
- Case 3: Denominator with linear factors and an irreducible quadratic CAL 2.2: 19
§6.3: 19, 20, 22, 23, 29
- Case 4: method of undetermined coefficients
CAL 2.2: 20
§6.3: 24, 25, 26, 31, 32, 33, 34
- Case 5 : Functions requiring long division
CAL 2.2: 21
§6.3: 7, 8, 16, 21, 30,
- Integrals that reduce to rational function integrals
CAL 2.2: 22, 22, 24, 25
§6.3: 41, 42
Trigonometric Integrals
- Forms $f(\sin x) \cos x$ and $f(\cos x) \sin x$ Read $\S 6.2$
CAL 2.2: 26
§6.2: 1-5, 13, 14, 16
- Forms $f(\tan x) / \cos ^{2} x$ and $f(\cot x) / \sin ^{2} x$ CAL 2.2: 27, 28, 29
- Forms $\tan ^{a} x / \cos ^{b} x$ and $\cot ^{a} x / \cot ^{b} x$ CAL 2.2: 30 §6.2: 17, 18, 21-25, 28
- Form: Products of trig functions CAL 2.2: 31 §6.2: 37, 38
- Form: Squares of trig functions

CAL 2.2: 32, 33, 34, 35
§6.2: 5-10

- Method of desperation (tangent substitution)
CAL 2.2: 36, 37
§6.2: 15, 19, 20, 26, 27, 29, 30, 36


## Rationalizing substitutions

- Form $\sqrt{a^{2}-b^{2} x^{2}}$

CAL 2.2: 38 §6.2: 42, 49, 50, 53

- Form $\sqrt{a^{2} x^{2} \pm b^{2}}$

CAL 2.2: 39
§6.2: 43-48, 51, 52, 54-58

- Form $\sqrt{a^{2} x^{2}+b x+c}$ with $a>0$

CAL 2.2: 40
§6.2: 61, 64

- Form $\sqrt{a^{2} x^{2}+b x+c}$ with $a<0$

CAL 2.2: 41
§6.2: 62, 63

## Improper Integrals

- Improper Integrals of the first kind §6.6: 5-22
CAL 2.3: 1
- Improper Integrals of the second kind
§6.6: 23-32
CAL 2.3: 2
- Convergence criteria for improper integrals
§6.6: 41-50
CAL 2.3: 3,4

Sequences

- Sequences and convergent sequences
Read $\S 8.1$
§8.1: 9-11, 16, 20, 23, 24, 30, 31
CAL 2.4: 1,2
- Divergent sequences

Read §8.1
§8.1: 17, 18
CAL 2.4: 3, 4

- Bounded sequences

Read $\S 8.1$
§8.1: 22, 25, 26, 28
CAL 2.4: 5,6,7

- Recursive sequences and monotonicity
Read lecture notes
§8.1: 42-44, 52b
CAL 2.4: 8
- Convergence and order

Read lecture notes
§8.1: 14, 32
CAL 2.4: 9,10

Series

- Series whose limit can be calculated Read $\S 8.2$
§8.2: 9-21, 22-28, 35-37
- Integral and comparison tests Read $\S 8.3$
§8.3: 6-8 (integral test)
§8.3: 9-10 (comparison test)
§8.3: 11, 12, 17-28
- Ratio and root test Read $\S 8.4$
§8.4: 19, 26, 27, 39, 40 (ratio test)
§8.4: 33, 34, 35 (root test)
- Alternating and absolute convergence tests
Read $\S 8.4$
§8.4: 20-24, 31, 32, 41, 42,44
Series approximation of functions
- Power series

Read $\S 8.5$
§8.5: 3-22

- Uniform convergence of power series
Read Lecture Notes
Study counterexamples in Online Lecture Notes
- Properties of power series Read Lecture Notes and $\S 8.6$ §8.6: 34-37
- Taylor expansion: 1. Term-by-term differentiation or integration Read $\S 8.6, \S 8.7$ §8.6: 3-12, 15-20, 21-24, 25-28
- Taylor expansion: 2. Via convergence theorem
Read $\S 8.7$
§8.7: 5-10, 27-32, 35, 43-46
- Taylor expansion: 3. Binomial series
Read $\S 8.7$
§8.7: 5 (2nd method), 23-26, 33, 34
§8.8: 25, 28 (for fun)
- Taylor expansion: 4. Product of series
Read $\S 8.7$
§8.7: 55, 58


## Parametric curves

- Definition of parametric curves Read §9.1
- Calculus on parametric curves Read $\S 9.2$ and lecture notes §9.2: 9-16, 22-26 (derivatives) §9.2: 27-31 (areas)
- Arclength of parametric curve Read $\S 9.2$ and lecture notes §9.2: 38-43, 49, 52
- Arclength of a polar curve Read §9.4
§9.4: 33-36

