

The University of Texas – Rio Grande Valley

Syllabus for Math 2318.01: Linear Algebra Fall 2019

EMAGC 1.208 MW 15:30-16:45

Contact information

Instructor: Dr. Eleftherios Gkioulekas, School of Mathematical and Statistical Sciences

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Course information

Prerequisites: MATH 2413 (or MATH 2487) with a grade of 'C' or better.

Corequisites: None.

Course Description: Topics include systems of linear equations, matrices and their algebraic properties, determinants, vectors, Euclidean n-space, linear transformations and their matrix representations, vector spaces, eigenvalues and eigenvectors, and applications to the sciences and business. Use of mathematical technology will be incorporated throughout the course.

Textbook

- (1) E. Gkioulekas (2012): "Lecture Notes on Linear Algebra", *Online Lecture Notes on Mathematics*, University of Texas Pan American, 232 pp.
 - Open Educational Resource
 - This document can be downloaded as a pdf file, at no cost, from <https://faculty.utrgv.edu/eleftherios.gkioulekas/Teaching/notes.html>

Other References

- (1) D.A. Santos (2006): "Linear Algebra", Community College of Philadelphia, 237 pp.
 - Open Educational Resource
 - This document can be downloaded as a pdf file, at no cost, from <https://faculty.utrgv.edu/eleftherios.gkioulekas/OGS/Santos/santos-linearalgebra.pdf>
- (2) W. Keith Nicholson (2019): "Linear Algebra with Applications", Open Edition, Lyryx Learning, 698 pp.
 - Open Educational Resource
 - This document can be downloaded as a pdf file, at no cost, from <https://lyryx.com/linear-algebra-applications/>

Outline of Topics

- | | |
|---|------------------------------------|
| • Brief introduction to logic and sets | Basic operations with matrices |
| Propositions and sets | Matrix multiplication |
| Predicates and quantified statements | Matrix inverses |
| • Brief introduction to proof | Matrix transpose |
| Negation and contrapositive of statements | Exam 1 |
| Methodology for writing proofs | Determinants |
| • Linear Algebra | Cofactor expansion of determinants |
| Matrices | Simplification of determinants |

$n \times n$ linear systems of equations	Groups
Matrix inverse	Vector spaces – definitions/properties
Cramer’s rule	Basic vector spaces
Gaussian elimination	Vector subspaces
• Eigenvalues and eigenvectors	Subspaces spanned by vectors
Definitions	Linear independence
How to find the eigenvalues	Linear independence in \mathbb{R}^n
How to find the eigenvectors	Exam 3
Characteristic polynomial	Basis and dimension of vector spaces
Cayley-Hamilton theorem	Dimension and canonical basis of \mathbb{R}^n
Exam 2	Basis of a vector space with known dimension
• Vector spaces	Dimension of $\text{span}(B)$
Internal operations	

Pedagogical objectives and expectations

The fundamental pedagogical objectives that students should strive for in every Mathematics course are:

- (1) To understand, learn, and remember the formal and rigorous mathematical *definition* for every concept covered in the course.
- (2) To understand, learn, and remember all the *theorems* and *propositions* that are applicable to previously defined concepts.
- (3) To understand, learn, and practice the *methods* for applying theorems in the solution of routine problems, and to be able to creatively synthesize techniques to solve problems that are non-routine and may require creative thinking.
- (4) To master *rigorous mathematical writing*, understand and use *logic and quantifier notation*, and realize and appreciate that every mathematical argument, from basic arithmetic, to advanced mathematics, with almost no exceptions, is a mathematical proof.
- (5) To master the course material to a level of excellence that will ensure sustained success in more advanced mathematics courses.

To be successful in this course, it is expected that you should:

- (1) Spend about 12 hours each week working homework problems, reviewing lecture notes, reading the textbook and online lecture notes, studying for exams, and seeking help from the tutors and instructor;
- (2) Complete all homework problems, check the correctness of your work, and understand the methods and principles they illustrate;
- (3) Master the designed course topics before each test, and if necessary, complete additional problems beyond those assigned and consult other sources if you find the assigned problems and text are insufficient;
- (4) Recognize that mastery of the solution to a problem is not demonstrated by simply obtaining the correct numerical answer, but only by a clear, systematic, and detailed solution that traces the given information to the final numerical answer and that employs knowledge developed in this and previous courses;
- (5) When you experience difficulty in the course, seek help from the tutors and instructor immediately;
- (6) Attend class meetings regularly, pay attention, and do not hesitate to ask questions; and
- (7) Write your solutions to homework, test, and quiz problems in an organized and legible way.

Final term paper guidelines

In lieu of a final exam, you will be expected to develop a course project and write a final term paper. A 1-2 page project proposal should be submitted by Week 7, but earlier is better than later. The term paper is due on the last day of class **and should be submitted by email**. If your paper happens to be original research, I will be happy to help you develop it further and publish it as a co-authored paper, with you as the primary author, after the class is over. You are also welcome to submit it as a single-author paper on your own initiative, and encourage you to do so, if you are already comfortable with the peer-review process. However, it is not required for this course to write a publishable paper.

It is recommended that the proposal and the paper be typeset with \LaTeX . Both should include an abstract and references, like a regular research paper. The project should be related to the topics covered in class and be a problem of interest to you and one that is fun for you to investigate. It is alright to choose a rather “easy” topic in order to be able to present it in a concise and self-contained paper. The paper should be as long as is necessary to develop your topic thoroughly. A normal paper is usually about 6-12 pages, and a long paper can go up to 20 pages.

You must edit your paper. Print your first version, go to a coffeeshop (optional), read it, mark changes on the paper. Make the changes, print again, and repeat. Remember that if this were a publishable paper, simple errors would be there to haunt you for eternity. Usual corrections are typos, equation errors, rephrasing, adding sentences/paragraphs, sometimes shuffling sentences/paragraphs around, etc. Be sure to punctuate your equations.

There are three possible types of papers you may write. Examples of these three types of papers will be posted to the course website. I do recommend that you look at them for ideas on how to express yourself in a tone that is appropriate for an academic publication.

- (1) **Application:** Use the concepts covered in class to study an application in science, engineering, or any other real-world problem. The problem may involve algebraic calculations, or computer calculations. It may also be a combination of both. Include computer source codes in an appendix, if applicable.
- (2) **Theory:** A theoretical paper extending the theory learned in this class. Do not simply regurgitate someone else’s explanation of your topic. Work out all the calculations step by step with your own hands, and strive to find a clear, detailed, elegant, and better way to present your topic.
- (3) **Review:** The paper reviews a body of literature on a given topic. The idea is to combine a number of papers into a coherent story, one that emerges only when combining the papers together, and tell us the story. You will find that papers tend to lead you to other papers and help you via their introduction to piece together your narrative. Publishable review papers may review about 100-300 references. A minimum of 10 references is expected for your term paper.

The usual structure of a paper is as follows:

- (1) **Title:** Your project should have a compelling title
- (2) **Abstract:** This is a short overview of the paper, a miniature version of about 100-200 words or so. Someone reading the abstract should get a good idea of what problem has been tackled, what types of techniques were used to solve it, and what sort of solution was found, and whether to actually.
- (3) **Introduction:** Review the relevant literature, present the problem, explain why it is interesting, and outline the argument of your paper. Go over what will be covered in the remaining sections of your paper.
- (4) **Setup:** Introduce notations, definitions, and review prior relevant results that you wish to use. For application papers, explain the application topic and formulate the mathematical questions that it reduces to that you are going to address in your paper. For theoretical papers, announce

your main results. For review paper give a general introduction of what the topic is all about that is accessible to a general audience. Briefly cite previous reviews or textbooks.

- (5) **Solution:** Establish your main results. Discuss the techniques used. For application papers, solve the mathematical problems stated in your setup algebraically or numerical. For theoretical papers, state and prove your main results. For review papers, explain what has been done in the literature. Remember the fundamental law of story telling: every story or substory has a beginning, a middle, and an end.
- (6) **Conclusion:** Summarize what you have done and what you have learned.
- (7) **Discussion:** Summarize your main results, assumptions made. How sensitive are your results if any of the assumptions are violated? Discuss if your results are realistic. If not, then why? How could the work be improved. (may be combined with **Conclusion** as one section)
- (8) **Appendices:** Use these to include source code, or detailed tedious calculations.
- (9) **References:** Bibliography. Refer to books or peer-reviewed articles only. \LaTeX can automate the creation of the bibliography for you.

In short, **take pride in your work** and have fun!

Evaluation of final term paper

The course project's final term paper will be evaluated on the following scale (out of 20):

20	Excellent
17	Above Satisfactory
13	Satisfactory
8	Below Satisfactory
4	Unsatisfactory
0	Not Submitted

The resulting score will be combined with the grades of the midterm exams to calculate your overall grade for the course.

Grading Policies

- **Grading:** There will be 3 major exams, and a final course project. The time and location of exams will be announced in class. Exams count for 60% and final course project for 40% of your grade. Combined, you get a numerical grade on a scale 0-20. Each exam question is graded on a 0-4 scale with $4 = A$, $3 = B$, $2 = C$, $1 = D$, $0 = F$. Combining all exam and project scores, as explained above, gives a weighted average score on a 0-20 scale. This score is then mapped to a letter grade as follows: A: 16-20; B: 12-16; C: 10-12; D: 7-10; F: 0-7. There will be no curve and no extra credit.
- **Explanation of grades:** Your exams are graded question by question on a 0-4 scale per question. Overall, if you are planning to take future Mathematics or STEM courses, I would like to see you score 3 or 4 on all questions on all major examinations. If you score less than that on any questions, it indicates weaknesses in understanding the material. You should be proactive about addressing these weaknesses.
- **Homework:** Homework will be assigned, but will not be collected or graded. Nevertheless, it is crucial to do the homework as part of your preparation for the exams. To keep up, I recommend that **after every lecture you should solve the homework problems corresponding to the material covered on that day's lecture. Thus you need to work on a continuous basis!** Maintain a well-organized written record of your homework solutions by **writing the statement of each problem** (so that your document is stand-alone and can be read by itself), followed by your detailed solution, and clearly indicate the problem, section, and chapter number of the question. Most homework problems require more than simply writing the answer, and so you must write

all steps of your solution and provide appropriate justification, as illustrated by the instructor's solved examples, as you would on a test. Write neatly and legibly, using rigorous mathematical notation. While you are encouraged to discuss homework problems with other students, tutors, your instructor, and other faculty, the write-up of your solutions must be your own work and not simply copied from another student or another source. Use a ring binder to collect your homework, and write with a black pen, as that will help you to later scan the ring binder as a PDF file, for possible future use, and as a form of backup. This will provide you with a readily available resource to prepare for tests and quizzes, as well as providing documentation of the homework problems should you have a question about a problem and seek help from the instructor or a tutor.

- **Attendance Policy:** Attendance will be taken during most class meetings. It is important to sign in the sign-up sheet if you are present. **The instructor has the prerogative to drop any student with four (4) or more unexcused absences.** Two (2) tardies will count as one (1) unexcused absence. A tardy is defined as entering the class late or leaving the class early. **If you miss any major exam, you will be dropped from the course.**
- **How to Excuse an Absence:** To excuse an absence, you must submit **in writing** the "Notification of a Scheduled Excused Absence form" stapled **with documentation**, before the date you will be absent, or no more than **three (3)** business days after the date. The form should be submitted in person in class, during office hours, or via the Mathematics Department secretary. The form can be downloaded from the course web page. If a major exam is missed during an excused absence, your score for that exam will be replaced with your final exam score. UTRGV's attendance policy excuses students from attending class if they are participating in officially sponsored university activities, such as athletics; for observance of religious holy days; or for military service.
- **Make-ups:** There are no make-up exams. In the case of **excused** absences the final exam will be used as a make-up exam. **Each student MUST take the final exam at the scheduled date and time.** There will be no make-ups for the final exam, after the official final exam date!
- **Regrading policy:** If you believe that a mistake in grading has been made you may request that your paper be regraded. Such request must be submitted **in writing** within one week from the day the graded test has been returned in class, and must be accompanied by the original (unaltered) paper. If you make any changes to the paper your request will be denied. Please note that if you request regrading, all problems are subject to review. Thus, your overall grade may be increased or decreased.

Other Policies

- **Course web page:** A course web page will be used to distribute the syllabus, assigned homework, solutions to exams, a copy of my lecture notes, and any other relevant material. A link to that page will be available from my main page at <http://faculty.utrgv.edu/eleftherios.gkioulekas/>
- **Exam pick-up:** If you do not pick up your test paper within 1 week from the test date, I will throw away your paper and keep record of your test score.
- **Calendar of Activities:** Information regarding important dates, such as, first day of classes, holidays, last day to drop a class before it appears on the transcript (the census date), last day to drop or withdraw with a DR grade, and final exam schedule are available at <http://my.utrgv.edu>, at the bottom of the screen, prior to login. Please be advised of these important dates, and feel free to inquire with the instructor about any questions you may have with regard to the academic calendar.
- **Calculators:** The problems you will encounter in my exams will not require a calculator, and you are better served in the long-term by minimizing your dependence on calculators. Don't use the calculator to approximate roots, exponentials, logarithms, etc. Mathematical problems

require **exact** answers. Approximations are reasonable **only** on application problems where the numbers given may be approximate themselves, and thus the best answer that can be deduced is unavoidably approximate.

- **Classroom Conduct:** Common courtesy requires that students arrive in class on time, and stay the entire class period. Turn your cellphones and pagers off. You are required to treat your classmate and instructor with respect and courtesy. Use of any electronic devices, except for calculators, is not allowed in class, and I reserve the option to remove you from the classroom without warning for any behaviour that I deem as disrespectful or disruptive. You agree to indemnify and hold harmless the professor with respect to all actions undertaken by the professor to enforce classroom conduct or to properly proctor exams. Taking my course implies your consent to this policy.
- **Revisions:** This syllabus may be revised at any time. The syllabus posted on the professor's course web site is the only copy guaranteed to incorporate all revisions that may be made under this policy and will thus supersede any other versions posted on other university websites.
- **Mandatory Course Evaluations:** Students are required to complete an ONLINE evaluation of this course, accessed through your UTRGV account (<http://my.utrgv.edu>); you will be contacted through email with further instructions. Students who complete their evaluations will have priority access to their grades.
- **Students with disabilities:** Students with a documented disability (physical, psychological, learning, or other disability which affects academic performance) who would like to receive academic accommodations should contact Student Accessibility Services (SAS) as soon as possible to schedule an appointment to initiate services. Accommodations can be arranged through SAS at any time, but are not retroactive. Students who experience a broken bone, severe injury, or undergo surgery during the semester are eligible for temporary services.
 - *Pregnancy, Pregnancy-related, and Parenting Accommodations:* Title IX of the Education Amendments of 1972 prohibits sex discrimination, which includes discrimination based on pregnancy, marital status, or parental status. Students seeking accommodations related to pregnancy, pregnancy-related condition, or parenting (reasonably immediate postpartum period) are encouraged to contact Student Accessibility Services for additional information and to request accommodations.
 - *Student Accessibility Services – Brownsville Campus:* Student Accessibility Services is located in 1.107 in the Music and Learning Center building (BMSLC) and can be contacted by phone at (956) 882-7374 or via email at ability@utrgv.edu.
 - *Student Accessibility Services – Edinburg Campus:* Student Accessibility Services is located in 108 University Center (EUCTR) and can be contacted by phone at (956) 665-7005 or via email at ability@utrgv.edu.
- **Scholastic dishonesty:** As members of a community dedicated to Honesty, Integrity and Respect, students are reminded that those who engage in scholastic dishonesty are subject to disciplinary penalties, including the possibility of failure in the course and expulsion from the University. Scholastic dishonesty includes but is not limited to: cheating, plagiarism (including self-plagiarism), and collusion; submission for credit of any work or materials that are attributable in whole or in part to another person; taking an examination for another person; any act designed to give unfair advantage to a student; or the attempt to commit such acts. Since scholastic dishonesty harms the individual, all students and the integrity of the University, policies on scholastic dishonesty will be strictly enforced (Board of Regents Rules and Regulations and UTRGV Academic Integrity Guidelines). All scholastic dishonesty incidents will be reported to Student Rights and Responsibilities.

- **Sexual misconduct and mandatory reporting:** In accordance with UT System regulations, your instructor is a “Responsible Employee” for reporting purposes under Title IX regulations and so must report to the Office of Institutional Equity & Diversity (oie@utrgv.edu) any instance, occurring during a student’s time in college, of sexual misconduct, which includes sexual assault, stalking, dating violence, domestic violence, and sexual harassment, about which she/he becomes aware during this course through writing, discussion, or personal disclosure. More information can be found at www.utrgv.edu/equity, including confidential resources available on campus. The faculty and staff of UTRGV actively strive to provide a learning, working, and living environment that promotes personal integrity, civility, and mutual respect that is free from sexual misconduct, discrimination, and all forms of violence. If students, faculty, or staff would like confidential assistance, or have questions, they can contact OVAVP (Office for Victim Advocacy & Violence Prevention) at 665-8287, 882-8282, or OVAVP@utrgv.edu.

Student Learning Outcomes

After completing this course students will be able to

- (1) Understand basic concepts of logic and proof needed to understand the theory of linear algebra.
- (2) Understand the definition of matrix operations, the matrix inverse, transpose, the properties governing them, and be able to solve problems using these concepts.
- (3) Understand the definition and properties of the determinant, and how to evaluate a determinant or show identities concerning determinants via the definition, via cofactor expansion, and via a simplification process.
- (4) Solve linear systems, including parametric linear systems, using the inverse matrix technique, Cramer’s rule, or Gaussian elimination, and understand the pros and cons of each technique.
- (5) Understand the definition of eigenvalues and eigenvectors of a square matrix, and know how to calculate them.
- (6) Understand the characteristic polynomial of a matrix and its basic properties, and how to take advantage of them.
- (7) Understand the Cayley-Hamilton theorem. and how to use it to calculate the inverse matrix or powers of a square matrix
- (8) Understand the definitions of internal operations, groups, vector spaces, their basic properties, and how to establish that a structure is a group or a vector space.
- (9) Understand the definitions related to subspaces, the properties of subspaces, and how to use them to establish that a structure is a vector space.
- (10) Understand the definition and properties of linear independence and how to establish linear dependence or independence.
- (11) Understand the definition and properties of the basis and dimension of vector spaces and how to use them to establish that a set of vectors is a basis to a vector space or to calculate the dimension of a vector space.